A new approach for non-linear analysis of power amplifiers

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A NEW APPROACH FOR NON-LINEAR ANALYSIS OF POWER AMPLIFIERS

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Abstract – This paper presents a new approach for non-linear analysis of power amplifiers based on the *n*-order polynomial modelling. A general expression for the fundamental frequency output is developed in this paper. It allows considering any order of polynomial model and obtaining analytical expression for the fundamental-frequency output without complex computations, using developed formulas. This model significantly simplifies non-linear analysis of power amplifiers and the process of designing linearizers. This paper also presents a simple approach for obtaining polynomial model from a frequency response of power amplifier. The proposed method is verified by experiments. Obtained polynomial model shows simulation results similar to the measurements for real amplifier. Developed *n*-order output modelling and proposed model extraction method can significantly simplify, optimize and enhance power amplifiers' non-linear analysis and behavioural modelling. Proposed modelling approach can be also used to design linearizers.

I. Introduction

New demands to communication systems require enhancement of power amplifiers' performances, such as widening dynamic range and even operation in saturation mode without distortion. To satisfy these requirements, linearizers are used [1]-[2]. While designing a linearizer for particular amplifier, non-linear analysis and output distortion modelling are carried out. Therefore, simple, quick and accurate methods for behavioural modelling and non-linear analysis are required.

Usually power amplifiers (PA) are represented by a Winner or Hammerstein model, which consist of a non-linear memoryless block and a linear FIR filter [3]. Filters are used to represent memory effects and polynoms in the memoryless blocks denominate PA non-linear distortions. The higher order of the polynom is considered, the more accurate PA behaviour model is, and the more efficient and precise linearizer can be designed. Therefore, there is a growing demand of straightforward methods for obtaining the output signal analytical model using high-order polynomial representation of PAs. Also, simple methods are required for extracting a PA polynomial model from the real measurements.

This paper presents a new approach for PA non-linear analysis, aimed at obtaining analytical expression for the output signal using high-order polynomial model. In literature 3rd and 5th –order polynoms are usually used for representing PA distortion [1]-[2]. Therefore, analytical model for output distorted signal for the 3rd and 5th orders are well-known. However, there is a lack of such knowledge for the high-order models. This paper aims to compensate for this shortage and develops a general *n*-order polynomial model for the PA output, where *n* can be as high as necessary for the required accuracy. The precise formulas up to 19th-order are presented in the paper.

Moreover, this paper describes a simple and accurate approach for obtaining PA polynomial model from its frequency response. This approach is verified by experiments and shows good performances. The obtained model produces simulation results close to the measured PA performances.

II. Proposed Approach for Polynomial Model Extraction

PA non-linear output is usually represented by a polynomial expression [1]-[3]:

$$V_{\mathrm{OUT}}(t) = \sum_{n=1}^{\infty} g_n \cdot V_{IN}^n(t) \tag{1}$$

Where V_{IN} is the input voltage of PA and g_1, g_2, g_3... are coefficients of the nonlinear terms. The model (1) allows obtaining mathematical formulas for distortion components of a signal, when PA operates in a non-linear mode.

Consequently, for the non-linear analysis, behavioural modelling and linearizing of PA, *g*-coefficients need to be extracted. This section describes a developed approach for obtaining polynomial model for a power amplifier from its frequency response.

For a tone input signal $V_{\mathrm{IN}}(t) = V\cdot\cos(\omega t)$. Considering the first three elements in (1) after applying trigonometric relations, PA output looks like:

$$V_{\mathrm{OUT}}(t) = \frac{g_2 V^2}{2} + (g_1 V + \frac{3}{4} g_3 V^3)\cos(\omega t) + \frac{g_2 V^2}{2}\cos(2\omega t) + \frac{3 g_3 V^3}{4}\cos(3\omega t). \tag{2}$$

According to (2) the output signal includes second and third harmonics apart from the fundamental tone. An example of such response is plotted in Figure 1.

