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# **Economic Growth and Regional Integration in Mexico**

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## **Abstract**

In this paper we examine the regional structure of output growth, volatility and prosperity in Mexico, focusing in particular on the degree of integration between both the regions and the individual states of the country. The results suggest that there is a high degree of similarity across the regions in the responses to domestic and international shocks affecting the economy, but there are also significant differences across the individual states within each region. We identify a positive relationship between output growth and volatility, but the relationship between growth and regional disparities appears to be negative, suggesting that higher (lower) growth is generally associated with lower (higher) regional dispersion in per capita GDP levels.

## **Keywords**

Regional Growth, Volatility, Regional Disparities, Exogenous Shocks, Cyclical Fluctuations, Co-Movement, Dynamics

## **JEL Classification Numbers**

R11 C22 E32 O11 O18

# Economic Growth and Regional Integration in Mexico

## 1. Introduction

Assessments of macroeconomic performance naturally focus on the behavior of aggregate national variables, such as GDP, employment and unemployment. It is important to recognize, however, that national indicators may conceal marked differences at the regional or local level and that a more detailed examination of regional performance can provide important insight about the degree of integration across the national economy and the causes of regional divergence. Previous empirical work on the Mexican economy has focused predominantly on aggregate macroeconomic performance, examining factors such as the structure of the business cycle, the degree of co-movement with the US economy, and the impact of various domestic and international events, such as the “Tequila crisis”, and major policy changes, such as those associated with joining the General Agreement On Tariffs And Trade (GATT) and the formation of the North American Free Trade Agreement (NAFTA) (see for example Castro *et al* 1997; Urzua *et al* 2000; Castillo and Diaz-Baustista 2002; Torres and Vela 2003; Hernandez 2004, 2006; Sosa 2008).

On the other hand, most of the debates about regional economic activity in Mexico have mainly focused on the identified process of regional convergence from 1940-1984 with the GATT entry, and the regional disparities registered from 1984-1995 which were accelerated with the NAFTA entry in 1995 mainly due to the economic benefits for the border and centre regions of the country (Rodríguez and Sánchez 2002; Esquivel and Messmacher 2002; Aguayo Téllez 2004; Rodríguez Orregia 2005; Chiquiar 2005; González Rivas 2007; Gómez and Ventosa 2007). These regions which developed a competitive export-oriented industrial sector attained accelerated growth rates that took them closer to the US economy but increased the gap with the poorest states within the country (South Mexico)

generating a process of regional divergence from 1985-2001. However the benefits derived from trade liberalization in the north and central regions of Mexico which made them more sensitive to the US economy cycles, were affected by the loss of international competitiveness after China's entry into the World Trade Centre (WTO) and the US market creating a new process of regional convergence in Mexico or at least less regional income disparities from 2001-2012 (Mendoza and Valdivia 2016).

As a result of different approaches in relation to the process of regional convergence / divergence in Mexico, a few number of studies have focused on either, the regional synchronization of economic cycles to the national cycle (Delajara 2011) or the degree of integration and synchronization of each of the 32 Mexican states to the US economic cycle (Mejía and Silva, 2014). Similar to previous findings about regional convergence, recent studies about economic cycles have found that entities of the northern border, north-central and central regions in Mexico featured more synchronized cycles with the United States (Mejia and Silva, 2014; Mejia and Campo, 2011) and that the degree of synchronization increased with NAFTA (Mejía, 2012). On the other hand, variations in the southern region are mostly related to specific shocks to the Mexican economy (Delajara 2012).

Evidence from previous studies have helped us to understand the behavior of the national and regional economy in Mexico, however more detailed regional analysis is needed to answer questions about the degree of integration across the economy and the nature of regional disparities. In this paper we examine the regional pattern of output growth in Mexico, focusing in particular on the cyclical co-movements across the regions and the nature of cross-state disparities in growth performance. The objectives are: *(i)* to identify the degree of regional integration in the Mexican economy and *(ii)* how regional disparities have evolved over time. Our analysis contributes to a growing literature concerned with the regional dimensions of macroeconomic performance, particularly in relation to growth

performance and business cycle structures. Representative examples of work in this area include: Artis and Zhang 1999; Martin 2001; Shepherd and Dixon 2002; Carlino and DeFina 1998, 2004; Gardiner *et al* 2005; Grimes 2005; Pons-Novell and Tirado-Fabregat 2006; Dow and Montagnolia 2007; Owyang *et al* 2009; Poncet and Barthelemy 2008; Wilkerson 2009; Dixon and Shepherd 2013. These studies typically examine different aspects of the trend and cyclical behavior of the macroeconomy, and its volatility, and attempt to determine the degree of integration or co-movement between different regions and subsectors, and the factors affecting regional disparities. In practice, the bulk of the existing work focuses on the behavior of developed countries, particularly the United States and the member states of the European Union. A distinctive feature of our contribution, in addition to the methodology employed, is that we utilize a rich data set to obtain results for an emerging economy which has experienced a diverse set of domestic and international shocks over the study period (1970-2009). The remainder of the paper is structured as follows. We begin by describing the database and the statistical procedure. Following on from this, we present our empirical analysis. We conclude by summarizing our results and their implications.

## **2. Data and Statistical Considerations**

### **2.1 Data**

The data for the study is a set of annual observations of real output and population in each of the 32 states of the United States of Mexico (i.e. Mexico), covering the period 1970-2009. From this primary data set we derive series for total and per capita national output (Mexico) and total and per capita output for each of the three broad regions of the country (North Mexico, Central Mexico and South Mexico) and for each of the individual states. The analysis also utilizes a series for US real GDP and various constructed dummy variables<sup>2</sup>.

