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Modelling and Forecasting International Interest Rate Spreads: UK, Germany, Japan and the US

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Abstract

The interest rate spread is of importance to policy makers and finance professionals in asset allocation and is a common measure of financial market stress. In this paper we model and forecast the interest rate spreads for a number of countries using two well known continuous time models and discrete time ARMA and ARFIMA models. We use monthly and weekly data which covers the recent global financial market crisis of 2007-2009 for Germany, Japan, UK and the US. We find that the continuous time Merton model outperforms all other model specifications in terms of the mean of the forecast errors, MAPE and RMSE.

KEYWORDS: Continuous Time, Gaussian Estimation, ARMA, ARIMA, ARFIMA, Time Series, Spreads, Forecasting, International.

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1 Introduction

The modelling and forecasting of interest rate spreads, or what is commonly referred to as the term spread or yield curve slope, is of importance for central bankers and other market practitioners. Interest rate spreads are simply the difference between long term and short term rates. Interestingly a change from a positive to a negative interest rate spread would indicate that a recession is likely in the future (see below for a more detailed discussion on this). Furthermore, the interest rate spread can be used to forecast future short-term interest rates, where Campbell and Schiller (1987), Fama (1984, 1990) and Hardouvelis (1988), amongst many others, document that interest rate spreads can predict the correct direction of future changes in short rates. Finally, interest rate spreads are of particular use to traders in international financial markets who trade the slope of the yield curve and fund managers who use this for asset allocation purposes.

The past twenty years has seen the emergence of the use of the term spread ability to predict output growth and recessions as an important area of research, where Wheelock and Wohar (2009) provide an excellent recent survey of these literature. Studies on predicting output growth for example include, Laurent (1988), Harvey (1988, 1989), Estrella and Hardouvelis (1991) provided some of the first empirical evidence on how interest rate spreads could be used to predict output growth in the US. This relation was confirmed by Shaaf (2000), Ang, Piazzesi, and Wei (2006), Aretz and Peel (2008) and Bordo and Haubrich (2008), although Bordo and Haubrich reported that spreads only improved forecast in three of the nine sub-periods studied. Looking at this relation in a more international context, Estrella, Rogrigues and Schich (2003), Duarte, Venetis and Paya (2005) and Nakaoto (2005) found similar results in Germany, the Euro area and Japan, respectively.

Looking at a different application of interest rate spreads, Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) demonstrated that the interest rate spread significantly outperforms other financial and macroeconomic variables in forecasting recessions in the US. This relation was again confirmed by Galvao (2006) and Rosenberg and Maurer (2008), while Bernard and Gerlach (1998) found a similar relation in eight industrialised countries. In Ivanova, Lahiri and Seitz (2000) they investigated the use of interest rate spreads as predictors of German inflation and business cycles. Interestingly, there is a relative dearth in the literature as to the best approach to model and forecast the interest rate spread itself. As a result, and to address this issue in this paper we look at a different aspect on spreads by modelling and forecasting international spreads using two different econometric modelling methodologies, one in continuous time and one in discrete time¹.

The international markets we consider are the UK, US, Japan and Germany. In particular for the UK market or the Gilt market (gilt-edged) is where Treasury bills and bonds are regularly auctioned by the UK Debt Management Office (DMO) on behalf of HM Treasury. A review of the market is given in DMO (2012). In the US market the securities issued are termed Treasury bills, notes and bonds issued by the US Treasury Department (see, for example, Treasury Bulletin (2013)). The Japanese Ministry of Finance (MOF) sells various kinds of Japanese Government Bonds (JGB) to the market over the short-medium and long term. Short term are Treasury bills, medium-term usually about 5 years and long term 10 years and over. An excellent overview is given in Yoshino (2008). Lastly the market for German government securities which are issued by the Finance Agency is divided into the short end market and capital market instruments from three months to 30 years. In the short end issue we have Treasury bills called “Bubills”. The other end of the market are Federal Treasury notes “Schaetze” with a maturity of two years, followed by five-year Federal notes “Bobls”.

¹ Other approaches to modelling include, for example, discrete time cointegration analysis and the use of vector Autoregressive (VAR) or Factor (or Fractionally Integrating Factor) Vector Autoregressive models (FI-F-VAR) models.

Lastly at the long end are Federal bonds “Bunds” with maturities of ten and 30 years. For Germany, Japan and the US we use as a proxy for the short end the 1-month interbank offered rate from the British Banking Association. The interbank rate represents the rate charged on short-term loans made between banks.

The modelling in continuous time in economics and finance has a long history. Bergstrom and Nowman (2007) provide a number of advantages of modelling in continuous time. The major problem with a continuous time model is that the data are only available at discrete time intervals. Thus a discretization of the continuous time model is needed. In this paper we use the discrete time model of Nowman (1997) (see Bergstrom and Nowman (2007)) for the estimation of the continuous time models. To estimate the parameters we use the general Gaussian estimation methods of Bergstrom (1983, 1990), Nowman (1997, 2001, 2003, 2006), Bergstrom and Nowman (2007) to estimate the well known continuous time models of Merton (1973) and Vasicek (1977).

We compare the forecasting performance of these models with the Autoregressive Integrated Moving Average (ARIMA) modelling of Box and Jenkins (1976) as well as the Autoregressive Fractionally Integrated Moving Average (ARFIMA) models, first proposed by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981), which have been developed and applied in forecasting time series. There is evidence in the literature of long memory in interest rates, where Shea (1991) found some evidence of long memory in interest rate spreads and Backus and Zin (1993) found evidence of long memory in interest rates using various time series. Supporting this argument, Crato and Rothman (1994) found similar results for bond yields when using the full maximum likelihood to estimate an ARFIMA(0;d;1) model.

The rest of the paper is organized as follows: Section 2 outlines the continuous time models and discrete time models used. Section 3 presents the data and empirical results are given in

Section 4. Forecasting performance is discussed in Section 5 and conclusions are presented in Section 6.

2 Continuous and discrete time spread modelling

We use the important continuous time interest rate models of Merton (1973) and Vasicek (1977) which have been widely used in financial markets in modelling the yield curve, bond pricing and options on bonds. The Vasicek (1977) model has the advantage of allowing for closed form solution of bond prices. It also incorporates the idea of mean reversion which incorporates the idea that when the interest rate is above its long run average it is pulled down and pulled upward when it is below the long run average. This is a common realistic assumption for the behaviour of interest rates and it is expected that interest rate spreads may also be pulled towards long run equilibrium. We will test in the empirical analysis both the drift element and mean reversion of interest rate spreads by the Vasicek model in the different international markets. Though we concentrate on the single factor case in this study other single factor models could also be used, for example, Cox, Ingersoll and Ross (1985) and Brennan and Schwartz (1980). Indeed multifactor models like the Generalized Vasicek model in Babbs and Nowman (1999) could also be used.

The models are presented below:

$$dr(t) = \alpha dt + \sigma \zeta(dt) \quad (t \geq 0) \quad (1)$$

$$dr(t) = \{\alpha + \beta r(t)\} dt + \sigma \zeta(dt) \quad (t \geq 0)$$

where $\{r(t), t > 0\}$ is the dependent variable the spread, α and β are the unknown drift and mean reversion parameters; σ is the volatility; $\zeta(dt)$ is a white noise error term precisely defined by Assumption below:

Assumption: ζ is a random measure defined on all subsets of the half line $0 < t < \infty$ with finite Lebesgue measure such that: $E[\zeta(dt)] = 0$ and $E[\zeta(dt)^2] = dt$ and

$E[\zeta(\Delta_1)\zeta(\Delta_2)]=0$ for any disjoint sets Δ_1 and Δ_2 on the half line $0 < t < \infty$. See Bergstrom (1984) for a discussion of random measures in econometrics.

To estimate the parameters using the discrete data we use the discrete model derived in Nowman (1997) for the estimation of continuous time models.

$$r(t) = e^\beta r(t-1) + \frac{\alpha}{\beta}(e^\beta - 1) + \eta_t \quad (t=1, 2, \dots, T), \quad (2)$$

$$E(\eta_s \eta_t) = 0 \quad (s \neq t), \quad (3)$$

$$E(\eta_t^2) = \frac{\sigma^2}{2\beta}(e^{2\beta} - 1) = m_t^2. \quad (4)$$

The Gaussian estimates are then obtained from the Gaussian likelihood function where $\theta = [\alpha, \beta, \sigma^2]$ and $L(\theta)$ is defined below (see Nowman (1997, Eq.7))

$$L(\theta) = \sum_{t=1}^T \left[2 \log m_t + \frac{\left\{ r(t) - e^\beta r(t-1) - \frac{\alpha}{\beta}(e^\beta - 1) \right\}^2}{m_t^2} \right], \quad (5)$$

where we can write

$$L(\theta) = \sum_{t=1}^T [2 \log m_t + \varepsilon_t^2], \quad (6)$$

The transformed residuals $\varepsilon_1, \dots, \varepsilon_T$ can be computed from $m_t \varepsilon_t = \eta_t$.

