Metrics for Energy-Aware Software Optimisation

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Abstract. Energy consumption is rapidly becoming a limiting factor in scientific computing. As a result, hardware manufacturers increasingly prioritise energy efficiency in their processor designs. Performance engineers are also beginning to explore software optimisation and hardware/software co-design as a means to reduce energy consumption. Energy efficiency metrics developed by the hardware community are often re-purposed to guide these software optimisation efforts.
In this paper we argue that established metrics, and in particular those in the Energy Delay Product (Et^n) family, are unsuitable for energy-aware software optimisation. A good metric should provide meaningful values for a single experiment, allow fair comparison between experiments, and drive optimisation in a sensible direction. We show that Et^n metrics are unable to fulfil these basic requirements and present suitable alternatives for guiding energy-aware software optimisation. We finish with a practical demonstration of the utility of our proposed metrics.

1 Introduction

Advances in processor design have delivered improvements in CPU performance for decades. As physical limits are reached, however, refinements to the same basic technologies are beginning to show diminishing returns [6]. One side-effect of this is an unsustainable rise in system power consumption, which has been identified as a primary constraint for exascale systems [20].

Moore's law, which states that transistor density doubles every 18-24 months, led to exponential increases in processor performance during a period often referred to as the "free lunch" [23]. More recently, the breakdown of Dennard scaling has meant that performance improvements are increasingly reliant on microarchitectural changes rather than increases in processor clock speed.

Hardware manufacturers are increasingly prioritising energy efficiency in their processor designs [15]. Research suggests that software modifications will be required to fully exploit the resulting improvements in modern architectures [21]. This has spurred interest in the possibility of optimising software for increased energy efficiency.

A fundamental aspect of performance engineering is *performance assessment*. To comment on the performance of a high performance computing system or a particular software package, we must first define an assessment metric. Metrics provide a means to evaluate a code or system based on some property of interest, allowing developers to perform high-level comparisons between different implementations and approaches. Some metrics also serve as fitness functions, combining various costs into a single *figure of merit* (FoM). Such metrics can be used to guide optimisation attempts and the search for better solutions [12].

New metrics which incorporate both energy and runtime costs will be required if developers are to identify and capitalise on new classes of energy-aware optimisations. Many early efforts have borrowed metrics developed by the hardware community, which has a long history of energy efficiency research. In particular, the Energy Delay Product (Et^n) family of metrics are frequently used for software optimisation.

In this paper we argue that Et^n and related metrics are not suitable for software optimisation. We discuss their shortcomings and provide examples of their failures in this domain. We then propose alternative metrics which address these shortcomings and compare their performance with Et^n. Finally, we demonstrate our metrics with an investigation of the energy efficiency of scientific codes.

Specifically, this paper makes the following contributions:

– We present a set of criteria that we believe are necessary for effective software optimisation metrics. Additionally, we introduce fitness landscape diagrams to visualise the behaviour of these metrics;
– We evaluate the Et^n family of metrics against our criteria. Our analysis highlights weaknesses in metrics commonly used in the energy efficiency optimisation literature;
– We propose two new metrics to measure software energy efficiency. We evaluate our proposals against the same criteria and describe how they improve on established metrics;
– Finally, we validate our proposed metrics with a study into the efficiency of codes from the Mantevo application suite.

The remainder of this paper is structured as follows: Section 2 presents a survey of related work; Section 3 lays the foundations for this work, providing formal definitions and criteria which we use to compare and assess different metrics; Section 4 uses these criteria to assess the suitability of Et^n metrics for software optimisation; Section 5 introduces our proposed metrics and evaluates them against the same criteria; Section 6 demonstrates the metrics discussed in previous sections by studying the energy efficiency of various applications; and finally Section 7 concludes this paper and describes upcoming research.

2 Related Work

Although energy consumption is becoming a constraint for scientific computing, minimising runtime is still an important optimisation objective. Optimising software according to multiple properties simultaneously is known as *Multi-Objective Optimisation* (MOO). MOO requires a balance to be struck between the potentially conflicting requirements imposed by different objectives.

The simplest approach to dealing with multiple optimisation criteria is to handle each one in isolation. A solution is said to be *Pareto-Efficient* if it is not dominated by any other solution across all objectives. Pareto-Efficiency yields a partial ordering, with a set of maximal elements but no ordering between them. The set of maximal elements delineates the *Pareto Front*, as shown in Fig. 1.

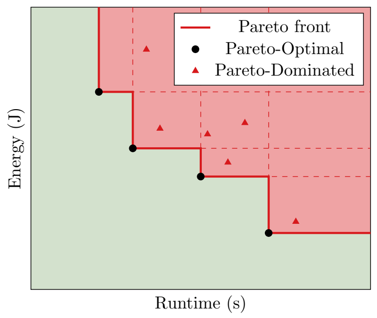


Fig. 1: Pareto-Efficiency

Pareto-Efficiency is useful when the relative importance of different requirements is unknown and the final choice of optimal solution can be deferred to the end user. For this reason it is often used by library developers who want their code to run efficiently in many different execution environments.

