A common analytical framework for assessing the life cycle greenhouse gas emissions and the economic value of product systems

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1. Introduction

This report has been produced as part of the EPSRC-funded research project, *Carbon Calculations over the Life Cycle of Industrial Activities* (CCaLC).[1] The objectives of the report are:

- to develop a common accounting framework that allows the life cycle carbon emissions and the economic value of a product system to be evaluated on a consistent basis;
- to investigate the relationship between aggregate measures of greenhouse gas emissions intensity and the life cycle emissions intensities of individual product systems.

The report comprises two sections. Following this brief introduction, the framework is developed in a formal analysis. The practical application of the framework is then explored using an illustrative example of a product system for a packaged good.

A pre-requisite for the framework is that it should be consistent with the principles and approaches prescribed in the standards that have been adopted for measuring life cycle greenhouse gas emissions and for measuring the value of economic activity in national accounts. The former are set out in the Publicly Available Specification (PAS 2050): *Specification for the assessment of the life cycle greenhouse gas emissions of goods and services* published by the British Standards Institution (BSI) in October 2008. The latter are set out in the *European System of National and Regional Accounts* (ESA95), which was adopted by the European Commission is June 1996 and applies to all EU member states.[2]

Life cycle greenhouse gas emissions

The objective of PAS 2050 is to provide a consistent method for assessing the life cycle greenhouse gas (GHG) emissions of goods and services – i.e. the emissions that are released by the processes of creating, modifying, transporting, storing, using, providing, recycling or disposing of goods and services. As such, it provides the basis for comparing the life cycle GHG emissions of different goods and services on a consistent basis, and for evaluating alternative product configurations, operational and sourcing options, etc. PAS 2050 sets out the requirements for defining the appropriate system boundary and for identifying the sources of GHG emissions within that boundary. It also specifies the data requirements and the methodology for using these to calculate the resultant carbon footprint.

Under the specified methodology, activity data for each process contributing to the production of the good (i.e. processes within the system boundary) is multiplied by an emission factor for the activity. These are then added together to obtain the life cycle emissions per functional unit of the good. For "business-to-consumer" goods, this includes emissions from the complete product life cycle, including those arising from its use. For business-to-business goods, it includes all of the GHG emissions that have occurred up to, and including, the point at which the input arrives at the user business.

Implicit in this methodology is the notion of an imbedded emissions value for a good or service. This represents the total emissions (per functional unit) of the production processes

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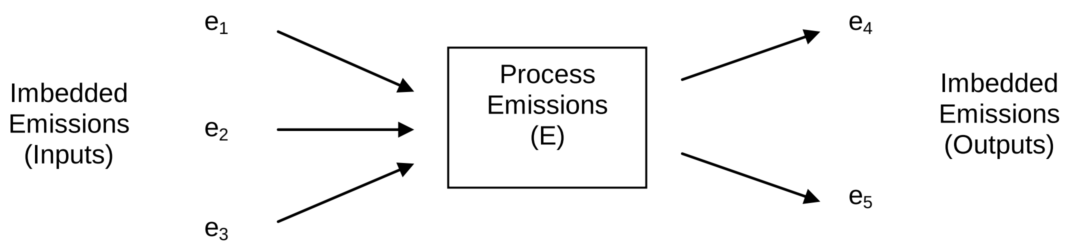
[1] EPSRC Research Project: EP/F003501/1

[2] Council Regulation (EC) No 2223/96, June1996.

that have contributed (directly or indirectly) to its production. For any given process, the value of the emissions imbedded in its outputs is equal to the direct emissions released by that process (i.e. process emissions), plus the sum of all of the emissions imbedded in its inputs. This is defined as the emissions balance equation for the process. This is illustrated in Figure 1.1, in which the process produces two outputs and uses three inputs. The coefficients a_1, a_2 and a_3 represent the quantities of each input used per unit of activity, while b_1 and b_2 represent the quantities of each output produced per unit of activity.

Figure 1.1 Process emissions balance



$$b_4 e_4 + b_5 e_5 = E + a_1 e_1 + a_2 e_2 + a_3 e_3$$

For business-to-consumer products, total lifecycle emissions are given by the imbedded emissions of the good consumed, plus the imbedded emissions of the resultant waste collection service required, plus any (process) emissions arising from its use.

Economic value

National accounts, or national account systems, provide a complete and consistent conceptual framework for measuring the economic activity of a nation (or other geographic area). As such, they provide – *inter alia* – information on the economic values of the various flows within the economy (i.e. production, expenditure and income) and the interactions with other countries (i.e. exports and imports).

In the United Kingdom (as in all other EU member states), the national accounts are produced in accordance the *European System of Accounts 1995* (ESA95)[3], which provides a coherent, consistent and integrated set of accounts and balance sheets based on internationally agreed concepts, definitions, classifications and accounting rules. ESA95 is itself based on – and is broadly consistent with – the principles set out in the *System of National Accounts 1993* (SNA93)[4], which was published jointly by the United Nations, the Commission of the European Communities, the International Monetary Fund, the Organisation for Economic Co-operation and Development, and the World Bank.

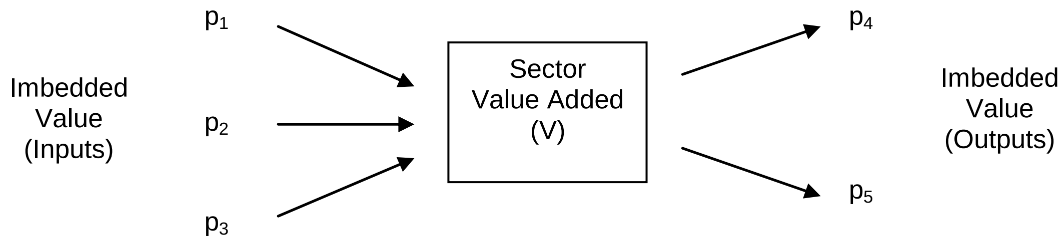
A key concept in the national accounts framework is that of gross value added (GVA). This is defined as the value of output for an individual producer, industry or sector, less the total value of its intermediate consumption (i.e. its expenditure on all inputs except for the primary factors, labour and capital). GVA is a balancing item and as such, it lacks dimensions – i.e. it

[3] See http://circa.europa.eu/irc/dsis/nfaccount/info/data/esa95/en/titelen.htm

[4] See http://unstats.un.org/unsd/sna1993/introduction.asp

does not have any corresponding quantity units. However, it can – conceptually at least – be expressed per unit of activity, with the values of the inputs and outputs also being expressed per unit of activity. The concept is illustrated in Figure 1.2 for a sector that produces two outputs and uses two inputs, where the prices of the inputs and outputs represent their respective imbedded values.

Figure 1.2 Sector gross value added



$$V = (b_4 p_4 + b_5 p_5) - (a_1 e_1 + a_2 e_2 + a_3 e_3)$$

GVA may be measured either at basic prices, or at producers' prices, depending on the price concept that is used to value the outputs (see Box 1 over page for definitions of different price concepts). Essentially, the two approaches differ in terms of how they treat product taxes and subsidies in the valuation of output; with the former excluding all product taxes and adding back any product subsidies, while the latter excludes only invoiced VAT and does not add back product subsidies. Both approaches are recognised under SNA93, although valuation at basic prices is preferred.[5] ESA95 is more prescriptive, specifying that output (and hence GVA) should be valued at basic prices.

Irrespective of the approach that is used to value outputs, all intermediate inputs are valued at purchasers' prices – i.e. prices including non-deductible taxes and any transport charges invoice separately. However, if VAT is completely deductible for intermediate consumption then the only difference between the purchasers' price of a good and the producer price is the cost of transportation invoiced separately. Consequently, the total value of intermediate consumption for an enterprise is the same whether it is valued at purchasers' prices or at producers' prices.[6] Under producers' prices, transportation services are unbundled from the goods and treated as a separate input. While this results in a different allocation of expenditures from that under purchasers' prices, it does not change the total value of the expenditures. It follows directly that the value of GVA at producers' prices is same as one which uses producers' prices to value both inputs and outputs.

[5]See SNA(1993), paragraph 15.33 .

[6] In practice, expenditures by enterprises on goods or services intended for intermediate use may include small amounts of non-deductible VAT which are excluded from the producers' prices. However, at the aggregate level, the discrepancy between the two approaches is very small and hence they are taken as being equivalent for the purposes of this analysis.

Box 1: Price concepts

There are three price concepts that are used to value outputs and inputs in the national accounts.

- *Basic price*

The amount receivable by the producer for a unit of a good or service produced as output, minus any tax payable, plus any subsidy receivable, on that unit as a consequence of its production or sale. It excludes any transport charges invoiced separately by the producer.

- *Producer's price*

The amount receivable by the producer for a unit of a good or service produced as output, minus any VAT (or similar deductible tax) invoiced to the purchaser. It excludes any transport charges invoiced separately by the producer.

- *Purchaser's price*

The amount paid by the purchaser, excluding any deductible VAT (or similar deductible tax) in order to take delivery of a unit of a good or service. It includes any transport charges paid separately by the purchaser.

2. Formal analysis

In this section a formal model is developed and used to derive expressions for the lifecycle emissions and economic values of individual products (i.e. goods and services) within an overall production / consumption system, and to analyse the relationship between the emissions intensity of individual products and aggregate measures of emissions intensity.

2.1 Model definition

It is assumed that the production system comprises N "produced" products; with each product being produced by a separate, corresponding process (i.e. product 1 is produced by process 1, etc). In addition, there are M resources, which may be either virgin or recycled. While these resources are not produced within the system, they may be generated as bi-products of production processes (e.g. as recycled production scrap).

Let **A** denote the partitioned matrix of technical input coefficients and let **B** denote the partitioned matrix of output coefficients. The (M×M) sub-matrices **A′** and **B′** represent the input / output coefficients for the products; while the (N×M) sub-matrices **A″** and **B″** represent the coefficients for the resources. The j^{th} column of **A** gives the input quantity of each product / resource required for one unit of "activity" by process *j*; while the corresponding column of B gives the output quantities of each product / resource. It is assumed that processes do not use their principle product as an input (i.e. the diagonal elements of **A** are all equal to zero) and that for each process, one unit of activity produces one unit of the principle product (i.e. the diagonal elements of **B** are all equal to one).

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}' & \mathbf{0} \\ \hline \mathbf{A}'' & \mathbf{0} \end{array}\right] \qquad \mathbf{B} = \left[\begin{array}{c|c} \mathbf{B}' & \mathbf{0} \\ \hline \mathbf{B}'' & \mathbf{I} \end{array}\right]$$

$$\mathbf{A}' = \begin{bmatrix} a'_{11} & \cdots & \cdots & a'_{1N} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ a'_{N1} & \cdots & \cdots & a'_{NN} \end{bmatrix} \qquad \mathbf{B}' = \begin{bmatrix} b'_{11} & \cdots & \cdots & b'_{1N} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ b'_{N1} & \cdots & \cdots & b'_{NN} \end{bmatrix}$$

$a_{ij} \geq 0$, $a_{ij} = 0$ for all $i = j$ $\qquad$ $b_{ij} \geq 0$, $b_{ij} = 1$ for all $i = j$

$$\mathbf{A}'' = \begin{bmatrix} a''_{11} & \cdots & \cdots & a''_{1N} \\ \vdots & & & \vdots \\ a''_{M1} & \cdots & \cdots & a''_{MN} \end{bmatrix} \qquad \mathbf{B}'' = \begin{bmatrix} b''_{11} & \cdots & \cdots & b''_{1N} \\ \vdots & & & \vdots \\ b''_{M1} & \cdots & \cdots & b''_{MN} \end{bmatrix}$$

$a_{ij} \geq 0$ $\qquad$ $b_{ij} \geq 0$

where **I** is the (M×M) identity matrix and **0** is a matrix of zeros (of appropriate dimension).

