On various low-hardware-complexity LMS algorithms for adaptive I/Q correction in quadrature receivers.

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ON VARIOUS LOW-HARDWARE-COMPLEXITY LMS ALGORITHMS FOR ADAPTIVE I/Q CORRECTION IN QUADRATURE RECEIVERS

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ABSTRACT

In this paper, the performance and convergence time comparisons of various low-complexity LMS algorithms used for the coefficient update of adaptive I/Q corrector for quadrature receivers are presented. We choose the optimum LMS algorithm suitable for low complexity, high performance and high order QAM and PSK constellations. What is more, influence of the finite bit precision on VLSI implementation of such algorithms is explored through extensive simulations and optimum wordlengths established.

1. INTRODUCTION

The homodyne/Zero-IF receivers provide high levels of integration. With this architecture, I/Q signal processing is used to downconvert the RF signal to baseband. However, this architecture, in common with all I/Q architectures, is vulnerable to mismatches between the I and Q channels. Both analog and digital methods for correcting the I/Q mismatches of homodyne receivers have been proposed in the literature [1]-[6]. This paper utilizes the unsupervised/blind adaptive DSP technique developed for the quadrature receivers in [5], [6]. Performance of various low complexity algorithms is analyzed and fixed point effects are taken into account.

This paper is organized as follows: Section 2 gives brief overview of the unsupervised adaptive I/Q corrector for quadrature receivers and shows the structure for implementing such algorithm. Section 3 looks in reduced complexity LMS algorithms. Simulation results for the performance and convergence rate are given for different algorithms. Section 4 gives information on the architecture and bit precision effects on the performance and concluding remarks are given in section 5.

2. UNSUPERVISED/BLIND ADAPTIVE I/Q MISMATCH CORRECTOR

In the presence of phase and gain mismatches in the quadrature downconverter, received baseband signal has in-phase and quadrature components given as:

$$\begin{aligned} r_1(k) &= \overbrace{(1+0.5\alpha_\varepsilon)\cos(\varphi_\varepsilon/2)}^{\psi} s_I(k) + \overbrace{(1+0.5\alpha_\varepsilon)\sin(\varphi_\varepsilon/2)}^{h_2} s_Q(k) \\ r_2(k) &= \underbrace{(1-0.5\alpha_\varepsilon)\sin(\varphi_\varepsilon/2)}_{h_1} s_I(k) + \underbrace{(1-0.5\alpha_\varepsilon)\cos(\varphi_\varepsilon/2)}_{\gamma} s_Q(k) \end{aligned} \quad (1)$$

where, φ_ε is the phase and α_ε is the gain mismatch between the I and Q channels, $s_I(k)$ and $s_Q(k)$ are the transmitted in-phase and quadrature signals. Also, ψ and γ are ≈ 1 and can be safely ignored. The in-phase signal $r_1(k)$ is corrupted by the quadrature signal $r_2(k)$ leaked due to phase and gain mismatches. A leakage from the quadrature signal into the in-phase signal also exists. Ideally the I and Q channels are not correlated with each other. However, in the presence of the quadrature phase and gain errors this relationship no longer exists and they are correlated. The proposed algorithm, depicted in Figure 1, acts as a decorrelator and tries to de-correlate the I and Q channels hence eliminating phase and gain errors. The source estimates, $c_1(z)$ and $c_2(z)$ can be expressed as:

$$\begin{aligned} c_1(z) &= (1-w_1h_2)s_I(z) + (h_1-w_1)s_Q(z) \\ c_2(z) &= (h_2-w_2)s_I(z) + (1-w_2h_1)s_Q(z) \end{aligned} \quad (2)$$

When the coefficients converge, i.e. $w_1 = h_1$ and $w_2 = h_2$ then the source estimates become:

$$\begin{aligned} c_1(z) &= (1-h_1h_2)s_I(z) \\ c_2(z) &= (1-h_1h_2)s_Q(z) \end{aligned} \quad (3)$$

As it can be seen from (3) the sources have been separated. Furthermore, $(1 - h_1h_2) \approx 1$ and can be safely ignored. The description blind or unsupervised implies that we do not know the mixing coefficients h_1, h_2, nor the probability distribution of the sources except that they are not correlated. LMS algorithm is used to update w_1 and w_2. A significant feature of the LMS algorithm is its simplicity. Moreover, it does not require the measurements of the correlation functions nor does it require matrix inversion. Indeed, it is the simplicity of the LMS algorithm that has mode it the standard against other algorithms are benchmarked [7].

