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Abstract

This paper explores the effects of a government tax policy in a growth model with economic transition and toxic housing bubbles applied to China. Such a policy combines taxing entrepreneurs with a one-time redistribution to workers in the same period. Under the tax policy, we find that the welfare improvement for workers is non-monotonic. In particular, there exists an optimal tax at which social welfare is maximized. Moreover, we consider the welfare effects of setting the tax at its optimum. We show that the tax policy can be welfare-enhancing, compare to the case without active policies. The optimal tax may also yield a higher level of welfare than the case even without housing bubbles. Finally, we calibrate the model to China. Our quantitative results show that the optimal tax rate is about 23 percent, and social welfare is significantly improved with such a tax policy.

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1 Introduction

Over the past decades, the Chinese housing sector has gone through a spectacular boom. In China's top tier cities, which include Beijing, Shanghai, Guangzhou, and Shenzhen, real housing prices have grown at an annual rate of 13.1 percent from 2003 to 2013 (Fang et al. 2015). Housing prices have soared in tier-2, tier-3, and tier-4 Chinese cities as well.[1] Yet the dramatic housing boom has been accompanied by a smaller increase in income: data for 35 major Chinese cities show the average real housing prices grew at 17 percent over the past decade, whereas the average income growth rate was only 11 percent (Chen and Wen 2017). This fact is at odds with the predictions of standard neoclassical model, which requires that housing prices grow at most as fast as aggregate income. If housing prices continue to grow, at a higher rate than that of average income, over time, housing would become increasingly unaffordable for Chinese households, especially for those living in top tier cities (Chen et al. 2018). In fact, for example, as of year 2016, a 90-square-meter apartment in Beijing or Shanghai costs more than 25 times average household income (Glaeser et al. 2017).

The dramatic size of the Chinese housing boom has left the global economic and policy communities a question that whether Chinese real estate is a bubble waiting to burst. Indeed, the fast rise of housing prices is widely viewed as a clear illustration of the dangers associated with speculative bubbles. Despite the housing boom, vacancy rates across Chinese major cities are large and persistent, at around one quarter, hence leading to a phenomenon dubbed as "ghost towns". These suggest the presence of housing bubbles.[2] In the literature, most models of rational bubbles adopt the over-

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[1]China's cities are typically divided into four categories or tiers, based on the level of economic development. See Fang et al. (2015) for a clear classification.

[2]In the small but growing literature studying China's housing boom, it is worth noting that some papers suggest Chinese cities may not experience housing bubbles. For example, Garriga et al. (2017) emphasize the importance of structural transformation and the resulting rural-urban migration in accounting for the upward trend in housing price movements in China. Also, Glaeser et al. (2017) analyze the determinants of demand and supply of housing in China and conclude that a housing

lapping generations framework (Samuelson 1958; Tirole 1985; Weil 1987; Grossman and Yanagawa 1993).[3] In terms of welfare implications, the conventional wisdom is that bubbles are welfare improving because of dynamic inefficiency. However, in terms of anecdotal evidence, bubbles can be potentially costly and are often accompanied with crisis and a sharp drop in household wealth when they burst. In fact, policymakers and researchers are more concerned about the welfare costs of bubbles.[4]

Specific to China's high housing price puzzle, an important paper is by Chen and Wen (2017), who propose a theory to explain the paradoxical housing boom in China–namely real housing prices outpacing income; high vacancies; and a high rate of return to capital. Their model framework is an extension of Song et al. (2011) with an intrinsically valueless housing asset,[5] which is shown to be important to study growing bubbles and to understand China's prolonged paradoxical housing boom. Their simulation results show that the model can quantitatively replicate China's housing price dynamics over the past decade fairly well and still be consistent with many other salient features of the Chinese economy. What's more, unlike many traditional bubble models where bubbles are welfare-improving, they show that housing bubbles are welfare-reducing and toxic. Specifically, in the model, the growing bubble crowds out productive investment, prolongs economic transition, and thus reduces social welfare.

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crash is not inevitable.

[3] It is well known that rational asset bubbles cannot arise in the simple infinite-horizon model because the transversality condition rules out exploding asset prices path (Santos and Woodford 1997). However, recent studies show that bubble can exist in the infinite-horizon economy under certain conditions, such as introducing trading frictions or borrowing constraints. Notable examples include Kocherlakota (1992, 2008), Caballero and Krishnamurthy (2006), Hellwig and Lorenzoni (2009), Farhi and Tirole (2012), Martin and Ventura (2012), Wang and Wen (2012), Miao and Wang (2014), and Hirano and Yanagawa (2017).

[4] In different model environments, potential costs may include volatility, fire sales after the collapse of bubbles (Caballero and Krishnamurthy 2006), and misallocation of resources in the presence of market distortions (Miao et al. 2015).

[5] Song et al. (2011) is an influential paper, which can endogenously generate and quantitatively account for some important features of China's economic transition, such as high output growth, sustained returns on capital, reallocation within the manufacturing sector, and a large trade surplus. Their growth model is therefor widely adopted in studying China's fast growth and economic transition, see, for example, Song et al. (2014), Chang et al. (2015), and Chen and Wen (2017).

Since a housing bubble reduces welfare, it is natural to ask what policies can be used to reduce the negative impact of housing bubbles and possibly improve social welfare? We find that a tax policy, which combines taxing entrepreneurs with a one-time redistribution to workers in the same period, may address this issue. The overarching objective of our study is to explore the welfare effects of a government tax policy in a growth model with economic transition and toxic housing bubbles applied to China.

One strand of literature discusses the impact of government policies on affecting bubbles in a classical Samuelson-Tirole bubble model (see Galí 2014 and Miao et al. 2015). For example, in an OLG model with nominal rigidities, Galí (2014) examines the impact of alternative monetary policy rules on a rational asset price bubble. He finds that a systematic increase in interest rates in response to a growing bubble is shown to enhance the fluctuations in the latter, through its positive effect on bubble growth. This calls into question the theoretical foundations of the case for "leaning against the wind" monetary policies. There is another strand of literature on China's housing price puzzle. However, most the theoretical works along this line focus on why the housing price level is so high (see Zhao 2015 and Garriga et al. 2017), instead of asking why housing prices have been able to grow faster than income.[6] Our analysis forms a bridge between the two strands of studies. To the best of our knowledge, our paper is the first to design and study the impact of government policies on reducing China's housing bubbles and improving welfare.