Balaprakash et al. use this approach to investigate the trade-offs between runtime and energy consumption for common kernels in scientific computing [2]. A similar technique has also been used to determine optimal checkpoint intervals for energy efficient fault tolerance [1].

A second MOO approach combines multiple objectives into a single scalar fitness function. This function then serves as a FoM metric for the overall utility of different solutions. Scalar fitness functions are in some sense fundamental to MOO; they can be used in isolation, but are also required by users choosing between solutions from a Pareto-efficient set.

Energy Delay Product was first proposed by Gonzalez et al. to measure the energy and runtime efficiency of microprocessors [9]. Martin et al. generalised this concept into the Et^n family of FoM metrics, with parameters E and t corresponding to energy and time [17]. They argue that Et^2 provides the best balance between the two optimisation objectives for microprocessor design. Srinivasan et al. reached the same conclusion, although for slightly different reasons [22].

Many authors have adopted these metrics originating from the hardware community and applied them to software optimisation problems. Vincent et al. describe a technique which minimises Et^1 using CPU throttling [8]. Bingham and Greenstreet use Et^n metrics to analyse runtime constraints imposed by a fixed energy budget for various algorithms [4]. Laros et al. use Et^n metrics to assess a number of production applications and state that Et^3 strikes the right balance between runtime and energy for high performance computing [16]. Et^1 has also been used extensively to quantify the efficiency of resource provisioning in a cloud computing environment [19,24].

Bekas and Curioni further generalised Et^n metrics to the form $E \cdot f(t)$, a product between energy and an application dependent function of time [3].

Another metric related to energy efficiency is FLOPS Per Watt, which relates the number of Floating Point Operations Per Second (FLOPS) and the rate of power consumption. Despite its name, this metric is quoted in units of Operations per Joule (1 Joule is defined as 1 Watt-Second). FLOPS/Watt measures how effective an application is at converting energy into floating point results.

Unlike Et^n, FLOPS/Watt does not measure application cost and hence cannot be used as a fitness function. This is analogous to metrics like branch misprediction rate, which may inform optimisation attempts but are not measures of utility. Branch misprediction can be eliminated by disabling speculative execution, but this does not result in better performance. Similarly, optimising for FLOPS/Watt may increase both runtime and energy consumption.

Heuristic models offer another source of optimisation guidance. Choi et al. proposed the Energy Roofline model to identify the algorithmic conditions needed for trade-offs between runtime and energy [5]. Similarly, in previous work we developed the Power Optimised Software Envelope model to assess the scope a code has for power optimisations on any given platform [18].

Some of our objections to existing metrics have been raised before, most notably by Hsu et al. [14]. They point out that Et^n and related metrics are unfairly biased towards massive parallelism and argue that there is a need for the development of new metrics.

We believe our work is timely and interesting because it offers a rigorous assessment of energy-aware software optimisation metrics. We show the flaws in current approaches and propose novel metrics which can be used as fitness functions to guide energy-aware software optimisation. We believe this work will be useful to practitioners in this nascent area of performance engineering.

3 Foundations

In this section we provide formal definitions which underpin later discussions and outline the desirable properties an optimisation metric should exhibit. We begin by formalising the notion of a code as a repeatable sequence of instructions which, when executed by a processor, incurs energy and runtime costs.

Definition 1. *All processors consume non-zero amounts of time and energy to run programs. The cost of a code θ is the pair $(E_\theta, t_\theta) \in \mathbb{R}_+ \times \mathbb{R}_+$ corresponding to the energy and runtime costs incurred by running it on a given platform.*

Definition 2. *Codes can be composed by concatenating their instruction sequences. The composition of codes θ and λ yields the following cost:*

$$\theta \circ \lambda = (E_\theta + E_\lambda, t_\theta + t_\lambda)$$

The goal of energy-aware software optimisation is to minimise the runtime and energy costs of a given application. Energy-aware optimisation metrics are functions of energy and time which capture the utility of a code.

Definition 3. *An energy-aware optimisation metric is an element-wise monotonic function M which combines energy and runtime costs into a scalar FoM:*

$$M\colon (E, t) \in \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$$

Element-wise monotonicity means that for all fixed $E_0, t_0 \in \mathbb{R}_+$, the functions $M(E_0, t)$ and $M(E, t_0)$ are monotonic. In other words, increasing one cost without a corresponding reduction in the other leads to a worse FoM.

Software optimisation can be modelled as a hill-climbing problem. Starting from an initial code θ, performance engineers make incremental changes and measure their impact using a FoM metric. Changes which improve performance against this metric are kept while those which reduce it are discarded. Whether a given code change represents an optimisation depends on the metric chosen.

Definition 4. *For logically equivalent codes θ and λ, the transformation $\theta \to \lambda$ is an optimisation with respect to metric M iff $M(\lambda)$ strictly dominates $M(\theta)$.*

By Definition 3, all valid metrics identify code changes which reduce both energy and time costs as optimisations. Similarly, all code changes leading to strictly worse performance will be disregarded. Energy-aware optimisation metrics only differ in cases where energy-time trade-offs are possible.