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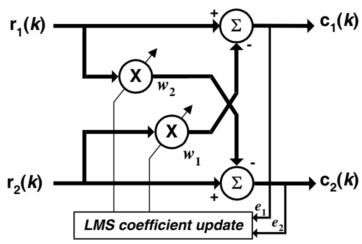


Figure 1 Adaptive I/Q corrector structure.

In spite of the computational efficiency of the LMS algorithm, additional simplifications may be necessary in some applications to reduce the computational requirements of the LMS algorithm. Next section will look into such reduced complexity LMS algorithms and compare them in terms of mean *Image Rejection Ratio* (IRR) [5] and convergence rate for different modulation formats and constellation sizes.

3. REDUCED/LOW COMPLEXITY LMS COEFFICIENT UPDATE

LMS coefficient update block occupies the large portion of the adaptive system. Set of simplifications to the LMS algorithm are found in the *sign algorithms* [8]. In these algorithms, the LMS coefficient update equation is modified by applying the *sign* operator to either the error $e(k)$, the data $x(k)$, or both the error and the data. Depending on which signal(s) the sign operator is applied to the *sign*-LMS algorithms can be divided into following sub-groups:

- Sign-Error (SE)
- Sign-Data (SD)
- Sign-error Sign-data (SS)

Coefficient update equations for above algorithms are given as [8]:

	Coefficient update equation
LMS	$\mathbf{W}(k+1)=\mathbf{W}(k)+2\mu e(k)\mathbf{r_2}(k)$
SE	$\mathbf{W}(k+1)=\mathbf{W}(k)+2\mu\,\mathrm{sgn}\{e(k)\}\mathbf{r_2}(k)$
SD	$\mathbf{W}(k+1)=\mathbf{W}(k)+2\mu e(k)\,\mathrm{sgn}\{\mathbf{r_2}(k)\}$
SS	$\mathbf{W}(k+1)=\mathbf{W}(k)+2\mu\,\mathrm{sgn}\{e(k)\}\,\mathrm{sgn}\{\mathbf{r_2}(k)\}$

Table 1 Coefficient update equations.

where,

$$\mathrm{sgn}\{e(k)\}=\begin{cases}1 & \text{if} \quad e(k)>0\\ 0 & \text{if} \quad e(k)=0\\ -1 & \text{if} \quad e(k)<0\end{cases} \tag{4}$$

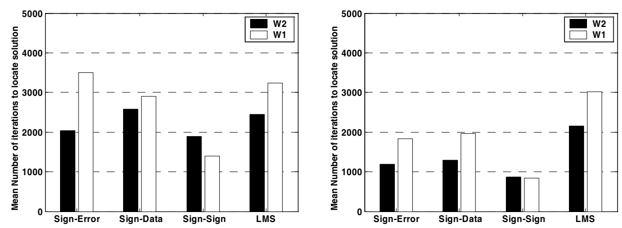
Figure 4, depicts the structures for implementing LMS and sign-LMS algorithms.

3.1. Performance comparison

In this section we will carry out extensive simulation studies to evaluate and compare the suitability of sign-LMS algorithms for adaptive I/Q mismatch correction in homodyne receivers. The performance is analysed considering 256-QAM and 32-PSK signals with ideal symbol rate sampling. AWGN channel is assumed. Phase and gain errors are randomly distributed between 0-30° and 1dB - 3 dB respectively. Results are averaged over 100 experiments. The performance is characterized by the IRR which can be expressed as [5]:

$$IRR(\alpha_\varepsilon,\varphi_\varepsilon)=10\log\left(\frac{2-2\cos\varphi_\varepsilon+0.5\alpha_\varepsilon{}^2(1+\cos\varphi_\varepsilon)}{2+2\cos\varphi_\varepsilon+0.5\alpha_\varepsilon{}^2(1-\cos\varphi_\varepsilon)}\right) \tag{5}$$