An alternate dimension is added through the estimation of discrete time models, namely the Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA) and Autoregressive Fractionally Integrated Moving Average (ARFIMA) models. These models differ in terms of their underlying assumptions regarding the degree of stationarity of the underlying data series, where the ARMA models assumes that the underlying data series are stationary, the ARIMA model that these are non-stationary, and the ARFIMA models that the data are fractionally integrated.

We begin with the ARMA($p; .q$), where this model will have p autoregressive, or lagged variable, and q moving average, or lagged error, terms, thus implying that both previous observations as well as previous disturbance terms have an influence on the current prevailing observation. The ARMA model is therefore specified as follows:

$$\phi(L) = \mu + \varepsilon_t - \theta(L)\varepsilon_t \quad (7)$$

where $\phi(L)$ and $\theta(L)$ denote the polynomials in the lag operator; hence $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p$, where p denotes the number of autoregressive terms in the model, and $\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \dots - \theta_q L^q$, where q denotes the number of moving average terms in the model. As stated above, one of the underlying assumptions for the ARMA models is that the underlying data series follows a stationary, i.e. $I(0)$, process; therefore, should one apply the ARMA model to a non-stationary data series, the results would be spurious. The reason for this is that, should one try to model a non-stationary data series using this one, when the dependent and independent variables are related over time, then one could find that the model has a high measure of fit, even if the two variables are completely unrelated.

The discrete time analysis continues with the ARIMA($p; .d; .q$) model, developed by Box and Jenkins (1976), which provides a contrast to the ARMA model by assuming that the underlying data series follows a non-stationary process. Once again this has p autoregressive and q moving average terms, as was the case of the ARMA model; however, this model extends the ARMA model in that it also has a d component, where this measures the number of times that the underlying data series has to be differenced in order to make the process stationary, where $d \geq 1$ and an integer. The ARIMA model is therefore specified as:

$$\phi(L) \left[(1-L)^d y_t \right] = \mu + \theta(L)\varepsilon_t \quad (8)$$

where $\phi(L)$ and $\theta(L)$ denote the polynomials in the lag operator, as described in the note to Expression (7) above; p and q denote the number of autoregressive and moving average terms, respectively; and $(1-L)^d = \Delta^d y_t$ is the d th difference of y_t .

The final alternative model is the ARFIMA($p;d;q$), first introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981), where the assumption is made that the underlying data series follow a mean reverting process, however, the Wold decomposition and the autocorrelation coefficients for this process will exhibit a very slow hyperbolic rate of decay, where, the higher the value of d , the slower the rate of decay. As was the case for the ARMA and ARIMA models, this model has p autoregressive and q moving average terms. Furthermore, like the ARIMA model, it also has a d component, where this measures the number of times that the underlying data series has to be differenced in order to make the process stationary, however, in this case $0 < d < 1$. The ARFIMA model parameterises the conditional mean of the series generating process as an ARFIMA($p;d;q$) process, which is specified as follows:

$$\phi(L)(1-L)^d (y_t - \mu) = \theta(L)\varepsilon_t \quad (9)$$

where $\phi(L)$ and $\theta(L)$ denote the polynomials in the lag operator, as described in the note to Expression (9) above, where all the roots of $\phi(L)$ and $\theta(L)$ lie outside the unit root circle; p and q denote the number of autoregressive and moving average terms, respectively; d denotes the fractional differencing parameter; and ε_t is white noise. This model is estimated using the Maximum Likelihood Estimation (MLE) method outlined in Sowell (1986, 1992), where the proposed log-likelihood function is then:

$$\ell = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \log|\Omega| - \frac{1}{2} Y' \Omega^{-1} Y \quad (10)$$

where $\{\Omega\}_{i,j} = \gamma_{|i-j|}$, where γ denotes the autocovariances of the ARFIMA process, and Y represents a T -dimensional vector of the observation on the process y_t . One should note that the Wold decomposition and the autocorrelation coefficients for this process will exhibit a very slow rate of decay, where the higher the value of d , the slower the decay. Furthermore, in the case of the first-difference, of the series, where $-0.5 < d < 0.5$, the process is covariance stationary, while, should $(0.5 < d < 1)$, the process would be fractionally integrated. This being said, as long as $d < 1$, the process will exhibit mean-reversion.

3 Data

The data sets used in this study for the UK comprise the monthly 15, 20 and 25-year Treasury yield spreads, relative to the 1-month yield obtained from the Bank of England². The monthly data covers the period January 1998 to December 2010. For Germany, Japan and the US weekly data on the 10-year Treasury spreads relative to the 1-month interbank offered rate over the period January 2000 to December 2010 are used³. The interbank short rate is published by the British Bankers Association (BBA) obtained from Datastream and is used as a proxy at the short end of the curve. The other bond yields were obtained from Datastream. Figures 1 to 3 provide graphs of the spreads and the change in the spreads for each data set respectively.

[Insert Figures 1-3 about here]

[Insert Tables 1-3 about here]

Table 1 and Table 2 provide summary statistics of both the levels and first-differences of the spreads, respectively. The average spread for the UK data varies from 0.233% to 0.397%, with standard deviations ranging from 1.647% to 1.818%. An interesting observation is that

² We use monthly data for the UK as the data is only available daily and monthly.

³ Many different types of spreads can be used. Smith (2012) looks at the spread between LIBOR and OIS rates, the LOIS spread. One could also use both uncollateralized and collateralized rates (repos) or even swap rates (e.g. EONIA). One could also model different interest rate spreads like only the very short rate (1 week) interest rate spreads.

the average value of the spread decreases while the standard deviation of the spread increases with the maturity of the spread, hence the 15-year spread is observed to have a higher average and lower standard deviation than the 25-year spread. For Germany the average spread is 1.202% with standard deviation of 0.926%, for Japan the average spread is 1.178% with standard deviation of 0.315% and for the US the average spread is 1.448% with standard deviation of 1.505%. For UK spreads it was found they exhibit significant autocorrelation across all maturities and lags. Furthermore, the US spread were found to exhibit significant autocorrelation up to the sixth lag, with mixed results for the Japanese spread and the German spread exhibiting no significant autocorrelation, with the possible exception of the first lag.

Having outlined the characteristics of the data, standard unit roots tests were performed on the respective spreads in order to determine the order of integration and thereby provide a preliminary indication of the most appropriate discrete time model, where the results of these tests are presented in Table 3. The results from the Augmented Dickey-Fuller (Dickey and Fuller (1981)) and Phillips-Perron (Phillips and Perron (1988)) unit root tests are unanimous in indicating that the levels of spreads are non-stationary, with the possible exception of the Japanese spread, where these are found to be significantly stationary at a 5% level of significance. When examining the results from the KPSS unit root tests (see Kwiatkowski, *et al.* (1992)), the results are somewhat more mixed. These results indicate that when examining UK spreads, these were found to only be non-stationary at the 5% and 10% levels of significance. Examining the results for the international spreads, both the German and Japanese spreads are found to be stationary, while the Japanese spread is only non-stationary at the 5% and 10% levels of significance.⁴ The fact that the results of these unit root tests are mixed gives a preliminary indication that interest rates spreads may follow a fractionally

⁴ One should note that the first-difference of all interest rate spreads is found to be stationary.

integrated process in that this lies somewhere in between the arguments of stationarity and non-stationarity.

4 Empirical results

The empirical results for the continuous time models of Merton (1973) and Vasicek (1977) are presented in Table 4⁵. For the UK data the Merton model drift is significant across the curve, while the Vasicek model does not display any evidence of mean revision. Turning to Germany for the Vasicek model the drift and mean revision are significant. For Japan in the Merton and Vasicek model the drift is significant but no evidence of mean revision in the Vasicek model. Lastly for the US in the Vasicek model the drift and mean revision are significant.

Having outlined the results from the continuous time estimation, we now move on to examine those from the discrete time models. Given the mixed unit root test results for these data series, as discussed in Section 3, $ARMA(0;0)$ through $ARMA(3;3)$ models were estimated using UK and international spreads data, where the best specification for each data series is selected on the basis of the log-likelihood value, Akaike Information Criterion (AIC) (see Akaike (1974)) and the Schwartz-Bayesian Information Criterion (SBIC) (see Schwartz (1978)). The results from these best specifications are presented in Table 5, where the best model is found to be an $ARMA(1;0)$ across all data series. Therefore one can conclude that UK and international interest rate spreads from the previous period will have a significant impact on current spreads.