## 2.2 Model specification

Our formal analysis is based on the assumption that the time-paths of real output ( $Y_t$ ) are driven by trend ( $\tau_t$ ), cycle ( $c_t$ ), exogenous ( $x_t$ ) and noise ( $\varepsilon_t$ ) components as follows:

$$Y_t = \tau_t + c_t + x_t + \varepsilon_t \quad (1)$$

With this representation, the sum of the cycle, exogenous and noise components can together be identified by extracting the trend from the data (de-trending):

$$y_t = Y_t - \tau_t = c_t + x_t + \varepsilon_t \quad (2)$$

The analysis follows a common impulse-propagation approach, due originally to Frisch (1933), Slutsky (1937) and Samuelson (1939), which supposes that shocks affecting the economy (or the regions) are the impulse factors that not only affect output directly, but can also potentially generate cyclical fluctuations via a propagation process that transforms the shocks in to a cyclical feature. In formal macroeconomic models, the propagation processes typically include factors such as imperfect wage and price flexibility and adjustment costs, which cause partial adjustment and cyclical persistence. In the empirical analysis of business cycle features, following the work of authors such as Sims (1972) and Sargent and Sims (1977), it is common to avoid a full structural (theoretical) specification of the factors that determine the adjustment process and utilize instead a ‘loosely-specified’ statistical model which represents the propagation process as an autoregressive (AR) or vector autoregressive (VAR) process. In this statistical approach, which is common in the empirical business cycle literature<sup>3</sup>, the propagation process is equivalent to a transfer function, which transforms the noise process into a cyclical process.

In order to apply this framework, we need to make assumptions about (i) the form of the trends in the data, which determines the appropriate de-trending procedure, (ii) the nature of the propagation process, which explains how the cyclical feature is generated, and (iii) the nature of the exogenous and noise components. Following the literature in this area, and

based on a prior examination of unit root tests, we assume that the trend component can be approximated as a random walk with drift<sup>4</sup> and that the cyclical component, if present, can be represented as a stationary autoregressive (AR) process. We also assume that the exogenous and noise terms are stationary components, reflecting the various domestic and international shocks affecting the economy. These assumptions imply that the observed  $Y_t$  can be represented as:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \dots + \alpha_k Y_{t-k} + \phi x_t + \varepsilon_t \quad (3)$$

where  $\alpha_1 = 1$  ;  $\sum_{i=2}^k \alpha_i < 1$  and  $x_t$  is a vector of I(0) exogenous variables.

After subtracting  $Y_{t-1}$  from both sides, equation (3) can be re-parameterized as a Dickey-Fuller-type regression:

$$\Delta Y_t = \mu + \beta_0 Y_{t-1} + \beta_1 \Delta Y_{t-1} + \dots + \beta_{k-1} \Delta Y_{t-k-1} + \phi x_t + \varepsilon_t \quad (4)$$

where  $\beta_0 = (\alpha_1 - 1)$ .

In the univariate case, the unit root assumption ( $\alpha_1 = 1$ ) means that the parameter on the levels term is zero ( $\beta_0 = 0$ ) and, with the trend effectively removed via the transformation to equation (4), the stationary components can be identified from an autoregressive model with exogenous components (ARX model) applied to the differenced series:

$$\Delta Y_t = \mu + \beta_1 \Delta Y_{t-1} + \dots + \beta_{k-1} \Delta Y_{t-k-1} + \phi x_t + \varepsilon_t \quad (5)$$

Assuming that the model is applied to the first differences of the logarithms of the data, equation (5) identifies variations in output growth as the sum of cyclical fluctuations and exogenous and noise factors. In this framework, the AR process can be regarded as a statistical representation of the cyclical component (see Engle and Kozicki 1993) and the nature and strength of the cycle is explained by the size and structure of the AR parameters.

In the multivariate context we are also considering,  $Y_t$  represents the vector of regional outputs and a proper understanding of movements in output growth requires some

consideration of the cointegration properties of the data as well as the autoregressive structure. In particular, the presence of a common stochastic trend would imply a long-run equilibrium relationship between the regional output variables and variations in output growth would then be explained in part by an equilibrium-correction component (representing the adjustment of the series to their common equilibrium trend) as well as any autoregressive feature, which generates a cycle around the equilibrium path. In this case, the appropriate procedure is to estimate (4) as a vector autoregressive model with exogenous components (VARX model), to test for cointegration, with an equilibrium correction term incorporated in (5) if a common stochastic trend is identified. In the absence of cointegration, no equilibrium-correction term is required and equation (5) can be applied in a univariate, multivariate or cross-section context as appropriate.

Our strategy is to examine first the path of output at the national level, and then across the broad regions of the economy, to determine whether common trend, cycle and noise features can be identified, with allowance for the impact of the exogenous domestic and international factors. The exogenous factors included in the model are: the impact of macroeconomic activity in the neighboring USA and a series of major shocks affecting the Mexican economy, associated with the economic crises of 1982, trade liberalization associated with GATT entry in 1986, the implementation of the NAFTA in 1994 and the Tequila crisis in 1995, the financial crisis affecting Latin America in 2001, and the international recession of 2009, initiated by the global financial crisis<sup>5</sup>. The indicator for US macroeconomic activity is US GDP growth, while the other exogenous factors are modeled by a set of constructed dummy variables, incorporated as one-period impulse dummies, to allow for transitory impacts, and as step dummies, with continuous impacts, to allow for the possibility that some of the exogenous factors may have exerted a permanent influence on



output growth. The noise process, which represents random shocks affecting the economy, is identified as the model residual.

### **3. National and Regional Output Growth**

Figure 1 plots the annual growth rates of real output for the national Mexico series and for the three broad regions considered (*i.e.* North, Central and South Mexico). The plots are suggestive of significant downturns in activity in the mid-1980s, the mid 1990s and perhaps the early 2000s. It is difficult to say much more from a visual inspection of the series, however, and a formal assessment of their structure and inter-relationship is required.