Specifically, we first show that the worker's lifetime utility is non-monotonically increasing with the tax policy. In particular, there exists a closed-form optimal tax rate, at which worker's welfare is maximized. This non-monotonic relationship between welfare and the tax rate is due to a trade-off effect of the tax policy. On the one hand,

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[6]At the same time, it is worth noting that there is a growing empirical literature on housing prices in China. For instance, see Wang (2011), Wu et al. (2012), Wang and Zhang (2014), Wu et al. (2014), Huang et al. (2015), Wu et al. (2015), and Wu et al. (2016), among others.

a positive tax rate increases workers' welfare as they receive real resources transferred to consumption every period. However, such a tax policy also implies a fall in capital accumulation, resulting in a decrease of productive resources. Second, we define social welfare as the sum of entrepreneur's and worker's lifetime utilities. Our results show that there exists an optimal tax rate, at which social welfare is maximized.

While it is immediately obvious that such a tax regime would reduce entrepreneur's welfare but improving worker's welfare, if the losses of entrepreneurs are dominated by the gains of workers, an active tax policy can improve social welfare by making the housing bubble less toxic to workers. We show analytically that under certain conditions, the tax policy can be welfare-enhancing, compare to the case without active policies. The optimal tax policy may also yield a higher level of welfare than the case even without housing bubbles. Finally, we calibrate the model to Chinese economy. Our quantitative results show that, under a large range of parameter values, the optimal tax rate changes from 21 percent to 23 percent. In addition and more importantly, the welfare gains by workers are quantitatively large whereas the welfare losses by entrepreneurs are relatively small. There are therefore always welfare gains when the tax policy is active.

Our paper is organized as follows. Section 2 describes a simple two-period growth model and characterizes the equilibrium, as well as the model's qualitative implications. Section 3 investigates the welfare implications of an explicit government tax policy, both on workers and entrepreneurs. Section 4 calibrates the model to China, and discusses quantitative implications of the tax policy. Section 5 concludes with remarks for further research. The Appendix contains proofs of all Lemmas and Propositions.

2 The benchmark model

This section describes our model economy and discusses its key assumptions. Our purpose is to study the Chinese housing market and the effects of an active tax policy on social welfare. Such a policy combines taxing entrepreneurs with a one-time redistribution in the same period. The redistribution is implemented by a lump-sum tax on entrepreneurs who are the marginal investors in the housing market and a lump-sum transfer to workers who are adversely affected by the housing bubbles.

Our analyses are based on the model of Chen and Wen (2017), which is consistent with the institutional background and stylized empirical facts about China and its housing market behavior. In particular, the model framework is an extension of Song et al. (2011) with an intrinsically valueless housing asset. For simplicity, the model excludes low-income households (workers) from the housing market because their participation has only a level effect but no growth effects on the housing prices. The housing bubbles are growing and more importantly, toxic, in that a housing bubble reduces aggregate consumption and the welfare of both entrepreneurs and workers. This market failure warrants policy interventions.

The model economy is populated by overlapping generations (OLG) of two-period lived agents who work in the first period and live off savings in the second period. Agents have heterogeneous skills. In each cohort, half of the population consists of workers without entrepreneurial skills and the other half consists of entrepreneurs. Entrepreneurial skills are inherited from parents; for simplification and without loss of generality, we do not allow transition between social classes. The total population, N_t, grows at an exogenous rate v; hence, $N_{t+1} = (1 + v)N_t$.

2.1 Technology

There are two production sectors and thus two types of firms, both requiring capital and labor. Labor is perfectly mobile across the two sectors but capital is not. The first sector consists of financially integrated neoclassical firms, F-firms, which, for simplicity, are owned by a representative financial intermediary (e.g., a state-owned bank).

The second sector is a newly emerging private sector composed of unconventional entrepreneurial firms, E-firms, operated by entrepreneurs. More specifically, E-firms are owned by old (parent) entrepreneurs, who are residual claimants on profits, and they hire their own children as managers. Workers can choose to work for either type of firm.

The key assumptions are that E-firms are more productive than F-firms but, due to asymmetric financial imperfections, E-firms are borrowing constrained—they cannot borrow from each other or from any other sources. As a result, E-firms must self-finance capital investment through their own savings. In contrast, F-firms can rent capital from their representative financial intermediary at a fixed interest rate. Accordingly, F-firms can survive in the short run despite inferior technology. Over time, however, labor will gradually reallocate from F-firms to E-firms as the capital stock of E-firms expands. Thus, the economy features a transition stage during which F-firms and E-firms coexist, but the F-sector is shrinking and the E-sector is expanding. When the transition ends, only E-firms exist and the economy becomes a representative-agent growth model with neoclassical features.[7] Matching the model specification to China, it is obvious that F-firms can be interpreted as state-owned enterprises, while E-firms are private firms.

The technology of F-and E-firms are described, respectively, by the following pro-

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[7]Note that, following Song et al. (2011) and Chen and Wen (2017), the term "transition" is different from the convention in the neoclassical growth model, where transition means the dynamic path from an initial state towards the steady state. In our paper, this conventional transition phase shows up after the F-sector disappears. To avoid confusion, we call this neoclassical transition period "post-transition" stage.

duction functions:

$$y_t^F = (k_t^F)^\alpha (A_t n_t^F)^{1-\alpha}, y_t^E = (k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha}$$

where y, k, and n denote per capita output, capital stock, and labor, respectively. The parameter $\chi > 1$ captures the assumption that E-firms are more productive than F-firms. Technological growth in both sectors is constant and exogenous, given by $A_{t+1} = (1+z)A_t$.