[Insert Table 5 about here]

Given the fact that the unit root tests presented in Section 3 provided mixed evidence of non-stationarity for the UK monthly and international spreads, $ARIMA(0;1;0)$ through

⁵ All estimates reported in the tables are based on annualized interest rate data.

ARIMA(3.;1.;3) models were estimated across all data series, where once again the best model is selected on the basis of the log-likelihood, AIC and SBIC measures. The results from these models are presented in Table 6. The best models for the UK spreads, presented in Panel A, are found to be either an ARIMA(1.;1.;0) for the 20-year spread, or an ARIMA(1.;1.;1), for the 15 and 25-year spreads. When looking at the international spreads, presented in Panel B, the best specification for the German 10-year spread was found to be an ARIMA(0.;1.;1) and for the US 10-year spread an ARIMA(1.;1.;0). None of the specifications for the Japanese 10-year spread are found to be significant at any conventional level of significance. One can therefore conclude, that with the exception of the German and Japanese spreads, there appears to be a significant autoregressive component in the first-difference of interest rate spreads, hence changes in the spreads during the previous period are found to have a significant impact on the current changes in the spreads. Furthermore, with the exception of the UK monthly 20-year spread and the Japanese and US spreads, the significant moving average coefficient for the first-difference indicates that shocks experienced during the previous period are found to have a significant impact on the current change in the interest rate spreads.

[Insert Table 6 about here]

As stated previously, the underlying assumption of the ARMA and ARIMA models is that the underlying data series follows either a stationary or non-stationary process, respectively. An interesting approach would be to extend this by arguing the interest rate spreads are fractionally integrated, hence shocks to interest rate spreads will not decay exponentially, as would be the case under the assumption of stationarity, nor would they persist indefinitely, as would be the case under the assumption of non-stationarity, instead, they would decay hyperbolically thereby indicating a delay in the mean reversion process. In

order to investigate this alternate hypothesis, ARFIMA(0.; d .;0) through ARFIMA(3.; d .;3) models are estimated across all data series, where, as was the case previously, the best model is selected on the basis of the log-likelihood, AIC and SBIC measures. The results from these best specifications are presented in Table 7.

[Insert Table 7 about here]

The results indicate that the value for the d -parameter is found to range between 0.620 and 1.007, hence, with the possible exception of Japanese spreads, which appear to exhibit non-stationary behaviour, all interest rate spreads are found to be fractionally integrated and mean reverting. The result for the Japanese spreads is not surprising in Japanese interests rates have remained suppressed since the collapse of Japanese banks in the early 1990s. Moving onto examine the lag structure, the results for the UK spreads are uniform in that prevailing interest rate spreads during the previous two periods are found to have a significant impact on current spreads, while past shocks are found to have no impact at all.

Examining the lag structure results for the international spreads, German 10-year spreads during the previous two periods are found to have a significant impact, while for the US 10-year spread, it is only the previous period that has a significant impact. In both cases past shocks are found to have no significant impact on interest rate spreads. In contradistinction, Japanese 10-year spreads are not only found to exhibit non-stationary behaviour, as discussed above, but these results indicate that while only the previous period has a significant impact on current prevailing spreads, as was the case for the US 10-year spread, however, unlike the other spreads, with exception of the UK 15-year daily spread, shocks over the past three periods are found to have a significant impact on the current prevailing spread.

5 Forecast Performance

Having outlined the in-sample characteristics of the interest spreads, above, we now compare the different methodologies based on their ex-post dynamic forecasts over out-of-

sample periods. Forecasts were performed for each of the models for the UK data over the period January 2011 and April 2011, while for Germany, Japan and the US over the period January 2011 and July 2011. Figures 4 to 6 provide a graphical comparison of the forecasts for each respective model with the actual value at that corresponding period. One interesting observation in the international markets is the forecasting performance of the Vasicek model is less satisfactory compared to the Merton model.

[Insert Figures 4-6 about here]

The forecasts from all models were then compared using the mean and variance of the forecast errors, to test for bias, and using the Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE) and Percentage Correct Direction Predicted (CDIR) forecast metrics, where the last three metrics are calculated as follows:

$$MAPE = 100 \times \left\{ \frac{1}{M} \sum_{i=1}^M \left| \left(\frac{r_i^a - r_i^f}{r_i^a} \right) \right| \right\} \quad (11)$$

$$RMSE = \sqrt{\frac{1}{M} \sum_{i=1}^M (r_i^a - r_i^f)^2} \quad (12)$$

$$CDIR = \frac{1}{M} \sum_{i=1}^M z_i \quad (13)$$

where r_i^a denotes the actual observed value at time i , r_i^f denotes the forecasted value at time i , M denotes the forecast horizon, and $z_i = 1$ if $(r_i^a - r_{i-1}^a)$ and $(r_i^f - r_{i-1}^a) > 0$ and $z_i = 0$.

The rationale behind the selection of these three forecast metrics is that while the RMSE provides a quadratic loss function, which would be useful if we were to consider large forecast errors as egregious, this would be viewed as a disadvantage if this was not the case. Dielman (1986) proposes that, in cases where there are outliers, the MAPE should be used, with Makridakis (1993) argue that the MAPE incorporates the best characteristics among the

various measures of forecast accuracy. This being said, in practical terms, the fact that forecast errors are minimised might not actually provide any guidance as to the potential profitability of employing the respective model in a market trading strategy (see Gerlow, et al. (1993) for a more detailed discussion on this matter). For this reason, Refenes (1995) proposed the use of the CDIR as indication of the best model.

[Insert Table 8 about here]

Table 8 presents the forecast metrics for each model and data set, respectively, with some interesting findings. The continuous time Merton model outperforms all other model specifications in terms of the mean of the forecast errors, MAPE and RMSE. In economic terms the RMSE are very small for the Merton model being less than one percent across the international markets. This compares favourably to other studies on forecasting rates (see for example, Nowman (2001), Nowman and Saltoglu (2003), Bergstrom and Nowman (2007)). Examining the variance of the forecast errors, the ARMA model was found to have the lowest variance for all UK spreads and the ARFIMA model for German spreads. For Japanese and US spreads the Merton model performed well. The final forecast metric, i.e. the CDIR, provided somewhat mixed results in that only in the case of the German and US spreads could we determine a clear winner, i.e. the ARMA model. Interestingly, the Vasicek model did not perform as well as expected, although this may be a result of there being insufficient data for the mean reversion characteristics of the interest spreads to be determined. Overall the continuous time Merton model forecast results estimated by the Gaussian estimation methods supports other empirical continuous time models using these methods, for example, Chambers (1993), Bergstrom, Nowman and Wymer (1992) and Bergstrom and Nowman (2007).

6 Conclusions

The study aimed at modelling and forecasting the interest rate spread using continuous time and discrete time modelling approaches. Two well known interest rate models were employed and tested against discrete time ARMA and ARFIMA models. The study looked at four major financial markets the UK, US, Japan and Germany. Using monthly and weekly data which covers the important recent global financial market crisis of 2007-2009 which affected all these markets we provided empirical evidence of the models fit historically. Extensive forecasting performance of the models was carried out. We find that the continuous time modelling methodology in terms of the Merton model using the Gaussian estimation method outperforms all other model specifications in terms of the mean of the forecast errors, MAPE and RMSE. Further research on other international spreads would be useful in the future with different methodologies.