[Figure 1 about here]

#### **3.1 National estimations**

We begin by examining the nature of output movements at the aggregate level, concentrating in particular on the degree of association with the US economy and the impacts of the various exogenous factors affecting the Mexican economy. Previous studies of the Mexican national economy point to a strong cyclical association with the United States, and the possibility that the two economies share a common trend (Castro *et al* 1997; Esquivel 1999; Castillo and Diaz Baustista 2002; Mejia Reyes 2003; Torres and Vela 2003; De Leon 2004; Hernandez 2004). Preliminary data tests indicate that Mexican and US real output can both be characterized as I(1) variables and we examined first whether a common trend process can be identified for the two economies. Following the procedure suggested by Johansen (1991), we tested for a common trend in a bivariate Vector Error Correction (VEC) model and the results failed to reject the null of no cointegrating relationship between Mexican and US output (with trace and maximum-eigenvalue test probabilities of 0.74 and

0.69 respectively). In the absence of cointegration with the United States, the analysis of Mexican national output movements can be conducted with an ARX model applied to the first differences (growth rates) of the series. We estimated the growth rate model allowing for a first-order autoregressive process with a set of exogenous regressors, comprising lagged US output growth ( $\Delta US_{t-1}$ ) and five one-period impulse dummy variables to capture the shocks of 1982, 1986, 1995, 2001 and 2009 (coded as  $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$ , and  $I_5$ ). We also allowed for the possibility of continued impacts (changes in growth behavior) arising from those shocks and associated policy changes (apart from the 2009 shock) by incorporating step dummies, with zeros before each date and ones afterwards (coded as  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$ ). The results that follow (see Table 1) show the parameter estimates of the model, the LM test for residual serial correlation (as an indicator of whether the dynamic structure of the model adequately represents the data), and the Jarque-Bera (JB) test for residual normality (which indicates whether the significance levels associated with the standard errors are reliable).

Our estimation procedure follows a conventional general to specific methodology (Sargan 1980; Hendry 1995). We first estimated a general model, encompassing all of the identified variables, and then tested down to the most likely parsimonious model. Table 1 presents our suggested model (*i.e.* Model 1) after some experimentation with the general model and deleting variables that were not significant at the 5% probability level.

[Table 1 about here]

The model is well-determined, with an  $R^2 = 0.75$  and the diagnostic tests suggest that the residuals are normally distributed and there is no significant residual serial correlation. In a second set of estimations (see Model 2) we also include one lagged for the dependent

variable. Interestingly, the estimation procedure showed no significant autoregressive structure in the growth rate model, with a test probability of 0.62 for the significance of a lagged dependent variable. This result indicates not only that the lagged dependent variable is insignificant, and adds nothing to the explanatory power of the model, but also that the parameter estimates of the most likely model are robust with respect to the inclusion of the additional lagged variable.

The absence of a significant autoregressive feature in the model suggests that there is no conventional business cycle feature in output growth. Although this result may reflect in part the use of annual data for estimation purposes, and there might well be a degree of persistence in quarterly output data, it should be noted that a significant association with lagged US output growth is identified, which indicates a degree of US-led cyclicity. Having said this, the results generally indicate that variations in Mexican output growth over the period have been dominated by significant one-off external and domestically-generated factors, rather than the cyclicity induced by an autoregressive process. The results further suggest that the shocks affecting the national economy are best modeled as having one-off impacts (impulse dummies  $I_2$ ,  $I_3$  and  $I_5$  are all significant at the 5% level) rather than continuing impacts, apart from the permanent downward (negative) adjustment in growth identified as occurring from 1982 onwards ( $S_1$  is significant at the 5% level, but  $S_2$ ,  $S_3$  and  $S_4$  are not significant).

### **3.2 Regional estimations**

The estimates for national output growth provide a useful benchmark against which to judge the broad regional output movements in North, Central and South Mexico. The matters we need to consider are: *(i)* is there a common trend across the regions, *(ii)* is there an autoregressive (cyclical) component in the growth rate, *(iii)* are regional output movements

linked to US growth, in a similar manner to the national economy, and *(iv)* are the impulse and step dummies significant for each region? If the answers for the three regions are the same as for Mexico at national level, then we can deduce that the regional dynamics are essentially a reflection of what is observed at the national level and that there is a high degree of integration in growth behavior. Of particular interest here is whether distance from the US, in terms of both geography and economic structure, implies a weaker association between domestic and US output movements. In this case, we would expect South Mexico to be the obvious candidate for a weaker association, not only because of its geographical distance, but also its lack of dependence on manufacturing exports to the United States.

The test for cointegration suggested that there may be a common stochastic trend for North and Central Mexico, but not for South Mexico. For the North-Central relationship, prior estimation of an unrestricted VAR in the levels of the series strongly suggested the presence of only a first-order lag structure, which implies that the associated VEC model contains no lagged difference terms and that the dynamics are explained solely by the equilibrium correction component and the exogenous factors. However, a more detailed examination of the VEC model for these two regions showed that the equilibrium-correction term was wrongly signed (suggesting divergent rather than convergent adjustment) and that it added only a trivial amount to the explained variation of the regional growth rates, in comparison with a model with no equilibrium-correction term included. Our interpretation of the result is that, while there may be a long-run equilibrium relationship between North and Central Mexico, for the sample period we are considering the evidence for cointegration is weak (or at least mixed) and the potential adverse consequence of omitting the equilibrium-correction term is likely to be minor. In view of this, our strategy is to proceed by examining independent ARX models for the three regions, with no cointegration imposed, focusing on the cycle, exogenous and noise components affecting output growth (see equation 5). An

advantage of this approach is that it allows straightforward comparisons with the results derived for the national series<sup>6</sup>. Following the same estimation procedure as for the national model, estimates of the ARX models for the three regions are shown in Table 2.

The regional results suggest that factors affecting output growth for North and Central Mexico are similar, with comparable impacts for lagged US output growth and the impulse dummies, and with only the  $S_1$  step dummy significantly affecting the growth path. In the case of South Mexico, while the results are similar to the other regions for the impulse and step dummies, no significant association with US output growth is identified. This confirms that co-movement with the US business cycle is not a feature of the growth path of the Southern states.

[Table 2 about here]

The results also confirm that, as for the national series, no autoregressive features are significant in any of the three regions, which implies that variations in regional output growth have been dominated by the various exogenous and noise factors, and that these factors have not generated any domestic cyclical persistence of the kind frequently identified in empirical business cycle models. For each of the regions it appears that around 75% of the variation in output growth can be explained by the various exogenous factors, with the remaining 25% accounted for by the noise component. The other point to note is that there are no significant step effects arising from the GATT, Tequila crisis and NAFTA step dummies, which indicates no continuing impacts from those events.

