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# A Mathematical, Computational and Symbolic Representation Framework towards Digital Marketing Planning

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**Abstract** - Digital marketing strategy formulation is examined using a hybrid mathematical, computational and symbolic presentation approach. The proposed framework is introduced as a new solution for conducting strategic analysis, producing digital marketing strategy alternatives, and making strategic choices. A software example is provided to demonstrate how the framework functions with preliminary results highlighting how the process and output of digital marketing strategy formulation may be improved.

**Keywords** - digital marketing strategy; computational framework; symbolic representation; Web-based decision support system; simulation; fuzzy logic; expert system

## I. INTRODUCTION

Rapid change, uncertainty and competition in the global marketing context, combined with rapidly changing consumer behaviours in relation to the use of digital platforms, digital media and channels, purchase and brand advocacy has made the digital marketing planning job more difficult and challenging with many campaigns failing to achieve their objectives. Whilst recent pioneering effort has been made to create a Web-based hybrid knowledge automation system to assist the digital marketing planning process [11] there is still a lack of formal mathematical formulation, computational modelling and knowledge automation specifications in this field. This study is amongst the first to set up and evaluate a hybrid mathematical, computational and symbolic knowledge representation framework for formulating digital marketing strategies.

## II. MATHEMATICAL AND COMPUTATIONAL MODELLING FOR DIGITAL MARKETING PLANNING

A definition for Web-based hybrid systems may be found in Li & Li [9, 10]. The proposed framework aims at assisting the key stages of digital marketing planning: 1) simulating and evaluating variables determining digital marketing strategies, and 2) performing approximate reasoning under uncertainty and advising strategic alternatives or options. The decision maker may provide

judgmental inputs to the analysis and simulation of relevant strategic variables. He or she may also apply intuition, experience and personal vision when providing judgemental inputs to variables, simulation procedure, and making the final choice on digital marketing strategies. The general system model is shown in Figure 1.

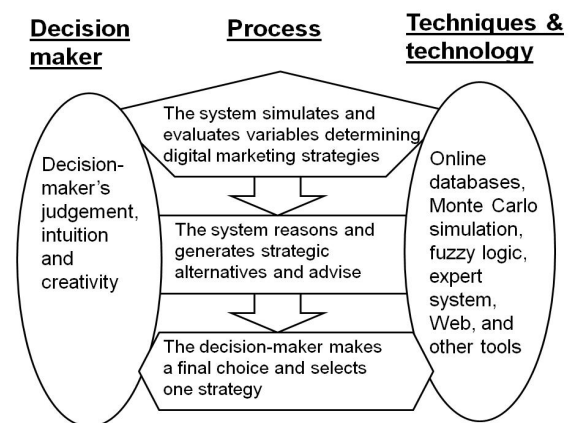


Figure 1. The process, supporting tools and the decision maker's interaction (Produced by Shuliang Li, the University of Westminster)

The following elements will be employed in our mathematical description:

The simplified digital marketing strategy formulation process focuses on the two key stages as shown in Figure 1: evaluating variables determining strategic decision making (E), and 2) generating strategic options (G); human judgement and creativity, J; decision support and artificial intelligence techniques and technologies, T, such as analytic hierarchy process (AHP), Monte Carlo simulation (MCS), fuzzy logic (FL), Web-based knowledge automation expert systems (KAES), on-line databases (DB), and other techniques (OT).

The digital marketing strategy development process may be formulated as:  $I = (E, G)$ . The supporting

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techniques and technologies can be defined as:  $T = (AHP, MCS, FL, KAES, DB, OT, Web)$ . Managerial judgement can be described as:  $J = (Judgement, Creativity)$ .

The outcome of the strategic decision making process can be expressed as the following equation:

$$OUTCOME = f_I(E, G) \oplus f_T(AHP, MCS, FL, KAES, DB, OT, Web) \oplus f_J(Judgement, Creativity)$$

where  $f_I$ ,  $f_T$  and  $f_J$  are implicit functions for the decision-making process, supporting techniques and human judgement, respectively. Here, symbol “ $\oplus$ ” indicates logical union or integration.

Within this framework, the Web-based hybrid intelligent support system, WHSS, is expressed as:

$$WHSS = AHP \Theta MCS \Theta FL \Theta KAES \Theta DB \Theta OT \Theta Web$$

where the symbol  $\Theta$  is a hybridisation and interaction operator and OT stands for other techniques and associated sub-systems.

