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Optimal Capital Income Taxation in a Two Sector Economy

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Abstract:

We extend the celebrated Chamley-Judd result of zero capital income tax and show that the steady state optimal capital income tax is nonzero, in general. In particular, we find that the optimal plan involves zero capital income tax in investment sector and a nonzero capital income tax in consumption sector. In a two sector neoclassical economy, interdependence of labour and capital margins allows the government to choose an optimal policy that involves nonzero tax on capital income. The distortion created by capital income tax in consumption sector can be undone by setting different rates of labour income taxes. The optimal plan thus involves zero capital income tax in both sectors only if optimal labour income taxes are equal. This may not be the optimal policy if marginal disutility of work is different across sectors and/or the social marginal value of capital is different across sectors. The difference in social marginal value of capital can be undone by setting different labour income taxes across sectors. We also show that if the government faces a constraint of keeping same capital and labour income tax rates across sectors, optimal capital income tax is nonzero.

Keywords: Optimal taxation, Ramsey problem, Primal approach, Two-sector model.

***JEL* Codes:** C61, E13, E62, H21.

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Introduction.

In this paper, we show that the optimal capital income tax in a standard two sector neoclassical economy is nonzero, in general. We follow Ramsey's (1927) methodology of optimal taxation and apply Ramsey's idea that consumers and firms react to changes in fiscal policy in a two sector dynamic model of taxation. We are motivated by the celebrated finding of Chamley (1986) and Judd (1985) that in a one sector neoclassical economy with competitive markets, the long run optimal policy involves zero tax on capital income. We examine the strength of this result in a broader class of dynamic general equilibrium models. We contribute by showing that the long run optimal policy involves zero capital income tax in investment sector and a nonzero capital income tax in consumption sector. The distortion created by nonzero capital income tax can be undone by setting different rates of labour income taxes. We find a set of conditions on labour income taxes and preferences for which our model recovers the Chamley-Judd result. The condition on labour income taxes is not confirmed by equilibrium conditions, and the preference restrictions are not general. We therefore conclude that Chamley-Judd result in our setting is a special and not the general case.

The dynamic general equilibrium approach to the optimal taxation problem established in literature follows Ramsey's (1927) seminal paper that formally recognized that consumers and firms react to changes in fiscal policy. Literature on optimal taxation of factor income in dynamic settings, ever since its advancement and sophistication, has established a set of celebrated substantive results. A comprehensive survey is presented in Erosa & Gervais (2001), and in Chari & Kehoe (1999). In the context of standard neoclassical growth model with infinitely-lived individuals, Chamley (1986) and Judd (1985) establish that an optimal income-tax policy entails taxing capital at confiscatory rates in the short run and setting capital income taxes equal to zero in the long run. This result is judicious since a positive tax on the return from today's savings effectively makes consumption next period more expensive relative to consumption in the current period. In an infinitely-lived agent's model, therefore, a positive tax on capital income in the steady state implies that the implicit tax rate of consumption in future has an unbounded increasing trend. This form of tax distortions is inconsistent with commodity tax principle, which is why taxing capital income cannot be optimal.

The current paper approaches the standard Ramsey problem using the *primal* approach in a dynamic general equilibrium set up of a two-sector model economy with demarcated features. The main result of this paper is based on the intuition that in a two sector economy, labour and capital margins are interdependent which is unlike a one sector economy. This interdependence implies that the optimal policy of capital income taxation depends on the optimal policy of labour income taxation. Thus in a two sector economy, the optimal policy may involve nonzero capital income tax rate since the distortion created by this tax can be undone by differential labour income taxes. When new capital is a final good and used in both sectors, the social marginal value of capital in the two sectors are very likely to be different. Additional investment in consumption sector is associated with a social marginal value of capital that is different from the social marginal value of capital in investment sector. The Ramsey planner's optimum satisfies two intertemporal equations for capital accumulation in two sectors; but capital is produced in one of them. The discounted returns from investment in both sectors are therefore equal to the social marginal value of capital in investment sector. We show that this equality is consistent with the general result of nonzero capital income tax rate in consumption sector. We argue that a nonzero capital income tax in consumption sector would not have potential compounding distortion effect, since economic agents have the option of shifting depreciated capital to the sector where its income is untaxed. The nonzero capital income tax in the consumption goods sector becomes, in terms of consequences, a tax which has uniform distortion pattern, similar to a period by period consumption tax, for example.

We find that if the optimal labour income taxes are equal across sectors, one can recover the Chamley-Judd result in our model. This may hold if the marginal disutility of work across sectors is same, implying that the before tax wages are same. Such preference restrictions are not general. Optimal labour income taxes may be equal across sectors if the social marginal value of capital is same across sectors, implying that relative price of consumption and relative price of new capital are equal. Our main result that capital income tax is nonzero in one sector is therefore based on the deviation of one relative price against the other. If relative price of investment goods is different from that of consumption goods, it is possible to tax/subsidize capital income because the distortion caused by the capital income tax might be undone by the relative price difference. If one assumes that the relative prices are equal, this would imply that the before tax rental rates of capital are same across sectors. The decentralized equilibrium is not consistent with this assumption. The nonzero capital income tax in consumption sector is therefore the general result. We consider the case where the government faces an *ex ante* constraint of keeping the two labour income tax rates and the two capital income tax rates equal across sectors. Restricting the government's choice of income taxes *ex ante* triggers an outcome with both nonzero capital income tax rates.

The set of policies which generates allocations that can be implemented as competitive equilibrium, as this paper advocates, prescribes that the optimal steady state capital income tax for capital goods sector is unambiguously zero, but the steady state optimal capital income tax for consumption goods sector is only conditionally zero. The set of conditions for which the celebrated Chamley-Judd result can be established, as characterized in three experiments using variants of utility functions, are neither inferred by the model nor justified by simple intuitions. In general, the steady state optimal capital tax for consumption goods sector can therefore be nonzero.

The Decentralized Economy.

Time is discrete and runs forever. The economy has two production sectors indexed by j, where $j = C, X$ denotes the consumption and investment sector, producing perishable consumption goods and new investment goods, respectively. There is a continua of measure one of identical, infinitely lived households, of identical firms in sector C that own a technology with which consumption goods can be produced, and of identical firms in sector X that own a technology with which new capital goods can be produced. The representative household is endowed with initial capital stock, with the property rights of the firms, and with one unit of time at each period. Firms combine capital and labor, the two factors of production, for final production. All households have identical preferences over intertemporal consumption and work. The representative household derives utility from consumption (c_t) and disutility from work. Working time in sector j is denoted by n_j. Household's preferences for consumption and labor service streams $\{c_t, n_{ct}, n_{xt}\}_{t=0}^{\infty}$, can be defined by the utility function over infinite horizon:

$$\mathbf{U}(c_0, c_1, \ldots\ldots, n_{c0}, n_{c1}, \ldots\ldots, n_{x0}, n_{x1}, \ldots.) = \sum_{t=0}^{\infty} b^t \mathbf{u}(c_t, n_{ct}, n_{xt}) \qquad (1)$$

where the subjective discount rate is b and $b \in (0,1)$. The utility function satisfies regularity conditions. The household purchases new capital goods and rents capital to the firms for one period. Capital decays at the fixed rate $d \in (0,1)$. Firms return the rented capital stock next period net of depreciation d, and pay unit cost of capital employed, equal to r_j. The consumption sector's technology is:

$$c_t + g_t \leq \mathbf{F}^{\mathbf{c}}(k_{ct}, n_{ct}) \tag{2.1}$$

where g_t is exogenously determined government consumption expenditure. The investment sector's technology is:

$$x_{ct} + x_{xt} \leq \mathbf{F}^{x}(k_{xt}, n_{xt}) \tag{2.2}$$

where x_{jt} denotes new investment goods. The technology $\mathbf{F}^{j}(k_j, n_j)$ satisfies standard regularity conditions and exhibits constant returns to scale. The government finances the exogenous stream of consumption expenditures $\{g_t\}_{t=0}^{\infty}$ solely by linearly taxing income from capital and labor employed in both sectors. Throughout the paper, the assumption that the government has access to some commitment device, or a *commitment technology* that allows the government to commit itself once and for all to the sequence of tax rates announced at time *0*, is maintained. The government taxes labor income and capital income at rates t_t^j per unit and q_t^j per unit, respectively. The government runs a balanced budget each period. The government's budget constraint for all time *t* can be written as:

$$g_t = t_t^c w_{ct} n_{ct} + t_t^x w_{xt} n_{xt} + q_t^c r_{ct} k_{ct} + q_t^x r_{xt} k_{xt} \tag{3}$$