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Figure 1: UK Interest Rate Spreads

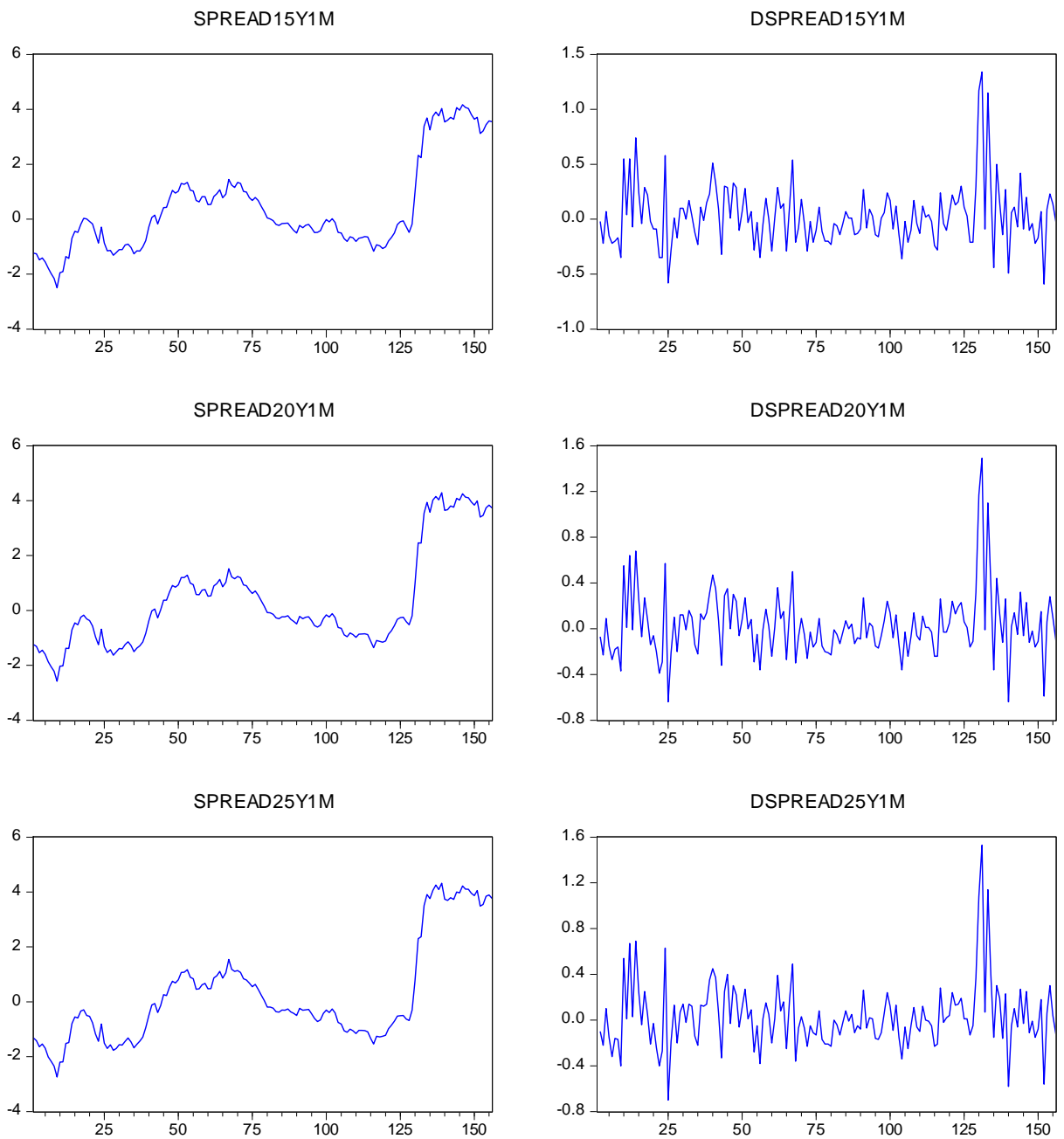


Figure 1: Monthly UK Spreads

Figure 2: US and German Interest Rate Spreads

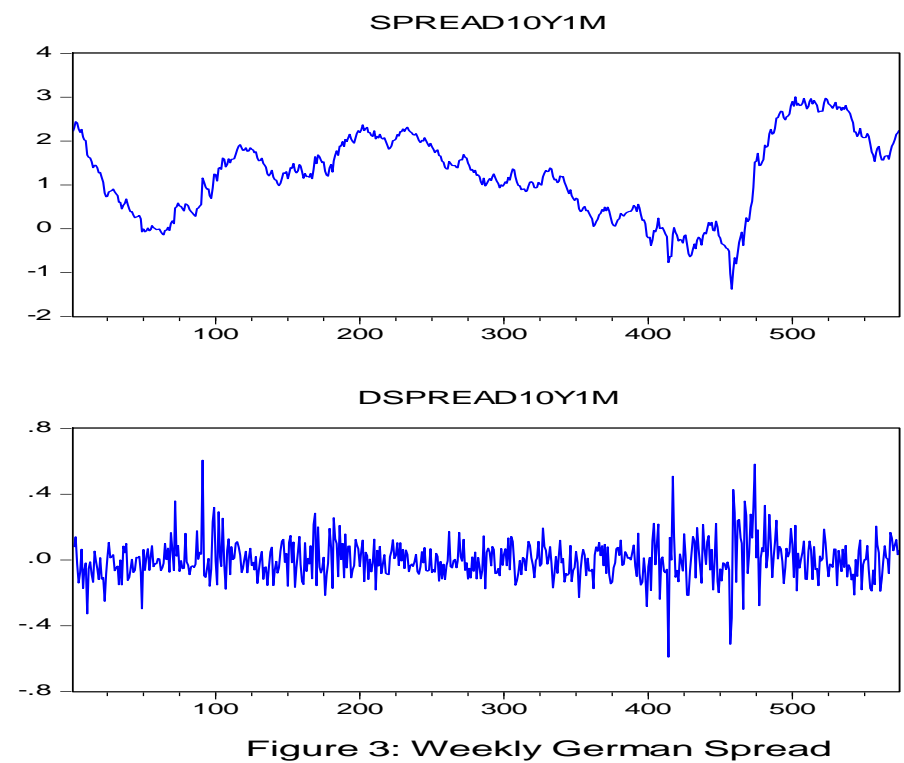
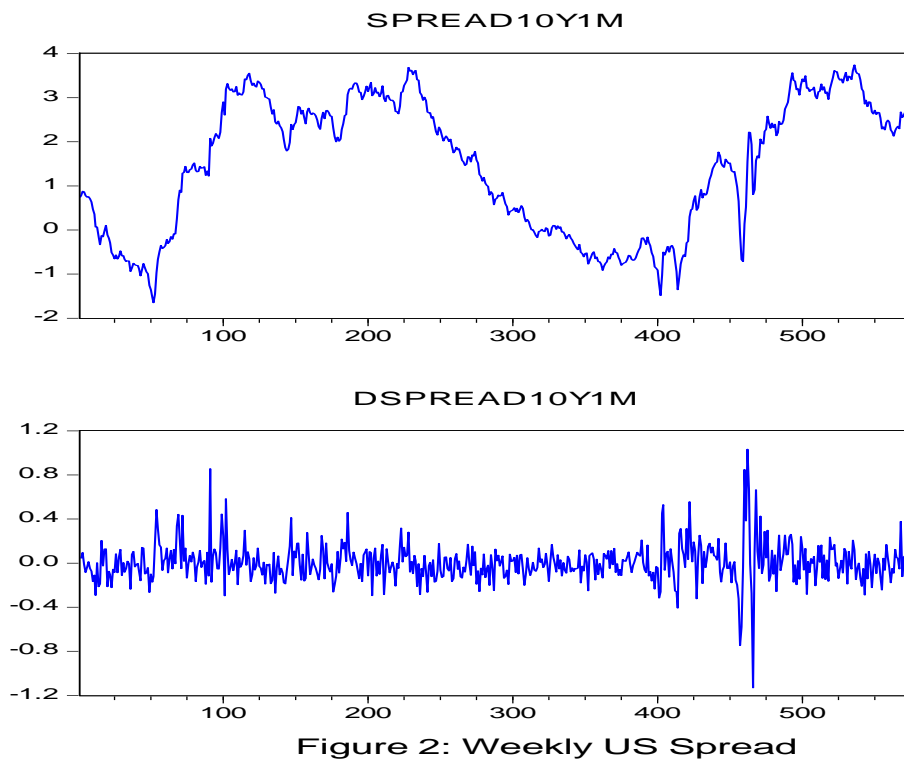