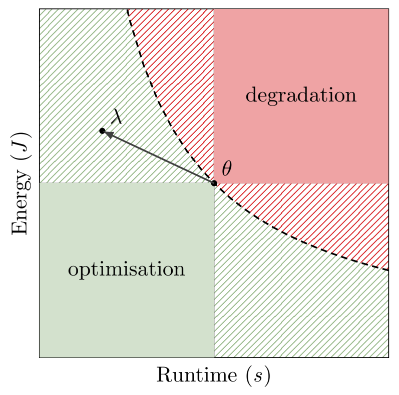
Fig. 2 shows how two valid metrics can disagree on whether the same code change $\theta \to \lambda$ is an optimisation. Lighter green areas correspond to optimisations and darker red areas to performance degradations. They are separated by a dashed *Isometric line* that connects all points with FoM values equal to $M(\theta)$. Both metrics agree on code changes in the solid shaded regions where costs change in tandem. Energy-time trade-offs are represented by cross-hatched quadrants. The MOO metric in Fig. 2a identifies $\theta \to \lambda$ as a valid energy-time trade-off, whereas Fig. 2b shows it is not an energy optimisation.

Energy-aware optimisation metrics ascribe a FoM to all (E, t) cost pairs. Returning to the hill-climbing analogy, we say that an optimisation metric defines a fitness landscape over the energy/time plane. Fig. 3 shows how plots similar to Fig. 2 can be used to visualise the fitness landscape of a metric.

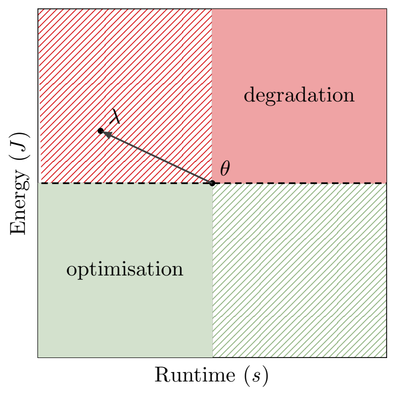
The isometric lines in Fig. 3 connect all points where the FoM is some multiple of a fixed value. Mathematically these lines represent level sets of our M function; intuitively they are contours in our fitness landscape. The closeness of these lines corresponds to the gradient of the fitness landscape.

Isotopic lines run perpendicular to isometric lines, and correspond to the path of fastest decent (steepest gradient) within the fitness landscape. Mathematically, these lines are orthogonal trajectories of a metric function M. Conceptually, they show the direction in which a metric drives optimisation.

Having formally defined what an energy-aware optimisation metric is and how it can be visualised, we now turn our attention to how it should behave.



(a) Multi-Objective Metric (Et^1)



(b) Single Objective Metric (Energy)

Fig. 2: Metric Optimisation Regions

The goal of an optimisation metric is to condense the utility of an application into a single, meaningful FoM. We have identified the following properties which an idealised optimisation metric should possess:

1. **Bounded:** A metric should bound regions of the optimisation space;
2. **Directed:** drive optimisation efforts in a sensible direction;
3. **Additive:** remain additive (linear) under code composition;
4. **Stable:** provide a stable definition of optimisation under code composition;
5. **Tunable:** be tunable to different application domains; and
6. **Intuitive:** correspond to a tangible and intuitive property of the system.

We explore these properties in more detail in the next section.

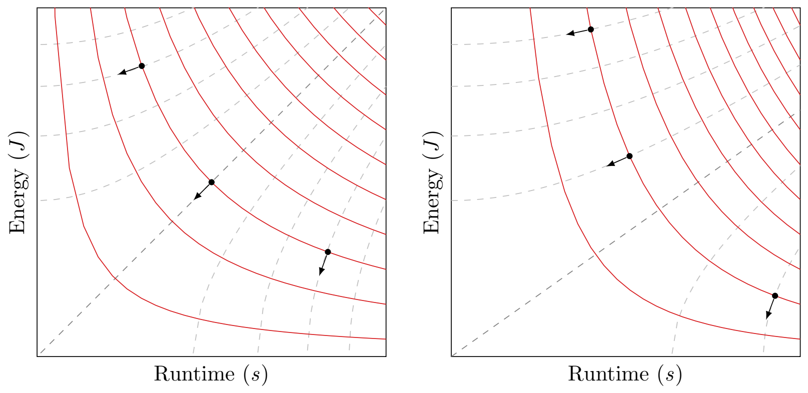
4 Et^n Evaluation

In the previous section we listed several desirable criteria for energy-aware optimisation metrics. We now use these criteria to evaluate the suitability of Et^n metrics for guiding software optimisation.

4.1 Analysis of Et^n

Bounded: Our first criteria states that energy-aware optimisation metrics should bound regions of the optimisation space. By this we mean that a metric should place upper limits on how much energy or runtime can be consumed under a given FoM. This requirement is met if the isometric lines described by a metric intercept both the energy and runtime axes.