The consumption good is the numeraire. Let p_t denote the relative price of a new capital good. The representative household chooses allocations in order to maximize discounted lifetime utility subject to:

$$c_t + p_t[k_{ct+1} + k_{xt+1}] \leq (1 - t_t^c) w_{ct} n_{ct} + (1 - t_t^x) w_{xt} n_{xt} + p_t[k_{ct} R_t^c + k_{xt} R_t^x] \tag{4}$$

where $R_t^j \equiv [p_t^{-1}(1 - q_t^j) r_{jt} + (1 - d)]$. The representative firm in sector j competitively maximizes profits. Competitive pricing ensures that returns are equal to their marginal value products. This implies that the equilibrium factor prices are $r_{ct} = F_{kc}^c(t), w_{ct} = F_{nc}^c(t), r_{xt} = p_t F_{kx}^x(t), w_{xt} = p_t F_{nx}^x(t)$.

Definition (Competitive Equilibrium): A competitive equilibrium is an allocation $(c, g, n_c, n_x, x_c, x_x, k_c, k_x)$, a price system (w_c, w_x, r_c, r_x, p), and a government policy (t^c, t^x, q^c, q^x) such that

(a) Given the price system and the government policy, the allocation $(c, n_c, n_x, x_c, x_x, k_c, k_x)$ solves the representative household's problem;

(b) Given the price system, the allocation (c, g, n_c, k_c) solves the problem of the representative firm in sector *C*;

(c) Given the price system, the allocation (x_c, x_x, n_x, k_x) solves the problem of the representative firm in sector *X*;

(d) The markets clear.

Given the assumption about the utility function, the household's budget constraint is satisfied with equality in equilibrium. The government policy, the household's budget constraint and the two resource constraints imply that the government budget constraint holds in equilibrium. Given total time endowment at each period for the household, define $\Im : \mathrm{R}_+^2 \to \mathrm{R}$ with $\Im$ (strictly) convex, such that the total time allocation constraint can be written as $\Im(n_{ct}, n_{xt}) \leq 1$. For (strict) convexity of the function $\Im : \mathrm{R}_+^2 \to \mathrm{R}$, imposing separability, the household's utility function is (non) linear in labour. Combining the necessary conditions derived from the representative household's problem, the necessary conditions derived from the firms' problems, the resource and time allocation constraints, it can be shown that the (competitive) equilibrium dynamics is characterized by the Transversality conditions together with the following system of equations in the set of unknowns $\{c_t, k_{ct}, k_{xt}, n_{ct}, n_{xt}, r_{ct}, r_{xt}, w_{ct}, w_{xt}, p_t, t_t^c, t_t^x, q_t^c, q_t^x\}$:

$$\Im(n_{ct}, n_{xt}) \leq 1 \tag{5a}$$

$$c_t + \overline{g}_t = \mathbf{F}^c(k_{ct}, n_{ct}) \tag{5b}$$

$$k_{ct+1} + k_{xt+1} = \mathbf{F}^x(k_{xt}, n_{xt}) + (1-d)(k_{ct} + k_{xt}) \tag{5c}$$

$$\mathbf{u}_{nc}(t) = -\mathbf{u}_c(t)(1 - t_t^c)w_{ct} \tag{5d}$$

$$\mathbf{u}_{nx}(t) = -\mathbf{u}_c(t)(1 - t_t^x)w_{xt} \tag{5e}$$

$$\frac{\mathrm{u}_c(t)}{\mathrm{u}_c(t+1)} = \frac{bp_{t+1}}{p_t} R_{t+1}^c \tag{5f}$$

$$\frac{\mathrm{u}_c(t)}{\mathrm{u}_c(t+1)} = \frac{bp_{t+1}}{p_t} R_{t+1}^x \tag{5g}$$

$$r_{ct} = \mathrm{F}_{kc}^c(t) \tag{5h}$$

$$w_{ct} = \mathbf{F}^{c}_{nc}(t) \tag{5i}$$

$$r_{xt} = p_t \, \mathrm{F}^{x}_{kx}(t) \tag{5j}$$

$$w_{xt} = p_t \, \mathbf{F}^{x}_{nx}(t) \tag{5k}$$

Equation (5a) represents the time allocation constraint. Equations (5b) and (5c) represent goods market clearing conditions. The rest of the equations are the set of equilibrium conditions derived from household's and firms' optimization problems. A few observations deserve attention here. Note (5f) and (5g) together imply that after tax returns from capital are equal in a competitive equilibrium, but not the before tax rental rates. Note also that with (5d) and (5e), a non-unitary marginal rate of substitution of labor across sectors would imply that after tax wage rates are not equal in equilibrium.

The Ramsey problem.

We follow the primal approach to the Ramsey problem, in which the government can be thought of as directly choosing a feasible allocation, subject to constraints that ensure the existence of prices and taxes such that the chosen allocation is consistent with the optimization behaviour of household and firms. This approach is similar to those of Lucas & Stokey (1983), Jones, Manuelli and Rossi (1993 & 1997), Chari & Kehoe (1999) and Ljungqvist & Sargent (2000). We introduce a single present-value budget constraint for the household. Note that in equilibrium $R_t \equiv R^c_t = R^x_t$. Consider, therefore, household's time T budget constraint:

$$c_T - (1 - t^{c}_{T}) w_{cT} n_{cT} - (1 - t^{x}_{T}) w_{xT} n_{xT} \le p_T R_T [k_{cT} + k_{xT}] - p_T [k_{cT+1} + k_{xT+1}] \tag{6a}$$

Let $\prod_{s=1}^{0} R_s \equiv 1$ be the numeraire. Divide (6a) by the period T term $p_T \prod_{s=1}^{T} R_s$ and evaluate the resulting expression at time T-1. Then add these two and evaluate the resulting expression at time T-2. Iterating this procedure (and finally adding the time 0 expression) and taking the limit of both sides of the sum as $T \to \infty$ results in the following expression:

$$\sum_{t=0}^{\infty} \frac{c_t - (1 - t^{c}_{t}) w_{ct} n_{ct} - (1 - t^{x}_{t}) w_{xt} n_{xt}}{p_t \prod_{s=1}^{t} R_s} \le R_0 [k_{c0} + k_{x0}] \tag{6b}$$

where $\lim_{t\to\infty} k_{jt+1}\left(\prod_{s=1}^{t} R_s\right)^{-1} = 0$ is already imposed since the present discounted value of the capital stock in sector j, j = C, X, in period t evaluated using period t market prices is asymptotically zero as $t \to \infty$. Expression (6b) is the household's present-value budget constraint, which says that the present value of consumption expenditures net of (net) labor earnings cannot exceed the value of the net initial assets. Assume that (6b) binds, i.e. there are no unused resources in the limit. Define the Arrow-Debreu price, $q_t^o \equiv p_t^{-1}\left(\prod_{s=1}^{t} R_s\right)^{-1}$ such that (6b) becomes:

$$\sum_{t=0}^{\infty} q_t^o c_t = \sum_{t=0}^{\infty} q_t^o (1-t_t^c) w_{ct} n_{ct} + \sum_{t=0}^{\infty} q_t^o (1-t_t^x) w_{xt} n_{xt} + R_0^c k_{c0} + R_0^x k_{x0} \tag{7}$$

with $q_0^o = p_0^{-1}$. The first order conditions from household's utility maximization problem with budget constraint (7) include:

$$q_t^o = \frac{b^t \,\mathrm{u}_c(t)}{p_0 \,\mathrm{u}_c(0)} \tag{8a}$$

$$(1-t_t^j) w_{jt} = \frac{-\mathbf{u}_{nj}(t)}{\mathbf{u}_c(t)} \tag{8b}$$

The formulation of the representative firms' problems is unchanged, implying that the necessary conditions from firms' problem are also unchanged. Use (8) to substitute out prices and taxes in (7) in order to derive:

$$\sum_{t=0}^{\infty} b^t [\mathbf{u}_c(t) c_t + \mathbf{u}_{nc}(t) n_{ct} + \mathbf{u}_{nx}(t) n_{xt}] - p_0 \,\mathbf{u}_c(0)[R_0^c k_{c0} + R_0^x k_{x0}] = 0 \tag{9}$$