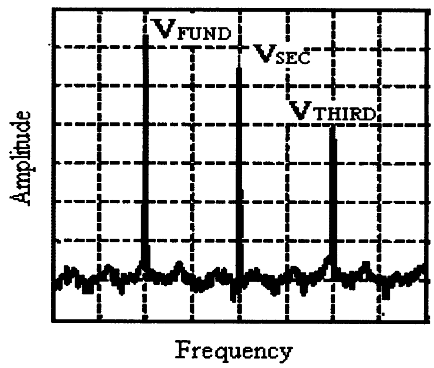

Figure 1. Typical frequency response of a PA in saturation

Using this graph and expression (2), it is easy to determine *g*-coefficients for the polynomial model (1). From the frequency response (Fig. 1) amplitudes of the harmonics can be obtained: V_{FUND}, V_{SEC}, V_{THIRD}... Than, using (2), the system of equations can be written:

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$$\begin{cases} g_1 V + \frac{3}{4} g_3 V^3 = V_{\text{FUND}}, \\ \frac{g_2 V^2}{2} = V_{\text{SEC}}, \\ \frac{3 g_3 V^3}{4} = V_{\text{THIRD}}. \end{cases} \quad (3)$$

From (3) the coefficients g_1, g_2, g_3... are obtained.

III. Proposed n-order Polynomial Output Modelling

Real power amplifiers can be presented by polynomial models, which have larger number of elements. This makes theoretical analysis of the distorted output products extremely complicated. In literature 3rd and 5th-order polynomial models are usually considered [1]-[2]. Therefore, analytical expressions for the 3rd and 5th-order of PA outputs are widely known. But there is a lack of such knowledge for the high-order polynomial models.

This section presents a developed general analytical expression for the fundamental-frequency output of *n*-order PA polynomial model.

The modulated input signal with variable amplitude $V_S(t)$ and phase $\varphi(t)$ can be written as:

$$V_{IN}(t) = V_S(t) \cdot \cos(\omega t + \varphi(t)) \quad (4)$$

Using *I* and *Q* components, (4) looks like:

$$V_{IN}(t) = V \cdot (I \cdot \cos(\omega t) - Q \cdot \sin(\omega t)) \quad (5)$$

Where: $I = \frac{V_S(t)}{V} \cdot \cos(\varphi(t))$, $Q = \frac{V_S(t)}{V} \cdot \sin(\varphi(t))$, $V = average(|V_S(t)|)$. Therefore, $average(I^2 + Q^2) = 1$.

After substituting (5) into (1) and applying trigonometric relations, the PA output looks like:

$$\begin{aligned} V_{OUT}(t) &= \{\frac{V^2 g_2 I^2}{2} + \frac{V^2 g_2 Q^2}{2}\} \\ &+ \{V g_1 (I \cos \omega t - Q \sin \omega t) \\ &+ \frac{3V^3 g_3 (I^2 + Q^2)}{4} (I \cos \omega t - Q \sin \omega t)\} \cdot \cos(\omega t) \\ &+ \{\frac{V^2 g_2 (I^2 - Q^2)}{2} \cos(2\omega t) - V^2 g_2 IQ \sin 2\omega t\} \\ &+ \{\frac{V^3 g_3 I(I^2 - 3Q^2)}{4} \cos 3\omega t \\ &- \frac{V^3 g_3 Q(3I^2 - Q^2)}{4} \sin 3\omega t\} \end{aligned} \quad (6)$$

Here, DC, second and third harmonics are not interesting for consideration because they can be easily filtered at the PA output. However, fundamental frequency distortion is very important:

$$\begin{aligned} V_{OUT}^{FUND}(t) &= V g_1 (I \cos \omega t - Q \sin \omega t) \\ &+ \frac{3V^3 g_3 (I^2 + Q^2)}{4} (I \cos \omega t - Q \sin \omega t) \end{aligned} \quad (7)$$