Fig. 3 shows that Et^n isometric lines do not intercept either axis. In theory, codes can be modified to consume an arbitrarily large amount of either time or energy while still improving their overall performance. We consider this to be a



(a) Et^1 (b) Et^2

Fig. 3: Et^n Metric Fitness Landscapes

flaw and assert that such changes should not count as optimisations. Another benefit of bounded metrics is that they limit the space in which to search for optimisations; something which Et^n cannot do.

Directed: Our second criteria requires metrics to guide optimisation in sensible directions. Intuitively, we wish to speed up slow codes and reduce the power consumption of energy intensive ones. On the contrary, Et^n disproportionately rewards speeding up fast codes and saving energy in frugal ones. As energy consumption increases, Et^n gives higher priority to runtime optimisation and vice versa. This fault was encountered by Hsu et al. when they noted that Et^n metrics are unfairly biased towards massive parallelism in HPC systems [14].

Our first two criteria are linked. It is necessary (but not sufficient) for a metric to be bounded in order for it to guide optimisation in a sensible direction. The isometric lines of an unbounded metric never touch either axis, meaning the corresponding isotopic lines must intersect the axes at right angles. As the energy or time cost of a code approaches zero, the path of fastest decent therefore tends exclusively towards further reductions in this already close-to-zero cost.

Additive: Our third criteria states that FoM metrics should be additive under code composition. Performance engineers focus their attention on expensive procedures within a code. This involves profiling the code to identify areas causing poor performance, based on the assumption that the cost of a code is the sum of the costs of its constituent parts. While true for simple metrics like energy and time, this is not generally the case for compound metrics.

Definition 5. *A metric is additive iff for code segments θ and λ:*

$$M(\theta \circ \lambda) = M(\theta) + M(\lambda)$$

Metric functions must be linear in terms of both time and energy in order to fulfil this requirement. This is not the case for Et^n, where the cost of a code tends to be much greater than the costs of its constituent parts. Profilers cannot be relied upon to identify targets for Et^n optimisation. Furthermore, this additional *non-local* cost depends on total application runtime and energy consumption. An Et^n FoM is therefore meaningless outside the context of a single fixed application.

Stable: Our fourth criteria requires metrics to provide a stable definition for optimisation. If the same code change alters the cost of two applications by the same amount, and it is an optimisation with respect to metric M for one of the codes, then it should count as an optimisation for both of them.

Definition 6. *A metric is stable iff for equivalent code segments* λ *and* λ'*:*

$$M(\lambda') < M(\lambda) \implies M(\theta \circ \lambda') < M(\theta \circ \lambda)$$

It is worth noting that linear metrics automatically fulfil this requirement. Linear metrics are inherently stable, however stable non-linear metrics also exist.

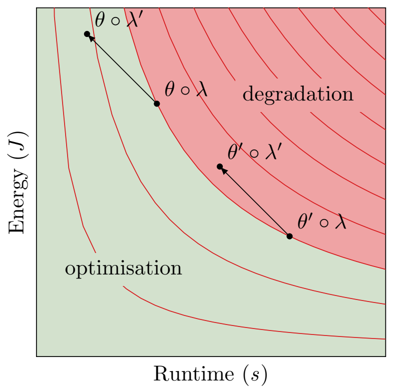
Et^n is an unstable metric as it does not provide a consistent definition of optimisation. Whether or not a code change counts as an optimisation under Et^n is *context sensitive*. Code changes can be counted as optimisations only when evaluated in the context of the full application. Targeted optimisation of particular subroutines is impossible, and all past optimisations must be re-evaluated every time a change is made to the application.

This failure of Et^n is best illustrated with an example. Suppose an application contains a procedure which consumes 10 J over 10 s to produce some result. This corresponds to an Et^1 FoM of $10 \times 10 = 100$. We then modify our procedure to produce the same result in 11 J and 9 s. This is a valid optimisation because although it increases energy consumption it reduces Et^n to $11 \times 9 = 99$.

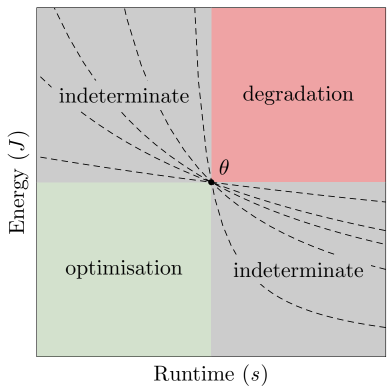
Once the procedure completes we are given the option to output results at a cost of (5 J, 10 s). Our un-optimised application could execute its tasks and output the results with an EDP of $(10 + 5) \times (10 + 10) = 300$. The same sequence of actions in the 'optimised' application results in a higher (worse) EDP of $(11 + 5) \times (9 + 10) = 304$. Under Et^n, choosing to save the results of our procedure retroactively invalidates our optimisation.

Fig. 4a shows how the same cost change applied to two codes with the same starting Et^n FoM may be considered either an optimisation or a performance degradation. Furthermore, Fig. 4b shows how any energy-time trade-off can be made to appear as an optimisation or a performance degradation depending on the context. Different ratios of E_θ and t_θ can shift the optimisation/degradation boundary to any point within the indeterminate quadrants.