2.2   Worker's problem

We now analyze agents' savings problem. Young workers earn a wage w and deposit their savings with the representative bank, receiving a gross interest rate R. In addition, with an active tax policy, the tax revenue T is transferred in a lump-sum manner to the current young for consumption. Without loss of generality, we also assume that workers do not speculate in the housing market, since allowing workers to invest in housing market does not change our main results.[8]

The worker's consumption-saving problem is:

$$\max \log c_{1t}^w + \beta \log c_{2t+1}^w$$

subject to $c_{1t}^w + s_t^w = w_t + T_t$ and $c_{2t+1}^w = s_t^w R$, where w_t is the market wage rate; c_{1t}^w, c_{2t+1}^w, and s_t^w denote consumption when young, consumption when old, and the worker's savings, respectively. This yields the optimal savings $s_t^w = \zeta^w(w_t + T_t)$, where $\zeta^w \equiv (1 + \beta^{-1})^{-1}$.

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[8]Indeed, in the quantitative analysis section later, we also consider the case where workers are allowed to speculate in the housing markets. We find that the growth rate of housing prices is unaffected. This is because of the equilibrium growth rate of housing prices in the model is determined by the rate of return to capital of the entrepreneurs, who act as the marginal investors in the "bubbly equilibrium".

2.3 The F-firm's problem

In each period, an F-firm maximizes profits by solving the following problem:

$$\max_{k_t^F, n_t^F} (k_t^F)^\alpha (A_t n_t^F)^{1-\alpha} - w_t n_t^F - R k_t^F$$

Profit maximization implies that R equals the marginal product of capital and that wages equal the marginal product of labor:

$$w_t = (1-\alpha)(\frac{\alpha}{R})^{\frac{\alpha}{1-\alpha}} A_t \tag{1}$$

Note that during the transition, wages per effective unit of labor, w_t/A_t, are constant due to a constant rental rate for capital and, accordingly, a constant capital-to-labor ratio, $(k_t^F/A_t n_t^F) = (\alpha/R)^{\frac{1}{1-\alpha}}$.

2.4 The E-firm's problem

Following Song et al. (2011), we assume that E-firms must hire a manager (i.e. the young entrepreneur) and pay him a fixed $\psi < 1$ fraction of the output produced as a compensation m_t, in order to satisfy an incentive-compatibility constraint.[9] The incentive constraint is important, because in its absence managers would be paid the workers' wage, and the equilibrium would feature no capital accumulation in E-firms and no transition from F-firms to E-firms. In addition, under an active tax policy, the E-firm, owned by an old entrepreneur, must pay a tax τ. Such a tax can be interpreted as various government policies that restrict the growth of private firms (e.g., entry

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[9]The managerial compensation must also exceed the workers' wage rate, i.e. $m_t > w_t$. We restrict attention to parameters and initial conditions such that the participation constraint is never binding in equilibrium.

barriers for private firms into "strategic" industries and a higher tax burden).[10] Since, in reality, tax policies affect the overall profitability of private firms, we assume that such a policy also applies to young entrepreneurs' managerial compensation. Accordingly, the lump-sum transfer to workers T_t is given by $T_t = \tau (k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha}$. The old entrepreneur's problem can be written as:

$$\max_{m_t, n_t^E} (1 - \tau)(k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha} - m_t - w_t n_t^E, \tag{2}$$

subject to the incentive constraint that $m_t \geq (1 - \tau)\psi(k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha}$, and arbitrage in the labor market implies that the wage is as in (1). The optimal contract implies that the incentive constraint is binding:

$$m_t = (1 - \tau)\psi(k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha} \tag{3}$$

Taking the first-order condition with respect to n_t^E and substituting in the equilibrium wage yield optimal level of employment as:

$$n_t^E = [(1 - \tau)(1 - \psi)\chi]^{\frac{1}{\alpha}} \left(\frac{R}{\alpha}\right)^{\frac{1}{1-\alpha}} \frac{k_t^E}{A_t \chi} \tag{4}$$

Plugging Equations (4) and (3) into Equation (2) yields the value of the firm:

$$\pi(k_t^E) = [(1 - \tau)(1 - \psi)]^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R k_t^E \equiv \rho_t^E k_t^E$$

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[10] As noted by Chen and Wen (2017), the Chinese government increases the tax rate of private firms over time. For example, in 2007, the state government issued a document (the 39th Decree), which requests a transition from preferential corporate income tax rates to legal tax rates. Accordingly, those who enjoyed a 15 percent corporate income tax rate before 2008 would face gradually progressive tax rates of 18 percent, 20 percent, 22 percent, 24 percent, and 25 percent for each year between 2008 and 2012, respectively. In this OLG framework, we interpret it as a one-off lump-sum tax accompanied by a one-time redistribution.

where ρ^E is the E-firm rate of return to capital. Following Song et al. (2011), we impose the following assumption about E-firms' relative productivity, such that an entrepreneur's return to capital is higher than the deposit rate, R, during the transition.

Assumption 1: $\chi > (1 - \psi)^{-\frac{1}{1-\alpha}}$

Given this assumption, young entrepreneurs would find it optimal to invest in the family business. If Assumption 1 were not satisfied, there would be no E-firms in equilibrium. Thus, a sufficiently large productivity difference is necessary to trigger economic transition.

2.5   The young entrepreneur's problem

As in Chen and Wen (2017), the young entrepreneur decides on consumption and portfolio allocations in housing investment, bank deposits, or physical capital investment. The rate of return to capital investment is simply ρ_t^E. We assume that the balanced growth rate, which equals the rate of return to housing investment at the steady state, is greater than the bank deposit rate—that is, $(1 + z)(1 + v) > R$. As a result, the entrepreneur will always prefer investing in housing to depositing funds in the bank. Given housing prices, P_t^H, the young entrepreneur faces a two-stage problem.