With $R_0^c = R_0^x$, the time 0 definition of R_t^j gives:

$$p_0 = \frac{(1-q_0^c)\,\mathrm{F}_{kc}^c(0)}{(1-q_0^x)\,\mathrm{F}_{kx}^x(0)} \tag{10}$$

such that (9) may be rewritten as:

$$\sum_{t=0}^{\infty} b^t[\mathbf{u}_c(t)c_t + \mathbf{u}_{nc}(t)n_{ct} + \mathbf{u}_{nx}(t)n_{xt}] - \Omega(c_0, n_{c0}, n_{x0}, q_0^c, q_0^x) = 0 \tag{11}$$

where $\Omega(c_0, n_{c0}, n_{x0}, q_0^c, q_0^x) \equiv \left[\frac{(1-q_0^c)\mathbf{F}_{kc}^c(0)}{(1-q_0^x)\mathbf{F}_{kx}^x(0)}\right]\mathbf{u}_c(0)[R_0^c k_{c0} + R_0^x k_{x0}]$

Expression (11) is, therefore, the intertemporal constraint that involves only allocations and initial capital income tax rates that can be implemented in a competitive equilibrium, and is known in literature as the implementability constraint of the corresponding Ramsey problem. The Ramsey problem for the government, therefore, is to choose allocations to maximize welfare subject to the two (binding) resource constraints and the implementability constraint. Let $\Phi \geq 0$ be the Lagrange multiplier on (11), and define[2]

$$\mathrm{V}(c_t, n_{ct}, n_{xt}, \Phi) \equiv \mathrm{u}(c_t, n_{ct}, n_{xt}) + \Phi[\mathrm{u}_c(t)c_t + \mathrm{u}_{nc}(t)n_{ct} + \mathrm{u}_{nx}(t)n_{xt}] \tag{12.1}$$

With $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$ as a sequence of Lagrange multipliers on the two resource constraints, for given government revenue target $\bar{g}_t$ and initial capital endowment k_0, the problem is therefore to fix initial capital income tax rates q_0^c and q_0^x and choose allocations to maximize welfare subject to (5b), (5c) and (11). The necessary conditions for an optimum for this problem due to changes in allocations are:

$$c_t : \mathrm{V}_c(t) = c_{1t}, \qquad \forall t \geq 1 \tag{12.2a}$$

$$n_{ct} : \mathbf{V}_{nc}(t) = -c_{1t}\,\mathbf{F}_{nc}^c(t), \qquad \forall t \geq 1 \tag{12.2b}$$

$$n_{xt} : \mathbf{V}_{nx}(t) = -c_{2t}\,\mathbf{F}_{nx}^x(t), \qquad \forall t \geq 1 \tag{12.2c}$$

$$k_{ct+1} : c_{2t} = b[c_{1t+1}\,\mathrm{F}_{kc}^c(t+1) + c_{2t+1}(1-d)], \qquad \forall t \geq 0 \tag{12.2d}$$

$$k_{xt+1} : c_{2t} = b\,c_{2t+1}[\mathrm{F}_{kx}^x(t+1) + (1-d)], \qquad \forall t \geq 0 \tag{12.2e}$$

$$c_0 : \mathrm{V}_c(0) = c_{10} + \Phi\Omega_{c0} \tag{12.2f}$$

$$n_{c0} : \mathrm{V}_{nc}(0) = -c_{10}\,\mathrm{F}_{nc}^c(0) + \Phi\Omega_{nc0} \tag{12.2g}$$

$$n_{x0} : \mathrm{V}_{nx}(0) = -c_{20}\,\mathrm{F}_{nx}^x(0) + \Phi\Omega_{nx0} \tag{12.2h}$$

[2]The following expression (12.1) is commonly referred to as the Pseudo utility function which combines the utility function and the infinite horizon part of the implementability constraint.

Consolidating (12.2) yields the following five equations:

$$\mathbf{V}_c(t)\frac{\mathbf{F}^c_{kc}(t)}{\mathbf{F}^x_{kx}(t)} = b\ \mathbf{V}_c(t+1)\frac{\mathbf{F}^c_{kc}(t+1)}{\mathbf{F}^x_{kx}(t+1)}[\mathbf{F}^x_{kx}(t+1)+(1-d)], \qquad \forall t \geq 1 \qquad (13.1a)$$

$$\mathbf{V}_{nc}(t) = -\mathbf{V}_c(t)\mathbf{F}^c_{nc}(t), \qquad \forall t \geq 1 \qquad (13.1b)$$

$$\mathbf{V}_{nx}(t) = -\mathbf{V}_c(t)\frac{\mathbf{F}^c_{kc}(t)}{\mathbf{F}^x_{kx}(t)}\mathbf{F}^x_{nx}(t), \qquad \forall t \geq 1 \qquad (13.1c)$$

$$\mathrm{V}_{nc}(0) = [\Phi\Omega_{c0} - \mathrm{V}_c(0)]\,\mathrm{F}^c_{nc}(0) + \Phi\Omega_{nc0} \qquad (13.1d)$$

$$\mathrm{V}_{nx}(0) = -\mathrm{V}_c(0)\frac{\mathrm{F}^c_{kc}(0)}{\mathrm{F}^x_{kx}(0)}\mathrm{F}^x_{nx}(0) + \Phi\Omega_{nx0} \qquad (13.1e)$$

Let N denote the set of policies for which a competitive equilibrium exists.

Definition (Ramsey Equilibrium): A Ramsey equilibrium is a policy h in N, an allocation rule $\Gamma(.)$, and a price system $\mathrm{P}(.) = \{w_j(.), r_j(.), p(.)\}$ for *j = C, X*, such that

(a) The policy h maximizes welfare subject to the resource constraints *(5b)* and *(5c)* and implementability constraint *(11)*.

(b) For every h', the allocation $\Gamma(h')$, the price system $\mathrm{P}(h')$, and the policy h' constitute a competitive equilibrium.

First, note that Ramsey equilibrium requires optimality by households and firms for all policies that the government might choose. Hence for a given value of the initial price level p_0 for which the Transversality condition is satisfied, an allocation $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$ and a multiplier F that satisfy the system of difference equations presented by (13.1) will characterize the Ramsey equilibrium. Using the resulting Ramsey allocation, one can then compute the Ramsey equilibrium values of all endogenous variables of the system.

The Steady State Optimal Policy.

Consider a case in which there is a $T \geq 0$ for which $g_t = \overline{g}$ for all $t \geq T$. Assume solution to the Ramsey problem converges to a time-invariant allocation, so that allocations are constant after some time. Then because $V_c(t)$ converges to a constant, the time invariant version of (13.1a) implies:

$$1 = b[\mathrm{F}^x_{kx} + (1-d)] \tag{14.1a}$$

Proposition 1: The steady state optimal tax rate on capital income from investment sector is zero.

Proof: Steady state version of (5g) is:

$$1 = b\mathbf{[}(1-q^x\mathbf{)}\mathbf{F}^x_{kx} + \mathbf{(}1-d\mathbf{)]} \tag{14.1b}$$

(14.1a) and (14.1b) together imply $q^x = 0$.

Proposition 1's finding is similar to what Judd (1985) and Chamley (1986) find using a one-sector model. This result is intuitive, since a nonzero tax rate on capital income in steady state would mean that distortions created by the tax evolves exponentially over time, contrary to a uniform distortion that might be created by simple labor or consumption taxes (see Judd (1999) for details). One cannot distort intertemporal margins because that leads to cumulative distortions. One way the current modelling approach differs from a conventional one-sector competitive model is how savings and capital accumulation occurs across sectors. Note that households pay a strictly positive relative price for the new capital goods and rent it out to firms in anticipation of income from investment. Firms return the rented capital stock net of depreciation. Of these two installed capital stocks, only k_x is required to produce future capital goods. Hence if capital income from k_x is taxed in a steady state, this will induce compounding nature of distortions.