Mini-applications are powerful tools in scientific computing [13]. They package relevant features of large production applications into smaller, more manageable codes. Performance engineers use them as test beds to search for optimisations which can be ported back to the original application. Sometimes



(a) Context Sensitive Optimisation



(b) Indeterminate Optimisations

Fig. 4: Et^n Optimisation Instability

optimisations which work at small scale will fail to improve the production application, signalling a discrepancy between the mini and production applications. Using Et^n metrics, however, optimisations to the mini-app may not count as optimisations to the production code even when they yield identical cost changes in both cases. This is further proof that Et^n metrics are incompatible with modern performance engineering techniques.

Tunable: Our penultimate criteria is that it should be possible to tune a metric to reflect the energy and time constraints of different domains. The Et^n metric meets this criteria via its n parameter. This parameter sets the 'exchange rate' at which small changes in runtime and energy can be traded against each other. This can be shown by equating the partial derivatives of Et^n as shown in Equation 1:

$$\frac{\partial}{\partial E}(Et^n) = t^n \qquad \text{and} \qquad \frac{\partial}{\partial t}(Et^n) = nEt^{n-1}$$
$$t^n \cdot \partial E = nEt^{n-1} \cdot \partial t$$
$$\frac{\partial E}{E} = n\frac{\partial t}{t} \tag{1}$$

Intuitive: Our final and most subjective criteria is that a metric should be intuitive. In practice, this means it should correspond to some tangible property of a system, ideally with values measured in meaningful units. Et^n does not meet this requirement.

The costs of an extra Joule or second are not fixed under Et^n; in fact, the cost of increasing each factor depends on the current magnitude of the other. This implies that a Joule consumed by a long running process somehow costs more than a Joule consumed by a short-lived one. Furthermore, real systems impose maximum and minimum rates of power consumption on a code which we refer to as P_{max} and P_{min}. Given that $P_{min} \cdot t < E < P_{max} \cdot t$, the growth rate of Et^n is $\Theta(t^{n+1})$. The FoM cost of an additional second or Joule grows polynomially, hindering comparison between different scales.

4.2 Justification of Et^n

The continued use of Et^n metrics despite their flaws is a testament to the need for standardised energy-aware optimisation metrics. In the absence of better alternatives, software engineers rely on Et^n because of its popularity and relative ease of use. Et^n metrics remain the de-facto standard technique for combining energy and runtime costs into a single FoM.

One factor which hides the problems with Et^n metrics is the small range of power consumption exhibited by modern hardware running HPC workloads as a result of high base power consumption and marginal differences under load [10]. Fig. 5a shows isometric lines for Et^1 and our proposed metrics. It shows how a small $[P_{min}, P_{max}]$ range confines (E_θ, t_θ) costs to a narrow envelope within the energy/time plane. This envelope limits the scope for divergence between different metrics. In the extreme case, when $P_{min} = P_{max}$, E_θ is a scalar multiple of t_θ and all energy-aware metrics become functions of time.

The scarcity of power-instrumented hardware means energy-aware optimisation is typically carried out at the level of individual nodes. Although single nodes exhibit narrow $[P_{min}, P_{max}]$ ranges, multi-node and system-level power draw is much less constrained. Fig. 5b shows two performance envelopes, with the larger having P_{min} and P_{max} values three times those of the smaller one. This models the effect of running the same code on a single node and over three nodes in parallel. Similar discrepancies would occur when running code on alternative architectures with significantly differing power characteristics, such as GPUs and FPGAs, that are emerging as candidate platforms for improved efficiency [7]. Even at this small scale the discrepancies between Et^n and other metrics become readily apparent.

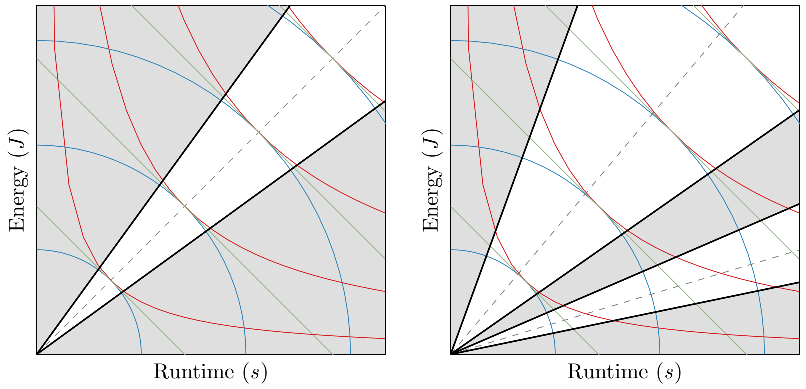
5 Proposed Metrics

In this section we propose two new FoM metrics for energy-aware software optimisation. These metrics have slightly different properties and the choice of which to use is left to the performance engineer. That said, they both significantly outperform Et^n metrics according to our assessment criteria.

Our first metric is a weighted sum of energy and runtime costs. Our second metric measures the cost of an application in terms of Euclidean distance from an 'optimal' point at the energy/time origin. The fitness landscapes for both metrics are shown in Fig. 6a and Fig. 6b respectively.