In the first stage, a young entrepreneur's consumption-saving problem is:

$$\max_{s_t^E} \log(m_t - s_t^E) + \beta \log R_{t+1}^E s_t^E$$

where $R_{t+1}^E \equiv \max\{\rho^E, P_{t+1}^H/P_t^H\}$ is the rate of return for the entrepreneur's savings and depends on the entrepreneur's portfolio choices. First-order conditions give the optimal savings of the young entrepreneur as $s_t^E = \frac{m_t}{(1+\beta^{-1})}$.

In the second stage, the young entrepreneur chooses portfolio allocations, given total savings, s_t^E. The fraction ϕ_t^E of savings is invested in capital, such that $K_{t+1}^E =$

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$\phi_t^E s_t^E N_t^E$, where $K_{t+1}^E = k_{t+1}^E N_{t+1}^E$ is total E-firm capital. The remaining $(1 - \phi_t^E)$ fraction of savings is invested in housing, such that $P_t^H H_t^E = (1 - \phi_t^E)s_t^E N_t^E$, where H_t^E is the total housing stock purchased by young entrepreneurs in period t. In addition, we ensure that there exists an interior solution for the portfolio choice, such that the following no-arbitrage condition holds:

$$\frac{P_{t+1}^H}{P_t^H} = \rho_{t+1}^E,$$

where $\rho_{t+1}^E = \rho^E$ is constant during the transition. Therefore, an old entrepreneur's income is simply $\rho^E s_t^E$. The above condition states that the entrepreneur's rate of return to housing and rate of return to capital must be equal in a bubbly equilibrium. That is, the young entrepreneur is indifferent between investing in the capital stock or in the housing asset. In addition, we assume that for each period the bank simply absorbs deposits from young workers, lends out to F-firms at interest rate R, and then invests the rest in foreign bonds with the same rate of return R, as in Song et al. (2011).

2.6   Law of motion

We now characterize the equilibrium dynamics during a transition in which there is positive employment in both E- and F-firms. There are two state variables in this model: K_t^E and A_t. Since the E-firm is self-financed, the law of motion for E-firm capital stock follows:

$$K_{t+1}^E = \phi_t^E \frac{\rho_t^E \psi}{(1 - \psi)\alpha} \frac{1}{1 + \beta^{-1}} K_t^E, \tag{5}$$

where $\rho_t^E = [(1 - \tau)(1 - \psi)]^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R$ for all periods during the transition stage. The entrepreneur's portfolio share in physical capital, ϕ_t^E, is assumed to be constant, which

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together with a constant ρ^E, implies that the growth rate of E-firm capital is constant during the transition. Similarly, the law of motion for housing demand is:

$$P_t^H \bar{H} = (1 - \phi_t^E) \frac{\rho_t^E \psi}{(1 - \psi)\alpha} \frac{1}{1 + \beta^{-1}} K_t^E, \tag{6}$$

where we have used the housing market-clearing condition, $H_t^E = \bar{H}$. Here, we assume that the housing supply is fixed, which reflects the fact that land available for home construction in China is strictly controlled by the government.

2.7 Post-transition equilibrium and the steady state

Once the transition is completed, F-firms disappear and all workers are employed by E-firms. Thereafter, the theory predicts standard OLG-model dynamics. Since $n_t^E = 1$, E-firm profit is:

$$\pi(k_t^E) = \alpha(1 - \tau)(1 - \psi)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}, \tag{7}$$

and the rate of return to E-firm capital is simply $\rho_t^E = \alpha(1-\tau)(1-\psi)(k_t^E)^{\alpha-1}(A_t\chi)^{1-\alpha}$.

The steady state of the economy is reached only in the post-transition stage. Since all per capita variables (except labor inputs and housing) grow at the rate A_t, we detrend them as $\hat{x}_t = x_t/A_t$. At the steady state, the law of motion for capital (5) implies:

$$\hat{k}^{E*} = \left[\frac{\psi \phi^{E*}(1 - \tau)\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + \upsilon)} \right]^{\frac{1}{1-\alpha}}. \tag{8}$$

Given that $\rho^{E*} = \alpha(1 - \tau)(1 - \psi)(\hat{k}^{E*}/\chi)^{\alpha-1}$, we have

$$\rho^{E*} = \alpha \frac{(1 - \psi)}{\psi} \frac{(1 + \beta^{-1})(1 + z)(1 + \upsilon)}{\phi^{E*}} \tag{9}$$

The equilibrium portfolio allocation ϕ^{E*} is solved by the no-arbitrage condition.

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Since the supply of housing is fixed, the growth rate of housing prices, denoted as $\rho_{t+1}^E = P_{t+1}^H / P_t^H$, equals the balanced growth rate, $(1 + z)(1 + v)$, in the steady state. As a result, the no-arbitrage condition implies that the E-firm steady-state portfolio share in physical capital is:

$$\phi^{E*} = \alpha(1 - \psi)(1 + \beta^{-1})/\psi$$

In addition, we follow Chen and Wen (2017) and impose the parameter restrictions to ensure the existence of housing bubbles.

Assumption 2: $\psi > \alpha(1 + \beta^{-1})/[1 + \alpha(1 + \beta^{-1})]$

This assumption implies that $\phi^{E*} < 1$, that is, a housing bubble exists in the equilibrium.

Assumption 3: $\psi < \alpha(1 + \beta^{-1})[\psi + \alpha(1 - \psi)]$

As shown by Chen and Wen (2017), if Assumption 2 and 3 are both satisfied, a housing bubble is toxic (welfare-reducing) in that it reduces aggregate consumption and the welfare of both entrepreneurs and workers. For entrepreneurs, in addition to foregone returns to capital, entrepreneurial housing investment reduces the lifetime income of future entrepreneurs and, thus, negatively impacts their consumption. For workers, in the post-transition stage, workers' lifetime utility decreases as a result of the housing bubble. This is because workers' wage income starts to depend positively on E-firm capital stock, while the rate of return to savings is still fixed.

3   Welfare analysis

In this section, we study the welfare implications of an explicit government tax policy. Specifically, we first analyze the asymmetric effects of the tax policy on both entrepre-

neurs and workers. We then examine for the existence of an optimal tax rate at which welfare is maximized. In addition and more importantly, we compare social welfare with and without an active tax policy and explore the necessary conditions for which welfare is improved. Let us start with entrepreneur's welfare and worker's welfare at different transition stages.