The optimal policy is different in general for capital income tax in consumption goods sector. For $t \to \infty$, $\frac{q^o_t}{q^o_{t+1}} \to \left[\mathbf{(}1-q^c\mathbf{)} \frac{\mathbf{(}1-t^x\mathbf{)}\mathbf{F}^x_{nx}\mathbf{u}_{nc}}{\mathbf{(}1-t^c\mathbf{)}\mathbf{F}^c_{nc}\mathbf{u}_{nx}} \mathbf{F}^c_{kc} + \mathbf{(}1-d\mathbf{)} \right]$ which implies that $1 = b\left[\mathbf{(}1-q^c\mathbf{)} \frac{\mathbf{(}1-t^x\mathbf{)}\mathbf{F}^x_{nx}\mathbf{u}_{nc}}{\mathbf{(}1-t^c\mathbf{)}\mathbf{F}^c_{nc}\mathbf{u}_{nx}} \mathbf{F}^c_{kc} + \mathbf{(}1-d\mathbf{)} \right]$ holds for $t \to \infty$. Together with (14.1a), this implies $q^c = 1 - \frac{\mathbf{F}^x_{kx}\mathbf{F}^c_{nc}}{\mathbf{F}^c_{kc}\mathbf{F}^x_{nx}} \left[\frac{\mathbf{(}1-t^c\mathbf{)}\mathbf{u}_{nx}}{\mathbf{(}1-t^x\mathbf{)}\mathbf{u}_{nc}} \right]$. The government's set of policies N for which a competitive equilibrium exists is therefore:

$$\mathrm{N} = \left\{ (t^c, t^x, q^c, q^x) \middle| q^x = 0, \frac{\mathbf{F}_{kx}^x \mathbf{F}_{nc}^c}{\mathbf{F}_{kc}^c \mathbf{F}_{nx}^x} \left[\frac{(1-t^c)\mathbf{u}_{nx}}{(1-t^x)\mathbf{u}_{nc}} \right] = 1 - q^c \right\} \tag{14.1c}$$

Proposition 2: If the utility function is separable in consumption and labour and linear in labour, and if the government sets the labour income tax rates equal across sectors, the steady state optimal tax rate on capital income from consumption sector is zero. Otherwise, it is not zero.

Proof: Consider $q^c = 1 - \frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x} \left[\frac{(1-t^c)u_{nx}}{(1-t^x)u_{nc}} \right]$, and recall $\frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x} = \frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}$ which is derived from the Ramsey equilibrium system defined by (13.1),.

Since $\mathrm{V}_{nc} = u_{nc} + \Phi[u_{cnc}c + u_{nc} + n_c u_{ncnc} + n_x u_{nxnc}]$

and, $\mathrm{V}_{nx} = u_{nx} + \Phi[u_{cnx}c + u_{nx} + n_c u_{ncnx} + n_x u_{nxnx}]$, the term $\frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x} \left[\frac{(1-t^c)u_{nx}}{(1-t^x)u_{nc}} \right] = 1$ if and only if (a) the utility function separable in consumption and labour and linear in labour, for which $\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}} = \frac{u_{nc}}{u_{nx}}$, and (b) the government sets labour income tax rates equal across sectors. Unless both conditions are satisfied simultaneously, $q^c \neq 0$.

Notice that for utility function defined by (1), it is not explicitly assumed that utility is linear in labour, and that the marginal rate of substitution of labor across sectors is unitary. The first simplification is common in literature that deals with similar models, which (together with separability of utility function in consumption and labour) dramatically simplifies the expressions of V_{nj} by ruling out the second and cross derivatives of labour services. The second simplification (unitary marginal rate of substitution of labour) would imply that after tax wages are equal across sectors. One way to abstract from this assumption is to assume that utility is derived from leisure, and that $u_{nc} = u_{nx} = u_n$. Such simplifications are not obvious where there exists some intratemporal adjustment cost of labor across sectors (see for instance, Huffman & Wynne (1999)). For such a class of utility functions where $\Im : \mathrm{R}_+^2 \to \mathrm{R}$ is strictly convex, $\frac{\mathrm{V}_{nc}}{\mathrm{V}_{nx}} = \frac{\mathrm{u}_{nc}}{\mathrm{u}_{nx}}$ does not necessarily hold. The more important (than preference specification) condition is one on optimal labour income taxes. Notice that if $\frac{\mathrm{V}_{nc}}{\mathrm{V}_{nx}} = \frac{\mathrm{u}_{nc}}{\mathrm{u}_{nx}}$, $1 - q^c = \frac{(1-t^c)}{(1-t^x)}$, and if $t_x > t_c$ capital income is subsidized. This is a classic result on optimal policy in the sense that distortion caused by one instrument can be undone with another one[3].

This particular analytical result has a very sharp intuition. Since capital is produced in a different sector, nonzero capital income tax in the consumption sector is similar, in terms of consequences, to a simple consumption tax which has uniform distortion pattern. Since capital is freely movable across sectors, and following proposition 1, it is feasible for the household to purchase new capital goods, invest the new capital k_x and both forms of the depreciated old capital goods in the capital goods sector. The next period capital to produce consumption goods is available through production of new capital goods. Hence, the depreciated capital good from consumption sector is transferred to investment sector for production. The household earns capital income from consumption sector in each period, gets taxed at a nonzero rate, and can avoid the compounding tax liabilities by shifting depreciated capital to the other sector.

The intuition also can be drawn from the deviation in social marginal values of capital in two sectors. To see this more clearly, consider the Ramsey problem, but through Chamley's (1986) approach. Using linear homogeneity property of the production functions, one can rewrite the government budget constraint:

$$g_t = \mathrm{F}^c(k_{ct}, n_{ct}) + p_t\, \mathrm{F}^x(k_{xt}, n_{xt}) - \tilde{r}_{ct}k_{ct} - \tilde{r}_{xt}k_{xt} - \tilde{w}_{ct}n_{ct} - \tilde{w}_{xt}n_{xt} \tag{15.1}$$

where $\tilde{r}_{jt} \equiv (1-q_t^j)r_{jt}$ and $\tilde{w}_{jt} \equiv (1-t_t^j)w_{jt}$. Thus the government's policy choice is constrained by (15.1), the two resource constraints and decentralized equilibrium conditions. The Lagrangian of the government's problem is:

[3] We keep the preference specification general in order to capture all possible results. Our main result is in no way driven by particular preference specification. Even if one assumes commonly used specification with $u_{nc} = u_{nx} = u_n$, our main result and main intuition are unchanged.

$$
\begin{aligned}
\overline{\mathrm{L}} = \sum_{t=0}^{\infty} b^{t} \{ & \mathrm{u}(c_t, n_{ct}, n_{xt}) \\
& + y_t [\mathrm{F}^c(k_{ct}, n_{ct}) + p_t \mathrm{F}^x(k_{xt}, n_{xt}) - \tilde{r}_{ct} k_{ct} - \tilde{r}_{xt} k_{xt} - \tilde{w}_{ct} n_{ct} - \tilde{w}_{xt} n_{xt} - g_t] \\
& + f_{1t}[\mathrm{F}^c(k_{ct}, n_{ct}) - c_t - g_t] \\
& + f_{2t}[\mathrm{F}^x(k_{xt}, n_{xt}) + (1-d)(k_{ct} + k_{xt}) - k_{ct+1} - k_{xt+1}] \\
& + m_{1t}[\mathrm{u}_{nc}(t) + \mathrm{u}_c(t)\tilde{w}_{ct}] + m_{2t}[\mathrm{u}_{nx}(t) + \mathrm{u}_c(t)\tilde{w}_{xt}] \\
& + m_{3t}[\mathrm{u}_c(t) - \frac{b}{p_t}\mathrm{u}_c(t+1)\{\tilde{r}_{ct+1} + p_{t+1}(1-d)\}] \\
& + m_{4t}[\mathrm{u}_c(t) - \frac{b}{p_t}\mathrm{u}_c(t+1)\{\tilde{r}_{xt+1} + p_{t+1}(1-d)\}] \}
\end{aligned}
\tag{15.2}
$$