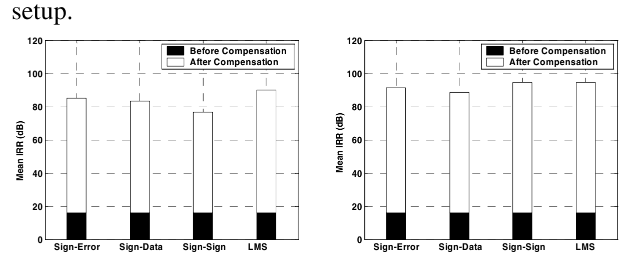
High IRR translates to small residual phase and gain errors after compensation. Mean number of iterations needed to locate solution for SE, SD, SS and LMS algorithms are depicted in Figure 2 for 256-QAM and 32-PSK modulation formats.



(a) 256-QAM, SNR=30 dB (b) 32-PSK, SNR=26.1 dB

Figure 2 Mean number of iterations to locate solution.

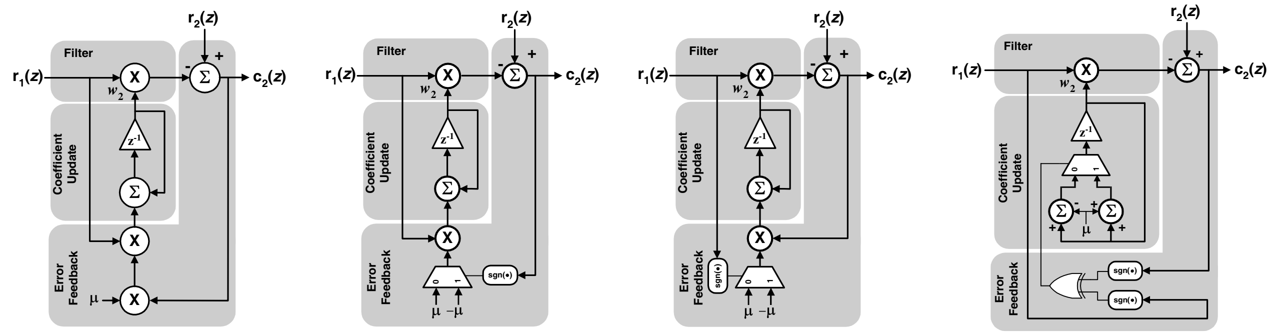
Generally, sign-sign algorithm is slower to convergence then the LMS. However, as can be observed, for our application, sign-sign LMS algorithm converges faster when compared to other algorithms. Slower convergence for 256-QAM compared to the 32-PSK can be explained by the fact that QAM constellation is more complex then PSK one. Figure 3 depicts the mean IRR for SE, SD, SS and LMS algorithms. Once again phase and gain errors are randomly distributed as in the previous simulation setup.



(a) 256-QAM, SNR=30 dB (b) 32-PSK, SNR=26.1 dB

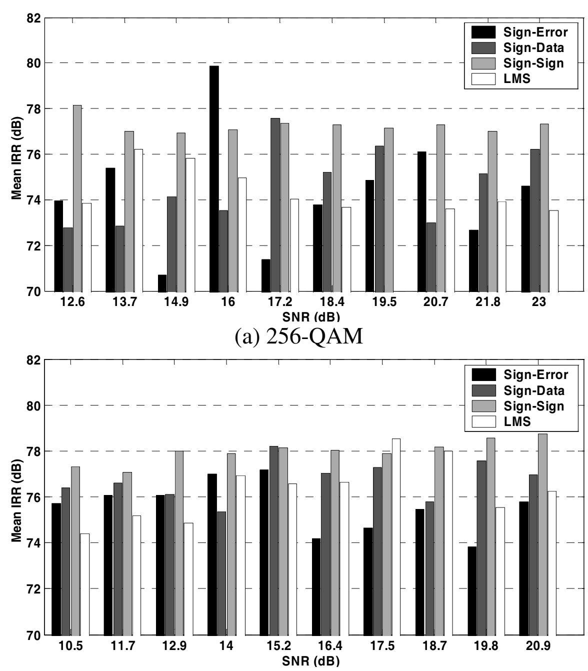
Figure 3 Mean IRR after compensation.