Lemma 1 summarizes the solutions to entrepreneur's welfare at the transition stage, the post-transition stage, and the steady state equilibrium. As discussed later, the welfare functions are also used to derive the optimal tax rate and explore the conditions under which an optimal policy is welfare-enhancing. All the proofs of lemmas and propositions are included in the Appendix.

Lemma 1 *Given $\alpha, \beta, \chi, \psi, \tau$, and k_t^E, A_t, the entrepreneur's lifetime utility function can be expressed as:*

1. For entrepreneurs born during the transition, the lifetime utility is:

$$
\begin{aligned}
& \log(m_t - s_t^E) + \beta \log \rho^E s_t^E \\
= \ & (1+\beta) \log \psi(1-\tau)^{\frac{1}{\alpha}}[(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}}(\frac{R}{\alpha})k_t^E + \beta \log[(1-\tau)(1-\psi)]^{\frac{1}{\alpha}}\chi^{\frac{1-\alpha}{\alpha}}R \\
& - \log(1+\beta) - \beta \log(1+\beta^{-1})
\end{aligned}
$$

2. For entrepreneurs born in the post-transition stage, but before reaching the steady state, the lifetime utility is:

$$
\begin{aligned}
& \log(m_t - s_t^E) + \beta \log \rho_{t+1}^E s_t^E \\
= \ & (\alpha + \alpha^2 \beta) \log k_t^E - (1-\alpha)\beta \log \phi_t^E + (1-\alpha)(1+\beta+\alpha\beta) \log A_t \chi \\
& + \log \frac{\psi(1-\tau)}{(1+\beta)} + \beta \log \frac{\alpha(1-\tau)^{1+\alpha}(1-\psi)\psi^\alpha(1+z)^{1-\alpha}}{(1+\beta^{-1})^\alpha(1+\upsilon)^{\alpha-1}}
\end{aligned}
$$

3. For entrepreneurs born after reaching the steady state, the lifetime utility is (after being detrended):

$$\log(\hat{m}^* - \hat{s}^{E*}) + \beta \log \rho^{E*} \hat{s}^{E*}$$
$$= (1 + \beta) \log \alpha \frac{(1 - \psi)(1 + \beta^{-1})}{\psi}(1 + z)(1 + \upsilon)\left[\frac{\psi(1 - \tau)\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + \upsilon)}\right]^{\frac{1}{1-\alpha}}$$
$$+ \left[\frac{\alpha(1 + \beta)}{1 - \alpha} - \beta\right] \log \phi^{E*} + \log\left(\frac{\psi}{(1 + \beta)\alpha(1 - \psi)}\right) + \beta \log(1 + \upsilon)$$

Note that the rate of return to capital is constant during the transition period, whereas it starts to depend negatively on capital stock when the economy enters the post-transition period. In particular, ρ^E is decreasing during the post-transition stage as capital accumulates; it is falling until the economy enters the steady state equilibrium.

Lemma 2 summarizes the solutions to worker's welfare at the transition stage, the post-transition stage, and the steady state equilibrium.

Lemma 2 *Given $\alpha, \beta, \chi, \psi, \tau$, and k_t^E, A_t, the worker's lifetime utility function can be expressed as:*

1. For workers born during the transition, the lifetime utility is:

$$\log(w_t - s_t^w + T_t) + \beta \log R s_t^w$$
$$= (1 + \beta) \log\left[(1 - \alpha)A_t(\frac{\alpha}{R})^{\frac{\alpha}{1-\alpha}} + \tau[(1 - \tau)(1 - \psi)\chi]^{\frac{1-\alpha}{\alpha}}(\frac{R}{\alpha})k_t^E\right]$$
$$+ \log \frac{1}{1 + \beta} + \beta \log \frac{R}{1 + \beta^{-1}}$$

2. For workers born in the post-transition stage, but before reaching the steady state, the lifetime utility is:

$$\log(w_t - s_t^w + T_t) + \beta \log R s_t^w$$
$$= (1 + \beta) \log\left[(1 - \tau)(1 - \alpha)(1 - \psi) + \tau\chi\right]A_t(k_t^E)^\alpha(A_t\chi)^{-\alpha}$$
$$+ \log \frac{1}{1 + \beta} + \beta \log \frac{R}{1 + \beta^{-1}}$$

3. For workers born after reaching the steady state, the lifetime utility is (after

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being detrended):

$$\log(\hat{w}^* - \hat{s}^{w*} + \hat{T}^*) + \beta \log R\hat{s}^{w*}$$
$$= (1+\beta) \log[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi][\frac{\psi\phi^{E*}(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+v)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha}$$
$$+ \log\frac{1}{1+\beta} + \beta\log\frac{R}{1+\beta^{-1}}$$

Note that the wage rate is constant during the transition when both E-firms and F-firms coexist. Unlike the rate of return to capital, the wage rate starts to depend positively on capital stock in the post-transition period. That is, it increases as E-firm's capital accumulates. Next, it is interesting to explore how such a tax policy affects the growth rate of housing prices at the different stages of the economy.

Lemma 3 *During both the transition and post-transition stages, the tax policy reduces housing bubbles.*

In this model, the housing bubble arises because high capital returns driven by resource reallocation are not sustainable in the long run. Rational expectations of a strong future demand for alternative stores of value can thus induce currently productive agents to speculate in the housing market. During both the transition and post-transition stages, the growth rate of housing prices equals the rate of return of capital by no-arbitrage condition, i.e. $\frac{P_{t+1}^H}{P_t} = \rho_{t+1}^E$. Specifically, during the transition stage, ρ^E is constant, $\rho^E = [(1-\tau)(1-\psi)]^{\frac{1}{\alpha}}\chi^{\frac{1-\alpha}{\alpha}}R$; during the post-transition stage, ρ_t^E is a function of two state variables, k_t^E and A_t, $\rho_t^E = \alpha(1-\tau)(1-\psi)(k_t^E)^{\alpha-1}(A_t\chi)^{1-\alpha}$. In either case, the housing growth rate is decreasing with a positive tax rate τ. We now explore the welfare implications of such a tax policy.