The solution to this problem gives the set of Ramsey equilibrium conditions. Consolidating and using decentralized equilibrium conditions, one derives:

$$
t_t^{c^*} = \frac{1}{y_t}\left[\frac{-\mathrm{u}_{nc}(t)}{\mathrm{F}^c_{nc}(t)} - f_{1t}\right] \tag{16.1}
$$

$$
t_t^{x^*} = \frac{1}{y_t p_t}\left[\frac{-\mathrm{u}_{nx}(t)}{\mathrm{F}^x_{nx}(t)} - f_{2t}\right] \tag{16.2}
$$

Notice first that optimal labour income tax rates in both sectors depends crucially on the social marginal value of capital, f_{1t} and f_{2t}. The Euler equation equivalents of Ramsey equilibrium are:

$$
f_{2t} = b\left\{y_{t+1}[p_{t+1}\mathrm{F}^x_{kx}(t+1) - \tilde{r}_{xt+1}] + f_{2t+1}[\mathrm{F}^x_{kx}(t+1) + (1-d)]\right\} \tag{17.1}
$$

$$
f_{2t} = b\left\{y_{t+1}[\mathrm{F}^c_{kc}(t+1) - \tilde{r}_{ct+1}] + f_{1t+1}\mathrm{F}^c_{kc}(t+1) + f_{2t+1}(1-d)\right\} \tag{17.1}
$$

for changes in k_{xt+1} and k_{ct+1}, respectively. These have straightforward interpretations. Condition (17.1) states that a marginal increment of capital investment in investment sector in period t increases the quantity of available capital goods at time (t+1) by the amount $[\mathrm{F}^x_{kx}(t+1) + (1-d)]$, which has social marginal value f_{2t+1}. In addition, there is an increase in tax revenues equal to $[p_{t+1}\mathrm{F}^x_{kx}(t+1) - \tilde{r}_{xt+1}]$, which enables the government to reduce other taxes by the same amount[4]. The reduction of this excess burden equals

[4] In equilibrium, note that $[p_{t+1}\mathrm{F}^x_{kx}(t+1) - \tilde{r}_{xt+1}] = q^{x}_{t+1} r_{xt+1}$.

$y_{t+1}[p_{t+1} F^{x}_{kx}(t+1) - \tilde{r}_{xt+1}]$. The sum of these two effects is period (t+1) is discounted back by discount factor b , and is equal to the social marginal value of the initial investment in investment sector in period t, given by f_{2t} .

Condition (17.2) states that a marginal increment of capital investment in consumption sector in period t increases the quantity of available consumption goods at time (t+1) by the amount $F^{c}_{kc}(t+1)$, which has social marginal value f_{1t+1}. This increment is adjusted by capital depreciation in investment sector, which has social marginal value f_{2t+1}. Thus the aggregate increment in the quantity of available consumption goods net of depreciation at time (t+1) in social marginal value is equal to $[f_{1t+1} F^{c}_{kc}(t+1) + f_{2t+1}(1-d)]$. The first term is due to an increase in capital in consumption sector, while the second terms stands for an indirect increase in production of consumption good through increase in depreciated capital in investment sector. Thus the social marginal values of capital in two sectors are in general different. The increased tax revenue equal to $[F^{c}_{kc}(t+1) - \tilde{r}_{ct+1}]$ enables the government to reduce other taxes by the same amount, and the reduction of this excess burden equals $y_{t+1}[F^{c}_{kc}(t+1) - \tilde{r}_{ct+1}]$. The sum of these two effects in period (t+1) is discounted back by the discount factor and is equal to the social marginal value of the available capital good in period t.

The steady state versions of (17.1) and (17.2) imply that

$$q^{c} = \frac{1}{y}\left[f_{2}\frac{F^{x}_{kx}}{F^{c}_{kc}} - f_{1} \right] \qquad (18)$$

and unless the term in parentheses is zero, the capital income tax in consumption sector is nonzero. Moreover, it is straightforward to show that if $u_{nc} = u_{nx} = u_{n}$, it is possible to undo the difference in social marginal value of capital by setting labour taxes equal across sectors. To see this, impose $u_{nc} = u_{nx} = u_{n}$ in (16.1) and (16.2), combine these with (13.1b) and (13.1c) and (18), which gives that $\left[f_{2}\frac{F^{x}_{kx}}{F^{c}_{kc}} - f_{1} \right] = 0 \Leftrightarrow t^{x} = t^{c}$.

Constrained tax choice.

The previous analysis concluded that the government's optimal choice of steady state capital tax rates may vary across sectors. Consider, for instance, a class of utility functions for which $\frac{V_{nc}}{V_{nx}} = \frac{u_{nc}}{u_{nx}}$ holds[5]. The government's set of policies for which a competitive equilibrium exists would then be:

$$\tilde{N} = \left\{ (t^c, t^x, q^c, q^x) \middle| q^x = 0, 1 - q^c = \frac{(1-t^c)}{(1-t^x)} \right\}$$

implying that the government sets a limiting zero tax on capital income from consumption sector if and only if it sets labour income tax rates equal across sectors. Hence given that particular class of utility functions, for any subset of Ramsey policy that prescribes varying labour income tax rates across sectors, the optimal steady state tax on capital income from consumption sector is nonzero. Here we consider a case where the government faces a constraint to keep these taxes equal across sectors, i.e. same labour income tax rates and same capital income tax rates across sectors. In principle, it is predictable that such additional constraints in the Ramsey problem would necessarily worsen Ramsey equilibrium outcome relative to the one proposed earlier. Our prescription of a nonzero tax on capital income from consumption sector is backed up by a clear intuition that such a capital tax will not have compounding distortion effects as long as the government keeps the other capital income tax zero. If the government's choice of capital income tax rates is constrained to be same ex ante, the only optimal rule for the government would be that both capital income tax rates are zero. Hence in a Ramsey problem with constrained tax choice, any nonzero optimal tax on capital income would be an outcome with lower welfare than the one proposed earlier.

To test it formally, note that since the after tax returns to capital are equal across sectors in a competitive equilibrium, constraining capital income taxes to be same is tantamount to constraining pre-tax returns to capital across sectors to be same. In other words, one can test the restriction of equal capital income taxes across sectors by incorporating the additional constraint $\mathbf{F}^c_{kc}(t) = p_t \mathbf{F}^x_{kx}(t), \forall t$ in the Ramsey problem. Substituting for the equilibrium

[5] One may consider the utility function as $u(.) = \ln c_t + [1 - n_{ct} - n_{xt}]$ which is supported by the lottery argument of Hansen (1985). This functional form is popular in real business cycle literature.

relative price of new capital goods, and imposing the constraint that government keeps the labor income tax rates same across sectors, the additional constraint becomes $\frac{\mathbf{F}_{kc}^{c}}{\mathbf{F}_{kx}^{x}} \cdot \frac{\mathbf{F}_{nx}^{x}}{\mathbf{F}_{nc}^{c}} = \frac{\mathbf{u}_{nx}}{\mathbf{u}_{nc}}$.

