Blind correlation-based DFE receiver for the equalization of single input multi output communication channels.

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***Abstract*—In this paper, the correlation-based Decision Feedback Equalizer (DFE), where the received data from multiple antennas are processed by a multi-dimensional matched filter and then combined prior to the equalization with a single input single output DFE, is discussed and its blind implementation is introduced. To perform the correlation-based DFE blindly, the multi-dimensional matched filter is replaced by an adaptive filter and the DFE filter weights are calculated via manipulating over the second order statistics of the received data. In the blind architecture, the adaptive filter converges to matched filter equivalents, therefore the matched filters of the corresponding communication channels are also blindly be estimated in addition to the blind equalization process. The mean-squared error of the estimation of matched filters and the equalization performance of the proposed blind architecture are also studied and simulated.**

I. Introduction

Decision Feedback Equalizer (DFE) is a non-linear channel equalizer, that consists of two filters, a feedforward $\boldsymbol{g}_k$ and a feedback filter $\boldsymbol{b}_k$, in addition to a decision-device in its feedback loop. The non-linearity of the equalizer is due to the decision-device, output of which is applied to the feedback filter. The non-linearity enables the equalizer to remove the portion of the Inter-Symbol Interference (ISI) caused by the previously detected symbols. Bit Error Rate (BER) analysis has shown that the DFE performance is better than linear equalizers (i.e. Minimum Mean Squared Error and Zero Forcing Equalizers) and close to the optimal receiver strategy, which is the Maximum Likelihood Sequence Estimation (MLSE) [1]. Therefore, in most cases where the complexity of the receiver is intended to be kept low, the DFE is preferable over MLSE for the channel equalization.

In our work we will study a DFE architecture where the signal reception is being conducted over multiple antennas at the receiver end. The system, comprising a single transmit antenna with multiple antennas employed at the receiver, is called Single Input Multi Output (SIMO). As long as the signal recovery is conducted with respect to the spatial condition of the channel, the communication over SIMO channels benefits the system performance with different gains, such as diversity gain and array gain. In addition to the performance improvement, that has been obtained by making use of multiple antenna reception, the system would benefit an extra performance gain, if matched filtering is made available at the receiver front end too. A matched filter simply maximizes the Signal-to-Noise Ratio (SNR), if it is matched to the impulse response of its corresponding communication channel. It is well known that the optimal detection strategy in a SIMO receiver is; passing the received signals through a multi-dimensional matched filter, which reduces the multiple streams to a single stream, and to perform MLSE detection over the resulting stream [1]. Having a closer BER performance, if DFE is preferred instead of MLSE, a near optimal receiver architecture is obtained, which is shown in Fig. 1.

The filtering blocks MF and DFE, shown with dashed borders, are respectively where the multi-dimensional matched filtering and DFE type equalization is performed. The use of the receiver in Fig. 1 is recently studied for underwater acoustics and detailed performance analysis have been conducted in [2], [3]. As it was named in [2], the architecture in Fig. 1 will be called the correlation-based DFE throughout the paper. The implementation of the correlation-based DFE needs the Channel Impulse Response (CIR), corresponding to each receive antenna $h_k^{(i)}$, to be estimated in advance before the matched filtering and the equalization processes.

Many techniques in the literature to establish the CIR of a SIMO channel depend on the transmission of training signals. However, the use of training symbols, which are carrying no valuable information at all, is inherently inefficient and wastes the bandwidth of the communication channel as well as resources at both ends. A blind scheme, where the communication is done without the need for any training symbols, would definitely improve the system efficiency. The well-known blind channel estimation methods in the literature involves implementation inefficient and expensive mathematics, such as Singular Value Decomposition (SVD) [4] Eigen Value Decomposition (EVD) [5] or the need for the use of Higher Order Statistics (HOS) of the received data [6]. In the work [7], Ozcelik et. al. has shown that; in a Single Input Single Output (SISO) communication scenario the matched filter can blindly be estimated via an adaptive filter being updated by a computationally simple algorithm, i.e. Constant Modulus

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Algorithm (CMA), if the matched filter is the sole unknown in the receiver. To build the multi-dimensional matched filter in our blind SIMO receiver, we have benefited from the findings of [7].