### 3.3 State estimations

The results from our previous section point to a high degree of integration between North and Central Mexico, with a lesser (but still relatively high) degree of integration between those regions and South Mexico. However, it should be recognized that the three broad regions contain numerous states and it is possible that the regional results may mask a higher degree of variation between the states. For completeness, we need to consider whether the apparently high degree of regional integration is mirrored across the individual states. To this end, we estimated a series of ARX models for each of the 32 individual states, following the same procedures applied to the national and regional series. Given the large number of estimated parameters, we present the state-level results in terms of Table 3, which records for each state whether or not a particular variable is significantly different from zero at the 5% significance level (recorded with a tick  $\checkmark$  if significant or a cross  $\times$  if not).

In all cases, we again failed to identify any significant autoregressive features, but the state equations were all well-determined and mirrored the regional results, in the sense that they identified strong impacts from the various exogenous factors. However, the results also point to considerable diversity across the states, with a significant degree of idiosyncratic behavior even for states within the same region. For example, there is evidence that almost all states were affected by the external shocks associated to trade openness in 1986 ( $I_2$ ) and the implementation of NAFTA in 1994 ( $I_3$ ), but the response to the 2001 shock is much more diverse, with a significant number of states unaffected. In the South Mexico in particular, there is evidence of idiosyncratic behavior in response to this shock, with none of the states affected on either a one-off or continuing basis. In contrast, the impacts of the shocks of the mid-80s and mid-90s appear to be quite uniform, with some exceptions in North Mexico in particular.

[Table 3 about here]

As an indication of the degree of diversity across the states, we can note that, while the results for North and Central Mexico both show a significant relationship with US growth, approximately half of the states in each of those regions show no such relationship; and in South Mexico, where no relationship with the US is identified, output growth in two of the six states (*i.e.* Quintana Roo and Yucatan) appears to be significantly related to US growth. More generally, a glance at Table 3 shows that there are some states for which the impacts of the exogenous factors exactly mirror the regional pattern, but there are others for which the pattern is very different. The differences between Baja California and Baja California Sur in North Mexico and Morelos and Nayarit in Central Mexico are just two examples which illustrate the point.

#### **4. Growth and Regional Volatility**

In this section we examine the volatility of output growth across the states, concentrating on two questions that have been discussed in the literature. The first question is whether there has been a structural change in the volatility of the shocks affecting output growth, and the second is whether there is any connection between the growth rate of output and its volatility.

Dealing first with the structural change question, there is a large body of evidence to suggest that many developed economies experienced a move to greater stability at some point in the mid-1980s or early 1990s, measured by a reduction in the volatility of output growth, and the evidence points to a reduction in the volatility of the shocks affecting economies as a major source of this volatility reduction (see for example Stock and Watson 2002; Ahmed *et al* 2004; Sensier and Van Dijk 2004; Summers 2005; Davis and Kahn 2008)<sup>7</sup>. The evidence for a reduction in volatility in developing countries is mixed, and to date there is little

evidence about the regional aspects of the matter, apart from studies of the US states and regions (Carlino 2007; Owyang *et al* 2008). The analysis of section 3 dealt with the impact of the major exogenous shocks (domestic and international) affecting the regions. In this section we consider whether there is any evidence to suggest a reduction in the volatility of the more general noise shocks affecting the regions.

[Figure 2 about here]

Figure 2 shows plots of the noise components of the regional ARX models identified earlier. A visual inspection of the plots points to a high degree of similarity in the shocks affecting the regions and this is confirmed by the cross correlations, which are 0.93 for North-Central Mexico, 0.79 for North-South Mexico, and 0.78 for South-Central Mexico. It is difficult to discern from the plots whether there is any structural change in volatility, although it does look as though there may been a reduction in volatility around the mid-1980s, particularly for South Mexico.

To investigate this matter further we applied the test for changes in variance suggested by Inclan and Tiao (1994). The procedure utilizes a test statistic derived from the behavior of the normalized (and centered) cumulative sum of squares of the residual noise series. Starting with the residual noise series  $e_t$  from the estimated model, where  $e_t$  is of length  $T$ , which in the present context is 128, the first step in the test procedure is to calculate the following series:

$$D_k = \frac{C_k}{C_T} - \frac{k}{T} \quad k = 1, \dots, T$$

Where  $C_k$  is the cumulative sum of the squared residuals and  $C_T$  is the final sum of the squared residuals:



$$C_k = \sum_{t=1}^k e_t^2 \quad \text{and} \quad C_T = \sum e_t^2$$

For the case of a single change in variance, Inclan and Tiao (1994) suggest that the most likely date of a change in volatility (if any change occurred) can be identified by searching for the point at which the modular value of  $D_k$  is maximized. The significance of the identified break is then determined with reference to the following test statistic:

$$TS(D_k) = \sqrt{T/2} D_k^* \quad D_k^* = \max_k |D_k|$$

Inclan and Tiao (1994) demonstrate that, under variance homogeneity  $\sqrt{T/2} D_k$  has an expected value equal to zero. With the aid of Monte Carlo replications, the authors calculate asymptotic and small sample critical values, and the null of no change in the series variance is rejected (at the chosen significance level) only if  $TS(D_k)$  exceeds the relevant critical value. The application of this procedure points to 1983/84 as the most likely date at which a reduction in volatility might have occurred, which matches the timing of the reduction in output volatility for the United States. However, the maximum test value for the regions is only 0.91, which indicates that the null of no change in variance can not be rejected at conventional significance levels<sup>8</sup>. This conclusion is confirmed by standard variance equality tests, which suggest no significant difference in the residual variance between the pre and post-1984 periods. The implication is that none of the regions of Mexico have experienced a reduction in the volatility of the noise shocks affecting the economy, of the kind which has been identified for the USA.