Proposition 1 *Given two state variables k_t^E and A_t, during the transition and post-transition stages, entrepreneur's welfare is monotonically decreasing with the tax policy; however, worker's welfare is non-monotonically increasing with the tax policy. Specif-*

ically, in the transition stage, the welfare-maximizing optimal tax rate $\tau^* = \alpha$; in the post-transition stage, the welfare-maximizing optimal tax rate $\tau^* = \frac{\chi(1-\alpha)-(1-\alpha^{T+1})(1-\alpha)(1-\psi)}{\chi(1-\alpha^{T+1})-(1-\alpha^{T+1})(1-\alpha)(1-\psi)}$, where $T \geq 1$ denotes the number of time periods since the economy enters the post-transition stage.

From Lemma 1, the entrepreneur's welfare is monotonically decreasing with a positive tax rate as the government directly tax both young and old entrepreneurs. For workers, although the welfare improvement with an active policy is positive, there exists a non-monotonic relationship between welfare and the tax rate. This is due to a trade-off effect of the tax policy. On the one hand, a positive tax rate increases workers' welfare as they receive real resources transferred for consumption every period. On the other hand, however, such a tax policy also implies a fall in capital accumulation, thus a decrease of productive resources. In the post-transition stage and steady state equilibrium, worker's wage rate is no longer constant and depends positively on the capital stock whereas their saving benefit is still fixed. Over time, the higher the tax rate, the lower real resources they can receive. Such a conflicting effect suggests that there may exist an optimal tax policy. The tractability of the model allows us to find a closed-form optimal tax rate at which worker's welfare is maximized.

Proposition 2 *In the bubbly steady state, the worker's lifetime utility is non-monotonically increasing with the tax policy. In particular, there exists an optimal tax rate $\tau^* = \frac{\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$, at which worker's welfare is maximized. If we define social welfare as the sum of entrepreneur's and worker's lifetime utilities, then there exists an optimal tax rate $\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$, at which welfare is maximized.*

Let us focus on the welfare effects of an active tax policy in the bubbly steady state. Proposition 2 states that there exists an optimal tax rate for workers, because of the trade-off effect that we highlight above. In addition and more importantly, there also exists an optimum at which social welfare (the sum of entrepreneur's and worker's

lifetime utilities) is maximized. This is still mainly because of the conflicting effect of tax policy on worker's welfare. However, the welfare losses of entrepreneurs have to be taken into consideration when it comes to the optimal tax rate for social welfare. Thus, one would expect a lower optimal tax, as discussed in Corollary 1.

Corollary 1 *The size of the optimal tax rate for social welfare is smaller than that for worker's welfare: $\tau^{**} < \tau^*$.*

If we compare the two optimal tax rates, we find that the size of the optimal tax for social welfare is smaller than that for worker's welfare. This is because taking into account of the welfare losses of entrepreneurs makes one to compensate for them, thus requiring a lower value for the tax rate. This, however, does not affect the existence of an optimal tax rate.

Definition 1 Social welfare is defined as the sum of entrepreneur's and worker's lifetime utilities, which gives $W = u^w + u^E = (\log c_{1t}^w + \beta \log c_{2t+1}^w) + (\log c_{1t}^E + \beta \log c_{2t+1}^E)$.[11]

Assumption 4 $\chi > 2(1 - \psi)$.

From Proposition 2, the optimal tax rate of social welfare is given by $\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$. As the tax rate is positive, we have $\chi > 2(1-\psi)$. Next, it would be interesting to explore whether and under what conditions that the optimal tax policy is welfare enhancing.

Proposition 3 *In the bubbly steady state, setting the tax rate at the optimal level* $\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$ *strictly improves worker's welfare by* $(1+\beta)\log(\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)})^{\frac{\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)}$, *whereas such a tax policy strictly reduces entrepreneur's welfare by* $(1+\beta)\log(\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)})^{\frac{1}{1-\alpha}}$.[12]

With agents having perfect foresight, if they anticipate that in each period the government implements the tax policy of combining an entrepreneurial tax and a lump-sum transfer to workers, they can evaluate the welfare effect of such a policy. Proposition

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[11] Note that we follow the assumption that workers and entrepreneurs are equally populated in the economy, as in Chen and Wen (2017).

[12] Here we focus on the equilibrium with and without the optimal tax policy.

3 essentially states that, if the tax policy is implemented at its optimum, the worker's welfare is strictly improved by $(1+\beta)\log\left(\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)}\right)^{\frac{\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)}$, whereas the entrepreneur's welfare is strictly reduced by $(1+\beta)\log\left(\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)}\right)^{\frac{1}{1-\alpha}}$. As such, if workers anticipate the effects of the tax policy, they would always prefer the tax rate to be set at the optimum.

Proposition 4 *Compare to the equilibrium without the tax policy, setting the tax rate at the optimum $\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$ improves social welfare, if $\left(\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)}\right)^{\frac{1+\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)} > 1$. Compare to the equilibrium even without housing bubbles, setting the tax rate at the optimal level improves social welfare, if $\left(\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)}\right)^{\frac{1+\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)}(\phi^{E*})^{\frac{2\alpha}{1-\alpha}-\frac{\beta}{1+\beta}} > 1$.*

Proposition 4 states the condition under which an optimal tax policy is welfare-enhancing, compare to the equilibrium without an active policy. The economic intuition is as follows. As mentioned above, housing bubbles are toxic (i.e. welfare-reducing) in this model as they reduce aggregate consumption and the welfare of both entrepreneurs and workers. For entrepreneurs, the tax policy is strictly welfare reducing. For workers, however, such a tax policy could compensate to some extent the welfare losses from a declining wage rate. In other words, an active tax policy makes the housing bubble less toxic to workers. If the benefits of workers dominate the costs of entrepreneurs, such a tax policy would improve social welfare. In addition, if the benefits from workers are large enough, it is possible that the optimal tax policy yields a higher level of social welfare than the case even without housing bubbles. In the next section, we quantify the welfare gains (or lack thereof) with an active tax policy, based on realistic parameterization of the Chinese economy.