For notational convenience, the following matrices are defined:

$$\mathbf{G} \quad = \quad \mathbf{B} - \mathbf{A} \quad = \quad \left[\begin{array}{c|c} \mathbf{B}' - \mathbf{A}' & \mathbf{0} \\ \hline \mathbf{B}'' - \mathbf{A}'' & \mathbf{I} \end{array} \right]$$

$$\mathbf{H} \quad = \quad [\, \mathbf{B} - \mathbf{A} \,]^{-1} \quad = \quad \left[\begin{array}{c|c} (\mathbf{B}' - \mathbf{A}')^{-1} & \mathbf{0} \\ \hline (\mathbf{A}'' - \mathbf{B}'')(\mathbf{B}' - \mathbf{A}')^{-1} & \mathbf{I} \end{array} \right]$$

It is assumed that the matrix **H** exists (i.e. **G** is non-singular). Since the diagonal elements of **G** are all equal to one, so to are the diagonal elements of **H**.

In addition to being used as inputs to production (i.e. intermediate consumption), the products may be consumed by households (i.e. final consumption) and /or invested in capital formation. They may also be exported and imported across the system boundary. Since virgin resources are not generated within the system, they must all be imported. It is assumed that all of the recycled resources generated as bi-products from production processes are exported (or added to stock); while all of the recycled resources used are imported (taken from stock).

It is assumed that the final consumption system comprises L consumption processes; with the partitioned matrix **C** denoting the input coefficients for these processes, where the l^{th} column of **C′** gives the input quantity of each product required for one unit of "activity" by consumption process *l*.

$$\mathbf{C} \quad = \quad \left[\begin{array}{c} \mathbf{C}' \\ \hline \mathbf{0} \end{array} \right] \qquad\qquad \mathbf{C}' = \begin{bmatrix} c'_{11} & \cdots & \cdots & c'_{1L} \\ \vdots & & & \vdots \\ c'_{N1} & \cdots & \cdots & c'_{NL} \end{bmatrix} \qquad\qquad c_{il} \geq 0,$$

2.2 Physical flows (mass balance)

By definition, in any given time period, the total supply of a product / resource must be equal to the total use, i.e.[7]

Output + Imports + Stock b/fwd ≡ Intermediate Consumption + Final Consumption + Exports + Stock c/fwd

Rearranging yields:

Output – Int. Consumption ≡ Final Consumption + (Exports – Imports) + (Stock b/fwd – Stock c/fwd)

where the left-hand side of the identity represents <u>net output</u>, and the right-hand side represents <u>net final demand</u>. This identity can be written in matrix notation as:

[7] For simplicity, gross fixed capital formation has been omitted from the uses. The inclusion of this would not change the results of the analysis in any way.

$$[\,\mathbf{B} - \mathbf{A}\,]\,\mathbf{z} \quad \equiv \quad \mathbf{d} \quad \equiv \quad \mathbf{C}\,\mathbf{v} + (\,\mathbf{x} - \mathbf{m}\,) + \mathbf{s} \qquad \text{... (1)}$$

where the vectors[8]

$\mathbf{z}$ = production process activity

$\mathbf{d}$ = net final demand

$\mathbf{v}$ = consumption process activity

$\mathbf{s}$ = change in stocks

$\mathbf{m}$ = imports

$\mathbf{x}$ = exports

The activity vector **z** must be non-negative – i.e. it is not possible to have a negative activity level. However, depending on the values of the input and output coefficients in **A** and **B**, it is possible that the net final demand vector may contain negative elements – indicating that either imports or reductions in stock outweigh the other two components.

It follows directly that:

$$\mathbf{z} \quad \equiv \quad \mathbf{H}\,\mathbf{d} \qquad \text{... (2)}$$

The activity level any given process j is provided by the j^{th} element (row) of **z**, i.e.

$$z_j \quad = \quad \mathbf{H}_j\,\mathbf{d} \quad = \quad \sum_i h_{ji}\,d_i \qquad \text{... (3)}$$

where $\mathbf{H}_j$ is the j^{th} row of matrix **H**. Thus, the elements of the j^{th} row of **H** represent the activity levels for process j that are required to support one unit of final demand for each product – e.g. h_{j1} represents the activity level required to support one unit of final demand for product 1, etc.

Denoting the output quantity of product i from process j by y_{ij}, and the input quantity of product k to that process by w_{kj}, it follows that:

$$y_{ij} \quad = \quad b_{ij}\,z_j \quad \text{and} \quad \mathbf{y} \quad = \quad \mathbf{B}\,\mathbf{z} \qquad \text{... (4)}$$

$$w_{kj} \quad = \quad a_{kj}\,z_j \quad \text{and} \quad \mathbf{w} \quad = \quad \mathbf{A}\,\mathbf{z} \qquad \text{... (5)}$$

where the vector

$\mathbf{y}$ = aggregate gross output

$\mathbf{w}$ = aggregate intermediate consumption

2.3 Emissions

By definition, the emissions balance equation for process j is:

[8] All vectors are column vectors unless stated otherwise.

$$\sum_i w_{ij} e_i \; + \; E_j z_j \quad \equiv \quad \sum_i y_{ij} e_i$$

Rearranging and noting (4) and (5) gives:

$$\sum_i (b_{ij} - a_{ij}) e_i \quad \equiv \quad E_j$$

or, in matrix form:

$$\mathbf{e} [\mathbf{B} - \mathbf{A}] \quad \equiv \quad \mathbf{E} \qquad \text{... (6)}$$

where

e is the row vector of imbedded emissions (per unit)

E is the row vector of production process emissions (per unit activity)

It follows directly that

$$\mathbf{e} \quad \equiv \quad \mathbf{E} [\mathbf{B} - \mathbf{A}]^{-1} \quad = \quad \mathbf{E} \, \mathbf{H} \qquad \text{... (7)}$$

The imbedded emissions for any given product *i* is provided by the i^{th} element (column) of the vector **e**, i.e.

$$e_i \quad \equiv \quad \mathbf{E} \, \mathbf{H}_i \quad = \quad \sum_j E_j h_{ji} \qquad \text{... (8)}$$

where $\mathbf{H}_i$ is the i^{th} column of matrix **H**. Thus, the elements of the i^{th} column of **H** represent the contributions of each process to the imbedded emissions of that product – e.g. h_{1i} represents the contribution of process 1, etc.

From the definition of **H**, it follows that for all resources, $e_i = E_i$. That is, emissions imbedded in the resource are equal to the emissions arising from its creation. Since these are assumed to be zero, imbedded emissions are also equal to zero for all resources.

Total emissions from production are given by:

$$\begin{aligned} \mathbf{E}\,\mathbf{z} \quad \equiv \quad \mathbf{e} [\mathbf{B} - \mathbf{A}] \mathbf{z} \quad &= \quad \mathbf{e}\,\mathbf{d} \\ &= \quad \mathbf{e} [\mathbf{C}\,\mathbf{v} + (\mathbf{x} - \mathbf{m}) + \mathbf{s}] \end{aligned} \qquad \text{… (9)}$$

Rearranging gives:

$$\mathbf{e}\,\mathbf{C}\,\mathbf{v} \quad \equiv \quad \mathbf{E}\,\mathbf{z} + \mathbf{e} (\mathbf{m} - \mathbf{x}) - \mathbf{e}\,\mathbf{s}$$

Thus, total production emissions attributable to consumption are equal to total production emissions plus emissions imbedded in net imports, less emissions imbedded in net stock increases. Note that both of the last two elements may be negative (e.g. if exports are greater than imports). Since imbedded emissions are equal to zero for all resources, it follows that total production emissions attributable to consumption include those arising from the reprocessing of recycled resources brought forward, but that no credit is given for any recycled resources carried forward. This is consistent with the treatment in PAS 2050.

If the emissions generated by consumption process l are denoted by E^c_l, then the imbedded emissions (per unit) for the output from that process are given by:

$$e^c_l = \sum_i e_i c_{il} + E^c_l = \sum_j E_j \left[\sum_i h_{ji} c_{il} \right] + E^c_l$$

Consequently, the total emissions attributable to consumption processes is given by:

$$\mathbf{e}^c \mathbf{v} = (\mathbf{e} \, \mathbf{C} + \mathbf{E}^c) \, \mathbf{v}$$

$$= (\mathbf{E} \, \mathbf{H} \, \mathbf{C} + \mathbf{E}^c) \, \mathbf{v} = \hat{\mathbf{E}} \, \mathbf{M} \, \mathbf{v} \qquad \ldots (10)$$

where

$\mathbf{e}^c$ is the row vector of imbedded emissions for the outputs of the consumer processes

$\mathbf{E}^c$ is the row vector of consumption process emissions

$\hat{\mathbf{E}}$ is the concatenated row vector $(\mathbf{E} \mid \mathbf{E}^c)$

$\mathbf{M}$ is the concatenated matrix $[\, \mathbf{H} \, \mathbf{C} \mid \mathbf{I} \,]$

2.4 Economic value

By definition, the total <u>value added</u> for process j is:

$$V_j^* z_j \equiv \sum_i y_{ij} (p_i - t_i) - \sum_i w_{ij} p_i \qquad \text{(at basic prices)}$$

$$V_j z_j \equiv \sum_i y_{ij} p_i - \sum_i w_{ij} p_i \qquad \text{(at producer prices)}$$

where V_j^* and V_j are the value added per unit of activity under the different price definitions; p_i is the producer price of product i; and t_i is the net (non-deductible) tax per unit applying to intermediate purchases of product i. Noting (4) and (5), it follows that:

$$\mathbf{V}^* \equiv \mathbf{p} \, [\, \mathbf{B} - \mathbf{A} \,] - \mathbf{t} \, \mathbf{B} \qquad \ldots (11a)$$

$$\mathbf{V} \equiv \mathbf{p} \, [\, \mathbf{B} - \mathbf{A} \,] \qquad \ldots (11b)$$

where $\mathbf{V}$, $\mathbf{V}^*$, $\mathbf{p}$ and $\mathbf{t}$ are all row vectors. It follows directly that:

$$\mathbf{p} \equiv \begin{cases} [\, \mathbf{V}^* + \mathbf{t} \, \mathbf{B} \,] \, [\, \mathbf{B} - \mathbf{A} \,]^{-1} = [\, \mathbf{V}^* + \mathbf{t} \, \mathbf{B} \,] \, \mathbf{H} & \ldots (12a) \\ \mathbf{V} \, [\, \mathbf{B} - \mathbf{A} \,]^{-1} = \mathbf{V} \, \mathbf{H} & \ldots (12b) \end{cases}$$

The producer price for any given product *i* is provided by the i^{th} element (column) of the vector **p**, i.e.

$$p_i \quad \equiv \quad \mathbf{V}\,\mathbf{H}_i \quad = \quad \sum_j V_j\, h_{ji} \qquad \text{... (13)}$$

where $\mathbf{H}_i$ is the i^{th} column of matrix **H**. Thus, analogous to imbedded emissions, the elements of the i^{th} column of **H** represent the contributions of each process to the value of that product – e.g. h_{1i} represents the contribution of process 1, etc.