The other question related to volatility is whether higher growth is typically associated with higher or lower volatility. The literature in this area has mainly focused on cross-country relationships for developed countries and there are divergent views about both the nature of the causal relationship<sup>9</sup>, if any, and whether the empirical relationship is positive or negative (see for example Ramey and Ramey 1995, and Blackburn and Pelloni 2004). In the present

context, we examined the cross-sectional evidence to determine whether the mean growth rates of the individual states over the sample period are associated with higher or lower volatility, where volatility is measured as the standard deviation of the growth rate for each state over the sample period.

Figure 3 plots the relationship between growth and volatility for the 32 Mexican States, together with the implied least squares regression line. The slope parameter of the least squares line is significantly different from zero at the 5% probability level and the correlation between growth and volatility is 0.64. This suggests that the states with the more volatile growth rates are also those with the higher mean growth rate. However, it should be noted that correlation does not necessarily imply causality and more research need to be done to better understand how volatility and/or different types of volatility (e.g. volatility induced by shocks or underlying structural changes) affect growth. This aspect goes beyond the aim of this paper but remains an interesting topic for future research and crucial to the evaluation of policy options (see, for example, Sahay and Goyal 2006).

[Figure 3 about here]

## **5. Growth and Regional Disparities**

The final matter we consider is the relationship between national output growth and disparities in regional prosperity, measured by the dispersion of per capita real GDP across the states. In this case, what we need to consider is whether higher national growth is associated with a rise or fall in regional income disparities. We measure the evolution of regional disparities by the time series of cross-state standard deviations of per capita GDP (the dispersion of per capita GDP levels), and the time series of the mean of the cross-state growth rates represents the movements in the national (average) growth rate. A positive

(negative) relationship between these variables would suggest that a higher mean growth rate is associated with greater (lesser) dispersion. A plot of the relationship between per capita GDP dispersion and the national growth rate is shown in Figure 4, together with the implied linear least squares fit. The plot suggests that higher national growth is associated with lower regional income dispersion. The correlation between the two variables is  $-0.35$ , which is significantly different from zero at the 5% probability level.

Alongside Figure 4, it is instructive to look at the time-path of per capita GDP dispersion, shown in Figure 5. This indicates that cross-state dispersion was rising over the period from the late 1970s to the late 1990s and then began to fall significantly from the early 2000s. Viewed in conjunction with the negative relationship between growth and dispersion, shown in Figure 4, this implies that the negative growth shocks identified as affecting the economy through the 1980s and 1990s had the additional consequence of causing regional income disparities to rise. Similarly, the absence of adverse growth shocks in the 2000s, apart from the shock at the end of the sample in 2009, would appear to have contributed to an associated reduction in regional disparities.

[Figure 4 about here]

[Figure 5 about here]

Our results concerning regional disparities are consistent with the findings of authors such as Rodriguez and Sanchez 2002; Esquivel and Messmacher 2002; Aguayo Téllez 2004; Rodríguez Orregia 2005; Chiquiar 2005; González Rivas 2007. These studies point to increasing regional disparities during the period 1985-2001 as a consequence of the entry of

Mexico to the GATT in 1984 and later to the NAFTA in 1994, partly due to the greater economic benefits enjoyed by the border states and some of the states of the central region, in comparison with the rest of the country. It is argued that these states developed an export-oriented industrial sector, and achieved consequently higher growth relative to other states, particularly because of closer integration with the US economy, but this also had the effect of increasing the income gap with the poorer states, particularly in South Mexico. Although our results generally support this argument, as Figure 5 suggests, from the early 2000s a new process of reduced regional income disparities seems to emerge. We have argued that this move towards reduced disparities is a consequence of the return to a more stable period of higher national growth. However, further work is needed to understand the details of this process and how it is related to broader national and international developments affecting Mexico.

## **6. Summary and Conclusions**

In this paper we have examined the regional structure of output growth in the Mexican economy and the extent to which adjustments in the national growth path are reflected in the regional and state growth paths. Our results suggest that the path of output growth across the economy has been dominated by a series of domestic and international shocks rather than the pattern of cyclical persistence observed in many developed economies. The results suggest that the main exogenous factors affecting the growth rate are the growth path of the neighboring USA and, most significantly, a series of one-off shocks associated with major international events, particularly associated with the Tequila crisis and the recent global recession. With the exception of a lasting (negative) step-change in the growth rate in the early 1980s, the results suggest that the exogenous shocks have exerted severe but only transitory impacts on the growth rate. The results for the broad regions suggest a high degree of similarity in the growth behavior of North and Central Mexico. The pattern for South

Mexico is more distinct and no association with the US growth is identified. A closer examination of the individual states suggests a much more diverse pattern of growth adjustments than the results for the broad regions might suggest. An important implication derived from our results is the need of further research to better understand not just the factors affecting growth but also the conditions that may trigger regional/state-growth. Also important is to consider differences in the estimation techniques, the variables used in the analysis and the source of the data.

An examination of regional output volatility suggests that there is no evidence to support the view that the Mexican economy experienced a reduction in the volatility of the noise shocks affecting output growth, of the kind of reduction identified for the USA and other developed economies. However, a more detailed examination of the individual states suggests that there is a positive relationship between the level of the growth rate and its volatility, with the faster growing states exhibiting greater volatility.

Our analysis of the individual states also suggests that there is a weak but significant relationship between growth and regional income dispersion, and that higher growth has generally been associated with lower regional disparities. Given the weak growth performance of the economy over much of the 1980s and 1990s, it is therefore not surprising that an increase in regional income disparities occurred over this period. The improved growth performance of the 2000s can similarly be regarded as a factor helping to reduce regional income disparities. Further research to better understand the association between growth and inequality across Mexican states and potential factors affecting this relationship (e.g. fiscal transfers, agglomeration economies and natural resource endowments) is needed for the design and implementation of growth policy measures according to state needs.