Consider, therefore, the Lagrangian form of Ramsey problem with constrained tax choice for the government,

$$
\begin{aligned}
Ɉ = \sum_{t=0}^{\infty} b^{t} \{ & \mathrm{V}(c_t, n_{ct}, n_{xt}, \overline{\Phi}) \\
& + c_{1t}[\mathrm{F}^{c}(k_{ct}, n_{ct}) - c_t - g_t] \\
& + c_{2t}[\mathrm{F}^{x}(k_{xt}, n_{xt}) + (1-d)(k_{ct} + k_{xt}) - k_{ct+1} - k_{xt+1}] \\
& + c_{3t}\left[\frac{\mathrm{F}_{kc}^{c}}{\mathrm{F}_{kx}^{x}} . \frac{\mathrm{F}_{nx}^{x}}{\mathrm{F}_{nc}^{c}} - \frac{\mathrm{u}_{nx}}{\mathrm{u}_{nc}}\right] \} - \overline{\Phi}\Omega(c_0, n_{c0}, n_{x0}, q_0^{c}, q_0^{x})
\end{aligned}
\tag{19.1}
$$

where $\{c_{1t}, c_{2t}, c_{3t}\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers on the two resource constraints and the additional tax choice constraint. The necessary conditions for an optimum for this problem for changes in consumption, labor supply and one period ahead capital stocks are:

$$c_t : \mathrm{V}_c(t) = c_{1t}, \qquad \forall t \geq 1 \tag{19.2a}$$

$$n_{ct} : \mathbf{V}_{nc}(t) = -c_{1t}\,\mathbf{F}_{nc}^{c}(t) - c_{3t}\left[\frac{\mathbf{F}_{nx}^{x}(t)}{\mathbf{F}_{kx}^{x}(t)}\left\{\frac{\mathbf{F}_{kcnc}^{c}(t)}{\mathbf{F}_{nc}^{c}(t)} - \frac{\mathbf{F}_{kc}^{c}(t)\mathbf{F}_{ncnc}^{c}(t)}{[\mathbf{F}_{nc}^{c}(t)]^2}\right\} - \left\{\frac{\mathbf{u}_{nxnc}(t)}{\mathbf{u}_{nc}(t)} - \frac{\mathbf{u}_{nx}(t)\mathbf{u}_{ncnc}(t)}{[\mathbf{u}_{nc}(t)]^2}\right\}\right], \quad \forall t \geq 1 \tag{19.2b}$$

$$n_{xt} : \mathbf{V}_{nx}(t) = -c_{2t}\,\mathbf{F}_{nx}^{x}(t) - c_{3t}\left[\frac{\mathbf{F}_{kc}^{c}(t)}{\mathbf{F}_{nc}^{c}(t)}\left\{\frac{\mathbf{F}_{nxnx}^{x}(t)}{\mathbf{F}_{kx}^{x}(t)} - \frac{\mathbf{F}_{nx}^{x}(t)\mathbf{F}_{kxnx}^{x}(t)}{[\mathbf{F}_{kx}^{x}(t)]^2}\right\} - \left\{\frac{\mathbf{u}_{nxnx}(t)}{\mathbf{u}_{nc}(t)} - \frac{\mathbf{u}_{nx}(t)\mathbf{u}_{ncnx}(t)}{[\mathbf{u}_{nc}(t)]^2}\right\}\right], \quad \forall t \geq 1 \tag{19.2c}$$

$$k_{ct+1} : c_{2t} = b\left\{c_{1t+1}\mathbf{F}_{kc}^{c}(t+1) + c_{2t+1}(1-d) + c_{3t+1}\left[\frac{\mathbf{F}_{nx}^{x}(t+1)}{\mathbf{F}_{kx}^{x}(t+1)}\left\{\frac{\mathbf{F}_{kckc}^{c}(t+1)}{\mathbf{F}_{nc}^{c}(t+1)} - \frac{\mathbf{F}_{kc}^{c}(t+1)\mathbf{F}_{nckc}^{c}(t+1)}{[\mathbf{F}_{nc}^{c}(t+1)]^2}\right\}\right]\right\} \tag{19.2d}$$

$$k_{xt+1} : c_{2t} = b\left\{c_{2t+1}[\mathbf{F}_{kx}^{x}(t+1) + (1-d)] + c_{3t+1}\left[\frac{\mathbf{F}_{kc}^{c}(t+1)}{\mathbf{F}_{nc}^{c}(t+1)}\left\{\frac{\mathbf{F}_{nxkx}^{x}(t+1)}{\mathbf{F}_{kx}^{x}(t+1)} - \frac{\mathbf{F}_{nx}^{x}(t+1)\mathbf{F}_{kxkx}^{x}(t+1)}{[\mathbf{F}_{kx}^{x}(t+1)]^2}\right\}\right]\right\} \tag{19.2e}$$

Consolidating (19.2) yields three necessary conditions for a Ramsey equilibrium, and the one of interest is:

$$V_c(t)\frac{F^c_{kc}(t)}{F^x_{kx}(t)} = b\left[V_c(t+1)\frac{F^c_{kc}(t+1)}{F^x_{kx}(t+1)}[F^x_{kx}(t+1)+(1-d)] + c_{3t+1}[\Theta_{t+1}]\right] - c_{3t}[\Lambda_t] \tag{19.3}$$

Where Θ_{t+1} and Λ_t are terms comprising derivatives of $F^c(.)$ and $F^x(.)$, evaluated at time t+1 and t, respectively, defined as[6]:

$$\Theta_{t+1} \equiv \left\{\frac{\mathbf{F}^x_{nx}}{\mathbf{F}^x_{kx}}\left[\frac{\mathbf{F}^c_{kckc}}{\mathbf{F}^c_{nc}} - \frac{\mathbf{F}^c_{kc}\mathbf{F}^c_{nckc}}{[\mathbf{F}^c_{nc}]^2}\right]\left[\frac{\mathbf{F}^x_{kx}+(1-d)}{\mathbf{F}^x_{kx}}\right] + \frac{\mathbf{F}^c_{kc}}{\mathbf{F}^c_{nc}}\left[\frac{\mathbf{F}^x_{nxkx}}{\mathbf{F}^x_{kx}} - \frac{\mathbf{F}^x_{nx}\mathbf{F}^x_{kxkx}}{[\mathbf{F}^x_{kx}]^2}\right]\left[\frac{d-1}{\mathbf{F}^x_{kx}}\right]\right\}$$

$$\Lambda_t \equiv \left\{\frac{\mathbf{F}^x_{nx}}{\mathbf{F}^x_{kx}}\left[\frac{\mathbf{F}^c_{kckc}}{\mathbf{F}^c_{nc}} - \frac{\mathbf{F}^c_{kc}\mathbf{F}^c_{nckc}}{[\mathbf{F}^c_{nc}]^2}\right] - \frac{\mathbf{F}^c_{kc}}{\mathbf{F}^c_{nc}}\left[\frac{\mathbf{F}^x_{nxkx}}{\mathbf{F}^x_{kx}} - \frac{\mathbf{F}^x_{nx}\mathbf{F}^x_{kxkx}}{[\mathbf{F}^x_{kx}]^2}\right]\right\}\frac{1}{\mathbf{F}^x_{kx}}$$

Recall the otherwise equivalent condition derived from Ramsey problem without tax choice constraint. For a $T \geq 0$ for which $g_t = \overline{g}$ for all $t \geq T$, and assuming convergence of the solution to the Ramsey problem to a time-invariant allocation, the time invariant version of (13.1a) implied $1 = b[F^x_{kx} + (1-d)]$, which acted instrumentally for the proof of proposition 1. With the current Ramsey problem, for $t \to \infty$, $1 = b\mathbf{[(}1-q^x\mathbf{)F}^x_{kx} + \mathbf{(}1-d\mathbf{)]}$ still holds in a Ramsey equilibrium. Unless $1 = b[F^x_{kx} + (1-d)]$ holds from the time invariant version of (19.3), it is trivial that $q^x \neq 0$ vis a vis $q^c \neq 0$. In proposition 3, it is formally proved that $1 = b[F^x_{kx} + (1-d)]$ does not hold in Ramsey equilibrium with constrained factor income tax. Consider a $T \geq 0$ for which $g_t = \overline{g}$ for all $t \geq T$, and assume that the solution to the Ramsey problem (19.1) converges to a time-invariant allocation. The time invariant version of (19.3) is:

$$V_c\frac{F^c_{kc}}{F^x_{kx}} = b\, V_c\frac{F^c_{kc}}{F^x_{kx}}[F^x_{kx} + (1-d)] + c_3\Sigma \tag{19.4a}$$

Where

$$\Sigma \equiv \left\{\frac{\mathbf{F}^x_{nx}}{\mathbf{F}^x_{kx}}\left[\frac{\mathbf{F}^c_{kckc}}{\mathbf{F}^c_{nc}} - \frac{\mathbf{F}^c_{kc}\mathbf{F}^c_{nckc}}{[\mathbf{F}^c_{nc}]^2}\right]\left[\frac{b\mathbf{[F}^x_{kx}+\mathbf{(}1-d\mathbf{)]}-1}{\mathbf{F}^x_{kx}}\right] + \frac{\mathbf{F}^c_{kc}}{\mathbf{F}^c_{nc}}\left[\frac{\mathbf{F}^x_{nxkx}}{\mathbf{F}^x_{kx}} - \frac{\mathbf{F}^x_{nx}\mathbf{F}^x_{kxkx}}{[\mathbf{F}^x_{kx}]^2}\right]\left[\frac{1-b\mathbf{(}1-d\mathbf{)}}{\mathbf{F}^x_{kx}}\right]\right\}$$

---

[6] The time notations attached to the derivatives are omitted in defining Θ_{t+1} and Λ_t, without loss of generality, just to avoid notational clutter.