5.1 Proposed Metric 1: Energy Delay Summation (EDS)

Energy and compute time are limited resources which have costs associated with their consumption. The primary cost of energy consumption is the purchase price



(a) Narrow Range Equivalence (b) Discrepancies at Scale

Fig. 5: Power-Limited Isometric Lines

of electricity. Environmental impact and other concerns can also be included. Runtime also has a monetary cost – the purchase costs of the machine amortised over its limited lifespan. Energy and runtime costs are captured by the α and β parameters in Equation 2.

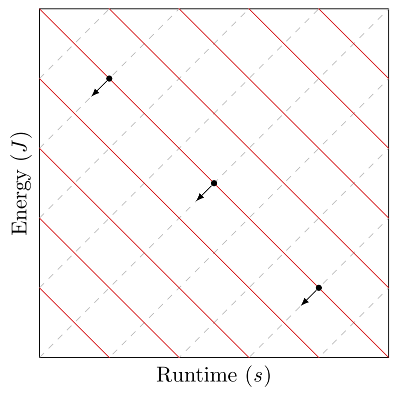
$$\begin{aligned} M(\theta) &= \alpha E_\theta + \beta t_\theta \\ &= (\alpha, \beta) \cdot (E_\theta, t_\theta) \end{aligned} \tag{2}$$

Bounded: Our first criteria requires metrics to bound regions of the energy/time space. The isometric lines in Fig. 6a intercept both axes, satisfying this criteria. An EDS FoM therefore places upper limits on energy and runtime costs. The runtime contribution to a metric is maximised when energy is minimised and vice versa, allowing us to deduce cost limits under a given FoM:

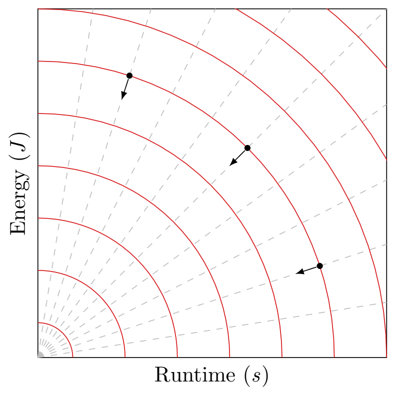
$$\begin{aligned} M(\theta) &= \alpha \cdot E_{max} + \beta \cdot 0 \\ \therefore\ E_{max} &= \frac{M(\theta)}{\alpha} \\ M(\theta) &= \alpha \cdot 0 + \beta \cdot t_{max} \\ \therefore\ t_{max} &= \frac{M(\theta)}{\beta} \end{aligned}$$

Performance engineers need not evaluate code changes with energy costs greater than E_{max}, or runtime costs greater than t_{max}. This is in stark contrast to the Et^n case, where any given energy or runtime cost could be considered an optimisation under the right circumstances.

Directed: Our second criteria requires metrics to guide optimisation in sensible directions. Fast, energy intensive codes are likely to require different optimisations to slow, energy efficient ones. As a linear function, EDS does not



(a) EDS



(b) EDD

Fig. 6: Proposed Metrics Fitness Landscapes

differentiate between these cases; the isotopic lines in Fig. 6a all run in parallel. Our metric still outperforms Et^n in this regard however as it does not introduce perverse optimisation incentives.

Additive and Stable: Our third and fourth criteria require metrics to be linear functions of time and energy and to provide stable definitions of optimisation. The function $\alpha E + \beta t$ is linear in both parameters. Linear functions are automatically stable; meaning this metric fulfils both criteria, providing stable definitions for optimisation and allowing for meaningful code profiling.

Tunable: Our penultimate criteria is that metrics be tunable to different application domains. Different energy and runtime costs can be specified via the α and β parameters. Unlike the exponential formulation of Et^n, it is immediately apparent how different values will alter the balance between energy and runtime.

A single scalar parameter would be enough to express any ratio of energy and time components. One property of this metric is that with appropriate tuning factors it can be used as a proxy for the monetary cost of running a code. This use-case is why we include two tuning parameters within this metric, to allow us to provide notional value results.

Intuitive: Our final criteria requires metrics to correspond to some meaningful property of the system. Given appropriate coefficients this metric can report results in terms of monetary cost. Monetary cost has meaningful units, allows for fair comparisons to be made between different platforms and architectures, and is useful during procurement.
Equation 2 provides a dot product formulation of the EDS metric which suggests a second geometric interpretation. Dot products correspond to the projection of one vector onto another – in this case of (E_λ, t_λ) onto (α, β).