## End notes

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<sup>2</sup> The series for Mexican real GDP per state are based on the methodology described in Mendoza (1997). The regional data is available only on an annual basis and so we are not able to conduct the analysis on a quarterly basis. The series for US real GDP was obtained from the Federal Reserve Bank of St Louis FRED data bank. The computations reported in the paper were undertaken in Matlab 7.

<sup>3</sup> See for example Engle and Kozicki, 1993; Backus, Kehoe and Kydland, 1995; Christodoulakis, Dimelis and Kollintzas, 1995; Hess and Shin, 1997; Hodrick and Prescott, 1997; Artis and Zhang, 1999; Shepherd and Dixon, 2008

<sup>4</sup> There is considerable uncertainty about the nature of the trend process in macroeconomic data and other processes, such as segmented linear trends are potentially admissible. Because of space limitations, we avoid any consideration of this issue and adopt the random walk trend assumption.

<sup>5</sup> A useful review of the earlier shocks affecting the Mexican economy through the 1980s and the 1990s can be found in Gould (1995).

<sup>6</sup> Although we do not impose cointegration in the regional estimates, we did allow for the possibility of cyclical interactions between the series (via the autoregressive terms) by estimating a three-region VAR model in the series differences. As for the national model, however, no significant autoregressive components are identified in any of the series, which implies that univariate ARX models are applicable.

<sup>7</sup> The period of reduced volatility came to an end with the onset of the global recession in 2008 and it remains to be seen what will happen to growth volatility after (if) a full recovery from the recession occurs.

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<sup>8</sup> The table of critical values reported by Inclan and Tiao does not actually include a sample size as small as the one used here. However, extrapolating from the reported critical values, it seems unlikely that the test value calculated for this sample indicates a significant change in variance. At the 5% significance level, the critical value of the Inclan and Tiao test falls from an asymptotic value of 1.35 to values of 1.30 and 1.27 for sample sizes of 200 and 100 respectively. In the present case, with a sample size of 40, the maximum test value is 0.91.

<sup>9</sup> For example, it has been argued that higher volatility implies greater uncertainty, which is detrimental to growth. On the other hand, it may be that there is no causal relationship and that periods of increased volatility are simply those in which disparities tend to rise, because higher (or lower) growth phases tend to be concentrated in a few regions or states.

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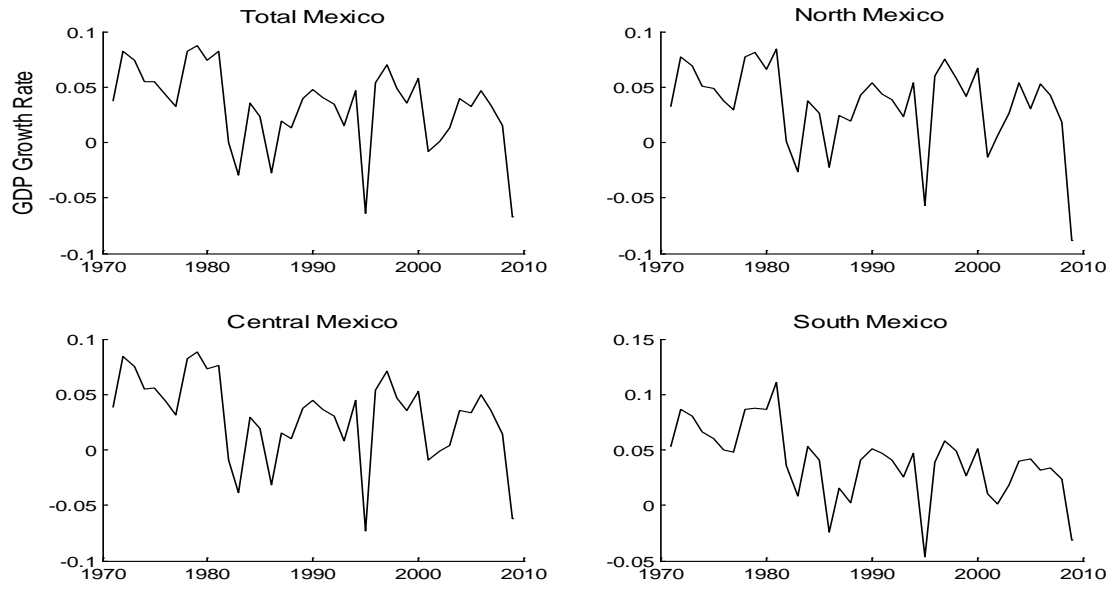
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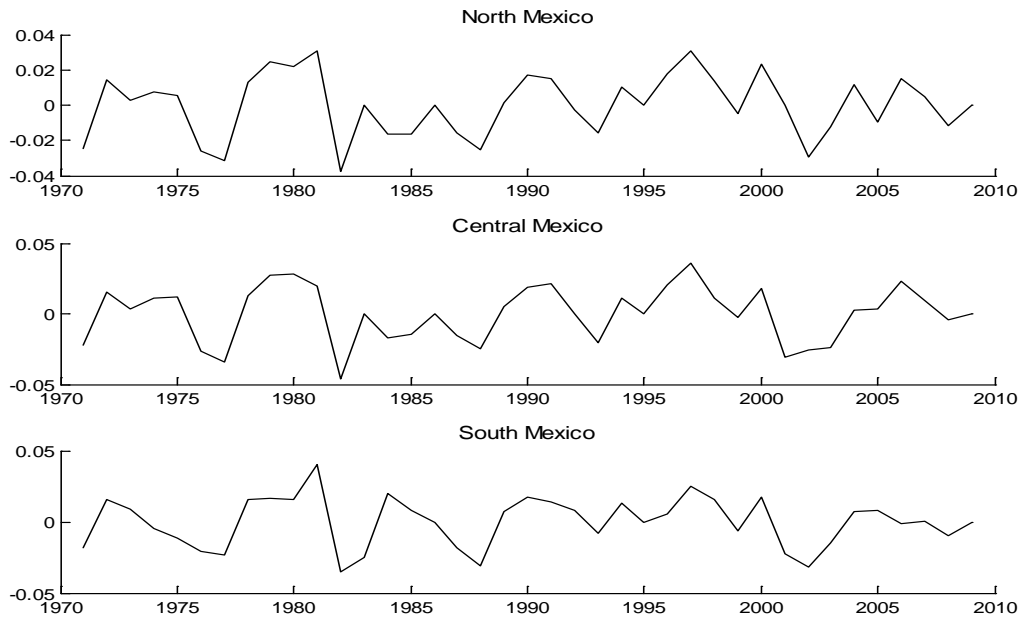
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**Figure 1.**  
**National and Regional GDP Growth in Mexico**  
**(1970-2009)**