In order to prove that both capital tax rates are nonzero, it is sufficient to prove that $\Sigma \neq 0$, which in turn implies $1 \neq b[\mathbf{F}_{kx}^{x} + (1-d)]$.

Proposition 3: For a steady state solution to the *Ramsey* problem *(19.1)* and a corresponding Ramsey allocation, the two associated steady state tax rates on capital income are nonzero.

Proof: Suppose not, and hence $\Sigma = 0$ such that *(19.4a)* implies $1 = b[\mathrm{F}_{kx}^{x} + (1-d)]$.

Since $\dfrac{\mathbf{F}_{nx}^{x}}{\mathbf{F}_{kx}^{x}}\left[\dfrac{\mathbf{F}_{kckc}^{c}}{\mathbf{F}_{nc}^{c}} - \dfrac{\mathbf{F}_{kc}^{c}\mathbf{F}_{nckc}^{c}}{[\mathbf{F}_{nc}^{c}]^{2}}\right] < 0$, $\dfrac{\mathbf{F}_{kc}^{c}}{\mathbf{F}_{nc}^{c}}\left[\dfrac{\mathbf{F}_{nxkx}^{x}}{\mathbf{F}_{kx}^{x}} - \dfrac{\mathbf{F}_{nx}^{x}\mathbf{F}_{kxkx}^{x}}{[\mathbf{F}_{kx}^{x}]^{2}}\right] > 0$ and $[1 - b(1-d)] > 0$, for $\Sigma = 0$, it must be that $b[\mathbf{F}_{kx}^{x} + (1-d)] - 1 > 0$, which is a contradiction.

Thus if the government faces a tax choice constraint, the Ramsey equilibrium outcome comprises taxing capital income from both sectors at a strictly nonzero rate. This policy cannot be optimal since it leaves no way to avoid compounding tax liabilities. With this tax plan in the scheme, the household will not be able to avoid the compounding tax liabilities by simply shifting depreciated capital.

Utility functions.

In this section we characterize the optimal steady state capital income tax for consumption sector associated with the Ramsey equilibrium (13.1) with a variant of commonly used utility functions. Huffman & Wynne (1999) propose a class of utility functions that captures the idea of intratemporal labour adjustment cost assuming that shifting labour across sectors is costly. Their proposed functional form characterizes strict convexity of the function $\Im : \mathrm{R}_{+}^{2} \rightarrow \mathrm{R}$ relevant to the current paper. Jones et. al (1997) present a useful specification of a utility function where the planner is unable to distinguish between income from two types of labour. We consider these utility function specifications for experimenting the key analytical results, acknowledging that there may be many other interesting cases to consider.

Equal marginal disutility of labour:

Consider the broader class of utility functions:

$$U(c_t, n_{ct}, n_{xt}) = \frac{[c_t \exp(1 - n_{ct} - n_{xt})]^{1-s} - 1}{1-s} \qquad (20.1a)$$

with $s \geq 0$, the inverse of elasticity of intertemporal substitution. Consider u(.) as a special case of $U(.)$ where $s \to 1$. As $s \to 1$, using l'Hôpital's rule, it is possible to show that

$$\mathbf{u}(c_t, n_{ct}, n_{xt}) = \ln c_t + (1 - n_{ct} - n_{xt}) \qquad (20.1b)$$

Specification (20.1b) that characterizes utility linear in labour services can be justified by the lottery argument of Hansen (1985). In the context of the current paper's analytical tractability, such utility functions simplify the expressions of V_{nj} by ruling out the second and cross derivatives of labour services. This specific form also exhibits unitary marginal rate of substitution of labour across sectors. While this simple assumption that workers receive equal marginal disutility from different sectors is typically held in a subset of multi-sector general equilibrium models, empirically, there is strong evidence against it for the case of the US industrial sector. The BLS survey 2002 reports suggests that injury related incidence per 100 worker varies greatly across different industrial sectors, and incidence rates are relatively higher in goods-producing sector as compared to the service producing sector. Hence, one can argue that such utility functions are increasingly stylized and ignores the empirically supported evidence of varying disliking for jobs across sectors.

The set of policies for the government which can be implemented in a competitive equilibrium, given (20.1b), is presented by:

$$\tilde{\mathrm{N}} = \left\{ (t^c, t^x, q^c, q^x) \middle| q^x = 0, 1 - q^c = \frac{(1-t^c)}{(1-t^x)} \right\}$$

which states that the optimal steady state capital income tax for consumption sector is zero if and only if the government keeps the two labour income tax rates equal across sectors. Now consider competitive equilibrium condition which states that the marginal rate of substitution of labour must equal the relative after tax wage rates. Given (20.1b), the marginal rate of substitution of labour across sectors is one. This implies the after tax wage rates across sectors are equal (and not the tax rates). Hence for $\tilde{\mathrm{N}}$, the government's optimal choice of labour income tax rates may or may not be equal across sectors, although both choices will generate allocations which can be implemented as a competitive equilibrium. In the particular policy choice where labour income tax rates vary, the government taxes capital income from consumption sector at a nonzero rate.

A possible extension to this specification may be to consider varying marginal disutility of labour across sectors maintaining the assumption that utility is linear in labour services. The simplest form that specifies this idea is perhaps $\mathrm{u}(c_t, n_{ct}, n_{xt}) = \ln(c_t) + [1 - n(n_{ct}, n_{xt})]$ where $n : \mathrm{R}_+^2 \rightarrow \mathrm{R}$ is a convex function and linear in its two arguments, such that $n_{n_j n_j} = 0$ and $n_{n_c n_x} = n_{n_x n_c} = 0$. In order to incorporate the non-unitary marginal rate of substitution of labour in this functional form, one can define a parameter $m > 0$ such that $n_{n_c} = mn_{n_x}$. Due to the empirical evidence from US industrial sector, it is sensible to assume that $m \neq 1$. Invoking this specification yields the same policy set for the government as given by $\tilde{\mathrm{N}}$, and same conclusion holds.

Intratemporal labour adjustment cost:

This functional form, as mentioned earlier, is in the spirit of Huffman & Wynne (1999). Assume there exist some intratemporal adjustment cost of labour across sectors, and consider the following utility function:

$$\mathbf{u}(c_t, n_{ct}, n_{xt}) = \mathbf{ln}(c_t) + \{1 - z\,[y n_{ct}^{-w} + (1-y) n_{xt}^{-w}]^{-\frac{1}{w}}\} \tag{20.1c}$$

Where $w \leq -1, z > 0$ and $1 \geq y \geq 0$. This specification of the utility function allows for the idea that it is costly to reallocate labour from one sector to the other. Note that with $w = -1, z = 2$ and $y = \frac{1}{2}$, (15.1c) reduces to $\mathbf{ln}(c_t) + \{1 - n_{ct} - n_{xt}\}$, which exhibits unitary marginal rate of substitution of labour across sectors, and is tantamount to saying that the household receives equal disutility from labour services from the two sectors. In the context of the current setting, the restrictions $w = -1$ and $y = \frac{1}{2}$ together imply that marginal rate of substitution of labour across sectors is equal to one. There is an issue, of course, that how these costs should be interpreted here, which I will not focus in detail.