The system model and the correlation based DFE is discussed in the next section. Section III describes how the non-blind correlation-based DFE receiver in Fig.1 is adapted for the blind use and also gives out the formulations, used to calculate and establish the multi-dimensional matched filter and the DFE blindly. The last section is dedicated to simulation results to represent the equalization and estimation performances for selected cases.

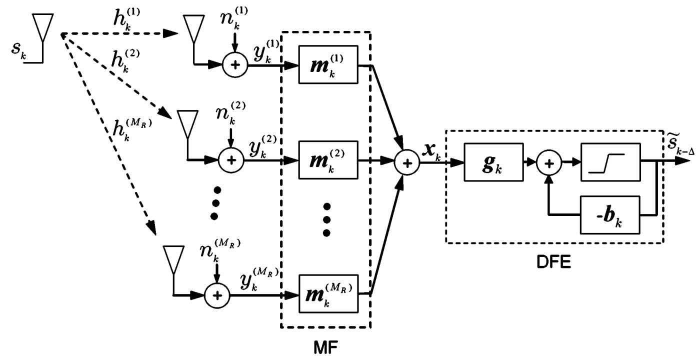

Fig. 1. Baseband communication model of Correlation-based DFE

II. System Model and Correlation based DFE

We assume a Single Input Multi Output (SIMO) communication channel with M_R receive antennas and a single transmit antenna $M_T = 1$. For our implementation, each channel between a receive antenna and the sole transmit antenna is assumed to be frequency selective linear time invariant. Fig. 1 shows the communication channel, where $h_k^{(i)}$ *for* $i = 1, \cdots, M_R$, represents the i^{th} transmission channel between the transmitter and the i^{th} receive antenna. Throughout the paper, the superscript $(\cdot)^{(i)}$ denotes the corresponding channel. The discrete baseband signal model of the communication can be written as follows

$$y_k^{(i)} = \sum_{\gamma=0}^{L} h_\gamma^{(i)} s_{k-\gamma} + n_k^{(i)} \qquad for\ i = 1, \cdots, M_R \tag{1}$$

where $y_k^{(i)}$ is the received signal at receive branch i, $n_k^{(i)}$ is the Additive White Gaussian Noise (AWGN), s_k is the transmitted symbol and the discrete channel length is $L+1$. Each matched filter, $\boldsymbol{m}_k^{(i)}$, is the conjugated time-reversed version of its corresponding channel, i.e. $m_k^{(i)} = (h_{L-k}^{(i)})^*$, where $(\cdot)^*$ represents the conjugation. Note that the ***bold*** characters throughout the paper and in the figures indicate the vectors of corresponding coefficient sets.

The signal x_k, which is the result of the combining operation after matched filtering, can be formulated as

$$\begin{aligned} x_k &= \sum_{i=1}^{M_R} \left(\boldsymbol{m}_k^{(i)} \star \boldsymbol{y}_k^{(i)} \right) \\ &= \sum_{i=1}^{M_R} \left(\sum_{l=0}^{L} {h_{L-k+l}^{(i)}}^* \left(\sum_{\gamma=0}^{L} h_\gamma^{(i)} s_{l-\gamma} \right) + \sum_{l=0}^{L} {h_{L-k+l}^{(i)}}^* n_l^{(i)} \right) \\ &= \underbrace{\sum_{l=0}^{2L} \left(\sum_{i=1}^{M_R} \sum_{\gamma=0}^{L} {h_{L+\gamma-l}^{(i)}}^* h_\gamma^{(i)} \right) s_{k-l}}_{signal\ component} + \underbrace{\sum_{i=1}^{M_R} \sum_{l=0}^{L} {h_{L-l}^{(i)}}^* n_{k-l}^{(i)}}_{noise(w_k)} \end{aligned} \tag{2}$$