#### A. The analytic hierarchy process for digital marketing strategy formulation

The analytic hierarchy process (AHP) [14, 15] can be applied to organise the problem in a hierarchical structure and perform pair-wise comparisons, and then calculate the relative importance of decision variables to obtain an overall ranking results [6, 19]. AHP can be used to estimate the weights for the factors or criteria determining digital marketing strategies.

#### B. Web-based Monte Carlo simulation for digital marketing

Monte Carlo simulation (MCS) is a technique that applies probability and random numbers to model and tackle uncertainty and stochastic permutation [13]. The algorithm for Monte Carlo simulation using uniformly distributed random numbers and the inverse function of a symmetric or asymmetric cumulative distribution of the triangular probability distribution, on the basis of the algorithm published by Brighton Webs Ltd. <http://www.brighton-webs.co.uk/distributions/triangular.asp> (Accessed on 01/12/2008).

#### C. Fuzzy logic and the space of strategic variables

“A fuzzy set is a class of objects with a continuum of grades of membership” [23, page 338]. Let  $U$  be a universe of discourse, a collection of objects  $\{u\}$ . A fuzzy set  $A$  in  $U$  is characterised by a membership or compatibility function  $\mu_A$  taking values in the interval  $[0, 1]$ .  $A$  in  $U$  is represented as [5, 24]:  $A = \{ (u, \mu_A(u)) | u \in U \}$

Based upon Fung et al [3]’s work, the set of variables (ranging from 1 to 10) that determine a dimension of a

strategic digital marketing grid/matrix can be expressed as an vector  $H$ :

$$H = (H_1, H_2, \dots, H_m)$$
 in the fuzzy space of  $U$

For the input fuzzy vector  $H$ , there exists a real vector  $w$  that represents the weights or relative importance for the strategic factors or variables, such that  $w = (w_1, w_2, \dots, w_m)$  which can be determined using the AHP method.

We then aggregate the values of  $H$  and  $w$  to work out the value of an object,  $u$ , using the following formula:

$$u = H_1 \cdot w_1 + H_2 \cdot w_2 + \dots + H_m \cdot w_m$$

#### D. Fuzzification of the variables or factors affecting digital marketing strategy formulation

Trapezoidal membership/compatibility functions are employed to fuzzify the aggregated scores for a strategic digital marketing grid dimension.. For real numbers  $a \leq b \leq c \leq d$ , the trapezoid  $\Gamma(a, b, c, d)$  with amplitude one is defined as [7, 8]:

$$\Gamma(u) = 0 \text{ if } u \leq a; \Gamma(u) = (u - a)/(b - a) \text{ if } a < u \leq b; \Gamma(u) = 1 \text{ if } b < u \leq c; \Gamma(u) = (d - u)/(d - c) \text{ if } c < u \leq d; \Gamma(u) = 0 \text{ if } d < u; \Gamma(a) = 1 \text{ if } a = b; \Gamma(d) = 1 \text{ if } c = d.$$

in which  $u \in U$ .

Figure 2 illustrates one example where trapezoidal membership functions are used. Other models may be fuzzified using a similar method.

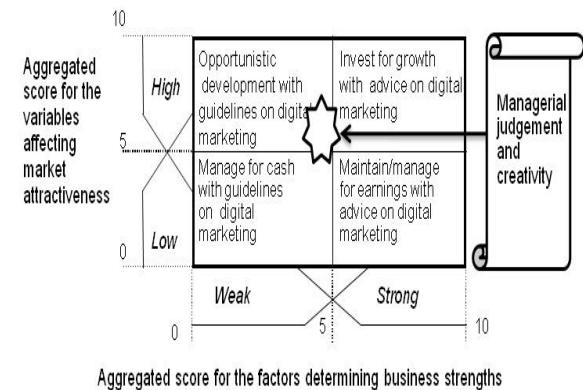


Figure 2. Extended McDonald [12]’s grid with digital marketing dimensions and guidelines (Produced by Shuliang Li, the University of Westminster)

#### E. Evaluation of the fuzzy rules for strategic models for recommending marketing strategies

Based on Zadeh [25]’s theory and principles for the management of uncertainty in expert systems, the mechanism for the evaluation fuzzy rules for international marketing strategy recommendation is proposed below. Let

$B_i$  stand for the dimension (e.g. competitive strengths) of a strategic matrix and  $K_i$  denote a list of strategic options.