From the definition of **H**, it follows that for all resources, $p_i = V_i$. That is, the producer price of a resource is equal to the value added during its creation. Unlike emissions however, this is not assumed to be zero, and hence resources generally have positive prices.

Aggregate value added is given by

$$\mathbf{V}^*\mathbf{z} \quad \equiv \quad \mathbf{p}\,[\,\mathbf{B} - \mathbf{A}\,]\,\mathbf{z} \;-\; \mathbf{t}\,\mathbf{B}\,\mathbf{z} \quad \equiv \quad \mathbf{p}\,\mathbf{d} \;-\; \mathbf{t}\,\mathbf{y} \qquad \text{… (14a)}$$

$$\mathbf{V}\,\mathbf{z} \quad \equiv \quad \mathbf{p}\,[\,\mathbf{B} - \mathbf{A}\,]\,\mathbf{z} \quad \equiv \quad \mathbf{p}\,\mathbf{d} \qquad \text{... (14b)}$$

Thus, aggregate gross value added at producer prices is equal to gross value added at basic prices plus the value of non-deductible product taxes:

$$\mathbf{V}\,\mathbf{z} \quad \equiv \quad \mathbf{V}^*\,\mathbf{z} \;+\; \mathbf{t}\,\mathbf{B}\,\mathbf{z} \qquad \text{... (15)}$$

Consider the expression for gross value added at producer prices (14b):

$$\mathbf{V}\,\mathbf{z} \quad \equiv \quad \mathbf{p}\,[\,\mathbf{C}\,\mathbf{v} \;+\; (\,\mathbf{x} - \mathbf{m}\,) \;+\; \mathbf{s}\,]$$

Denoting the row vectors of product taxes on final consumption and on imports by $\mathbf{t}_c$ and $\mathbf{t}_m$ respectively, then it follows that:

$$\mathbf{V}\,\mathbf{z} \;+\; \mathbf{t}_c\,\mathbf{c} \;+\; \mathbf{t}_m\,\mathbf{m} \quad \equiv \quad (\,\mathbf{p} + \mathbf{t}_c\,)\,\mathbf{C}\,\mathbf{v} \;+\; \mathbf{p}\,\mathbf{x} \;-\; (\,\mathbf{p} - \mathbf{t}_m\,)\,\mathbf{m} \;+\; \mathbf{p}\,\mathbf{s} \qquad \text{... (14)}$$

This is one of the GDP identities from the national accounts framework. The left-hand side is the output definition of GDP – i.e. gross value added at purchaser prices, plus non-deductible VAT, plus net taxes on imports. The right-hand side is the expenditure definition of GDP – i.e. final consumption and changes in stocks at their respective purchaser prices, plus exports at f.o.b. prices, minus imports at f.o.b. prices.

If the shadow price (per unit) for the output from consumption process *l* is denoted by q_l, then the value added by that process is given by:

$$V^c_l \quad = \quad q_l \;-\; (\,\mathbf{p} + \mathbf{t}_c\,)\,\mathbf{C}_l$$

where $\mathbf{C}_l$ is the l^{th} column of the input coefficient matrix **C**.

Consequently, the total value added attributable to consumption is given by:

$$\mathbf{q}\,\mathbf{v} \quad = \quad (\,(\,\mathbf{p} + \mathbf{t}_c\,)\,\mathbf{C} + \mathbf{V}^c\,)\,\mathbf{v}$$

$$= \quad (\mathbf{V\,H\,C} + (\mathbf{V}^c + \mathbf{t}_c\,\mathbf{C})\,)\,\mathbf{v} \quad = \quad \hat{\mathbf{V}}\,\mathbf{M}\,\mathbf{v} \qquad \ldots (15a)$$

where

- $\mathbf{q}$ is the row vector of shadow prices for the outputs of the consumer processes.
- $\mathbf{V}^c$ is the row vector of consumption process value added
- $\hat{\mathbf{V}}$ is the concatenated row vector $(\mathbf{V} \mid \mathbf{V}^c + \mathbf{t}_c\,\mathbf{C}\,)$
- $\mathbf{M}$ is the concatenated matrix $[\,\mathbf{H\,C} \mid \mathbf{I}\,]$

The process value added ($\mathbf{V}^c$) is included for completeness and it can be interpreted as the (shadow) value of the time required by the consumer to undertake each consumption activity. However, this means that it will vary between consumers, and hence that the average shadow price will depend on the mix of consumers – which is clearly problematic. Furthermore, the definition of GDP in the national accounts framework does not recognise the value of time spent on consumption (under the expenditure definition), nor the value added by consumption processes (under the production definition). Consequently, it is assumed that value added is equal to zero for all consumption processes and hence:

$$\mathbf{q\,v} \quad = \quad (\,\mathbf{p} + \mathbf{t}_c)\,\mathbf{C\,v} \quad = \quad \hat{\mathbf{V}}\,\mathbf{M}\,\mathbf{v} \qquad \ldots (15b)$$

2.5 Emissions intensity

Production

The aggregate emissions intensity of production is typically defined as the ratio of aggregate production emissions to aggregate GVA (i.e. production emissions per unit GVA). Using the definition of GVA at producer prices, aggregate emissions intensity is therefore given by:

$$\varepsilon \quad = \quad \frac{\mathbf{E\,z}}{\mathbf{V\,z}} \quad = \quad \frac{\mathbf{e\,d}}{\mathbf{p\,d}}$$

$$= \quad \sum_{i=1} \left(\frac{e_i}{p_i} \right) \beta_i \qquad \text{where} \quad \beta_i \quad = \quad \frac{p_i\,d_i}{\mathbf{p\,d}} \qquad \ldots (16)$$

That is, aggregate production emissions intensity is equal to the weighted average of the individual product emission intensities (ε_i), where the weights are equal to their respective shares of aggregate net demand. The above formulation requires that all products / resources have strictly positive prices (i.e. $p_i > 0$), otherwise the emissions intensity is not defined. While this is a reasonable assumption for all of the produced products, it may not always be so for the non-produced resources. However, provided that imbedded emissions are also equal to zero for these resources (which is assumed to be the case for all resources), then the emissions intensity can be defined to be zero and expression (15) continues to hold. In turn,

$$\varepsilon_i \quad = \quad \frac{e_i}{p_i} \quad = \quad \frac{\mathbf{E\,H}_i}{\mathbf{V\,H}_i}$$

$$= \sum_{j} \left(\frac{E_j}{V_j} \right) \gamma_j \qquad \text{where} \quad \gamma_j = \frac{V_j \, h_{ji}}{\mathbf{V}\,\mathbf{H}_i} \qquad \ldots (17)$$

That is, the individual emissions intensity of each product is equal to the weighted average of the individual production emission intensities of the processes, where the weights are equal to their respective contributions to the product's unit value – i.e. their respective shares of the production value chain for that product. As with the expression for aggregate emissions intensity, the expression for the individual product emission intensities requires that value added is non-zero for all processes.[9] While it is likely that value added would be strictly positive for most processes, it is possible that it could be equal to zero. However, again, provided that the process emissions are also equal to zero, then the emissions intensity can be defined to be zero for that process and expression (16) continues to hold.

Consumption

An alternative definition of emissions intensity compares aggregate emissions attributable to consumption with the total value of consumption; where the former includes emissions generated during consumption.

$$\varepsilon^c = \frac{(\mathbf{e}\,\mathbf{C} + \mathbf{E}^c)\,\mathbf{v}}{(\mathbf{p} + \mathbf{t}_c)\,\mathbf{C}\,\mathbf{v}} = \frac{\mathbf{e}^c\,\mathbf{v}}{\mathbf{q}\,\mathbf{v}}$$

$$= \sum_{l=1} \left(\frac{e^c_l}{q_l} \right) \beta_l \qquad \text{where} \quad \beta_l = \frac{q_l \, v_l}{\mathbf{q}\,\mathbf{v}} \qquad \ldots (18)$$

That is, aggregate emissions intensity is equal to the weighted average of the individual "consumption service" emission intensities (ε^c_l), where the weights are equal to their respective shares of aggregate consumption value. Again, the above formulation requires that all consumption services have strictly positive shadow prices (i.e. $q_l > 0$), otherwise the emissions intensity is not defined. However, unlike the case of production intensities, this is guaranteed since the shadow price is equal to the marginal utility of consumption (divided by the marginal utility of income), which is strictly positive.[10]

$$\varepsilon^c_l = \frac{e^c_l}{q_l} = \frac{\hat{\mathbf{E}}\,\mathbf{M}_l}{\hat{\mathbf{V}}\,\mathbf{M}_l}$$

$$= \sum_{j} \left(\frac{\hat{E}_j}{\hat{V}_j} \right) \gamma_j \qquad \text{where} \quad \gamma_j = \frac{V_j \, m_{jl}}{\hat{\mathbf{V}}\,\mathbf{M}_l} \qquad \ldots (19)$$

That is, the emissions intensity of each "consumption service" is equal to the weighted average of the individual emission intensities of the production and consumption processes, where the weights are equal to their respective contributions to the service's shadow value –

[9] Note that, unlike prices, the value added by a process could be negative – i.e. it could be loss-making.

[10] As will be seen in section 3, when consumption processes are decomposed to create artificial processes with no output, the emissions intensity for these processes is not defined and expression (18) fails to hold.

i.e. their respective shares of the total value chain for that service. While it is reasonable to assume that the process intensity is defined for all consumption processes given the (almost) universal coverage of taxes on final consumption, as noted above, it is possible that value added could be equal to zero. However, provided that the process emissions are also equal to zero, then the emissions intensity can be defined to be zero for that process and expression (16) continues to hold.

2.6 Partial system

The preceding analysis includes all of the products / resources and processes in the production system. However, in practice, most analyses will only consider a subset of the system. In this section, the implications of this for calculating the production emissions and the value added of products are considered.