where $\boldsymbol{y}_k^{(i)}$ is the vector of received symbols at the i^{th} antenna. $(\star)$ is the convolution operator. We can conclude from the derivation of the above that the effective channel over the DFE block of the architecture in Fig. 1 is $q_l = \sum_{i=1}^{M_R} \sum_{\gamma=0}^{L} {h_{L+\gamma-l}^{(i)}}^* {h_\gamma}^{(i)}$ or similarly $q_l = \sum_{i=1}^{M_R} \sum_{\gamma=0}^{L} {m_{l-\gamma}^{(i)}}^* {h_\gamma}^{(i)}$ *for* $l = 0, \cdots, 2L$, which states that the effective channel is simply the sum of the autocorrelations of all channel impulse responses. The derivation in (2) also shows that the noise w_k is colored due to the matched filtering operation at the receiver. Therefore a whitening filter, $\boldsymbol{g}_k$, has to be incorporated into the receiver architecture not only to overcome the ISI but also to whiten the noise. The communication over the effective channel can be formulated as;

$$\begin{pmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k+T-1} \end{pmatrix} = \underbrace{\begin{pmatrix} q_{2L} & \cdots & q_0 & & \\ & \ddots & \ddots & \\ & & q_{2L} & \cdots & q_0 \end{pmatrix}}_{Q} \begin{pmatrix} s_{k-2L} \\ \vdots \\ s_k \\ \vdots \\ s_{k+T-1} \end{pmatrix} + \begin{pmatrix} w_k \\ w_{k-1} \\ \vdots \\ w_{k+T-1} \end{pmatrix} \tag{3}$$

which is simply $\boldsymbol{x}_k = Q\boldsymbol{s}_k + \boldsymbol{w}_k$ in vector notation format. The DFE is to equalize the effective channel response $\boldsymbol{q}_l$. Therefore, the DFE filters $\boldsymbol{g}_k$ and $\boldsymbol{b}_k$ have to be calculated with respect to the communication channel matrix Q, formulated in (3). Q is a Toeplitz matrix of dimension Tx$(T+2L)$, which diagonally contains the effective channel vector q_l and the rest of its elements are all zeros. We define two vectors; one is the concatenated vector of feedforward and feedback filters $\tilde{\boldsymbol{g}}_k = (\boldsymbol{g}_k, -\boldsymbol{b}_k)$ and the other is the corresponding vector of inputs to these filters, $\tilde{X}_k = (\boldsymbol{x}_k, \boldsymbol{s}_{F,\Delta})$ [1]. 1xT is the feedforward filter length and 1xF is the length for the feedback filter. Both concatenated vectors $\tilde{\boldsymbol{g}}_k$ and $\tilde{X}_k^T$ are of length 1x$(F+T)$, where $(\cdot)^T$ represents vector transpose. For a single channel DFE, using the MMSE criterion, the filter weights can be found as follows

$$\tilde{\boldsymbol{g}}_k = \underset{\boldsymbol{g}_k, \boldsymbol{b}_k}{argmin}\ E\{|\tilde{\boldsymbol{g}}_k \tilde{X}_k - s_{k-(2L+1)+\Delta}|^2\} \tag{4}$$

the minimization of which leads to $\tilde{\boldsymbol{g}}_k = R_{s_{k-(2L+1)+\Delta}\tilde{X}}(R_{\tilde{X}\tilde{X}})^{-1}$, where Δ is the equalization delay. The vector of previously detected symbols is $\boldsymbol{s}_{F,\Delta} = (\ s_{k-(2L+1)+\Delta-F} \ \cdots \ s_{k-(2L+1)+\Delta-1}\ )$ [1]. If the channel is already known by the receiver, the autocorrelations $R_{\tilde{X}\tilde{X}} = E\{\tilde{X}_k\tilde{X}_k^H\}$ and the crosscorrelation $R_{s_{k-(2L+1)+\Delta}\tilde{X}} = E\{s_{k-(2L+1)+\Delta}\tilde{X}_k^H\}$ can be calculated in a straightforward fashion instead of performing expectation operations over the received symbols. For this reason we define matrix P which is