For a two-dimensional strategic matrix, the general form of a fuzzy inference rule can be expressed as:

$R_i$ : If ( $B_{i1}$  is  $b_{i1}$  and  $B_{i2}$  is  $b_{i2}$ ), then  $K_i$  is  $k_i$

where  $b_{i1}$  and  $b_{i2}$  are the linguistic variables corresponding to the specific values (e.g. weak or strong) of the two-dimensions in a strategic grid and  $k_i$  is the linguistic variable corresponding to a particular strategic option.

#### F. Evaluation of the premise of a fuzzy rule for digital marketing strategies

The grade of certainty or level of confidence of the predicates in the premise (condition part) of the rule  $R_i$  is given by:

The level of confidence of ' $B_{i1}$  is  $b_{i1}$ ' is  $y_{i1}$

The level of confidence of ' $B_{i2}$  is  $b_{i2}$ ' is  $y_{i2}$

The overall grade of certainty in the premise of  $R_i$  can be expressed in the form of conjunctive rule:

$$y_i = (y_{i1}) \wedge (y_{i2}) = \min(y_{i1}, y_{i2})$$

In real-world decision-making situations, to avoid same levels of confidence for different strategic options caused by the above expression, we can apply Bayes theorem to a fuzzy extension [22]. Because the two dimensions of a strategic grid are independent to each other, the overall degree of confidence or certainty factor in the premise of  $R_i$  may also be denoted as a joint fuzzy probability:

$$y_i = (y_{i1}) (\cdot) (y_{i2})$$

#### G. Determining the grade of certainty for the consequent of the rule

For fuzzy rule  $R_i$ , the grade of certainty or level of confidence of its consequent (conclusion part of the rule) will be the same as the overall degree of confidence of its premise. The grade of certainty of the consequent ' $K_i$  is  $k_i$ ' also equals  $y_i$ .

#### H. Strategic recommendations for digital marketing strategies

After execution of the fuzzy rules, the automated system produces one or more strategic alternatives in the form of:

$K_j$  is  $k_{j1}$ :  $y_{j1}$

$K_j$  is  $k_{j2}$ :  $y_{j2}$

... ..

$K_j$  is  $k_{jn}$ :  $y_{jn}$  in which  $n \leq 4$  for a four-cell or nine-cell strategic grid/matrix.

The outputs are not aggregated as we want to give a list of strategic options with associated degrees of confidence. The decision-makers then evaluate the system's recommendations and choose a particular digital marketing strategy.

#### I. Online databases in support of digital marketing strategy formulation

Following Silberschatz, Korth and Sudarshan [17]'s mathematical notations, relational algebra representation, and specifications on manipulative operations on the relations of databases, the target hybrid systems mainly include Web-enabled insertion, deletion and updating data for the process of digital marketing planning.

Selecting tuples and displaying data from a Web-enabled relation,  $r$ , can be expressed as:

$\sigma_{selection\ predicate}(r)$  in which  $\sigma$  stands for selection manipulation.

A deletion manipulation can be stated as:

$$r \leftarrow r - E$$

Insertion into a Web-based relation can be represented in the following form:

$r \leftarrow r \cup E$  where  $\cup$  is the union of sets and  $E$  is an expression.

Choosing tuples from Web-based relation,  $r$ , and modify them when necessary:

$$r \leftarrow \prod_{F_1, F_2, \dots, F_n} (\sigma_P(r)) \cup (r - \sigma_P(r))$$

in which each  $F_i$  is the  $i$ th data field of  $r$  while  $P$  symbolizes the condition that looks up which tuples to alter. Here,  $\prod$  is the projection operator on Web-based database server via the Internet.

#### J. Making a final choice and selecting one digital marketing strategy

The decision makers are required to provide their judgemental inputs, intuition and preferences to the variables determining the strategies. They should also apply their intuition and creativity when making a final choice on the basis of the system-generated strategic alternatives for digital marketing decisions, looking at different grades of certainty.