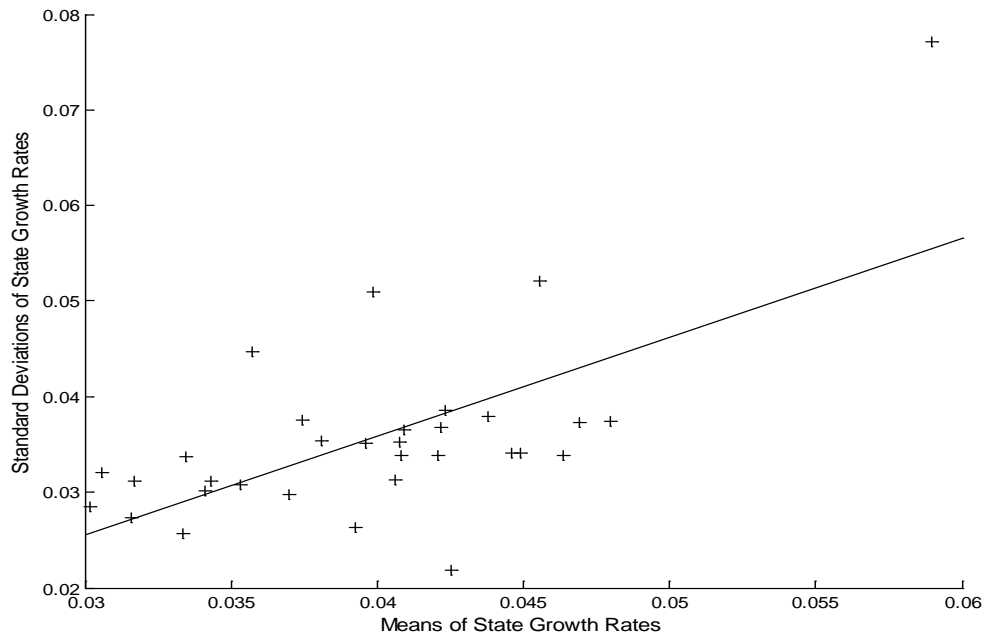
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**Figure 2.**  
**Regional ARX Model Noise Components**



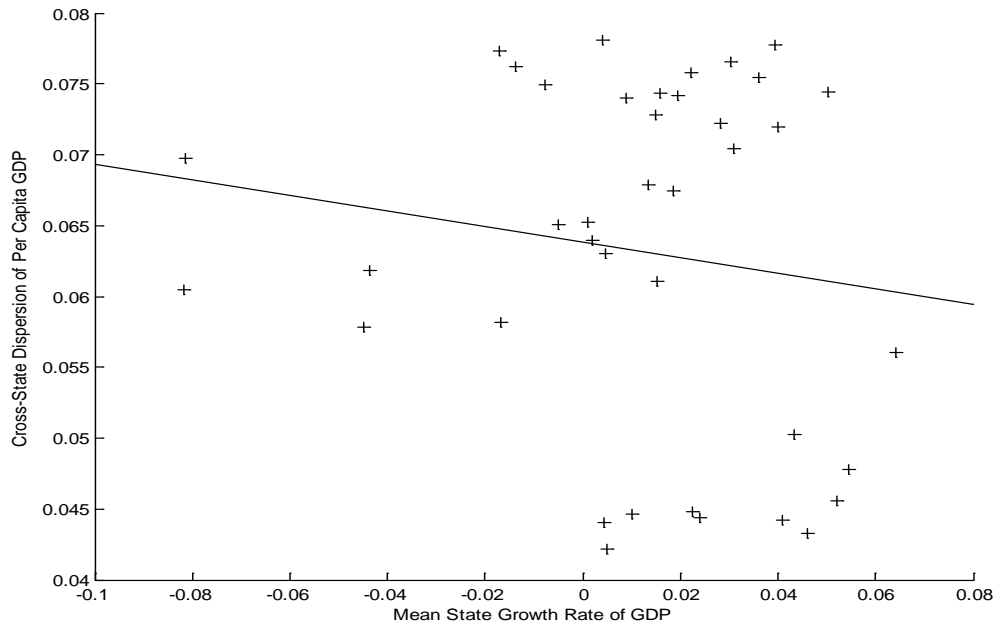
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**Figure 3.**  
**Growth and Volatility Across the States**



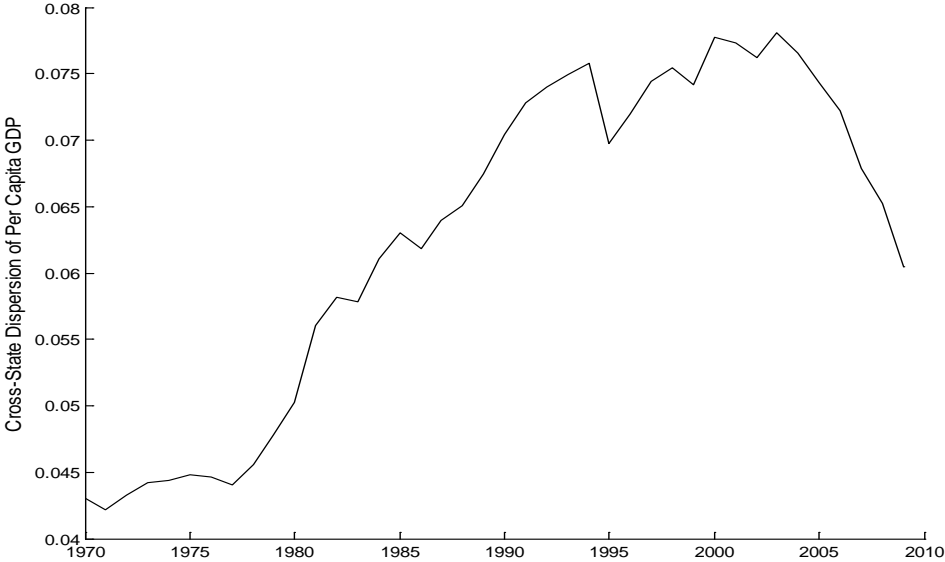
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**Figure 4.**  
**Growth and Regional Dispersion**