TABLE I

MEMBERS OF THE DEVELOPED SERIES

b_3	b_5	b_7	b_9	b_{11}	b_{13}	b_{15}	b_{17}	b_{19}
$\frac{3}{4}$	$\frac{5}{8}$	$\frac{35}{64}$	$\frac{63}{128}$	$\frac{231}{512}$	$\frac{429}{1024}$	$\frac{6435}{16384}$	$\frac{12155}{32768}$	$\frac{46189}{131072}$

In (7) distortion components produced by the even terms of the PA model are not present, therefore distortion at the fundamental frequency is caused only by the odd terms. (7) can be rewritten in another way:

$$V_{OUT}^{FUND}(t) = g_1 \cdot V_{IN}(t) + V_{3DIST}^{FUND}(t) \quad (8)$$

Where:

$$V_{3DIST}^{FUND}(t) = \frac{3V^2 g_3 (I^2 + Q^2)}{4} \cdot V_{IN}(t) \quad (9)$$

Because only odd-terms produce distortion at the fundamental frequency, the even ones will not be considered further. Carrying out similar mathematical calculations for the 5th-order polynomial model, it is possible to obtain the next term:

$$V_{5DIST}^{FUND}(t) = \frac{5V^4 g_5 (I^2 + Q^2)^2}{8} \cdot V_{IN}(t) \quad (10)$$

The 7th-order term will produce:

$$V_{7DIST}^{FUND}(t) = \frac{35V^6 g_7 (I^2 + Q^2)^3}{64} \cdot V_{IN}(t) \quad (11)$$

Having considered the third (9), fifth (10) and seventh-order (11) distortion components, it is possible to find regularity in their expressions. Using mathematical induction the general formula can be found:

$$V_{(2k+1)DIST}^{FUND}(t) = b_{2k+1} V^{2k} g_{2k+1} (I^2 + Q^2)^k \cdot V_{IN}(t) \quad (12)$$

Here k=1, 2, 3... and b_{2k+1} is a member of the mathematical series, which will be described bellow.

Using (8)-(12) the accurate formula for PA output at the fundamental frequency can be found for the *n*-order polynomial model:

$$\begin{aligned} V_{OUT}^{FUND}(t) &= g_1 \cdot V_{IN}(t) \\ &+ (\sum_{k=1}^{n} b_{2k+1} V^{2k} g_{2k+1} (I^2 + Q^2)^k) \cdot V_{IN}(t) \end{aligned} \quad (13)$$

Here, the first term is undistorted linear output and the second one represents distortion. b_{2k+1} is a mathematical series of coefficients near the corresponding terms. Using Matlab the first nineteen members of the *b*-series has been found. Tab. I presents the odd terms. Series b_{2k+1} and formula (13) allow considering as many polynomial elements as necessary for the required accuracy.

IV. Experimental Verification of the Proposed Approach

In order to verify accuracy of the proposed modelling method, experiments with power amplifier in saturation using harmonic test and 16-QAM digital modulated signal were carried out. Experimental equipment, shown on

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Fig. 2, consist of power amplifier ZFL500, analog signal generator Hewlett Packard A6488, spectrum analyzer Hewlett Packard 8594EM, digital modulated signal generator Agilent E4433B and vector signal analyzer VSA Agilent E4406A.