The marginal rate of substitution of labour across sectors for this specification, for all permissible values of w , is:

$$\frac{\mathbf{u}_{nc}}{\mathbf{u}_{nx}} = \frac{y\, z n_c^{-w-1}[y\, n_c^{-w} + (1-y\,)n_x^{-w}]^{-\frac{1}{w}-1}}{(1-y\,)z n_x^{-w-1}[y n_c^{-w} + (1-y\,)n_x^{-w}]^{-\frac{1}{w}-1}}$$

For any $w < -1$, which can be interpreted as the adjustment cost parameter, the optimal steady state tax rate for capital income from consumption sector is:

$$q^c = 1 - \left[\frac{y z n_c^{-w-1}[y n_c^{-w} + (1-y\,)n_x^{-w}]^{-\frac{1}{w}-1}(1+\Phi) - \Phi[n_c\, \mathbf{u}_{ncnc} + n_x\, \mathbf{u}_{nxnc}]}{(1-y\,)z n_x^{-w-1}[y n_c^{-w} + (1-y\,)n_x^{-w}]^{-\frac{1}{w}-1}(1+\Phi) - \Phi[n_c\, \mathbf{u}_{ncnx} + n_x\, \mathbf{u}_{nxnx}]}\right]\left[\frac{(1-t^{\,c})\mathbf{u}_{nx}}{(1-t^{\,x})\mathbf{u}_{nc}}\right]$$

With $\mathbf{u}_{ncnc} \neq 0$, $\mathbf{u}_{nxnx} \neq 0$, $\mathbf{u}_{ncnx} \neq 0$ and $\mathbf{u}_{nxnc} \neq 0$. This implies the set of policies at the government's choice which can be implemented in competitive equilibrium comprises of q^c which is nonzero, even in the case when the government sets labour income tax rates equal across sectors.

Two types of labour:

This particular functional form where labour services are of two specific types is due to Jones et. al (1997), and is intended to represent the case where the planner is unable to distinguish between income from two types of labour. A probable rationale for this utility function may be the often realized and empirically supported fact that producing capital goods is typically more skill-intensive than producing manufacturing consumption goods. The example considered therefore features one household that sells two types of labour in the market. Jones et. al (1997) invoke this specification with an ex ante restriction on the choice of labour income tax rates. I will consider the unconstrained version. Consider the following utility function:

$$\mathbf{u}(c_t, 1-n_{ct}, 1-n_{xt}) = \frac{c_t^{1-s}\,(1-n_{ct})^{g_c}\,(1-n_{xt})^{g_x}}{1-s} \qquad (20.1d)$$

with $s \geq 0$, and $g_j < 0$. The marginal rate of substitution of labour across sectors is:

$$\frac{\mathbf{u}_{nc}}{\mathbf{u}_{nx}} = \frac{g_c(1-n_x)}{g_x(1-n_c)}$$

Since now the utility function has cross derivatives of consumption and labour supply, it is useful to state the following expression:

$$\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}} = \frac{g_c}{g_x}\left[\frac{\Phi\{\mathbf{Y}\} - (1+\Phi)(1-n_x)^{g_x}(1-n_c)^{g_c-1}}{\Phi\{\mathbf{Z}\} - (1+\Phi)(1-n_c)^{g_c}(1-n_x)^{g_x-1}}\right]$$

where

$$\mathbf{Y} \equiv n_c(1-n_x)^{g_x}(g_c-1)(1-n_c)^{g_c-2} + n_x(1-n_x)^{g_x-1}g_x(1-n_c)^{g_c-1} - (1-s)(1-n_x)^{g_x}(1-n_c)^{g_c-1}$$

$$\mathbf{Z} \equiv n_c(1-n_x)^{g_x-1}g_c(1-n_c)^{g_c-1} + n_x(1-n_x)^{g_x-2}(g_x-1)(1-n_c)^{g_c} - (1-s)(1-n_x)^{g_x-1}(1-n_c)^{g_c}$$

It is straightforward to notice that for all permissible values of the parameter g, the condition $\frac{\mathrm{V}_{nc}}{\mathrm{V}_{nx}} = \frac{\mathrm{u}_{nc}}{\mathrm{u}_{nx}}$ does not hold. This implies the set of policies at the government's disposal for which a competitive equilibrium exists (i.e. which can be implemented in a competitive equilibrium), $\mathrm{N} = \left\{(t^c, t^x, q^c, q^x) \middle| q^x = 0, \frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}\left[\frac{(1-t^c)\mathbf{u}_{nx}}{(1-t^x)\mathbf{u}_{nc}}\right] = 1-q^c\right\}$, prescribes that an ex post choice of equal labour income tax rates is not sufficient to guarantee zero steady state tax on capital income from consumption goods sector.

Concluding remarks.

The paper formulated a two-sector neoclassical production model with infinitely-lived agents in order to analyze the optimal income taxation problem (the Ramsey problem) and examine celebrated optimal capital income taxation principle. The extension of one-sector model to a two-sector version with endogenous capital good's price makes it convenient to scrutinize sector specific optimal capital income taxes in the steady state. The analysis reached a startling conclusion. We find that the optimal capital income tax is nonzero, in general, and the nonzero tax rate is optimal since its distortions can be undone by setting different labour income tax rates. We find that while it is optimal to set a long run zero tax on capital income from investment sector, the optimal steady state capital income tax for consumption sector is nonzero in general. For a standard class of utility functions that has desirable properties, this result holds, and the set of conditions for which this tax rate is zero is in no way inferred by the equilibrium conditions. We also find that if the government faces a constraint to keep factor income tax rates same across sectors, the optimal capital income tax is nonzero.

Our main result is based on the intuition that if capital is produced as a final good, its relative price is different than that of consumption goods. This is tantamount to having different social marginal value of capital in two sectors where it is used. This difference allows the government to tax/subsidize capital income in one sector and undo the distortion by setting different labour income tax rates. This paper thus advocates that the government's long run tax policy comprises of three income tax instruments --- the two labour income tax rates and nonzero tax on capital income in the consumption sector --- all of which have uniform distortion pattern. Capital income from consumption sector can be taxed at a nonzero rate optimally without creating compounding distortions in the long run as long as the other capital income tax is set at zero. This allows economic agents to shift depreciated capital to the untaxed sector and avoid the compounding capital tax liabilities.

Bibliography.

Chamley, C. (1986). 'Optimal Taxation of Capital Income in general Equilibrium with Infinite Lives', *Econometrica*, Vol. 54, pp. 607-622.

Chari, V. V. & Kehoe, P. J. (1999). 'Optimal Fiscal and Monetary Policy', in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics, Vol. 1*. Amsterdam: North-Holland, 1999, pp.1671-1745.

Erosa, A. & Gervais, M. (2001). 'Optimal Taxation in Infinitely-Lived Agent and Overlapping Generations Models: A Review', Federal Reserve Bank of Richmond *Economic Quarterly*, Vol. 87/2, pp. 23-44.

Hansen, G. D. (1985). 'Indivisible Labour and the Business Cycle', *Journal of Monetary Economics*, Vol. 16, pp. 309-328.

Huffman, G. W. & Wynne, M. A. (1999). 'The Role of Intratemporal Adjustment Costs in a Multisector Economy', *Journal of Monetary Economics*, Vol. 43, pp. 317-350.

Jones, L. E., Manuelli, R. E. & Rossi, P. E. (1993). 'Optimal Taxation in Models of Endogenous Growth', *The Journal of Political Economy*, Vol. 101, pp. 485-517.

Jones, L. E., Manuelli, R. E. & Rossi, P. E. (1997). 'On the Optimal Taxation of Capital Income', *Journal of Economic Theory*, Vol. 73, pp. 93-117.

Judd, K. A. (1999). ‘Optimal Taxation and Spending in General Competitive Growth Models’, *Journal of Public Economics*, Vol. 71, pp. 1-26.

Judd, K. L. (1985). ‘Redistributive Taxation in a Simple Perfect Foresight Model’, *Journal of Public Economics*, Vol. 28, pp. 59-83.

Ljungqvist, L. & Sargent, T. J. (2000). *Recursive Macroeconomic Theory*. London: The MIT Press, 2000.

Lucas, R. E., Jr., & Stokey, N. (1983). ‘Optimal Fiscal and Monetary Policy in an Economy without Capital’, *Journal of Monetary Economics*, Vol. 12, pp. 55-93.

Ramsey, F. P. (1927). ‘A Contribution to the Theory of Taxation’, *The Economic Journal*, Vol. 37, pp. 47-61.