Define the processes / products that are within the partial system as "internal" and those outside as "external". Partition the input and output coefficient matrices A and B

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}$$

such that the sub-matrix:

- A_{11} / B_{11} represents the input / output coefficients for internal products by internal processes
- A_{12} / B_{12} represents the input / output coefficients for external products by internal processes
- A_{21} / B_{21} represents the input / output coefficients for internal products by external processes
- A_{22} / B_{22} represents the input / output coefficients for external products by external processes

Similarly, partition the matrices G and H

$$\mathbf{G} = \mathbf{B} - \mathbf{A} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix}$$

$$\mathbf{H} = [\mathbf{B} - \mathbf{A}]^{-1} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix}$$

By definition,

$$\begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix}$$

Consequently, it follows that:

$$\mathbf{G}_{11}\mathbf{H}_{11} + \mathbf{G}_{12}\mathbf{H}_{21} = \mathbf{I} = \mathbf{H}_{11}\mathbf{G}_{11} + \mathbf{H}_{12}\mathbf{G}_{21} \quad \ldots (19a)$$

$$\mathbf{G}_{11}\mathbf{H}_{12} + \mathbf{G}_{12}\mathbf{H}_{22} = \mathbf{0} = \mathbf{H}_{11}\mathbf{G}_{12} + \mathbf{H}_{12}\mathbf{G}_{22} \quad \ldots (19b)$$

$$\mathbf{G}_{21}\mathbf{H}_{11} + \mathbf{G}_{22}\mathbf{H}_{21} = \mathbf{0} = \mathbf{H}_{21}\mathbf{G}_{11} + \mathbf{H}_{22}\mathbf{G}_{21} \quad \ldots (19c)$$

$$\mathbf{G}_{21}\mathbf{H}_{12} + \mathbf{G}_{22}\mathbf{H}_{22} = \mathbf{I} = \mathbf{H}_{21}\mathbf{G}_{12} + \mathbf{H}_{22}\mathbf{G}_{22} \quad \ldots (19d)$$

Denote the (row) vectors of imbedded emissions and process emissions for internal products / processes by $\mathbf{e}_1$ and $\mathbf{E}_1$ respectively, and denote the corresponding vectors for external products by $\mathbf{e}_2$ and $\mathbf{E}_2$. Then

$$\mathbf{e} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \end{bmatrix}$$

From (12b):

$$\begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \end{bmatrix} \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \quad \ldots (20)$$

which implies that

$$\mathbf{e}_1 = \mathbf{E}_1\mathbf{H}_{11} + \mathbf{E}_2\mathbf{H}_{21} \quad \ldots (21a)$$

$$\mathbf{e}_2 = \mathbf{E}_1\mathbf{H}_{12} + \mathbf{E}_2\mathbf{H}_{22} \quad \ldots (21b)$$

Thus, the elements of the i^{th} column of $\mathbf{H}_{11}$ represent the contributions of each internal process to the imbedded emissions of internal product i, while the column of $\mathbf{H}_{21}$ represent the contributions of each external process – e.g. h_{1i} represents the contribution of process 1, etc.

Similarly, from (11b)

$$\begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 \end{bmatrix} \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix}$$

which implies that

$$\mathbf{E}_1 = \mathbf{e}_1\mathbf{G}_{11} + \mathbf{e}_2\mathbf{G}_{21} \quad \ldots (22a)$$

$$\mathbf{E}_2 = \mathbf{e}_1\mathbf{G}_{12} + \mathbf{e}_2\mathbf{G}_{22} \quad \ldots (22b)$$

The whole system is therefore defined by either (21a/b) or (22a/b), with each pair of equations implying the other. In contrast, the partial system is defined only by (22a), with the values of $\mathbf{e}_2$ being taken as exogenous. Rearranging yields the following expression for $\mathbf{e}_1$:

$$\mathbf{e}_1 = [\, \mathbf{E}_1 - \mathbf{e}_2\mathbf{G}_{21} \,]\, \mathbf{G}_{11}^{-1} \quad \ldots (23)$$

A necessary condition for this to be equivalent to (21a) is:

$$[\, \mathbf{E}_1 - \mathbf{e}_2\mathbf{G}_{21} \,]\, \mathbf{G}_{11}^{-1} = \mathbf{E}_1\mathbf{H}_{11} + \mathbf{E}_2\mathbf{H}_{21}$$

Rearranging yields:

$$\mathbf{e}_2\,\mathbf{G}_{21}\,\mathbf{G}_{11}^{-1} \quad = \quad \mathbf{E}_1\,[\,\mathbf{G}_{11}^{-1} - \mathbf{H}_{11}\,] \;-\; \mathbf{E}_2\,\mathbf{H}_{21}$$

$$\mathbf{e}_2 \quad = \quad \mathbf{E}_1\,[\,\mathbf{G}_{11}^{-1} - \mathbf{H}_{11}\,]\;\mathbf{G}_{11}\;\mathbf{G}_{21}^{-1} \;-\; \mathbf{E}_2\,\mathbf{H}_{21}\,\mathbf{G}_{11}\,\mathbf{G}_{21}^{-1}$$

$$\mathbf{e}_2 \quad = \quad \mathbf{E}_1\,[\,\mathbf{I} - \mathbf{H}_{11}\,\mathbf{G}_{11}\,]\,\mathbf{G}_{21}^{-1} \;-\; \mathbf{E}_2\,\mathbf{H}_{21}\mathbf{G}_{11}\mathbf{G}_{21}^{-1}$$

But note from (19a) that $\mathbf{I} - \mathbf{H}_{11}\mathbf{G}_{11} = \mathbf{H}_{12}\mathbf{G}_{21}$ and from (19c) that $\mathbf{H}_{21}\mathbf{G}_{11} = -\mathbf{H}_{22}\mathbf{G}_{21}$. Therefore:

$$\mathbf{e}_2 \quad = \quad \mathbf{E}_1\,\mathbf{H}_{12}\,\mathbf{G}_{21}\,\mathbf{G}_{21}^{-1} \;+\; \mathbf{E}_2\,\mathbf{H}_{22}\,\mathbf{G}_{21}\mathbf{G}_{21}^{-1}$$

$$= \quad \mathbf{E}_1\,\mathbf{H}_{12} + \mathbf{E}_2\,\mathbf{H}_{22}$$

Thus, provided that the imbedded emissions of the external products ($\mathbf{e}_2$) satisfies (21b) – i.e. they fully reflect the emissions generated as a result of their production – including any emissions generated by internal processes, then the imbedded emissions calculated for the internal products in the partial system are the same as they would be if calculated within the whole system.[11]

Similarly, a necessary condition for the prices of the internal products in the partial system

$$\mathbf{p}_1 \quad = \quad [\,\mathbf{V}_1 - \mathbf{p}_2\,\mathbf{G}_{21}\,]\,\mathbf{G}_{11}^{-1} \qquad \ldots (24)$$

to be the same as those calculated under the whole system definition is that the exogenous prices of the external products ($\mathbf{p}_2$) fully reflect the value added by the processes that contribute to their production – including any value added by internal processes, i.e.

$$\mathbf{p}_2 \quad = \quad \mathbf{V}_1\,\mathbf{H}_{12} + \mathbf{V}_2\,\mathbf{H}_{22} \qquad \ldots (25)$$

[11] A direct corollary of this is that equations (23) and (21b) define the whole system.

3. Illustrative Example

The methodology set out in the previous section is illustrated using the simple product system shown in Figure 3.1. Following the guidelines provided in PAS 2050, the process map for the system is divided into five steps; starting with raw materials production and moving through manufacture, distribution and retail, to consumer use and finally disposal and / or recycling.

In this simple example, a final packaged good (FG) is manufactured from two intermediate goods (IG1 and IG2). The first intermediate good – the packaging – is made from a combination of a virgin material (VM1) and a recycled material (RM1) in a fixed proportion; with the former being produced from a virgin resource (VR1). The second intermediate good – the content – is produced from another virgin material (VM2), which in turn is produced from a corresponding virgin resource (VR2). The final good passes through the retail chain before being consumed. After consumption, the discarded packaging is collected by the waste collection authority. A fixed proportion of the collected packaging is separated out from the waste stream and sent for reprocessing, with the remainder being sent to landfill for disposal. In addition to the post-consumption waste, the production of the packaging (IG1) generates scrap material which is sent for reprocessing; while the production of the contents generates residues which are sent to landfill. With the exception of the flows through the consumption process, all of the other flows between processes entail physical transportation – which is assumed to be by road.

Of course, this representation of the product system for a packaged good is a considerable simplification of what is a much more complex process in reality. In particular, it omits a number of important inputs, such as energy. Nevertheless, it captures the salient features of the product system and serves to illustrate the methodology.

Figure 3.1 shows the physical flows through the system, from the production of the raw materials from virgin resources, to the final disposal of the waste packaging. However, for the purposes of the analysis, it is useful to make some amendments to the representation of the product system. These are shown in Figure 3.2. The first amendment is to move the reprocessing of waste packaging (RM1 production) from the final process step (i.e. disposal and recycling) to the first step (i.e. raw materials), with the separated waste packaging being treated as a recovered resource (RR1). Thus, the reprocessing of the recovered resource is treated as being analogous to the processing of the virgin resources. This is consistent with the treatment of recycling set out in PAS 2050, where lifecycle emissions include those arising from recycled material inputs and those arising from disposal of waste material, with no credit being given for material recycled at the end of the product's life.[12]

The second amendment is to reverse the direction of the flows for waste collection and disposal, so that these now represent the flows of waste collection services (WCS) provided by the waste collection authority, and waste disposal services (WDS) provided by the landfill operator. However, these flows continue to be measured in the original physical units (e.g. tonnes). Thus, for every tonne of waste packaging collected, one tonne of waste collection service is "consumed", with the "production" of this service requiring the "input" of waste disposal services. The input quantity required depends on the proportion of waste packaging that is recovered, with the latter being treated as a production bi-product.

[12] Annex D, PAS 2050: 2008, Specification for the assessment of the life cycle greenhouse gas emissions of goods and services