Figure 3: Japan Interest Rate Spread

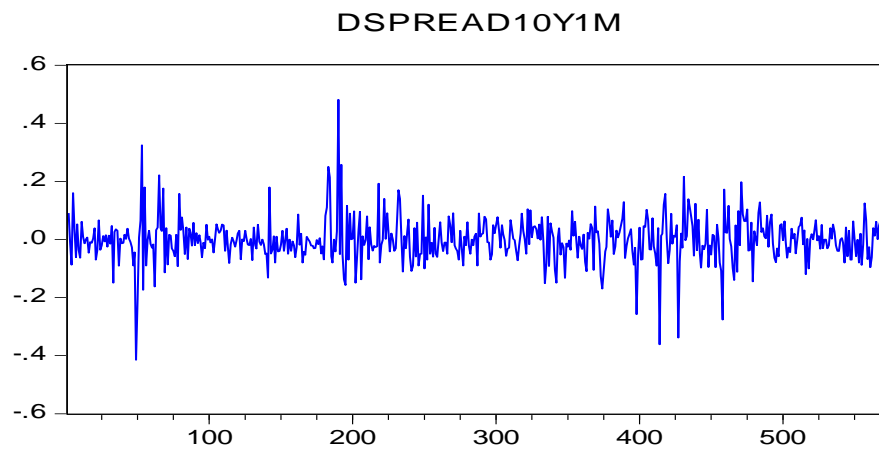
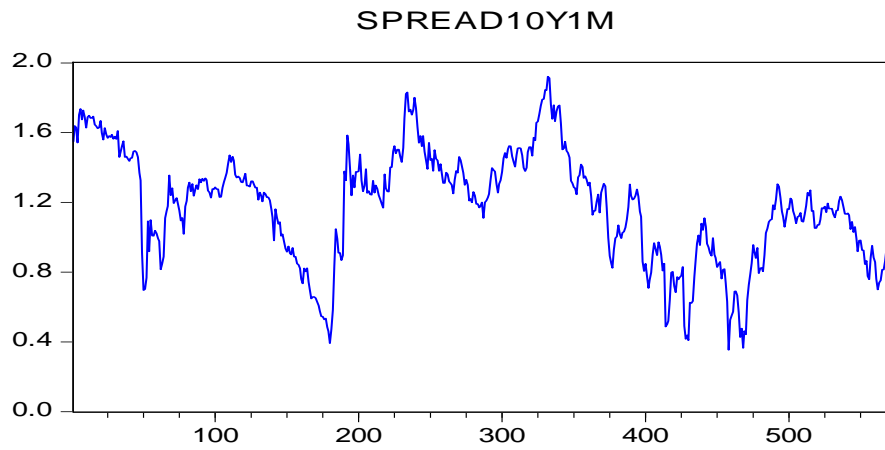
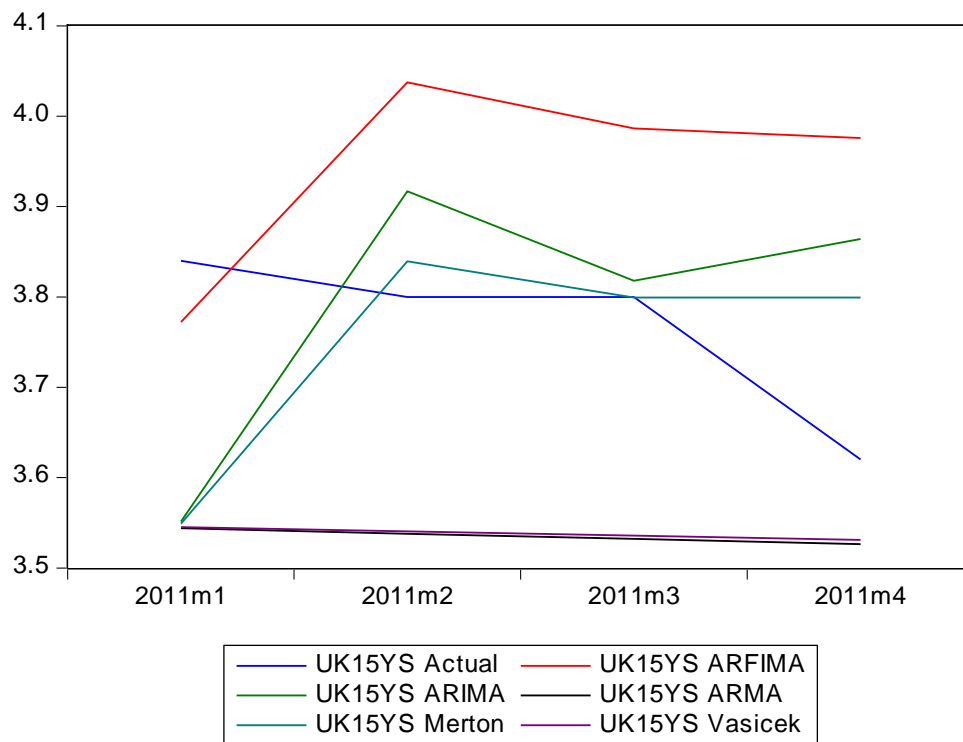
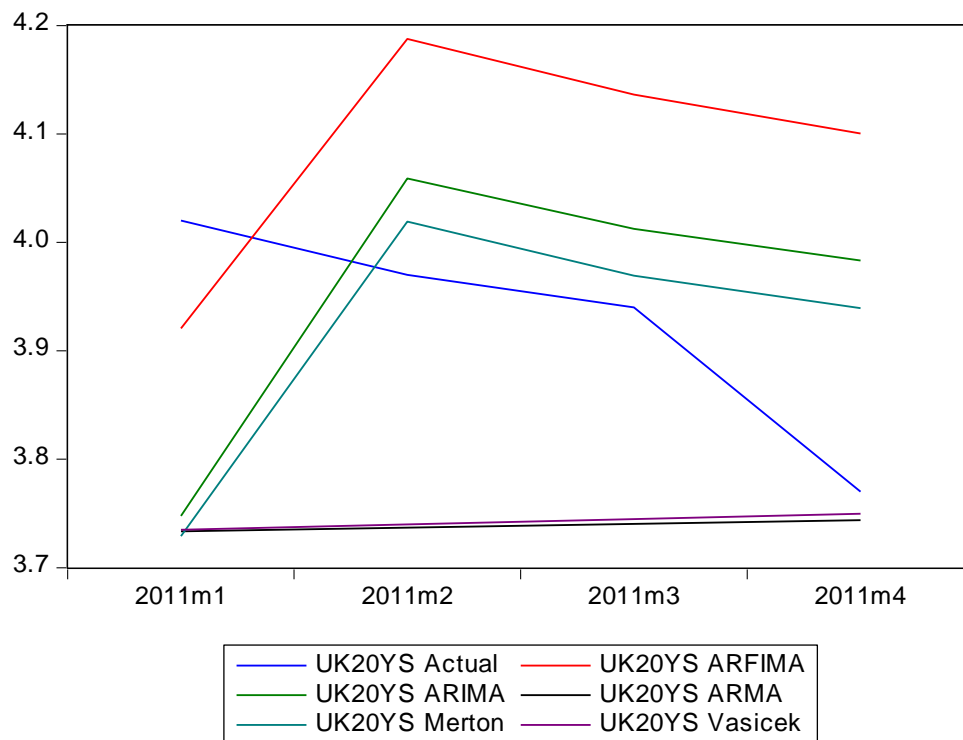