As can be seen from Figure 3, all algorithms more or less perform similarly in terms of IRR. There is not much to choose between them. Another comparison is carried out



(a) Normal LMS (b) Sign-error (c) Sign-data (d) Sign-Sign

Figure 4 LMS and sign-LMS structures.



(a) 256-QAM

(b) 32-PSK

Figure 5 Mean IRR for low-SNR. Phase error-randomly distributed between and 0-30°, gain error randomly distributed between 1dB - 3 dB for (a) 256-QAM and (b) 32-PSK.

in establishing how the algorithms perform under low-SNR environments. Results are depicted in Figure 5. As can be seen, sign-sign algorithm performance is better then the rest under low-SNR conditions. Careful observation of the results indicate that sign-sign LMS is most suited for our application in terms of speed and IRR that can be achieved with very small hardware overhead. Next sections of this paper will look into fixed point implementation and performance analysis of the fixed point sign-sign LMS algorithm.

4. SIGN-SIGN LMS ARCHITECTURE AND IMPLEMENTATION

Sign-sign LMS update equation can be further simplified by observing the fact that sgn{●} is the signum function which is +1 or –1 for a positive or negative argument, respectively. The coefficient update then reduces to:

$$\mathbf{W}(k+1) = \mathbf{W}(k) \pm 2\mu \tag{6}$$

depending on the product of sgn{$e(k)$}.sgn{$\mathbf{r_2}(k)$}. In the implementation, either $\mathbf{W}(k)$+2μ or $\mathbf{W}(k)$-2μ are computed, while sgn{$e(k)$}.sgn{$\mathbf{r_2}(k)$} is computed using an eXclusive-OR (XOR) operation on the sign bits of both the error and the input to the respective tap. Depending on the outcome of the XOR operation, $\mathbf{W}(k)$+2μ or $\mathbf{W}(k)$-2μ is computed. Hence, the multiplications required in the normal LMS algorithm reduce to a single XOR operation, which results in significant power and area savings. Figure 6 shows the area and power efficient, low-complexity 2-cycle time-multiplex implementation of the *sign-sign* LMS based adaptive I/Q corrector.

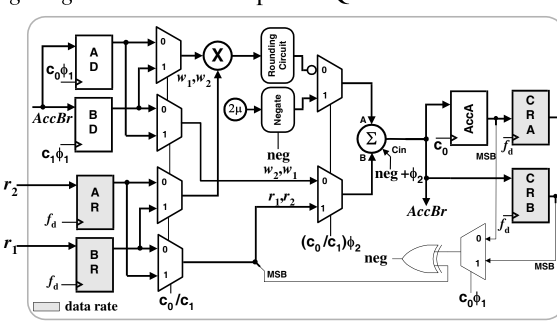


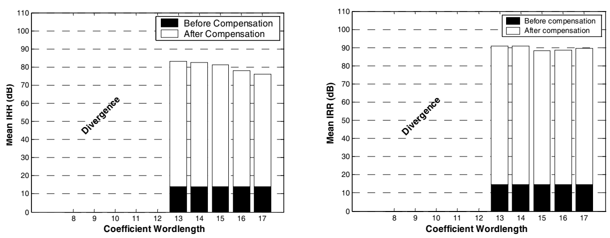
Figure 6 sign-sign LMS based adaptive I/Q corrector architecture.

The architecture consists of 7 registers, 4 operating at the data rate and rest operating at twice the data rate, a single multiplier and an adder with negate circuitry implemented as a bank of xor gates with common negate signal (*neg*) and 2-1 muxes to route the data to specific registers. Schedule of the processor is depicted in Table 2. At the first half of CO cycle (ϕ_1), r_2w_1 and $w_2(k+1)$ are computed and then $c_1=r_1- r_2w_1$ is computed in the second half of C0 (ϕ_2). This is followed by r_1w_2 and $w_1(k+1)$ computation for the next stage and $c_2=r_2- r_1w_2$ for the current stage. Processor takes two cycles to compute c_1 and c_2. Since processor operates two times the data rate, it can operate in real time producing outputs at the data rate.