4 Quantitative analysis

We have focused so far on qualitative predictions of the tax policy. A numerical analysis based on realistic parameters is also interesting, both to quantify the welfare gains (or lack thereof) associated with the tax policy and to give a sense of how the model predictions are sensitive to changes in the different parameters.

4.1 Parameters

Our parameterization largely follows those of Song et al. (2014), who calibrate a similar two-period OLG model for China. Each period in our model corresponds to 30 calendar years. In terms of technology parameters, the capital income share is set $\alpha = 0.5$, consistent with Bai et al. (2006). We assume an annualized world interest rate of 1.75 percent, following Song et al. (2011), that is, $R = (1.0175)^{30}$. The land supply is normalized to unity, $\bar{H} = 1$. On the demand side, β is calibrated to $(0.98)^{30}$ so as to generate a 35.3 percent household savings rate (in the absence of activist policies), which meets the average urban household savings rate in 1998–2012 (Chen and Wen 2017). In addition, ψ and χ are calibrated to match two empirical moments: (i) the capital-output ratio of Chinese F-firms is around three times larger than that of E-firms (average 1998–2005) (Song et al. 2011); (ii) the rate of return to capital is around 18 percent (Chen and Wen 2017). This yields $\psi = 0.625$ and $\chi = 8.24$. Finally, as in Chen and Wen (2017), the rate of labor-augmented technological growth is set to 3.8 percent, that is, $z = (1.038)^{30} - 1$, to meet the average 10 percent growth rate of GDP during 1998–2012. The population growth rate is set to three percent, consistent with the average urban population growth rate in China during the same period, implying that $\upsilon = (1.03)^{30} - 1$.

4.2 Results

We now report the quantitative result of the optimal tax rate, and more importantly, quantify whether setting the tax rate at its optimum improves social welfare. Following Proposition 2, we can calculate the optimal tax rate $\tau^{**} = 23.25\%$, at which social welfare is maximized. That is, the optimal tax policy requires the setting of a 23% tax on entrepreneurs with a one-time redistribution to workers. This policy would of course reduce entrepreneur's welfare while strictly improve worker's welfare, according to Proposition 3. What is of most interest to us is whether such a tax policy improves total welfare, and if so, by how much we can gain compare to the case without active policies.

Proposition 4 suggests that if $\left(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)}\right)^{\frac{1+\alpha}{1-\alpha}} \frac{\frac{1}{2}\chi}{(1-\psi)} > 1$, setting an optimal tax is welfare-improving. Given our calibration, our calculation indicates that, $\left(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)}\right)^{\frac{1+\alpha}{1-\alpha}} \frac{\frac{1}{2}\chi}{(1-\psi)} = 4.9664$, and social welfare is improved by 1.0757 (or in relative term, 19.6%).[13] Specifically, in accordance to Proposition 3, having normalized the initial welfare value to be the same for the two groups, we find a large increase in the level of welfare for workers (by 1.431, or in relative term, 36.6%), whereas the drop in entrepreneur's welfare is relatively moderate (by 0.3553, or in relative term, 22.3%). In addition, we try to evaluate the welfare gains (or losses) with the optimal tax rate, compared to the case even without housing bubbles. Based on Proposition 4, if $\left(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)}\right)^{\frac{1+\alpha}{1-\alpha}} \frac{\frac{1}{2}\chi}{(1-\psi)} (\phi^{E*})^{\frac{2\alpha}{1-\alpha} - \frac{\beta}{1+\beta}} > 1$, the optimal tax policy would be welfare-enhancing. We show that, $\left(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)}\right)^{\frac{1+\alpha}{1-\alpha}} \frac{\frac{1}{2}\chi}{(1-\psi)} (\phi^{E*})^{\frac{2\alpha}{1-\alpha} - \frac{\beta}{1+\beta}} = 3.80$ in this case, and social welfare is improved by 0.896 (or in relative term, 16.8%). In sum, our simple numerical experiments indicate that, using realistic parameter values specific to Chinese economy, a government tax policy can yield a higher level of social welfare,

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[13]The relative welfare improvement might seem large as we are only doing the steady state analysis of a two-period OLG model.

compared to the case without active policies or even without housing bubbles. The relative gains are from workers (albeit small losses for entrepreneurs) for whom such an active tax policy would compensate the welfare losses from toxic housing bubbles and a declining wage income.

4.3 Sensitivity analysis

This section performs sensitivity analysis with regard to some key parameters. As shown in the previous section, most parameter values are standard in the literature or consistent with Chinese data. We are interested in evaluating two parameters, ψ and χ, as they are not explored in the literature despite having important policy implications on social welfare. ψ reflects the managerial cost involved in managing private firms, while χ measures the productivity difference between E-firms and F-firms. Note that in our model, ψ and χ are simultaneously solved for the capital-output ratio of the two types of firms and the rate of return of capital ρ^E. That is, (i) denote κ_F, κ_E as capital to output ratio for F-firms and E-firms, given α, $\kappa_F/\kappa_E = [(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}}$; (ii) given α and R, $\rho^E = (1-\psi)^{\frac{1}{\alpha}}\chi^{\frac{1-\alpha}{\alpha}}R$. In general, F-firms (as state-owned enterprises) have a higher capital-to-output ratio, which means $\kappa_F/\kappa_E > 1$, and ρ^E is also greater than one. In this section, we first set χ at its benchmark value and perform sensitivity analysis with respect to ψ. Then, we set ψ at its benchmark value and perform sensitivity analysis with respect to χ. Results are shown in Table 1 and Table 2, respectively.