$$P = \begin{pmatrix} & Q & \\ 0_{F\text{x}T+2L-F} & & I_{F\text{x}F} \end{pmatrix} \tag{5}$$

where $0_{F\text{x}T+2L-F}$ is the zero matrix of size $F\text{x}T+2L-F$ and $I_{F\text{x}F}$ is the identity matrix of size $F\text{x}F$. With the definition of matrix P, the autocorrelation matrices can directly be calculated by simply manipulating on P, i.e. $R_{\tilde{X}\tilde{X}} = E\{\tilde{X}_k\tilde{X}_k^H\} = \sigma_s^2PP^H + \sigma_w^2MM^H$ and $R_{s_{k-\Delta}\tilde{X}} = 1_{\Delta,T+2L}P^H$, where $1_{\Delta,T+2L}$ is a $T+2L$ length vector where its Δ^{th} element is 1 [8]. The matrix M is the Toeplitz matrix of true matched filters, instead of which an identity matrix of I_{T+F} is preferred for the blind usage.

III. Blind Correlation-based DFE

Assuming linearity is maintained, the position of the filter blocks in a receiver can be exchanged whilst providing the equalization. Nevertheless, the non-linear equalizer, i.e. DFE, should be converted into a linear form in order to maintain the linearity of the receiver in the first place. The linearity in a DFE can be obtained by carrying the decision device out of the feedback loop and placing it at the end of the receiver. In the z-domain, the equation for the output of the linear-DFE $Y(z)$ is $Y(z) = G(z)X(z) - B(z)Y(z)$, where the capital letter representations are the z-domain transforms of their discrete time-domain counterparts. It can be shown that the transfer function of the linear-DFE is $D(z) = Y(z)/X(z) = G(z)/(1+B(z))$, which can simply be implemented with a linear equalizer. The use of the linear form will obviously affect the performance of the DFE, however it is needed for the linearization.

Applying the equalization at the end of the system structure we expect the overall system impulse response to be $\boldsymbol{d}_k \star \boldsymbol{q}_k \cong \delta_{k-\Delta}$. For simplicity, if we call the linear DFE filter impulse response $\boldsymbol{d}_k$ and choose its length to be the same with the channel, the following amendment in the overall system response is true.

$$\begin{aligned} q_k^{(eff)} = \boldsymbol{d}_k \star \boldsymbol{q}_k &= \sum_{l=0}^{2L} d_{k-l} \sum_{i=1}^{M_r}\left(\sum_{\gamma=0}^{L} m_{l-\gamma}^{(i)} h_{\gamma}^{(i)}\right) \\ &= \sum_{i=1}^{M_r}\left(\sum_{l=0}^{2L} m_{k-l}^{(i)}\left(\sum_{\gamma=0}^{L} d_{l-\gamma} h_{\gamma}^{(i)}\right)\right) \end{aligned} \tag{6}$$

which shows that, although the matched filtering is performed after the linear DFE, the equalization can still be made. As can be seen in (6), this time each channel response $h_k^{(i)}$ needs to get filtered with the linear DFE, $\boldsymbol{d}_k$. The Correlation based DFE, therefore, can be implemented as shown in Fig. 2. Because the same filter is to be used at every antenna, a single equalizer $\boldsymbol{d}_k$ is enough. For making use of the single equalizer, two switches are utilized, working at a frequency of T_i/M_R, where T_i is the symbol period.

In order to implement the architecture in Fig.2, the linear-DFE filter $\boldsymbol{d}_k$ and matched filter of each channel, $\boldsymbol{m}_k^{(i)}$, should be estimated prior to the equalization. In the following two sub-section we will show how to estimate the unknown parameters of the receiver blindly.