Figure 3.1 Process map – physical flows

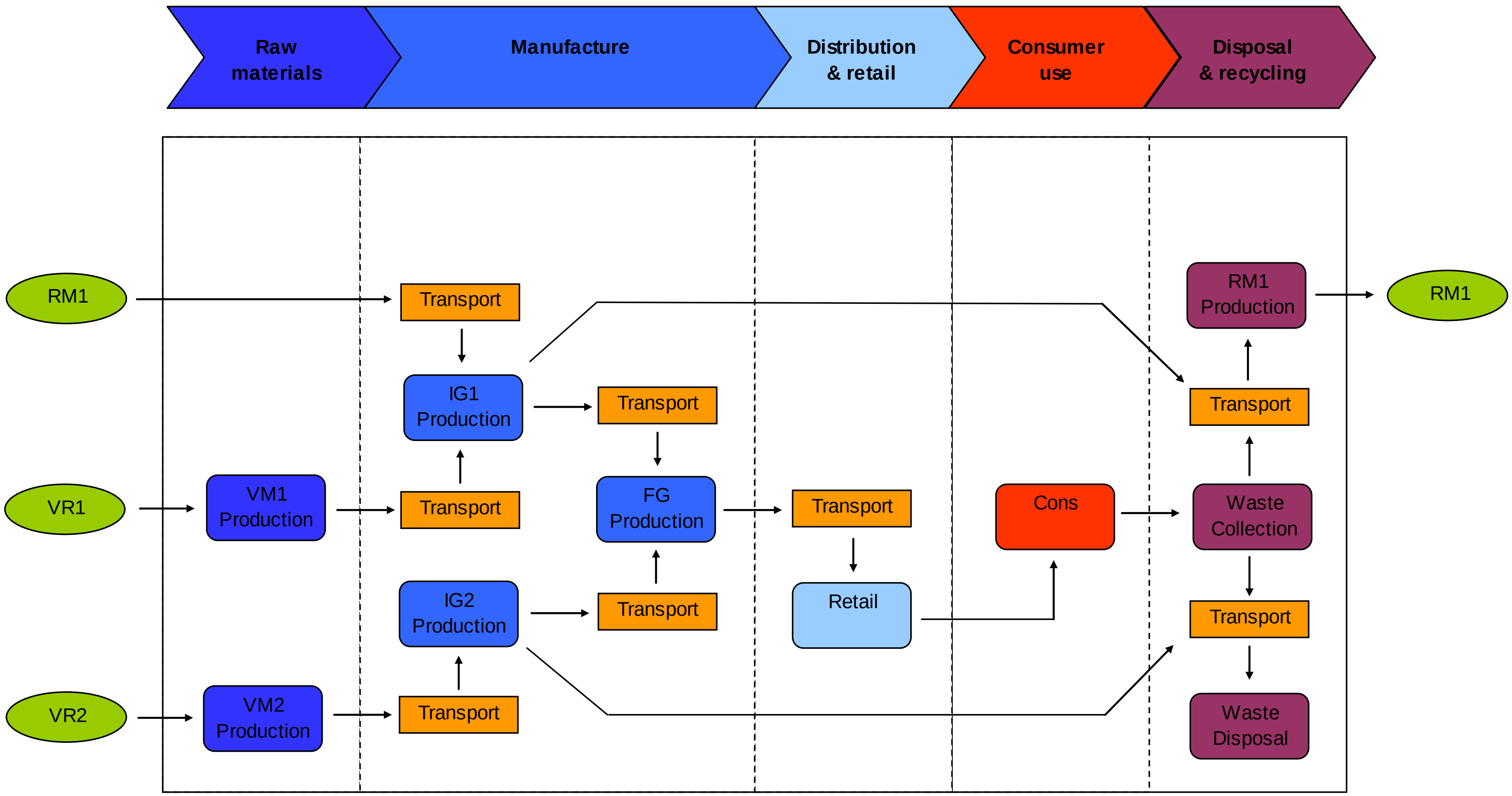
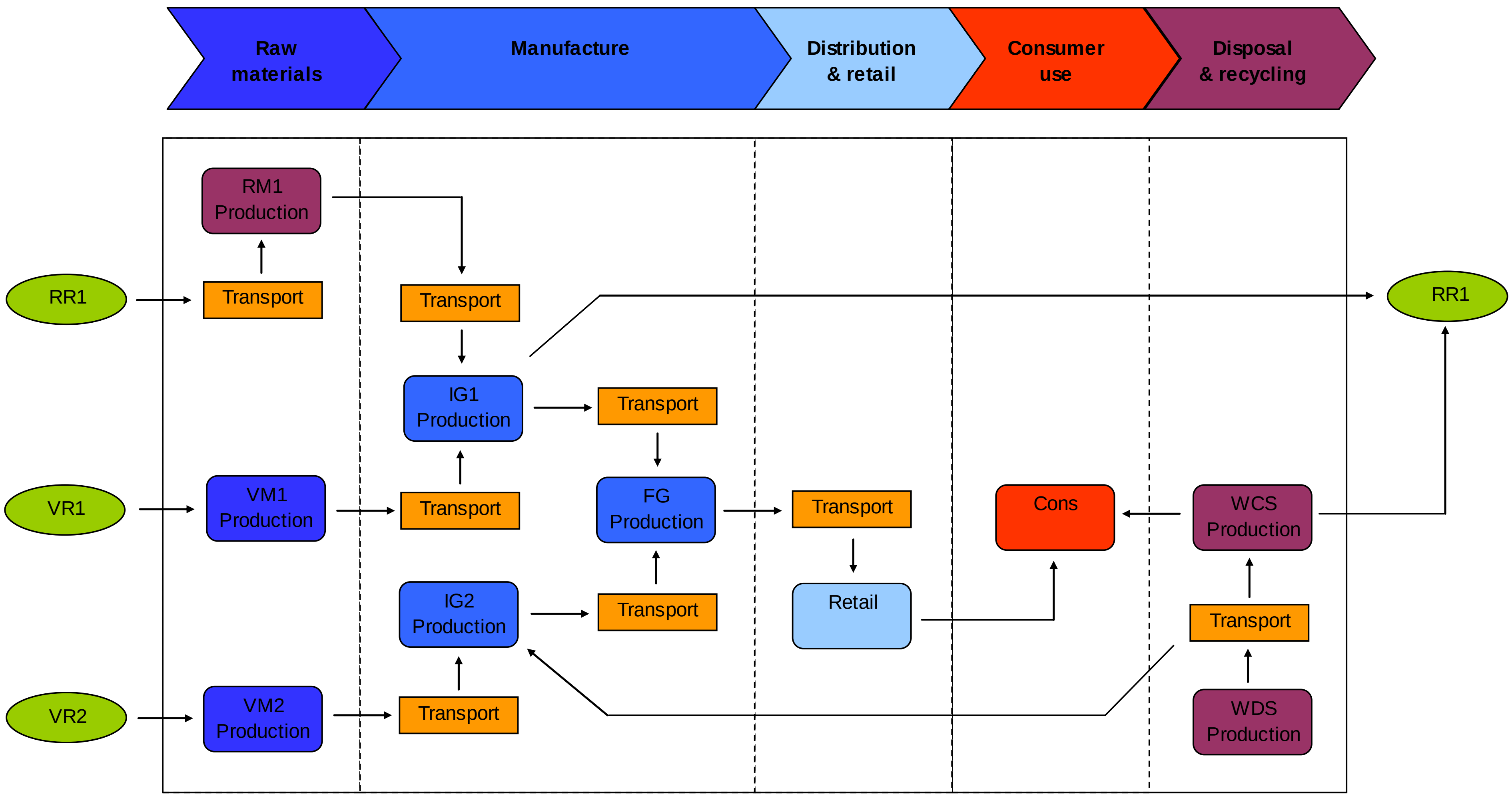


Figure 3.2 Process map -



The values of the output coefficients (B) and the input coefficients (A) that have been used for this illustrative example are given in Table 3.2 and Table 3.3 respectively.[13] The values represent the quantities of outputs produced / inputs used per unit of activity for each process, with the dimensions reflecting the units of measurement (as shown in Table 3.1). The first ten rows / columns relate to the products and services produced within the system; the last three rows / columns relate to the non-produced resources. Table 3.4 shows the resultant net output matrix (B-A), with Table 3.5 showing the inverse of this matrix.

Table 3.1 Units of measurement

Product		Units
RM1	Recycled material (packaging)	Tonnes
VM1	Virgin material (packaging)	Tonnes
VM2	Virgin material (content)	Tonnes
IG1	Packaging	Tonnes
IG2	Content	Litres (000)
FG	Packaged good (manufactured)	Bottles (000)
RG	Packaged good (retailed)	Bottles (000)
WCS	Waste collection service	Tonnes
WDS	Waste disposal service	Tonnes
TS	Transportation service	m^3 km (000)
VR1	Virgin resource (packaging)	Tonnes
VR2	Virgin resource (content)	Tonnes
RR1	Recycled resource (packaging)	Tonnes

With the exception of the production of packaging (IG1) and waste collection services (WCS), each production process generates only its respective primary product; with one unit of activity producing one unit of output. These two processes also generate the recovered resource (RR1) as a bi-product of production in addition to their respective primary products. The inputs to each process reflect the flows shown in Figure 3.2, with the transportation of products used as inputs to a particular process being treated as an input of transport services. As was noted in section 2, the input coefficients for the three resources are all equal to zero, reflecting the fact that they are not produced within the system.

Packaging production uses 0.8 tonnes of virgin material (VM1) per unit activity and 0.4 tonnes of recycled material (RM1) – i.e. the recycled content of packaging is 33%. It produces 1 tonne of packaging (IG1) per unit activity, plus 0.2 tonnes of recovered resource (RR1). The production of waste collection services (WCS) generates 0.6 tonnes of recovered resource as a bi-product – i.e. an end-of-life recycling rate of 60%. Consequently, it requires the input of only 0.4 tonnes of waste disposal services (WDS) per unit activity. The production of the content (IG2) requires the input of 0.2 tonnes of waste disposal services (WDS) for the production residues.

Table 3.6 shows the resultant production and intermediate consumption quantities for the various products and resources for a given vector of final demands, together with the activity

[13] The values have been chosen purely to illustrate the methodology and are not intended to be representative of reality.

levels of the corresponding processes.[14] Final consumption of the packaged good is assumed to be 100,000 bottles. The values of the input coefficients for final good production imply that the weight of each bottle is 0.5 kg. Consequently, the consumption of waste collection services is 50 tonnes. Net imports are equal to zero for all of the produced products and services (i.e. there are no flows across the system boundary for these). For the three resources, the value of net imports is set so that the respective activity levels are equal to zero. The two virgin resources are imported. In contrast, the recycled resource is exported, reflecting the fact that the quantity generated (i.e. 40 tonnes) is greater than the amount used in the production of recycled material (i.e. 20 tonnes).[15]

Also shown in Table 3.6 are the imbedded emissions (e) and producer price (p) for each product, service and resource that are calculated from the exogenous input values for the process emissions (E) and value added (V) per unit activity for the corresponding processes.[16] It is assumed that the waste packaging and the residues from the content production are both inert and hence there are no emissions generated by their disposal. As was noted in the previous section, it is assumed that process emissions are also equal to zero for the three resources, which implies that so too are the corresponding imbedded emissions values. However, it is assumed that all three resources have a market value, and hence their respective processes generate positive value added.

For each of the produced products and services, the imbedded emissions value represents the aggregation of all of the process emissions that have contributed (directly or indirectly) to its production, including all of the emissions generated by the transportation of products up to the point of production. As such, it is an "output value" and it does not include the emissions generated during the product's transportation to succeeding processes. If one wished to calculate "input values" for the imbedded emissions of products / services used in a particular process, this can be done by reallocating the emission imbedded in its input of transportation services. For example, the input of transportation services for packaging production (IG1) is 600 m^3 km per unit of activity (i.e. the input coefficient is 0.6). If this is split proportionally between the two material inputs, then the transportation emissions per tonne of material input is the same for both materials – at 2 kg CO_2 per tonne, and hence the input values of imbedded emissions for recycled and virgin materials are equal to 6 kg CO_2 per tonne and 7 kg CO_2 per tonne respectively.[17] Similarly, transportation costs can be reallocated to the material inputs to give the input price including transportation (i.e. the purchaser's price).

Total production emissions are equal to 3,020 kg CO_2, while the total value added is £48,000. In each case, the value is the same whether it is derived from the emissions / value added of the component processes, or the imbedded emissions / prices of the product final demands. The resultant aggregate production emissions intensity is equal to 0.063 kg CO_2 per £.

[14] Strictly speaking, process activity levels are dimensionless. However, it is convenient to denominate them in the units of their respective principle products.

[15] In the context of this example, exports may be to another product system rather than another country. They may also represent stock-building in a "closed loop" system where the recycled content of packaging is increasing over time – i.e. this year's recovery is equal to next year's use, which is greater than this year's use.