5.2 Proposed Metric 2: Energy Delay Distance (EDD)

Our first metric measured code performance in terms of separable energy and time costs. This definition fulfilled all but one of our criteria; as a linear function it was not able to direct the optimisation of codes according to their starting costs. Our second metric remedies this by defining the cost of a code as its distance from the most optimal point on our fitness landscape – the origin.

$$M(\theta) = \sqrt{{E_\theta}^2 + (\beta t_\theta)^2}$$

EDD can also be expressed as the magnitude of a weighted cost vector:

$$M(\theta) = \|(E_\theta,\, \beta \cdot t_\theta)\|$$

Bounded: The isometric lines in Fig. 6b follow semi-circular trajectories which intercept the axes. This satisfies our first criteria, meaning this metric also limits E_{max} and t_{max} for a given FoM. We can derive these limits as follows:

$$\begin{aligned} M(\theta) &= \sqrt{{E_{max}}^2 + \beta \cdot 0} \\ \therefore\ E_{max} &= M(\theta) \\ M(\theta) &= \sqrt{0 + \beta \cdot {t_{max}}^2} \\ \therefore\ t_{max} &= \frac{M(\theta)}{\beta} \end{aligned}$$

Directed: The isometric lines for this metric form concentric ellipse segments centred about the origin. As a result, the corresponding isotopic lines converge on the origin. Fig. 6b makes it clear that as a result this metric prioritises optimisations which minimise whichever cost is greater.

Additive: The formula for EDD is non-linear, meaning the overall FoM of a code is not equivalent to the sum of its parts. This is an unavoidable consequence of being a directed metric, and means that EDD is not well suited for accurate code profiling. Unlike Et^n, the discrepancy between the sum of component FoMs and the overall code FoM for EDD is bounded. As EDD is defined in terms of vector magnitude it obeys the triangle inequality. As energy and time costs are always positive, we have:

$$\sqrt{M(\theta)^2 + M(\lambda)^2} < M(\theta \circ \lambda) \leq M(\theta) + M(\lambda)$$

Stable: EDD does not meet our stability criteria. Fig. 7 shows a case where $M(\lambda') < M(\lambda)$, yet $M(\theta \circ \lambda') > M(\theta \circ \lambda)$. The runtime axis is scaled so that isometric lines remain concentric for all values of β. That said, EDD instability is bounded by $M(\theta) + M(\lambda) - M(\theta \circ \lambda)$ as this metric obeys the following inequality:

$$M(\lambda') < M(\lambda) \implies M(\theta \circ \lambda') < M(\theta) + M(\lambda)$$

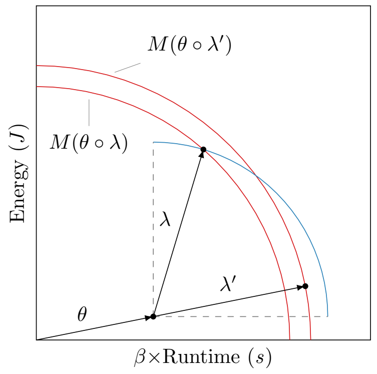

$M(\theta \circ \lambda')$
$M(\theta \circ \lambda)$
Energy (J)
λ
λ'
θ
$\beta\times$Runtime (s)

Fig. 7: EDD Instability

Tunable: This metric is tunable via the β parameter. A single parameter is sufficient to achieve any ratio of energy to runtime contribution.

Intuitive: This metric has a direct geometric interpretation as the Euclidean distance to the origin. It does not treat energy and runtime as separate and distinct costs; in reality they are inseparable. In general, reducing the runtime of a code will also reduce its energy consumption. EDD defines the cost of a code in terms of how far away it is from being perfectly optimal.

6 Case Study

In this section we investigate the energy-efficiency characteristics of codes in the Mantevo [13] mini-application benchmark suite. Our results show that the issues with Et^n become more evident at larger scales.

We carried out our experiments on the Taurus system at TU Dresden, which is equipped with High Density Energy Efficiency Monitoring (HDEEM) instrumentation [11]. Taurus is a heterogeneous cluster with several classes of node. This work was carried out on the largest of these classes, with each node featuring two 12-core Intel Xeon E5-2680 v3 CPUs and 64 GB of memory.

All codes were compiled with ICC version 15.0.3. Application parameters were based on default values, with problem sizes tuned where necessary to ensure reasonable run times on single nodes. Results were averaged over 5 runs to minimise the impact of random variations in runtime and energy.
We use Et^3 in these experiments because Laros et al. found that this strikes the right balance between runtime and energy for high performance computing [16]. This implies that a 1% reduction in runtime is approximately three times more valuable than the same reduction in energy consumption.

In order to facilitate comparison we have based our EDS and EDD parameterisation on the same 3:1 ratio. Whereas the Et^n parameter operates in a relative fashion, however, EDS and EDD parameters are based on absolute costs of consumption. The power drawn by active Taurus nodes ranges between 207.68 W and 345.33 W [18], meaning the magnitude of energy costs will be around 300 times greater than that of runtime. In order to compensate for this effect we scale the runtime cost by a factor of 300 before applying the 3:1 ratio, resulting in the parameters $\alpha = 1$ and $\beta = 3 \times 300 = 900$.

In practice we would prefer to adopt a more fine-grained parameterisation which reflects real-world costs incurred by HPC systems. That said, exact cost figures are seldom made available in the public domain.

For our first test we measured the runtime and energy consumption of various codes running on a single node. The results for this test are presented in Table 1.