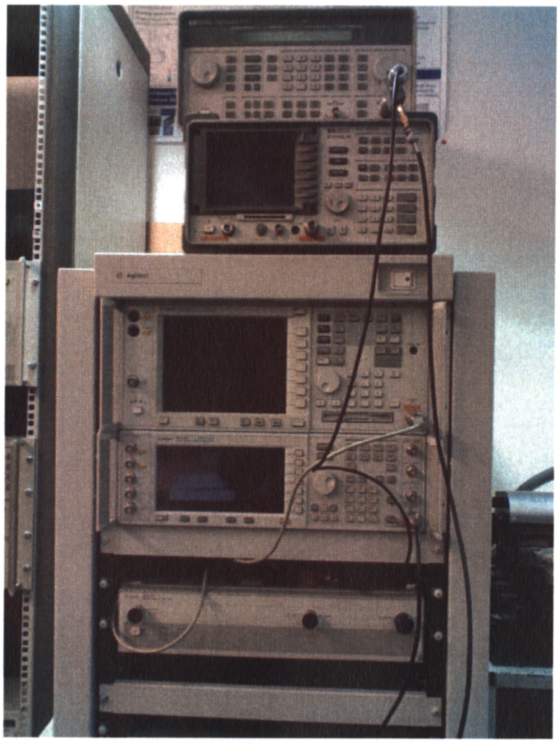

Figure 2. Experimental test bench

Initially, an RF tone with frequency 500 MHz has been generated and passed thought the PA. According to the PA data sheet, ZFL500 has 21 dB gain and a 1-dB compression point at 10 dBm output.

In order to obtain *g*-coefficients for the amplifier model, a frequency response has been generated (Figure 3). Measured amplitudes of the harmonics for different power levels are presented in Table II. From Table II the initial coefficients for the amplifier model were obtained according to procedure described above (3):

$$g_1 = 10.5;\ g_3 = -4.5;\ g_5 = -1.$$

At fundamental frequency for the 5[th] order polynomial model, (13) gives general expression for the output:

$$V_{OUT}^{FUND}(t) = g_1 \cdot V_{IN}(t) + \frac{3}{4} g_3 \cdot V^3{}_{IN}(t) + \frac{5}{8} g_5 \cdot V^5{}_{IN}(t) \quad (14)$$

Therefore, output distortion model for ZFL500 looks like:

$$V_{OUT}^{FUND}(t) = 10.5 \cdot V_{IN}(t) - 3.3 \cdot V^3{}_{IN}(t) - 0.6 \cdot V^5{}_{IN}(t) \quad (15)$$

This model has been verified by comparison of the measured and modelled characteristics of the power amplifier: transfer function (Figure 4) and the 16-QAM output spectrum (Figures 5-6).

TABLE II

MEASURED OUTPUT HARMONICS FOR ZFL500 VS P_{INPUT}

P_{INPUT}, dBm	V_{FUND}, dBm	V_{SEC}, dBm	V_{THIRD}, dBm	V_{FOURTH}, dBm	V_{FIFTH}, dBm
-10	10.6	0	0	0	0
-5	15.5	-25	0	0	0
0	20.4	-16	-23	0	0
3	23.2	-9	-15	0	0
5	24.8	-3	-8	-24	0
7	26.0	4	-2	-16	0
8	26.5	6	0	-14	-22

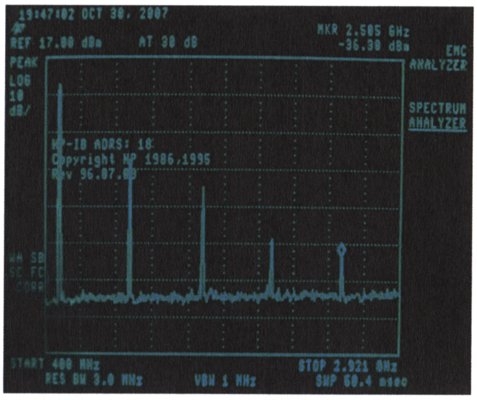

Figure 3. Measured frequency response of ZFL500 at 8 dBm output

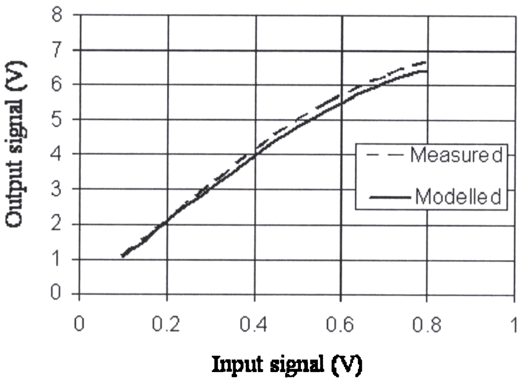

Figure 4. Transfer characteristic for ZFL500: measured and obtained from the developed model