[16] $\mathbf{e} = \mathbf{HE}$ and $\mathbf{p} = \mathbf{HV}$

[17] The allocation does not necessarily have to be proportional – any allocation between inputs is possible. Indeed, if inputs are measured in different units, a proportional split is not meaningful. Under proportional reallocation, transportation emissions per unit of material input are equal to $(a_{TS} / (a_{RM} + a_{VM})) \times e_{TS}$.

Table 3.2 Output coefficient matrix (B)

Product	Process												
	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod
RM1	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
TS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
VR1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
VR2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
RR1	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	1.0

Table 3.3 Input coefficient matrix (A)

Product	Process												
	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod
RM1	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0
TS	0.5	0.0	0.0	0.6	0.6	1.6	0.2	0.4	0.0	0.0	0.0	0.0	0.0
VR1	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VR2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RR1	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 3.4 Net output coefficient matrix (G = B – A)

	Process												
Product	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod
RM1	1.0	0.0	0.0	(0.4)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	1.0	0.0	(0.8)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	1.0	0.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	1.0	0.0	(0.5)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	1.0	(3.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	1.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	(0.2)	0.0	0.0	(0.4)	1.0	0.0	0.0	0.0	0.0
TS	(0.5)	0.0	0.0	(0.6)	(0.6)	(1.6)	(0.2)	(0.4)	0.0	1.0	0.0	0.0	0.0
VR1	0.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
VR2	0.0	0.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
RR1	(1.0)	0.0	0.0	0.2	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	1.0

Table 3.5 Inverse of net output coefficient matrix ($H = G^{-1}$)

	Process												
Product	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod
RM1	1.0	0.0	0.0	0.4	0.0	0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	1.0	0.0	0.8	0.0	0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	1.0	0.0	1.0	3.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	1.0	0.0	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	1.0	3.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	0.2	0.6	0.6	0.4	1.0	0.0	0.0	0.0	0.0
TS	0.5	0.0	0.0	0.8	0.6	3.8	4.0	0.4	0.0	1.0	0.0	0.0	0.0
VR1	0.0	1.0	0.0	0.8	0.0	0.4	0.4	0.0	0.0	0.0	1.0	0.0	0.0
VR2	0.0	0.0	1.0	0.0	1.0	3.0	3.0	0.0	0.0	0.0	0.0	1.0	0.0
RR1	1.0	0.0	0.0	0.2	0.0	0.1	0.1	(0.6)	0.0	0.0	0.0	0.0	1.0

Table 3.6 Physical flows, emissions (kg CO_2) and value (£)

Product	Units	Activity Level (z)	Gross Prod (y)	Int Cons (w)	Final Cons (c)	Net Imports (m-x)	Final Demand (d)
RM1	Tonnes	20	20	20	0	0	0
VM1	Tonnes	40	40	40	0	0	0
VM2	Tonnes	300	300	300	0	0	0
IG1	Tonnes	50	50	50	0	0	0
IG2	Litres (000)	300	300	300	0	0	0
FG	Bottles (000)	100	100	100	0	0	0
RG	Bottles (000)	100	100	0	100	0	100
WCS	Tonnes	50	50	0	50	0	50
WDS	Tonnes	80	80	80	0	0	0
TS	m^3 km (000)	420	420	420	0	0	0
VR1	Tonnes	0	0	40	0	40	(40)
VR2	Tonnes	0	0	300	0	300	(300)
RR1	Tonnes	0	40	20	0	(20)	20

Product	Unit emissions Process (E)	Unit emissions Product (e)	Unit value Process (V)	Unit value Product (p)
RM1	2.0	4.0	10.0	130.0
VM1	5.0	5.0	5.0	25.0
VM2	1.0	1.0	10.0	60.0
IG1	1.0	9.0	38.0	114.0
IG2	2.0	5.4	6.0	100.0
FG	0.5	27.6	86.0	507.0
RG	0.5	28.9	92.0	607.0
WCS	1.0	2.6	46.0	22.0
WDS	0.0	0.0	50.0	50.0
TS	4.0	4.0	40.0	40.0
VR1	0.0	0.0	20.0	20.0
VR2	0.0	0.0	50.0	50.0
RR1	0.0	0.0	100.0	100.0

Total emissions	
E z	e d
3,020	3,020

Total Value	
V z	p d
48,000	48,000

Intensity (ε)	
0.063	0.063

Table 3.7 shows the emission intensities for the various products and services (the penultimate row) and for the corresponding production processes (in the final column). By definition, the product and process intensities for the three resources are all equal to zero. The columns in the table show the weighting factors that are applied to the process intensities to calculate the emission intensity for each product / service; while the final row gives the weighting factors that are applied to these intensities to calculate the aggregate production intensity.

In this example, the virgin material for packaging (VM1) has by far the highest process intensity (and product intensity). However, it contributes very little to the emissions intensity of the final packaged good (FG) – having a weighting factor of only 0.4%. In contrast, transportation services has a weighting factor of 30%, making it the largest contributor to the emissions intensity of the final packaged good (accounting for over 50%) despite its relatively low process intensity.

The total emissions and value added attributable to consumption are shown in Table 3.8. It is assumed that there are two consumption processes; each using 2,000 bottles of the packaged good and 1 tonne of waste collection services per unit of activity. Consequently, the imbedded input emissions are the same for each process – at 60 kg CO_2 per unit of activity. However, it is assumed that the emissions per unit of activity generated by the first process – at 40 kg CO_2 per unit of activity – are four times greater those generated by the second, and hence the lifecycle emissions per unit activity are higher for this process.[18] The first process also has a higher activity level, resulting in it accounting for almost 70% of the total life cycle emissions attributable to consumption – which is equal to 4,420 kg CO_2.

By definition, neither of the consumption processes generates any value added, and hence the lifecycle value per unit activity is the same for both processes – at £1,236 per unit activity. The resultant total value of consumption is £61,800, giving an aggregate consumption intensity of 0.072 kg CO_2 per £. The intensity of the first consumption process is higher, reflecting its higher rate of process emissions.

Finally, Table 3.9 show the summary intermediate use table. This shows how total intermediate consumption (TIC) at producer prices is broken down between products and processes. The sum of the columns provides the total expenditure on each product / service / resource; while the sum of the rows provides the total expenditure by each process. Also provided are the values of gross output and GVA for each process, together with the values of GVA per unit activity – which are the same as those in Table 3.6. There is no expenditure on the retail good (RG) or waste collection services (WCS), as these are consumed solely by the final consumer. Neither the production of waste disposal services (WDS), nor transport services (TS), entails any expenditure on inputs, although this just reflects the simplified system definition used for this example.[19] Aggregate gross output is £194,600, while total intermediate consumption amounts to £146,600, yielding an aggregate value for GVA of £48,000 – again, the same as in Table 3.6.

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[18] The life cycle emissions per unit activity for each process are equal to the sum of the respective values for imbedded input emissions and process emissions.

[19] As note above, a number of important inputs have been omitted in the interests of simplicity.

Table 3.7 Product and production process emissions intensities

Process	Product													ε
	RM1	VM1	VM2	IG1	IG2	FG	RG	WCS	WDS	TS	VR1	VR2	RR1	
RM1	7.7%	0.0%	0.0%	3.5%	0.0%	0.4%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.20
VM1	0.0%	20.0%	0.0%	3.5%	0.0%	0.4%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.00
VM2	0.0%	0.0%	16.7%	0.0%	10.0%	5.9%	4.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.10
IG1	0.0%	0.0%	0.0%	33.3%	0.0%	3.7%	3.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.03
IG2	0.0%	0.0%	0.0%	0.0%	6.0%	3.6%	3.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.33
FG	0.0%	0.0%	0.0%	0.0%	0.0%	17.0%	14.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.01
RG	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	15.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.01
WCS	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	209.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.02
WDS	0.0%	0.0%	0.0%	0.0%	10.0%	5.9%	4.9%	90.9%	100.0%	0.0%	0.0%	0.0%	0.0%	0.00
TS	15.4%	0.0%	0.0%	28.1%	24.0%	30.0%	26.4%	72.7%	0.0%	100.0%	0.0%	0.0%	0.0%	0.10
VR1	0.0%	80.0%	0.0%	14.0%	0.0%	1.6%	1.3%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.00
VR2	0.0%	0.0%	83.3%	0.0%	50.0%	29.6%	24.7%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.00
RR1	76.9%	0.0%	0.0%	17.5%	0.0%	2.0%	1.6%	-272.7%	0.0%	0.0%	0.0%	0.0%	100.0%	0.00
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	
ε	0.031	0.200	0.017	0.079	0.054	0.054	0.048	0.118	0.000	0.100	0.000	0.000	0.000	0.063
	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	126.5%	2.3%	0.0%	0.0%	-1.7%	-31.2%	4.2%	100.0%

Table 3.8 Consumption process emissions and value added

Product	Final Cons (c)
RM1	0
VM1	0
VM2	0
IG1	0
IG2	0
FG	0
RG	100
WCS	50
WDS	0
TS	0
VR1	0
VR2	0
RR1	0

	Input coefficients	
	Process	
Product	A	B
RM1	0.0	0.0
VM1	0.0	0.0
VM2	0.0	0.0
IG1	0.0	0.0
IG2	0.0	0.0
FG	0.0	0.0
RG	2.0	2.0
WCS	1.0	1.0
WDS	0.0	0.0
TS	0.0	0.0
VR1	0.0	0.0
VR2	0.0	0.0
RR1	0.0	0.0
Activity Level (v)	30	20

	Unit emissions		
	Process		
	A	B	
E^c	40	10	
e C	60	60	
(E+eC) v	3,012	1,408	4,420

	Unit value		
	Process		
	A	B	
V^c	0	0	
p C	1,236	1,236	
(V+pC) v	37,080	24,720	61,800
Intensity (ε^c)	0.081	0.057	0.072
	60.0%	40.0%	100.0%

Table 3.9 Intermediate Use (£)

Product	Process													TIC
	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod	
RM1	0	0	0	2,600	0	0	0	0	0	0	0	0	0	**2,600**
VM1	0	0	0	1,000	0	0	0	0	0	0	0	0	0	**1,000**
VM2	0	0	0	0	18,000	0	0	0	0	0	0	0	0	**18,000**
IG1	0	0	0	0	0	5,700	0	0	0	0	0	0	0	**5,700**
IG2	0	0	0	0	0	30,000	0	0	0	0	0	0	0	**30,000**
FG	0	0	0	0	0	0	50,700	0	0	0	0	0	0	**50,700**
RG	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
WCS	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
WDS	0	0	0	0	3,000	0	0	1,000	0	0	0	0	0	**4,000**
TS	400	0	0	1,200	7,200	6,400	800	800	0	0	0	0	0	**16,800**
VR1	0	800	0	0	0	0	0	0	0	0	0	0	0	**800**
VR2	0	0	15,000	0	0	0	0	0	0	0	0	0	0	**15,000**
RR1	2,000	0	0	0	0	0	0	0	0	0	0	0	0	**2,000**
TIC	**2,400**	**800**	**15,000**	**4,800**	**28,200**	**42,100**	**51,500**	**1,800**	**0**	**0**	**0**	**0**	**0**	**146,600**
Gross Output	2,600	1,000	18,000	6,700	30,000	50,700	60,700	4,100	4,000	16,800	0	0	0	**194,600**
GVA	200	200	3,000	1,900	1,800	8,600	9,200	2,300	4,000	16,800	0	0	0	**48,000**
GVA / Unit	10.0	5.0	10.0	38.0	6.0	86.0	92.0	46.0	50.0	40.0				

In the preceding example, the life cycle emissions and value added were derived for a packaged good. However, in some cases one may be interested in the emissions and value added for one of the individual components (e.g. the packaging), or in comparing these for different component specifications (e.g. different packaging materials). In the latter case, the relative values can be determined by comparing the emissions and valued added for different versions of the packaged good system (with alternative packaging specifications). However, this does not provide any information about the absolute emissions and value added of the packaging under either specification. In order to determine these, it is necessary to separate out the two components in the product system. This is illustrated in Figure 3.3 (over page), in which the flows relating to the packaging are shown in black and the flows relating to the content are shown in green.