Figure 4: Weekly Japan Spread

Figure 4: Actual vs. Forecasted UK 15-Year and UK 20-Year Spreads

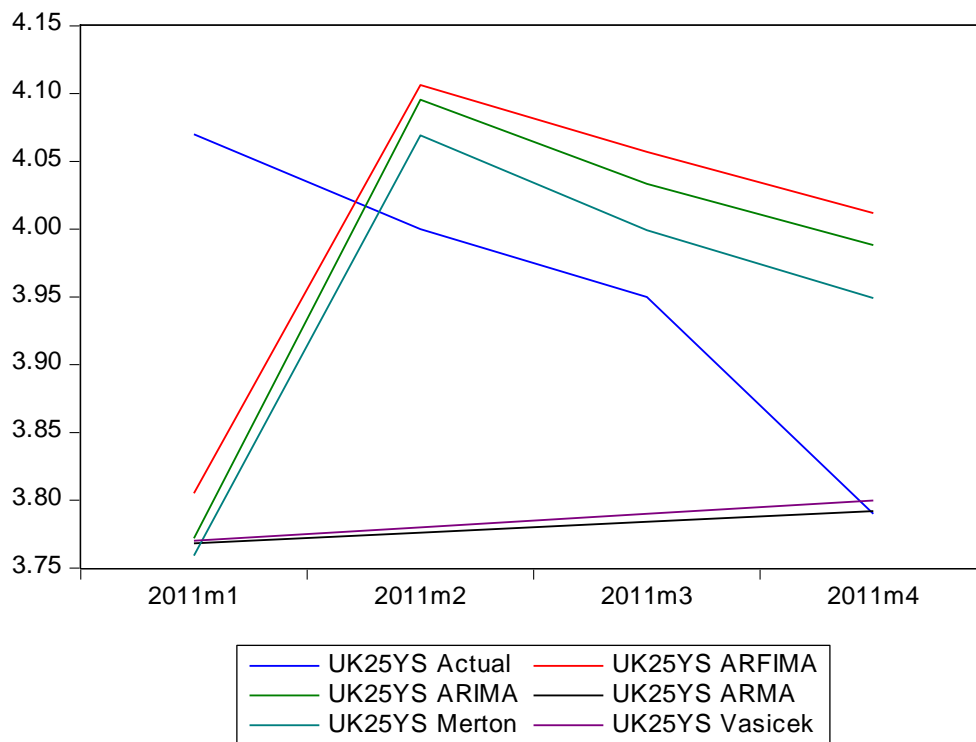


UK 15-Year Treasury Yield Spreads

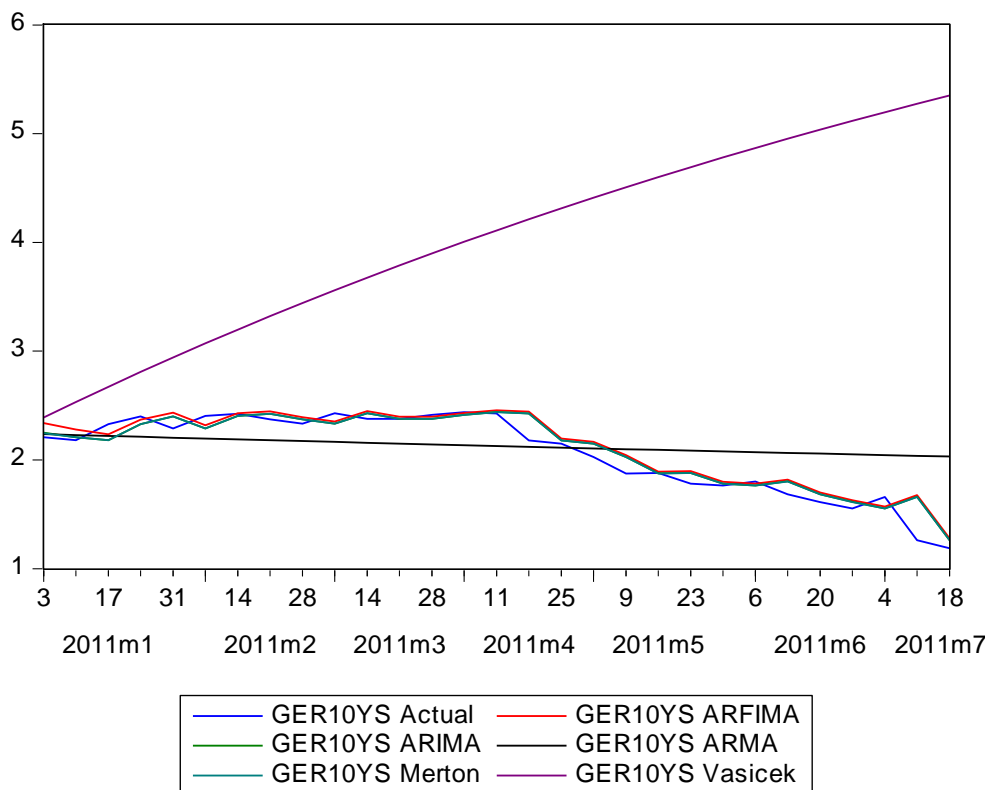


UK 20-Year Treasury Yield Spreads

Figure 5: Actual vs. Forecasted UK 25-Year and German 10-Year Spreads

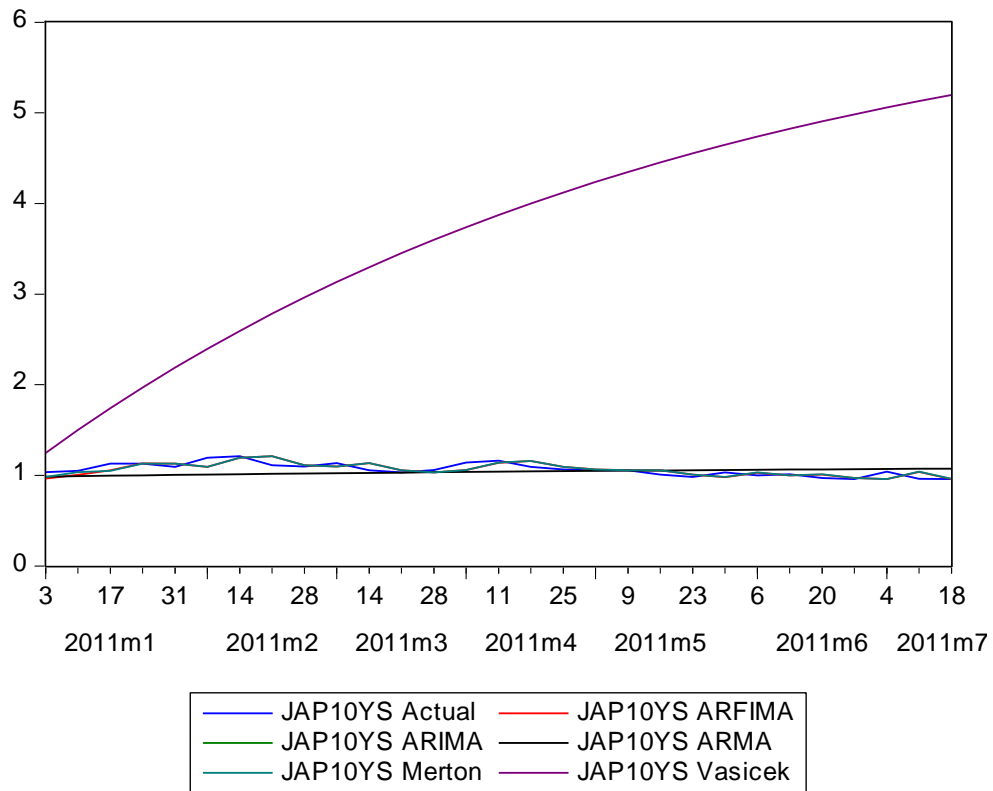


UK 25-Year Treasury Yield Spreads

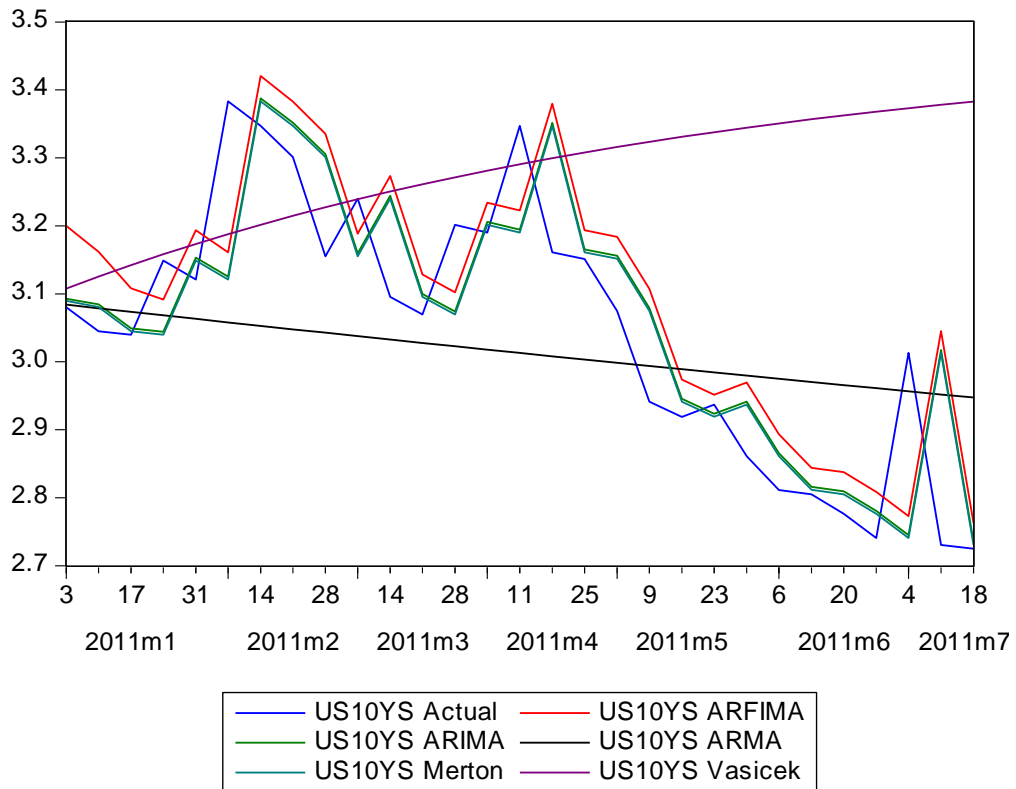


German 10-Year Treasury Yield Spreads

Figure 6: Actual vs. Forecasted Japanese and US 10-Year Spreads



Japanese 10-Year Treasury Yield Spreads



US 10-Year Treasury Yield Spreads

Table 1: Summary Data Statistics for Spread Levels

Panel A - UK Spreads										
	Obs.	Mean	Variance	SD	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$\rho(6)$
UK15YS	156	0.397	2.712	1.647	3.541*** (0.060)	6.008** (0.050)	15.524* (0.001)	15.685* (0.003)	15.695* (0.008)	16.282** (0.012)
UK20YS	156	0.331	3.099	1.760	5.261** (0.022)	9.779* (0.008)	20.295* (0.000)	20.364* (0.000)	20.395* (0.001)	20.537* (0.002)
UK25YS	156	0.233	3.305	1.818	7.179* (0.007)	14.811* (0.001)	25.542* (0.000)	25.714* (0.000)	25.730* (0.000)	25.772* (0.000)
Panel B – International Spreads										
	Obs.	Mean	Variance	SD	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$\rho(6)$
GER10YS	574	1.202	0.857	0.926	3.728*** (0.053)	3.979 (0.137)	5.507 (0.138)	6.541 (0.162)	6.906 (0.228)	7.044 (0.317)
JAP10YS	574	1.178	0.099	0.315	0.514 (0.473)	6.986** (0.030)	10.226** (0.017)	-0.075 (0.161)	28.042* (0.000)	28.043* (0.000)
US10YS	574	1.448	2.264	1.505	25.088* (0.000)	26.866* (0.000)	26.899* (0.000)	32.003* (0.000)	39.823* (0.000)	41.772* (0.000)

Note: This table provides the descriptive statistics for the Treasury yield spreads for the period between January 1998 and December 2010, for the UK spreads, and January 2000 and December 2010, for the international spreads. The UK spreads are calculated as the spread between the 1-month yield and 15-year, 20-year and 25-year yields on UK Treasuries, denoted UK15YS, UK20YS, UK25YS, respectively. The international spreads are calculated as the spread between the 1-month interbank rate and 10-year yield on German, Japanese and US Treasuries, denoted GER10YS, JAP10YS and US10YS, respectively. Figures in parentheses denote the respective p -values and (*, **, ***) denote a coefficient that is significant at the 1%, 5% and 10% levels, respectively.

Table 2: Summary Data Statistics for First-Differences of Spreads

Panel A - UK Spreads										
	Obs.	Mean	Variance	SD	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$\rho(6)$
UK15YS	155	0.031	0.080	0.282	0.122 (0.727)	0.815 (0.665)	9.920** (0.019)	10.779** (0.029)	10.811*** (0.055)	11.552*** (0.073)
UK20YS	155	0.032	0.082	0.286	0.247 (0.619)	1.642 (0.440)	11.064** (0.011)	12.063** (0.017)	12.090** (0.034)	12.299*** (0.056)
UK25YS	155	0.033	0.081	0.285	0.460 (0.498)	3.201 (0.202)	11.109** (0.011)	11.223** (0.024)	11.300** (0.046)	11.444*** (0.076)
Panel B – International Spreads										
	Obs.	Mean	Variance	SD	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$\rho(6)$
GER10YS	573	0.000	0.015	0.121	0.001 (0.981)	0.584 (0.747)	2.110 (0.550)	3.083 (0.544)	3.643 (0.602)	3.777 (0.707)
JAP10YS	573	-0.001	0.006	0.077	0.005 (0.942)	6.992** (0.030)	10.512** (0.015)	25.028* (0.000)	27.313* (0.000)	27.316* (0.000)
US10YS	573	0.004	0.030	0.174	0.006 (0.939)	0.162 (0.922)	0.162 (0.983)	3.470 (0.483)	8.568 (0.128)	9.869 (0.130)