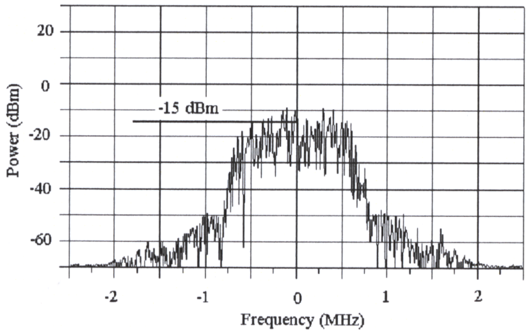

Figure 5. Simulated 16-QAM output spectrum for the developed ZFL500 model

Simulations of the PA model (Figures 4, 5) have been carried out using Matlab-ADS co-simulation system. Here, proposed PA model was implemented in Matlab and then called from ADS during the simulation. Comparison of the simulated and measured results (Figures 4-6) allows making a conclusion that the developed polynomial behavioural model of power amplifier is accurate and can be used for simulations in place of the real device.

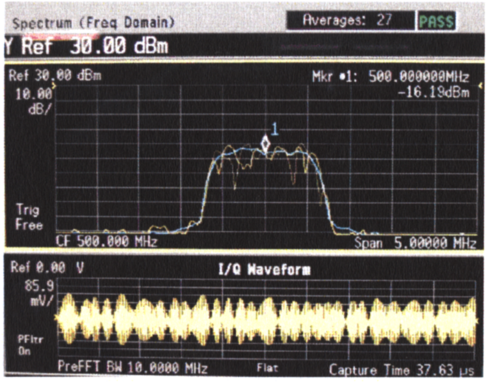

Figure 6. Measured 16-QAM output spectrum for ZFL500

V. Conclusion

This paper presented a new approach in PA behavioural modelling and non-linear analysis. The simple and reliable method of polynomial model extraction from a power amplifier frequency response has been presented. A new approach in modelling PA output has been developed. General n-order polynomial model for the output signal has been presented. The obtained expression allows making quick, simple and accurate analytical prediction of the output distortion at fundamental frequency by analytical formulas, which can be implemented to any power amplifier and any order of polynomial model. The new mathematical series b_n, representing coefficients in output signal series has been developed. Obtaining its general or recursive formula will be one of the directions for future work. This series can be used in a non-linear analysis of PA. It gives straightforward prediction of the PA output signal in a saturation mode. The proposed modelling approach has been verified by experiments. The experimental results for a tone input signal as well as 16-QAM modulated signal proved the accuracy of the proposed method.

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НОВЫЙ МЕТОД НЕЛИНЕЙНОГО АНАЛИЗА УСИЛИТЕЛЕЙ МОЩНОСТИ

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Аннотация – Даная статья описывает новый метод нелинейного анализа усилителей мощности, основанный на полиномиальном моделировании n-го порядка. В статье разработан общий вид формулы, описывающей выходной сигнал на основной частоте. Метод позволяет рассматривать любую степень полиномиальной модели и получать аналитическое выражение для выходного сигнала основной частоты без сложных вычислений, используя предложенную формулу. Данная модель существенно упрощает нелинейный анализ усилителей мощности и процесс проектирования линеаризаторов. В статье также предложен простой подход для получения полиномиальной модели из частотной характеристики усилителей мощности. Предложенный метод подтвержден экспериментально. Результаты моделирования полученной полиномиальной модели совпадают с результатами измерений характеристик усилителя мощности. Разработанная выходная модель n-го порядка и предложенный метод извлечения модели могут существенно упростить, оптимизировать и расширить возможности нелинейного анализа усилителей мощности и бихевиориального моделирования. Описанный подход моделирования может быть также применен для проектирования линеаризаторов.

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