Blind Calculation of DFE weights: It has previously been shown that the calculation of $R_{s_{k-\Delta}\tilde{X}}$ and $R_{\tilde{X}\tilde{X}}{}^{-1}$ needs the effective channel $\boldsymbol{q}_l$ to be known, which was found as the sum of the autocorrelations of the M_R responses $h_k^{(i)}$. In practice the autocorrelation of a channel can blindly be estimated using sample averages, written as

$$\hat{q}_k^{(i)} = \sum_{j=0}^{K} y_j^{(i)} y_{j+k}^{(i)\,*} \,/\, \left(K\sigma_s^2\right) \qquad for\ \ k \neq 0 \tag{7}$$

which enables the calculation of the autocorrelation of the channel by manipulating the channel output without the need for a priori channel information, where K is the number of data symbols to be used for the estimation of the autocorrelation vector. The symbol variance σ_s^2 and K can be a few thousands for a good estimate. If $k = 0$, the noise variance σ_n^2 should also be taken into account. Therefore $\hat{q}_k^{(0)}$ can be calculated using (7) by subtracting σ_n^2 from the result.

Blind Calculation of the Matched Filters: As being mentioned earlier in the Introduction, the unknown matched filter can be estimated blindly by the use of the CMA, if the matched filter is the sole unknown [7]. The said condition is satisfied, once the DFE weights are calculated with sample averaging. The block with dashed lines in Fig. 2 (i.e. MF) is where the multi-dimensional matched filtering is performed. It can be viewed as a system of M_R discrete Finite Impulse Response (FIR) filters, the output of which are added down into a single outputs after filtering. For our work, MF is assumed to be a single FIR filter of length $M_R\text{x}(L+1)$, the coefficients of which are adaptively updated by the CMA algorithm. Calling

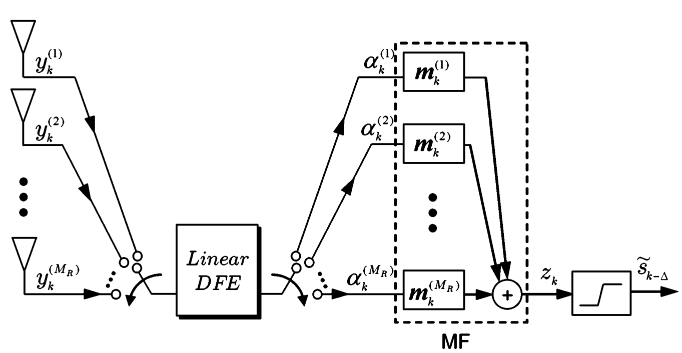

Fig. 2. Novel Architecture for Correlation-based DFE

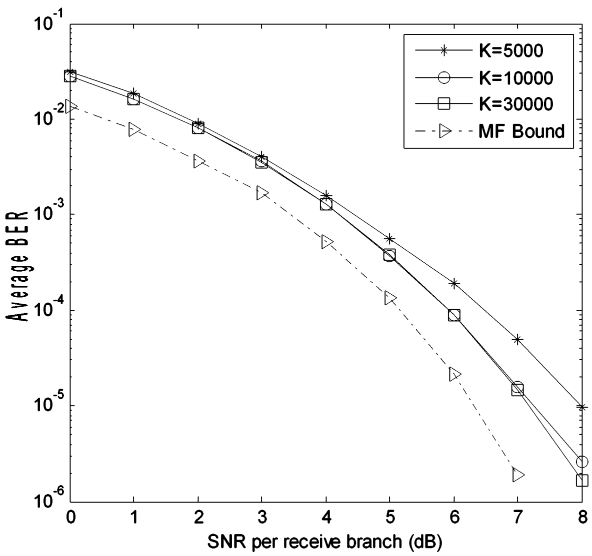

Fig. 3. BER performance of the blind Correlation-based DFE with different number of CMA updates (M_R=4, L=2)

the MF vector $\mathbf{M}$, the CMA can be used for its update as formulated below [9]

$$\mathbf{M}(n+1) = \mathbf{M}(n) + \mu\alpha\left(k\right)z\left(n\right)\left(\left|z\left(n\right)\right|^2 - 1\right)^2 \qquad (8)$$