Note: This table provides the descriptive statistics for the change in the Treasury yield spreads for the period between January 1998 and December 2010, for the UK spreads, and January 2000 and December 2010, for the international spreads. The UK spreads are calculated as the spread between the 1-month yield and 15-year, 20-year and 25-year yields on UK Treasuries, denoted UK15YS, UK20YS, UK25YS, respectively. The international spreads are calculated as the spread between the 1-month interbank rate and 10-year yield on German, Japanese and US Treasuries, denoted GER10YS, JAP10YS and US10YS, respectively. Figures in parentheses denote the respective p -values and (*, **, ***) denote a coefficient that is significant at the 1%, 5% and 10% levels, respectively.

Table 3: Unit Root Tests on Spread Levels

Panel A - UK Spreads			
	UK15YS	US20YS	UK25YS
Augmented Dickey-Fuller Test Statistic	-1.407 (0.578)	-1.425 (0.569)	-1.482 (0.540)
Phillips-Perron Test Statistic	-0.917 (0.781)	-0.844 (0.804)	-0.858 (0.799)
KPSS Test Statistic	0.670**	0.680**	0.674**
Panel B – International Spreads			
	GER10YS	JAP10YS	US10YS
Augmented Dickey-Fuller Test Statistic	-1.554 (0.506)	-2.881** (0.048)	-1.563 (0.501)
Phillips-Perron Test Statistic	-1.699 (0.432)	-3.003** (0.035)	-1.388 (0.589)
KPSS Test Statistic	0.239	0.657**	0.322

Note 1: This table presents the results of the Augmented Dickey-Fuller (Dickey and Fuller (1981)), Phillips-Perron (Phillips and Perron (1988)) and KPSS (Kwiatkowski, *et al.* (1992)) unit root tests on the Treasury yield spreads for the period between January 1998 and December 2010, for the UK spreads, and January 2000 and December 2010, for the international spreads. The UK spreads are calculated as the spread between the 1-month yield and 15-year, 20-year and 25-year yields on UK Treasuries, denoted UK15YS, UK20YS, UK25YS, respectively. The international spreads are calculated as the spread between the 1-month interbank rate and 10-year yield on German, Japanese and US Treasuries, denoted GER10YS, JAP10YS and US10YS, respectively. Figures in parentheses denote the respective p -values, critical values for the KPSS test are 1%: 0.739, 5%: 0.463 and 10%: 0.347; and (*, **, ***) denote a test coefficient that is significant at the 1%, 5% and 10% levels, respectively.

Note 2: The results of the same unit root tests on the changes in interest rate spreads (the first-difference of the series) are not presented for reasons of brevity, but are available upon request.

Table 4: Continuous Time Results for the UK and International Spreads

Panel A – UK Spreads			
	UK15YS	UK20YS	UK25YS
Merton			
α	-0.0008* (0.0002)	-0.0008* (0.0002)	-0.0008* (0.0002)
σ^2	0.00001 (0.00001)	0.00001 (0.00001)	0.00001 (0.00001)
Vasicek			
α	0.0003 (0.0002)	0.0003 (0.0002)	0.0003 (0.0002)
β	-0.0098 (0.0157)	-0.0067 (0.0144)	-0.0053 (0.0141)
σ^2	0.00001 (0.00001)	0.00001 (0.00001)	0.00001 (0.00001)
Panel B – International Spreads			
	GER10YS	JAP10YS	US10YS
Merton			
α	0.00001 (0.00005)	-0.0001** (0.00004)	0.00004 (0.00007)
σ^2	0.000001 (0.00001)	0.00001 (0.00001)	0.00001 (0.00001)
Vasicek			
α	0.0020* (0.0002)	0.0032* (0.0004)	0.0017* (0.0002)
β	-0.0234*** (0.0137)	-0.0489 (0.0393)	-0.0489* (0.0087)
σ^2	0.00001 (0.00001)	0.00001 (0.00001)	0.00001 (0.00001)

Note: This table provides the results for the Merton and Vasicek models of the Treasury yield spreads for the period between January 1998 and December 2010, for the UK spreads, and January 2000 and December 2010, for the international spreads. The UK spreads are calculated as the spread between the 1-month yield and 15-year, 20-year and 25-year yields on UK Treasuries, denoted UK15YS, UK20YS, UK25YS, respectively. The international spreads are calculated as the spread between the 1-month interbank rate and 10-year yield on German, Japanese and US Treasuries, denoted GER10YS, JAP10YS and US10YS, respectively. Figures in parentheses denote the respective standard errors. *, **, *** denote a coefficient that is significant at the 1%, 5% and 10% levels, respectively.