Compared to the original system definition (shown in Figure 3.2), the decomposed system differs only in relation to those processes that produce, or use, the packaged good. Final good production now produces two outputs – the packaging and the content – in fixed proportions; with both products passing through the retail chain to final consumption. After consumption, the discarded packaging is collected by the waste collection authority and either recovered or sent to landfill – as in the original system.

The easiest way to incorporate this into the analysis is to amend the output coefficient matrix (**B**) so that final good production and retailing both produce the packaging (IG1) as a bi-product along with their primary product (which is redefined to be the content) – with the output coefficient being set equal to the input coefficient for packaging in the final good production process (i.e. 0.5). Correspondingly, the input coefficient matrix (**A**) is amended to reflect the fact the packaging is now used as an input by retailing – with the input coefficient again being set equal to 0.5. The amended matrices, together with the resultant net output coefficient matrix and its inverse are shown in Tables 3.10 – 3.13. Note that the only effect of these amendments on the net output matrix is to change the packaging coefficient final good production from – 0.5 to zero.

In addition to the changes to the input-output matrices, the final consumption vector also has to be amended to reflect the fact that 50 tonnes of packaging are now consumed in additions to the 100,000 “bottles” of content. The resultant production and intermediate consumption quantities are shown in Table 3.14, along with the activity levels of the various production processes. Comparing these with the corresponding quantities in Table 3.6, it is clear that the only differences relate to the gross production and intermediate consumption of packaging (IG1); with the former increasing by 100,000 tonnes and the latter increasing by 50,000 tonnes. All of the other quantities and all of the activity levels are completely unchanged.

Turning to the imbedded emissions and prices – also shown in Table 3.14, the only differences relate to the final good (FG) and the retail good (RG), where both the emissions and prices are reduced to reflect the fact that they now only comprise the content. For example, the emissions imbedded in the final good decline from 27.6 to 23.1 kg CO_2, while its price falls from £0.61 per bottle to £0.45 per “bottle”. However, these changes make no difference to total emissions or total value added, which remain at 3,020 tonnes CO_2 and £48,000 respectively.

Figure 3.3 Amended process map (decomposed final good)

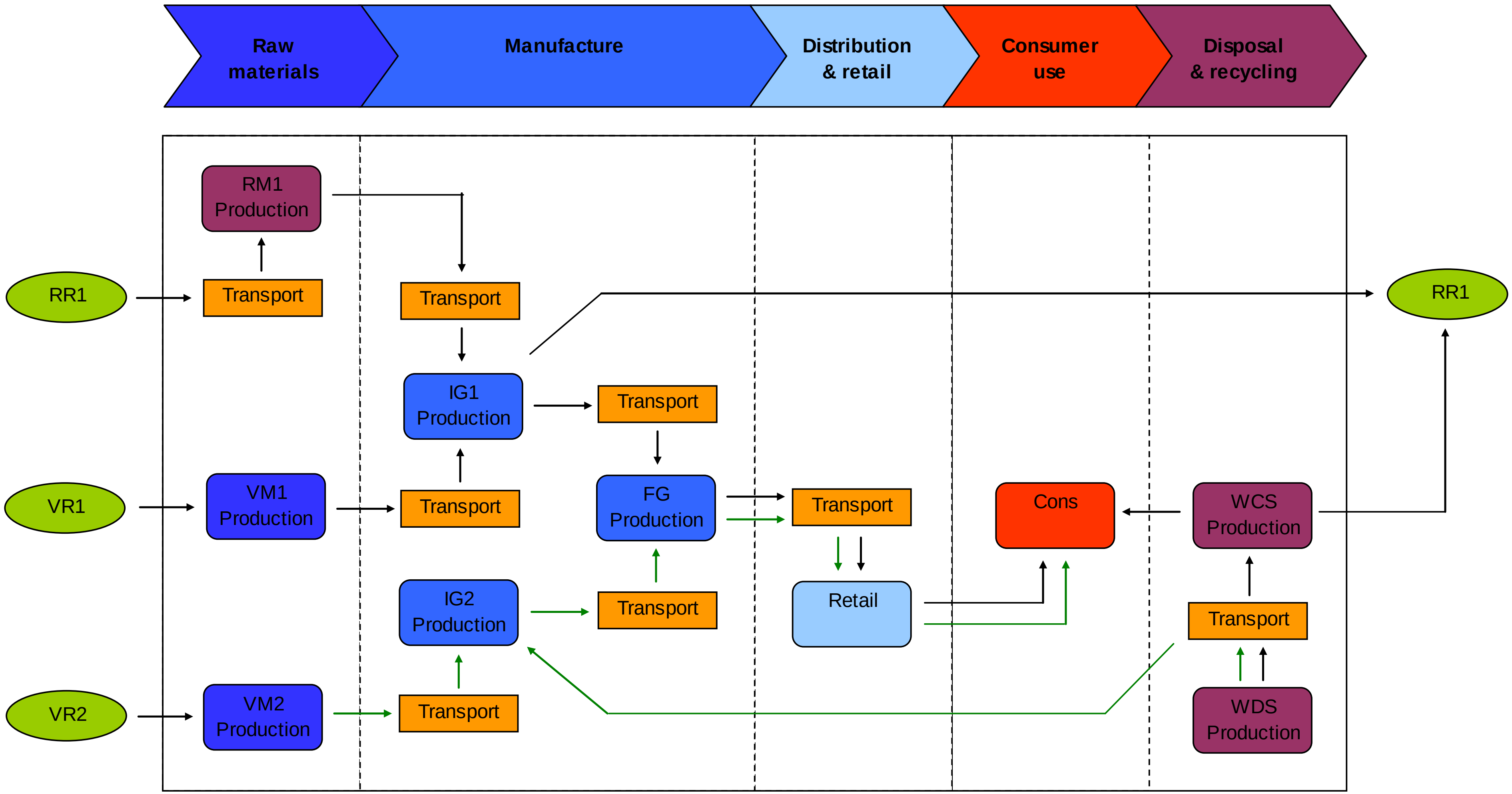

Table 3.10 Output coefficient matrix (B)

	Process												
	RM1	VM1	VM2	IG1	IG2	FG	RG	WCS	WDS	TS	VR1	VR2	RR1
Product	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod
RM1	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	1.0	0.0	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
TS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
VR1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
VR2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
RR1	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	1.0

Table 3.11 Input coefficient matrix (A)

	Process												
	RM1	VM1	VM2	IG1	IG2	FG	RG	WCS	WDS	TS	VR1	VR2	RR1
Product	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod	Prod
RM1	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	0.0	0.0	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0
TS	0.5	0.0	0.0	0.6	0.6	1.6	0.2	0.4	0.0	0.0	0.0	0.0	0.0
VR1	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VR2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RR1	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 3.12 Net output coefficient matrix (G = B – A)

	Process												
Product	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod
RM1	1.0	0.0	0.0	(0.4)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	1.0	0.0	(0.8)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	1.0	0.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	1.0	(3.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	1.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	(0.2)	0.0	0.0	(0.4)	1.0	0.0	0.0	0.0	0.0
TS	(0.5)	0.0	0.0	(0.6)	(0.6)	(1.6)	(0.2)	(0.4)	0.0	1.0	0.0	0.0	0.0
VR1	0.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
VR2	0.0	0.0	(1.0)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
RR1	(1.0)	0.0	0.0	0.2	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	1.0

Table 3.13 Inverse of net output coefficient matrix ($H = G^{-1}$)

	Process												
Product	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod
RM1	1.0	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM1	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VM2	0.0	0.0	1.0	0.0	1.0	3.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
IG1	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
IG2	0.0	0.0	0.0	0.0	1.0	3.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
FG	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
RG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
WCS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
WDS	0.0	0.0	0.0	0.0	0.2	0.6	0.6	0.4	1.0	0.0	0.0	0.0	0.0
TS	0.5	0.0	0.0	0.8	0.6	3.4	3.6	0.4	0.0	1.0	0.0	0.0	0.0
VR1	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
VR2	0.0	0.0	1.0	(0.0)	1.0	3.0	3.0	(0.0)	0.0	0.0	0.0	1.0	0.0
RR1	1.0	0.0	0.0	0.2	0.0	0.0	0.0	(0.6)	0.0	0.0	0.0	0.0	1.0

Table 3.13 Physical flows, emissions (kg CO_2) and value (£)

Product	Units	Activity Level (z)	Gross Prod (y)	Int Cons (w)	Final Cons (c)	Net Imports (m-x)	Final Demand (d)
RM1	Tonnes	20	20	20	0	0	0
VM1	Tonnes	40	40	40	0	0	0
VM2	Tonnes	300	300	300	0	0	0
IG1	Tonnes	50	150	100	50	0	50
IG2	Litres (000)	300	300	300	0	0	0
FG	Bottles (000)	100	100	100	0	0	0
RG	Bottles (000)	100	100	0	100	0	100
WCS	Tonnes	50	50	0	50	0	50
WDS	Tonnes	80	80	80	0	0	0
TS	m^3 km (000)	420	420	420	0	0	0
VR1	Tonnes	0	0	40	0	40	(40)
VR2	Tonnes	(0)	(0)	300	0	300	(300)
RR1	Tonnes	0	40	20	0	(20)	20

Unit emissions	
Process (E)	Product (e)
2.0	4.0
5.0	5.0
1.0	1.0
1.0	9.0
2.0	5.4
0.5	23.1
0.5	24.4
1.0	2.6
0.0	0.0
4.0	4.0
0.0	0.0
0.0	0.0
0.0	0.0

Unit value	
Process (V)	Product (p)
10.0	130.0
5.0	25.0
10.0	60.0
38.0	114.0
6.0	100.0
86.0	450.0
92.0	550.0
46.0	22.0
50.0	50.0
40.0	40.0
20.0	20.0
50.0	50.0
100.0	100.0

Total emissions	
E z	e d
3,020	3,020

Total Value	
V z	p d
48,000	48,000

Intensity (ε)	
0.063	0.063

Table 3.14 shows the emission intensities for the various products and services (the penultimate row) and for the corresponding production processes (in the final column). While there is no change to the overall emissions intensity of production – which remains at 0.063 kg CO_2 per £, nor to the process intensities (given in the final column), there are changes to both the weighting factors and the product intensities (given in the penultimate row) for the final good (FG) and the retail good (RG). In both cases the product emission intensities decline. However, this is offset by a change in the mix of final demand (given in the final row) to leave the overall emission intensity unchanged.