providing that a data symbol constellation having a constant squared-modulus of one (i.e. $|s\left[k\right]|^2 = 1$) is preferred. In equation (8), n represents the iteration number. $\mathbf{M}(n)$ is the MF vector at the n^{th} iteration and similarly $z(n)$ is the output of filter $\mathbf{M}$ at the n^{th} iteration. μ is a small positive step size and $\alpha(k)$ is the concatenated vector of processed symbols by the DFE filters corresponding to the MF output at the n^{th} iteration;

$$\alpha\left(k\right) = \Big[\alpha_k^{(1)}, \alpha_{k-1}^{(1)}, \cdots, \alpha_{k-L-1}^{(1)}, \alpha_k^{(2)}, \alpha_{k-1}^{(2)}, \cdots, \alpha_{k-L-1}^{(M_R)}\Big] \qquad (9)$$

It should also be noted that; If the iterations to update $\mathbf{M}$ are made periodically at every symbol period so that n is incremented as k does, then the two index variables k and n can be regarded as the same.

IV. Simulations

Two simulation sets are provided in this section, first of which is to illustrate the Bit Error Rate (BER) performance of the novel blind equalizer and the second is for the Mean Squared Error (MSE) performance of the multi-dimensional matched filter estimation. For the simulations, unit energy QPSK symbol transmission is assumed from the single transmit antenna (M_T) to multiple receive antenna (M_R). The channels of each receive branch are real values and in unit norm, satisfying the normalization $\sum_{k=0}^{L}|h_k^{(i)}|^2 = 1$. The noise added to the received signals is assumed to be of the white Gaussian distributed. The simulation parameters were chosen as; M_R=4, L=2, T=8, F=2, Δ=4 (Delay for DFE), μ=0.01 (Step size for the CMA). Note that, the reason as to why we kept the size of the channel so short (i.e. M_R=4, L=2) was to realize the diversity gain over the BER curve more easily. For channels with higher or lower orders, our methods converges without problems.

In order to understand how well the novel technique equalizes a SIMO channel, three BER results have been plotted on Fig. 3. The plots differ in the number of symbols to be used to update the multi-dimensional matched filter by CMA. Respectively $K = 5\text{x}10^3$, $K = 10\text{x}10^3$ and $K = 30\text{x}10^3$ symbols are used to plot these three curves. For the simulations, randomly generated SIMO channels have been tested and the BER average has been taken for each set of simulations. In order to plot the matched filter bound, a single shot transmission is assumed in order to realize the ISI-free performance of the system. [1].

To realize the MSE performance, four plots have been provided in Fig. 4. The MSE is defined as $MSE = (\sum_{i=1}^{M_R}||\hat{\mathbf{h}}_k^{(i)} - \mathbf{h}_k^{(i)}||_2^2)\,/(\sum_{i=1}^{M_R}||\mathbf{h}_k^{(i)}||_2^2)$, where $\hat{\mathbf{h}}_k^{(i)}$ is the channel vector estimated by the novel method and $\mathbf{h}_k^{(i)}$ is the vector of true channel coefficients. $||\cdot||_2$ represents the Euclidean norm operation. As can be seen on Fig. 4, the MSE decrease with respect to K is shown using the same simulation parameters given above. Similar to the BER plots, the results are the averaged MSE of several randomly generated channels. As expected, the MSE rate decreases with the use of more symbols for update. For the case where $SNR = 7dB$, two plots have been provided. Note that the choice of μ is also defines the convergence speed and MSE of the estimation.