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**Figure 5.**  
**The Time Path of Regional Dispersion**



**Table 1.**  
**National Output Growth**  
**ARX Model**

Variables	Model 1		Model 2	
	Coefficient	Prob.	Coefficient	Prob.
$\mu$	0.05	0.00	0.05	0.00
$\Delta US_{t-1}$	0.41	0.03	0.39	0.04
$I_2$	-0.06	0.00	-0.06	0.01
$I_3$	-0.09	0.00	-0.10	0.00
$I_4$	-0.04	0.00	-0.04	0.04
$I_5$	-0.08	0.00	-0.08	0.00
$S_1$	-0.03	0.00	-0.03	0.00
$\Delta Y_{t-1}$			<b>0.06</b>	<b>0.62</b>
R-squared	0.75		0.76	
LM test	0.71	0.50	0.73	0.49
Jarque-Bera test	1.15	0.56	1.10	0.57

Note: Parsimonious model showing significant variables at the 5% probability level. No significant coefficients are in bold text.

**Table 2.**  
**Regional Output Growth**  
**ARX Model**

Variables	North Mexico		Central Mexico		South Mexico	
	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
$\mu$	0.05	0.00	0.05	0.00	0.07	0.00
$\Delta US_{t-1}$	0.4	0.04	0.41	0.04	<b>0.21</b>	<b>0.20</b>
$I_2$	-0.06	0.01	-0.06	0.01	-0.05	0.00
$I_3$	-0.09	0.00	-0.10	0.00	-0.08	0.00
$I_4$	-0.05	0.00				
$I_5$	-0.11	0.00	-0.07	0.00	-0.07	0.00
$S_1$	-0.02	0.00	-0.04	0.00	-0.05	0.00
R-squared	0.74		0.71		0.76	
LM test	0.59	0.56	0.85	0.44	1.89	0.17
Jarque-Bera test	0.83	0.66	1.52	0.47	0.12	0.94

Note: Parsimonious models showing significant variables at the 5% probability level.  
No significant coefficients are in bold text.

**Table 3.**  
**State Impacts of Exogenous Variables**  
**ARX Model**

State/Variable	$\Delta US_{t-1}$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$S_1$	$S_2$	$S_3$	$S_4$
<b>North Mexico</b>	√	√	√	√	√	√	√	$x$	$x$	$x$
Aguascalientes	$x$	√	√	√	$x$	√	$x$	$x$	$x$	√
Baja California	$x$	√	√	√	√	√	$x$	$x$	√	√
Baja Calif. Sur	√	$x$	$x$	$x$	$x$	√	√	√	$x$	$x$
Chihuahua	$x$	√	√	√	√	√	$x$	$x$	√	√
Coahuila	√	√	√	√	√	√	√	$x$	$x$	$x$
Durango	√	$x$	√	√	$x$	$x$	√	$x$	$x$	$x$
Nuevo Leon	√	√	√	√	√	√	√	$x$	$x$	$x$
San L. Potosi	√	√	√	√	√	√	√	$x$	$x$	$x$
Sinaloa	$x$	√	√	√	√	$x$	√	$x$	$x$	$x$
Sonora	$x$	√	√	√	√	$x$	$x$	$x$	$x$	$x$
Tamaulipas	√	$x$	√	√	√	√	√	√	$x$	$x$
Zacatecas	$x$	$x$	$x$	$x$	$x$	√	√	$x$	$x$	$x$
<b>Central Mexico</b>	√	√	√	√	$x$	√	√	$x$	$x$	$x$
Colima	$x$	$x$	√	√	√	√	√	$x$	$x$	$x$
Mexico City	√	√	√	√	$x$	√	√	$x$	$x$	$x$
Guanajuato	$x$	√	√	√	$x$	√	√	$x$	√	√
Guerrero	√	√	$x$	√	$x$	$x$	√	$x$	$x$	$x$
Hidalgo	√	√	$x$	√	$x$	$x$	√	$x$	$x$	$x$
Jalisco	√	√	√	√	$x$	√	√	$x$	√	√
Mexico State	√	√	√	√	$x$	√	√	$x$	$x$	$x$
Michoacán	√	√	$x$	√	$x$	√	√	$x$	$x$	$x$
Morelos	$x$	√	√	√	$x$	√	$x$	$x$	$x$	√
Nayarit	√	$x$	$x$	√	$x$	$x$	√	$x$	$x$	$x$
Puebla	$x$	√	√	√	$x$	√	√	$x$	√	√
Queretaro	$x$	√	√	√	√	√	√	$x$	$x$	$x$
Tlaxcala	$x$	√	√	√	$x$	√	$x$	√	√	√
Veracruz	√	√	$x$	√	$x$	√	√	$x$	$x$	$x$
<b>South Mexico</b>	$x$	$x$	√	√	$x$	√	√	$x$	$x$	$x$
Campeche	$x$	$x$	√	√	$x$	$x$	$x$	$x$	$x$	√
Chiapas	$x$	$x$	√	√	$x$	$x$	$x$	√	√	√
Oaxaca	$x$	√	√	√	√	$x$	$x$	√	$x$	$x$
Quintana Roo	√	$x$	$x$	√	$x$	$x$	√	√	√	$x$
Tabasco	$x$	√	√	√	√	$x$	√	$x$	$x$	$x$
Yucatan	√	$x$	√	√	$x$	$x$	√	√	$x$	$x$