Cycle	Mult. o/p	Accumulator o/p	
		ϕ_1	ϕ_2
C0	$r_2 w_1$	$w_2(k+1)= w_2(k)\pm 2\mu$	$c_1=r_1- r_2 w_1$
C1	$r_1 w_2$	$w_1(k+1)= w_1(k) \pm 2\mu$	$c_2=r_2-r_1 w_2$

Table 2 2-cycle architecture schedule

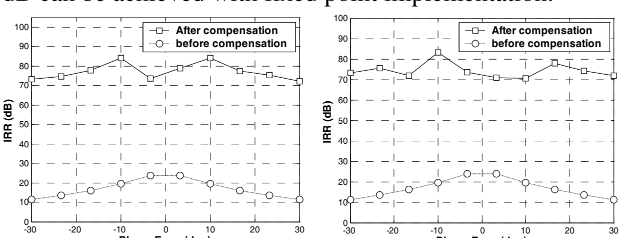
Simulation results depicted on Figure 7 show the mean IRR than can be achieved as a function of coefficient wordlength. I/O wordlength is chosen to be 16-bits 2's complement sign-fraction number. Coefficient wordlength less then 13 caused the sign-sign LMS algorithm to diverge. Once again phase and gain errors are randomly distributed between 0 – 30° and 1dB – 3 dB



(a) 256-QAM, SNR=30 dB (b) 32-PSK, SNR=26.1 dB

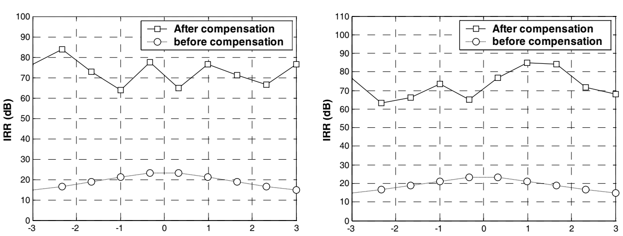
Figure 7 sign-sign LMS performance estimation of bit precision for coefficient.

Figures 8 and 9 depict the performance of the fixed-point sign-sign LMS algorithm for varying phase and gain errors respectively. As can be observed mean IRR of 75 dB can be achieved with fixed point implementation.



(a) 256-QAM, SNR=30 dB (b) 32-PSK, SNR=26.1 dB

Figure 8 IRR before and after compensation for varying phase error -30 – 30° and fixed gain error of 3 dB.



(a) 256-QAM, SNR=30 dB (b) 32-PSK, SNR=26.1 dB

Figure 9 IRR before and after compensation for varying gain error -3 – 3 dB and fixed phase error or 30°.

5. CONCLUDING REMARKS

In this paper we compared different low-hardware-complexity LMS algorithms for adaptive I/Q correction in quadrature receivers. According to results, sign-sign LMS algorithm is best suited for the required application. Fixed point design of the sign-sign LMS algorithm is carried out and optimum wordlengths established. Time-multiplex low-complexity 2-cycle architecture that operates twice the data rate is proposed. This is followed by extensive simulation studies to establish the performance of the fixed point implementation. Different modulation formats and constellation sizes are used in the process.

In normal LMS implementation, multipliers occupy large portion of the coefficient update. The sign-sign LMS approach eliminates the multipliers in the coefficient update attaining low hardware complexity and low power dissipation. Simulation results show that fixed-point sign-sign LMS satisfy the performance and low hardware complexity requirements for our application. In conclusion, sign-sign LMS algorithm is the optimum solution for adaptive I/Q mismatch correction having low hardware complexity suitable for any modulation format and constellation sizes, large and small. Next phase of the work is to prototype this architecture on FPGA board and run it in real-time and compare the performance that can be achieved with the simulated ones.

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