Table 5: ARMA Results for the UK and International Spreads

Panel A - UK Spreads			
	UK15YS	UK20YS	UK25YS
α	5.509 (12.482)	9.118 (32.628)	13.717 (71.844)
$\phi(1)$	0.994* (0.014)	0.996* (0.013)	0.998* (0.013)
Log-Likelihood	-23.157	-25.109	-24.910
AIC	-2.526	-2.501	-2.503
SBIC	-2.461	-2.436	-2.438
Panel B – International Spreads			
	GER10YS	JAP10YS	US10YS
α	1.205** (0.595)	1.146* (0.105)	2.150 (1.377)
$\phi(1)$	0.992* (0.005)	0.970* (0.010)	0.994* (0.005)
Log-Likelihood	399.892	664.794	191.132
AIC	-4.230	-5.155	-3.502
SBIC	-4.208	-5.133	-3.479

Note: This table provides the results for the ARMA model of the Treasury yield spreads for the period between January 1998 and December 2010, for the UK spreads, and January 2000 and December 2010, for the international spreads. The UK spreads are calculated as the spread between the 1-month yield and 15-year, 20-year and 25-year yields on UK Treasuries, denoted UK15YS, UK20YS, UK25YS, respectively. The international spreads are calculated as the spread between the 1-month interbank rate and 10-year yield on German, Japanese and US Treasuries, denoted GER10YS, JAP10YS and US10YS, respectively. Figures in parentheses denote the respective standard errors and (*, **, ***) denote a coefficient that is significant at the 1%, 5% and 10% levels, respectively.

Table 6: ARIMA Results for the UK and International Spreads

Panel A - UK Spreads			
	UK15YS	UK20YS	UK25YS
α	0.034 (0.035)	0.033 (0.028)	0.037 (0.040)
$\rho(1)$	0.742* (0.210)	0.183** (0.080)	0.748* (0.157)
$\theta(1)$	-0.599** (0.252)	----- (-----)	-0.551* (0.198)
Log-Likelihood	-20.020	-22.812	-18.309
AIC	-2.558	-2.529	-2.580
SBIC	-2.460	-2.463	-2.482

Panel B – International Spreads			
	GER10YS	JAP10YS	US10YS
α	0.000 (0.005)	-0.001 (0.003)	0.004 (0.009)
$\rho(1)$	----- (-----)	0.270 (0.558)	0.208* (0.041)
$\theta(1)$	0.085** (0.042)	-0.226 (0.565)	----- (-----)
Log-Likelihood	400.635	659.872	202.292
AIC	-4.233	-5.140	-3.542
SBIC	-4.211	-5.107	-3.519

Note 1: This table provides the results for the ARIMA model of the Treasury yield spreads for the period between January 1998 and December 2010, for the UK spreads, and January 2000 and December 2010, for the international spreads. The UK spreads are calculated as the spread between the 1-month yield and 15-year, 20-year and 25-year yields on UK Treasuries, denoted UK15YS, UK20YS, UK25YS, respectively. The international spreads are calculated as the spread between the 1-month interbank rate and 10-year yield on German, Japanese and US Treasuries, denoted GER10YS, JAP10YS and US10YS, respectively. Figures in parentheses denote the respective standard errors and (*, **, ***) denote a coefficient that is significant at the 1%, 5% and 10% levels, respectively.

Note 2: The ARIMA models run on the Japanese spreads were not found to be significant, but are presented for convenience and consistency.

Table 7: ARFIMA Results for the UK and International Spreads

	Panel A - UK Spreads			Panel B - International Spreads		
	UK15YS	UK20YS	UK25YS	GER10YS	JAP10YS	US10YS
d	0.620* (0.066)	0.631* (0.113)	0.822* (0.072)	0.659* (0.052)	1.007* (0.038)	0.652* (0.034)
α	0.117 (0.089)	0.118 (0.095)	0.064 (0.055)	0.017 (0.013)	-0.001 (0.003)	0.032 (0.019)
$\rho(1)$	0.513* (0.079)	0.519* (0.079)	0.350* (0.079)	0.467* (0.041)	0.623* (0.148)	0.623* (0.032)
$\rho(2)$	0.234* (0.076)	0.245* (0.076)	0.244* (0.074)	0.140* (0.035)	----- (-----)	----- (-----)
$\rho(3)$	----- (-----)	----- (-----)	----- (-----)	----- (-----)	----- (-----)	----- (-----)
$\theta(1)$	----- (-----)	----- (-----)	----- (-----)	----- (-----)	-0.630* (0.148)	----- (-----)
$\theta(2)$	----- (-----)	----- (-----)	----- (-----)	----- (-----)	0.143* (0.049)	----- (-----)
$\theta(3)$	----- (-----)	----- (-----)	----- (-----)	----- (-----)	-0.207* (0.041)	----- (-----)
Log-Likelihood	-19.788	-19.762	-18.009	390.929	670.177	196.450
AIC	-2.561	-2.562	-2.584	-4.200	-5.172	-3.520
SBIC	-2.463	-2.463	-2.486	-4.166	-5.117	-3.498

Note 1: This table provides the results for the ARFIMA model of the Treasury yield spreads for the period between January 1998 and December 2010, for the UK spreads, and January 2000 and December 2010, for the international spreads. The UK spreads are calculated as the spread between the 1-month yield and 15-year, 20-year and 25-year yields on UK Treasuries, denoted UK15YS, UK20YS, UK25YS, respectively. The international spreads are calculated as the spread between the 1-month interbank rate and 10-year yield on German, Japanese and US Treasuries, denoted GER10YS, JAP10YS and US10YS, respectively. Figures in parentheses denote the respective standard errors and (*, **, ***) denote a coefficient that is significant at the 1%, 5% and 10% levels, respectively.

Table 8: Forecast Metrics for the UK and International Spreads

Panel A - Mean of Forecast Errors						
	UK15YS	UK20YS	UK25YS	GER10YS	JAP10YS	US10YS
ARMA Model	0.2299	0.1864	0.1725	-0.0673	0.0263	0.0347
ARIMA Model	-0.0225	-0.0256	-0.0198	-0.0370	N/A	-0.0169
ARFIMA Model	-0.1779	-0.1612	-0.0426	-0.0622	0.0009	-0.0523
Merton Model	0.0183	0.0108	0.0083	-0.0364	-0.0007	-0.0126
Vasicek Model	0.2268	0.1826	0.1675	-1.9623	-2.5807	-0.2260

Panel B - Variance of Forecast Errors						
	UK15YS	UK20YS	UK25YS	GER10YS	JAP10YS	US10YS
ARMA Model	0.0085	0.0127	0.0166	0.1047	0.0079	0.0279
ARIMA Model	0.0516	0.0434	0.0476	0.0122	N/A	0.0142
ARFIMA Model	0.0319	0.0336	0.0450	0.0117	0.0027	0.0143
Merton Model	0.0390	0.0387	0.0430	0.0122	0.0026	0.0142
Vasicek Model	0.0087	0.0131	0.0173	1.4880	1.5194	0.0649

Panel C - Mean Absolute Percentage Error						
	UK15YS	UK20YS	UK25YS	GER10YS	JAP10YS	US10YS
ARMA Model	6.060%	4.690%	4.317%	15.093%	7.075%	4.611%
ARIMA Model	4.450%	4.128%	4.263%	4.511%	N/A	2.812%
ARFIMA Model	5.686%	5.425%	4.432%	5.012%	3.916%	3.518%
Merton Model	3.394%	3.426%	3.703%	4.496%	3.780%	2.737%
Vasicek Model	5.977%	4.593%	4.295%	110.552%	248.701%	8.948%

Panel D - Root Mean Squared Error						
	UK15YS	UK20YS	UK25YS	GER10YS	JAP10YS	US10YS
ARMA Model	0.2433	0.2103	0.2054	0.3249	0.0913	0.1678
ARIMA Model	0.1980	0.1822	0.1899	0.1147	N/A	0.1183
ARFIMA Model	0.2357	0.2263	0.1885	0.1233	0.0514	0.1285
Merton Model	0.1719	0.1706	0.1797	0.1146	0.0505	0.1178
Vasicek Model	0.2408	0.2077	0.2025	2.2995	2.8508	0.3372

Panel E - Percentage Correct Direction Predicted						
	UK15YS	UK20YS	UK25YS	GER10YS	JAP10YS	US10YS
ARMA Model	50.000%	25.000%	25.000%	65.517%	44.828%	75.862%
ARIMA Model	25.000%	75.000%	75.000%	48.276%	N/A	48.276%
ARFIMA Model	50.000%	75.000%	75.000%	48.276%	44.828%	48.276%
Merton Model	0.000%	50.000%	50.000%	48.276%	41.379%	48.276%
Vasicek Model	50.000%	25.000%	25.000%	34.483%	44.828%	24.138%

Note 1: This table provides the forecast metrics for the various models employed to forecast the Treasury yield spreads. The forecast period extended from January 2011 till April 2011, for the UK spreads, and from 3 January 2011 till 18 July 2011, for the international spreads.

Note 2: Forecast metrics are not provided for the ARIMA model in the case of the Japanese spread as the results for this model were found to be insignificant in the in-sample period estimations.