Table 1: Single Node Code Costs

Code	Runtime (s)	Energy (J)	Et^3	EDS	EDD
TeaLeaf	323.8	99,810.3	3,388,489,410,000	391,230	100,280
PathFinder	337.1	71,943.9	2,755,945,330,000	375,334	72,646
CloverLeaf	214.3	57,861.2	569,447,289,000	250,731	58,214
CloverLeaf3D	153.1	43,755.9	157,022,581,000	181,546	43,991
MiniMD	125.5	31,162.1	61,596,822,000	144,112	31,387
CoMD	105.6	24,837.8	29,248,540,000	119,878	25,037
MiniFE	36.7	8,465.6	418,461,937	41,496	8,536
HPCCG	36.5	8,059.5	391,910,164	40,910	8,133

The first thing to note is that Et^n results rapidly become unwieldy even for relatively short runtimes and low node counts. The runtime of HPCCG is around 11.4% that of TeaLeaf, and it also exhibits a slightly lower rate of power draw. This translates to a four orders of magnitude difference in their Et^n values. Adding a single second to the runtime of TeaLeaf would further increase its FoM by 8613 times the total Et^n of HPCCG.

Another thing to note is that despite large variations in values, all metrics assign the same efficiency ordering to these codes. As previously mentioned, the limits of single-node power draw limit the scope for metrics to disagree.

For our second test we measured the runtime and energy consumption of MiniMD running at scale. The results for this test are presented in Table 2.

These results show how biased Et^n metrics are in favour of massive parallelism. The efficiency of MiniMD according to Et^n improves as the node count increases to 18. It is only at the point when adding nodes delivers little or no reduction in runtime that this trend reverses.

EDS identifies 4 nodes as the optimal node count. This configuration delivers roughly twice the runtime performance of a single node at the cost of doubling the energy consumption. Adding nodes beyond this point results in energy costs increasing faster than runtime performance improves.

EDD identifies 1 node as the optimal node count. This corresponds to the intuition that parallelism introduces overhead. As the parallel overhead grows, so too does inefficiency as measured by this metric.

Et^3 gives the impression that below-linear speed-ups coupled with above-linear rises in energy consumption represent efficiency gains. Conversely, our

Table 2: MiniMD Multi-Node Costs

Nodes	Runtime (s)	Energy (J)	Et^3	EDS	EDD
1	125.5	31,162.4	61,597,424,000	144,112	31,388
2	94.2	44,999.0	37,614,512,300	129,779	45,086
4	66.8	63,166.0	18,828,375,900	123,286	63,190
6	55.2	76,400.0	12,850,216,400	126,080	76,412
8	54.0	99,032.6	15,594,067,100	147,633	99,043
12	44.0	119,008.9	10,137,658,200	158,609	119,011
16	39.8	145,198.3	9,154,006,200	181,018	145,197
18	37.8	152,380.5	8,230,099,000	186,401	152,376
24	36.0	191,056.9	8,913,951,100	223,457	191,046
28	37.2	231,525.5	11,918,663,500	265,006	231,516
32	37.5	258,054.5	13,608,342,900	291,805	258,041
64	39.4	518,748.6	31,728,187,600	554,209	518,713
128	46.2	1,203,476.1	118,676,068,000	1,245,056	1,203,410

EDS and EDD metrics conform to a more conventional understanding of energy efficiency. They identify optimal configurations which can be justified intiutively.

7 Conclusion

In this paper we argue that the Et^n family of metrics are not appropriate for energy-aware software engineering. We propose alternative metrics which can be used to measure the cost of applications and guide their optimisation. Finally, we compare the performance of our metrics against established techniques by studying codes taken from the Mantevo mini-application suite.

We began by showing how Et^n metrics are unable to provide meaningful values for individual experiments, cannot be compared between experiments and do not support optimisation efforts. Improving the Et^n FoM of a section of code can degrade overall performance. Et^n metrics drive optimisation efforts in counterproductive directions, encouraging developers to speed up already fast code and seek energy efficiency gains in energy efficient codes. Finally, these metrics provide no meaningful definition of an optimisation. In total, Et^n was able to fulfil only one of our seven criteria for software optimisation metrics.

We then proposed EDS and EDD, novel metrics which outperform Et^n against all of our assessment criteria. EDS is appropriate for measuring the cost of applications, while EDD is well suited to guiding application optimisation. Both our metrics fulfil the majority of our criteria, and EDS fulfils the maximum number possible.

Our paper finishes with a study into the energy-efficiency costs of several popular applications. This study shows how the flaws of Et^n have managed to remain hidden in small-scale optimisation studies. It also demonstrates that these flaws will prevent Et^n from being employed at scale. As a result, new metrics like EDS and EDD will be required to support performance engineers as interest in energy optimisation continues to grow.

7.1 Future Work

The properties of our metrics makes them particularly well suited to comparing codes running at different scales and on different architectures. We intend to use EDS and EDD to investigate the power optimisation characteristics of various codes running on accelerator-based technologies. Our ultimate aim is to demonstrate how the correct metric can facilitate the discovery of energy-aware software optimisations. In our ongoing work we focus our search towards GPU and FPGA platforms as promising candidates for energy optimisation.

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