Revisiting the BER results in Fig. 3, it can be said that the blind Correlation-based DFE performs equalization close to

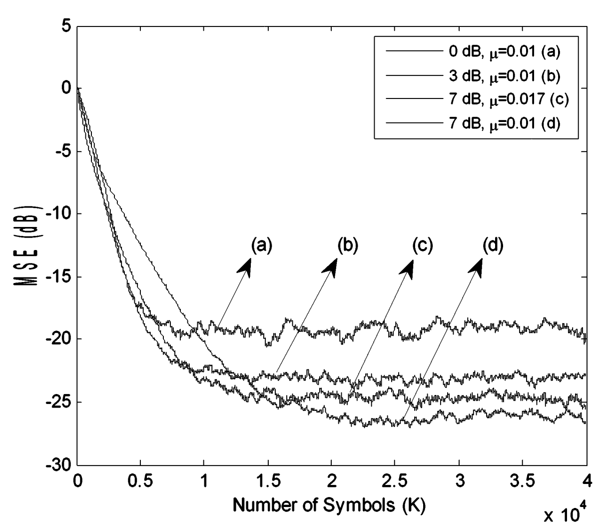

Fig. 4. MSE performance of the blind Correlation-based DFE with different number of CMA updates (M_R=4, L=2)

the matched filter bound (for low SNRs, 0.5dB is achieved). The diversity gain, which can be presumed from the slope of the BER curve, is almost fully employed by the blind architecture. It should also be mentioned that our novel techniques has its shortcomings too. This is due to the multi-dimensional matched filter coefficients, being found by blind estimation and thus in estimation error, which in high SNR values resulting a loss in the diversity gain for our novel technique. Especially for the case, where the update is done over $5\text{x}10^3$ symbols, the loss in the diversity gain is more apparent. Furthermore, because a linear model for the DFE is preferred rather than that of a non-linear structure, the BER plots are further from the MF Bound. We can also conclude from the figure that, even a relatively small number of symbols are used for the multi-dimensional matched filter update, the BER performance is still preferable over the other cases where K is large.

V. Conclusion

In this paper, we have presented the blind implementation of the correlation-based DFE for the equalization of SIMO channels. It has been shown that; without the need of any computational inefficient state-of-the-art SIMO channel estimation method, the required matched filters for the correlation based-DFE can blindly be estimated by making use of a computational convenient method, i.e. CMA. How the BER and the MSE are changed over the number of data, used for matched filter update, is provided with the paper for a selected case of SIMO communications. The BER performance of our proposed method is shown to be close to the MF bound therefore is preferable over non-blind schemes. For the selected simulation parameters, it has been shown that the number of symbols to be used to estimate the unknown matched filters are in the order of thousands. For a communication channel, where the coherence time is greater than the symbol time in the orders of 10^3, the proposed method is preferable instead of using training data.

References

[1] A. Paulraj, R. Nabar and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2005.

[2] T. C. Yang, *Correlation-based decision-feedback equalizer for underwater acoustic communications*, IEEE J. Ocean. Eng., vol. 30, issue 4, pp. 865-880, Oct. 2005

[3] Hee C. Song, W. S. Hodgkiss, W. A. Kuperman, W. J. Higley, K. Raghukumar, T. Akal, and M. Stevenson, *Spatial diversity in passive time reversal communications*, Journal of the Acoustical Society of America, vol. 120, issue 4, pp. 2067-2076, July 2006

[4] G. Xu, H. Liu, L. Tong and T. Kailath, *A least squares approach to blind channel identification*, IEEE Trans. Signal Processing, vol 43, pp. 2982-2993, Dec. 1995.

[5] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue, *Subspace methods for the blind identification of multichannel FIR filters*, IEEE Trans. Signal Processing, vol. 45. pp. 516-525, Feb. 1995.

[6] J.K. Tugnait, *Blind equalization and estimation of fir communications channels using fractional sampling*, IEEE Trans. Commun., vol. 44, pp. 324-336, Mar. 1996

[7] I. Ozcelik, I. Kale, B. Baykal, *Blind adaptation of a matched filter using the Constant Modulus Algorithm coupled with an optional correction method*, in IEEE ISSPA'03, vol. 2, pp. 291-294, July 2003.

[8] J. Cioffi, *EE379a Digital Communications: Signal Processing class reader*, Stanford University, Stanford, CA, 2008.

[9] G. Li, Z. Ding, *New blind/semi-blind equalization algorithms for GSM systems*, in IEEE SPAWC '99, pp. 66-69, May 1999.