### III. EXAMPLE: THE WebDigital SYSTEM

As an application example, the WebDigital system [11], is included in this section to validate and demonstrate the computational algorithms, symbolic knowledge representation and approximate reasoning framework discussed in previous sections. WebDigital is a research prototype that is designed to assist the main stages of digital marketing strategy development illustrated in Figure 1. It was built and created by the first and second-named authors on the basis of the client-server architecture, with server-side programming and software implementation.

In line with the algorithms and specifications provided in this paper, a Web-enabled *Monte Carlo simulation module* was developed to model and simulate the ambiguities and variations in relation to the digital marketing variables. *Fuzzy logic* was utilised and coded to symbolise pertinent variables, and compute the grades of certainty for digital marketing factors using trapezoidal membership/compatibility functions. A *knowledge base* was constructed to accommodate “if ... then ...” rules and fuzzy rules for representing relevant strategic digital marketing grids, international e-marketing models, e-mail marketing expertise and other domain knowledge obtained from the literature. *Inference functions* were programmed to carry out forward chaining under uncertainty and generate digital marketing strategy alternatives with various levels of confidence. A *Web-server database component* was created to store simulation results, and saves and retrieves the user’s judgemental inputs and data entries. The *Web-based user interface* was coded to aid the dialogue between the user and the WebDigital system.

The knowledge was acquired from the literature. McDonald[12]’s four-box marketing strategy development directional policy matrix is adapted and extended to cover digital marketing dimensions and elements including expertise and guidelines obtained from Varadarajan and Yadav’s [20], Sultan and Rohm [18], Chaffey’s [1], Chaffey, Ellis-Chadwick, Johnston and Mayer [2], Gay, Charlesworth and Esen [4], and Watson and Zinkhan [22]. Knowledge about e-marketing strategies in international markets is collected from Sheth and Sharma [16].

Business case-based evaluation with associated questionnaire survey was undertaken in November and December of 2009, and September 2010, with four company directors and managers, and three university course leaders participated. The results revealed that WebDigital is extremely efficient in improving the speed of decision-making and overcoming time zone and geographical barriers. Respondents also reported that it is very effective in increasing the awareness of the variables that affect digital marketing strategies. It was reported to be helpful in linking Web-enabled strategic analysis models, digital marketing knowledge automation with managerial judgement and intuition. Importantly, the WebDigital system was considered as having a positive impact on

enhancing the quality of digital marketing strategy formulation.

According to the respondents, WebDigital “would help in the process of taking the correct strategy in local and global markets”. It provides “checklist of available appropriate actions for relevant strategy based on automated assessment”. “The system helps improve the outcomes of my company’s digital marketing strategy as it considers the various factors that impact on company performance by the use of recognised and relevant models, techniques and technologies”. It offers “more advice and routes for digital marketing” and “boosts our confidence in making the right decisions”.

### IV. CONCLUSIONS

In this paper we have proposed a hybrid mathematical and computational framework with symbolic knowledge representation for digital marketing strategy formulation. According to extensive literature search, this is the first study to establish a comprehensive computational and knowledge automation framework for digital marketing planning. We have developed a systematic method for modelling and automating the process of digital marketing planning, through integrating the strengths of online databases, Monte Carlo simulation, expert systems, fuzzy rules and approximate reasoning to complement human intuition and creativity, and support the key stages of the planning process. A software example has been presented to demonstrate the value of such a framework.

The findings of this paper indicate that the digital marketing planning process can be described mathematically and supported effectively. The usefulness of applying this Web-based hybrid intelligent framework combined with relevant software systems is that the decision making procedure in digital marketing strategy development can be made more efficient and effective.

In addition, the definitions of pertinent concepts, mathematical description, computational algorithms, and symbolic knowledge representation will provide a basis for designing and tailoring Web-based hybrid decision support systems in relevant fields.

Given the increasing complexity of competitive environments and consumer interaction with digital media, digital marketing decision making will require more effective decision support systems through Web-based delivery combined with hybrids of conventional decision support techniques, intelligent software agents, Web analytics, and circumstance-sensitive features. Such a solution will be essential for the timely and efficient detection and reaction to contextual changes and more importantly, it still incorporates managerial judgement to play a leading role in selecting and identifying variables which influence the strategic decisions and making the final strategic choice.

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