The total emissions and value added attributable to consumption are shown in Table 3.15. Unlike its treatment in the production system, the emissions and value added attributable to packaging use are isolated by introducing a third, "dummy" consumption process (C), which does not produce any output. The original two processes now just use the content (RG) to produce their respective consumption services, with the packaging (IG1) and the waste disposal service (WDS) being used as inputs by the dummy process. Assuming that the dummy consumption process does not generate any emissions, the result of the change is to reduce the emissions attributable to the original two processes by 348 kg CO_2 and 232 kg CO_2 respectively. However, these reductions are exactly offset by the emissions attributable to packaging use, leaving total emissions unchanged at 4,420 kg CO_2.

The introduction of the dummy variable has no impact on the shadow value of the output from the original two consumption processes. However, it does reduce the cost of the inputs by the combined value of the packaging and waste collection services. Consequently, these processes now generate positive value added (at £136 per unit activity). Again, this is offset by the negative value added generated by packaging use, so that in aggregate, consumption does not generate any value added. The net result of these changes is to leave the total value added attributable to each of the original processes unchanged. With the total value added attributable to packaging use equal to zero, the total value of consumption is also unchanged, at £61,800.

Since there is no impact on either total emissions or total consumption value, aggregate emissions intensity remains unchanged at 0.072 kg CO_2 per £. However, because emissions intensity is undefined for the dummy process (due to zero divide condition), it is no longer equal to the weighted average of consumption process intensities.

Finally, Table 3.16 shows the summary intermediate use table. The only changes versus the original version (see Table 3.9) are an increase in expenditure on packaging (IG1) by retailing (and in aggregate) and a corresponding reduction in expenditure on the final good (FG). All of the other values remain unchanged. In particular, there is no impact on the values of TIC, gross output or GVA for any of the production processes.

Table 3.14 Product and production process emission intensities

Process	Product													ε
	RM1	VM1	VM2	IG1	IG2	FG	RG	WCS	WDS	TS	VR1	VR2	RR1	
RM1	7.7%	0.0%	0.0%	3.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.20
VM1	0.0%	20.0%	0.0%	3.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.00
VM2	0.0%	0.0%	16.7%	0.0%	10.0%	6.7%	5.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.10
IG1	0.0%	0.0%	0.0%	33.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.03
IG2	0.0%	0.0%	0.0%	0.0%	6.0%	4.0%	3.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.33
FG	0.0%	0.0%	0.0%	0.0%	0.0%	19.1%	15.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.01
RG	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.01
WCS	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	209.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.02
WDS	0.0%	0.0%	0.0%	0.0%	10.0%	6.7%	5.5%	90.9%	100.0%	0.0%	0.0%	0.0%	0.0%	0.00
TS	15.4%	0.0%	0.0%	28.1%	24.0%	30.2%	26.2%	72.7%	0.0%	100.0%	0.0%	0.0%	0.0%	0.10
VR1	0.0%	80.0%	0.0%	14.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.0%	0.00
VR2	0.0%	0.0%	83.3%	0.0%	50.0%	33.3%	27.3%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	0.00
RR1	76.9%	0.0%	0.0%	17.5%	0.0%	0.0%	0.0%	-272.7%	0.0%	0.0%	0.0%	0.0%	100.0%	0.00
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	
ε	0.031	0.200	0.017	0.079	0.054	0.051	0.044	0.118	0.000	0.100	0.000	0.000	0.000	0.063
	0.0%	0.0%	0.0%	11.9%	0.0%	0.0%	114.6%	2.3%	0.0%	0.0%	-1.7%	-31.2%	4.2%	100.0%

Table 3.15 Consumption process emissions and value added

Product	Final Cons (c)
RM1	0
VM1	0
VM2	0
IG1	50
IG2	0
FG	0
RG	100
WCS	50
WDS	0
TS	0
VR1	0
VR2	0
RR1	0

	Input coefficients		
	Process		
Product	A	B	C
RM1	0.0	0.0	0.0
VM1	0.0	0.0	0.0
VM2	0.0	0.0	0.0
IG1	0.0	0.0	1.0
IG2	0.0	0.0	0.0
FG	0.0	0.0	0.0
RG	2.0	2.0	0.0
WCS	0.0	0.0	1.0
WDS	0.0	0.0	0.0
TS	0.0	0.0	0.0
VR1	0.0	0.0	0.0
VR2	0.0	0.0	0.0
RR1	0.0	0.0	0.0
Activity Level (v)	30	20	50

	Unit emissions			
	Process			
	A	B	C	
E^c	40	10	0	
e C	49	49	12	
(E+eC) v	2,664	1,176	580	4,420

	Unit value			
	Process			
	A	B	C	
V^c	136	136	-136	
p C	1,100	1,100	136	
(V+pC) v	37,080	24,720	0	61,800
Intensity (ε^c)	0.072	0.048	n/a	0.072
	60.0%	40.0%	0.0%	100.0%

Table 3.16 Intermediate Use (£)

Product	Process													TIC
	RM1 Prod	VM1 Prod	VM2 Prod	IG1 Prod	IG2 Prod	FG Prod	RG Prod	WCS Prod	WDS Prod	TS Prod	VR1 Prod	VR2 Prod	RR1 Prod	
RM1	0	0	0	2,600	0	0	0	0	0	0	0	0	0	**2,600**
VM1	0	0	0	1,000	0	0	0	0	0	0	0	0	0	**1,000**
VM2	0	0	0	0	18,000	0	0	0	0	0	0	0	0	**18,000**
IG1	0	0	0	0	0	5,700	5,700	0	0	0	0	0	0	**11,400**
IG2	0	0	0	0	0	30,000	0	0	0	0	0	0	0	**30,000**
FG	0	0	0	0	0	0	45,000	0	0	0	0	0	0	**45,000**
RG	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
WCS	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
WDS	0	0	0	0	3,000	0	0	1,000	0	0	0	0	0	**4,000**
TS	400	0	0	1,200	7,200	6,400	800	800	0	0	0	0	0	**16,800**
VR1	0	800	0	0	0	0	0	0	0	0	0	0	0	**800**
VR2	0	0	15,000	0	0	0	0	0	0	0	0	0	0	**15,000**
RR1	2,000	0	0	0	0	0	0	0	0	0	0	0	0	**2,000**
TIC	**2,400**	**800**	**15,000**	**4,800**	**28,200**	**42,100**	**51,500**	**1,800**	**0**	**0**	**0**	**0**	**0**	**146,600**
Gross Output	2,600	1,000	18,000	6,700	30,000	50,700	60,700	4,100	4,000	16,800	0	(0)	0	**194,600**
GVA	200	200	3,000	1,900	1,800	8,600	9,200	2,300	4,000	16,800	0	(0)	0	**48,000**
GVA / Unit	10.0	5.0	10.0	38.0	6.0	86.0	92.0	46.0	50.0	40.0				

4. Conclusions

A common accounting framework has been developed that allows the life cycle GHG emissions and the economic value of a product system to be evaluated on a consistent basis. The framework is consistent with methodology for assessing GHG emissions specified in PAS 2050, and with the national accounts concepts and principles set out in ESA95 / SNA93.

In the calculation of the economic value of a product system, process outputs may be valued either at basic prices (i.e. excluding all product taxes and subsidies), or at producer prices (i.e. including non-deductible product taxes such as the climate change levy). While the first approach has the advantage of being consistent with the approach adopted for the valuation of output and gross value added in the national accounts (under ESA95), the second is more consistent with the approach used for the calculation of lifecycle emissions. Since, the concept of gross value added at producer prices is recognised by SNA93 (albeit not widely used), the consistency with the calculation of lifecycle emissions would seem to justify the use of producer prices.

Transportation of goods between processes is treated as a separate input to the process that uses the goods. While this is different to the treatment in the national accounts framework – in which transport margins are included in the purchasers' prices of goods – it has no impact on aggregate value added, or on the value added of individual processes. Retailing and distribution processes are treated in exactly the same way as production processes, in that they "produce" their own output goods using the manufactured goods as inputs. This is different to their treatment in the national accounts framework, where the output of these sectors is defined to be the gross margin that they make from buying and re-selling the manufactured goods.[20] However, again this makes no difference to the value added of individual processes, or to aggregate gross value added – although it does increase the input and output values for these sectors.

The imbedded emissions value (e) for a good or service represents the sum of the emissions generated by all of the processes that have contributed (directly or indirectly) to its production. For business-to-business carbon footprints, this represents the "cradle-to-gate" emissions value for the good or service. For business-to-consumer carbon footprints, provided that the functional unit is defined in terms of an output from a consumption process (i.e. the ultimate service enjoyed by the consumer), the lifecycle emissions are given by the imbedded emissions value for that output. This is equal to the sum of the imbedded emissions for all inputs (including any waste collection and disposal services), plus the emissions generated by the consumption process itself.

Similarly, the producer price (p) of a good or service represents the "cradle-to-gate" value added by all of the processes contributing to its production, while the shadow price of the output from the consumption process represents the lifecycle value-added. Under the convention that consumption processes do not generate any value-added, this is equal to total value of the inputs used by the process – including the cost of any waste collection and disposal services.

[20] In the national accounts framework, the final consumer purchases the product from the manufacturer and purchases a separate service from the retailer, represented by the retail margin – with the latter being added to the price of the product. Thus the purchaser's price of the manufactured product for the final consumer includes the retail margin as well as any transport costs from the manufacturer to the retailer.

The aggregate emissions intensity of production – defined as aggregate production emissions divided by aggregate GVA (at producers' prices) – is equal to the weighted average of the individual product emission intensities (i.e. ratio of imbedded emissions to producer price), where the weights are equal to the products' respective shares of aggregate net demand. In turn, the emissions intensity of each product is equal to the weighted average of the individual production emission intensities of the processes 9i.e. the ration of process emissions to value added), where the weights are equal to their respective contributions to the product's unit value – i.e. their respective shares of the production value chain for that product.

Analogously, the aggregate emissions intensity of consumption – defined as aggregate lifecycle emissions divided by aggregate consumption expenditure at purchasers' prices – is equal to the weighted average of the emission intensities of the individual "consumption services", where the weights are equal to their respective shares of aggregate consumption expenditure. Each of these intensities is in turn equal to the weighted average of the emission intensities of the production and consumption processes, where the weights are equal to their respective contributions to the service's shadow value – i.e. their respective shares of the lifecycle value chain for that service.

The basic framework can be used to compare the lifecycle emissions and economic value of alternative specifications for the product system. For example, the lifecycle impact of changing the packaging material used for a packaged good can be assessed by comparing the emissions and valued added for different versions of the product system for the packaged good (with alternative packaging materials). However, while this provides information on the "cradle-to-gate" emissions and value added of each packaging material, up to the point at which the packaging is combined with its content, it does not provide any information about the lifecycle emissions and value added of either packaging material. In order to determine these, it is necessary to separate out the two components in the product system. A simple method for doing this is demonstrated in the illustrative example, where the packaged good is decomposed into the packaging and the content. This allows the isolation of the lifecycle emissions and value added for the packaging; the latter being equal to zero by definition, with positive value added by the production processes being offset by a negative value added in consumption.