In Table 1, we vary the value of ψ from 0.15 to 0.65,[14] while keeping χ constant at its benchmark, and we report also the optimal tax rate and welfare gains, compare to the case without such a tax policy. Our results show that as ψ increases, the optimal tax rate decreases and so do the welfare gains.[15] The optimal tax rate however does

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[14]A value higher than 0.65 would yield a capital return rate ρ^E that is smaller than 1, which means the economy is not having a bubble. These scenarios are therefore not examined.

[15]It is immediately clear that both κ_F/κ_E and ρ^E are negatively related to ψ and positively related

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not change much, ranging from about 21% to 23%. More importantly, we find that there are always welfare gains with an active tax policy. In other words, the lower the managerial cost involved, the more room the economy gains in terms of social welfare.

Similar results hold when we vary the value of χ from 7 to 12,[16] as shown in Table 2. Both the optimal tax rate and welfare are increased with a higher χ, and the welfare gains are always positive in the range. In essence, these mean the larger the initial productivity difference between the two types of firms, the higher the optimal tax rate is, though this has no effect on the welfare-enhancing property of the one-off tax policy. The above experiments imply that, under the tax policy, the benefits for workers are generally large enough to compensate the losses of entrepreneurs, which in turn yields a higher level of social welfare.

Indeed, we also explore a case where workers are allowed to speculate in the housing market. This involves assuming the workers to put a fixed fraction of their savings, ϱ, in the housing market and receive the same housing returns as those of the entrepreneurs, ρ_t^E. In this instance, in the second period, on top of the interest return, R, workers receive also the housing returns, and therefore have second-period consumption as $c_{2t+1}^w = s_t^w[(1 - \varrho)R + \varrho\rho_t^E]$.[17] While it turns out that this specific case does not allow for the analytical derivation of a closed-form optimal tax rate as in Proposition 1, we verified that Propositions 2-4 remain largely true, in that, setting the tax rate at the optimal level τ^{**} strictly improves social welfare in the bubbly steady state, despite the negative effect on housing speculations.[18]

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to χ.

[16]A value smaller than 7 would yield a capital return rate ρ^E small than 1.

[17]In fact, given that $\rho_t^E > R$, workers will always invest in housing up to ϱ share of their savings.

[18]Derivations of this experiment are available upon request.

5 Concluding remarks

Housing prices have soared in Chinese cities over the last decades, which raises growing concerns across the world regarding the housing bubbles. Based on a growth model with economic transition and toxic housing bubbles applied to China, this paper designs and studies the welfare implications of an active government tax policy designed to curb housing bubbles. Such a policy combines taxing entrepreneurs with a one-time redistribution to workers in the same period. The aim of the study is to explore whether and how this tax policy can reduce the negative effects of a bubble and improve social welfare.

Under the tax policy, we find that the welfare improvement for workers is non-monotonic. In particular, there exists an optimal tax rate at which worker's welfare is maximized. Defining social welfare as the sum of entrepreneur's and worker's lifetime utilities, we also find a closed-form tax at which social welfare is maximized. Moreover, we consider the welfare effects of setting the tax at its optimum. We show that the tax policy can be welfare-enhancing, compared to the case without active policies. The optimal tax may also yield a higher level of welfare than the case even without housing bubbles. Finally, we calibrate the model to China. Our quantitative results show that the optimal tax rate is about 23 percent, and social welfare is significantly improved with such a tax policy. To the best of our knowledge, our paper is the first to investigate the impact of government policies on reducing the negative effects of housing bubbles and improve social welfare in a model environment with Chinese characteristics.

It is also worth highlighting some directions in which our analysis can be extended. For tractability, the model has focused only on the demand side of housing market, and in particular, speculative motives. It however ignores the power of supply factors over housing prices. Physical construction costs and the price of land may have significant

influence on shaping housing prices (Glaeser 2013). According to Glaeser et al. (2017), supply remains elastic in many Chinese cities, especially outside the first tier cities. Thus, a complete account for both demand and supply factors may improve our understandings of the great real estate boom in China. In addition, our paper considers only a simple tax policy. For future research, however, it would be interesting to analyze other types of government policies which are actively used in numerous countries, such as regulation on speculation (Hirano and Yanagawa 2017), loan-to-value (LTV) policy (Miao et al. 2015), home-purchase restriction (Du and Zhang 2015), and leverage and collateral restriction (Zhao 2015; Ikeda and Phan 2016). These are all important issues for our future research.

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Table 1

Sensitivity analysis with respect to ψ

	κ_F/κ_E	ρ^E	optimal tax rate	welfare gains
ψ				
0.150	7.0040	6.0576	0.2092	0.5868
0.250	6.1800	4.7161	0.2142	0.6579
0.350	5.3560	3.5423	0.2192	0.7412
0.450	4.5320	2.5362	0.2241	0.8407
0.550	3.7080	1.6978	0.2289	0.9627
0.625 (the benchmark)	3.0900	1.1790	0.2325	1.0757
0.650	2.8840	1.0271	0.2337	1.1189

Source: authors' calculations

Table 2

Sensitivity analysis with respect to χ

	κ_F/κ_E	ρ^E	optimal tax rate	welfare gains
χ				
7.00	2.6250	1.0016	0.2294	0.9746
8.00	3.0000	1.1447	0.2320	1.0573
8.24 (the benchmark)	3.0900	1.1790	0.2325	1.0757
9.00	3.3750	1.2878	0.2340	1.1310
10.00	3.7500	1.4309	0.2357	1.1974
11.00	4.1250	1.5739	0.2370	1.2579
12.00	4.5000	1.7170	0.2381	1.3134

Source: authors' calculations

1 Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1

For entrepreneurs born during the transition, the lifetime utility function is:

$$
\begin{aligned}
& \log(m_t - s_t^E) + \beta \log \rho^E s_t^E \\
= \ & \log(m_t - \frac{m_t}{1+\beta^{-1}}) + \beta \log(\frac{m_t}{1+\beta^{-1}}) + \beta \log \rho^E \\
= \ & \log(\frac{m_t}{1+\beta}) + \beta \log(\frac{m_t}{1+\beta^{-1}}) + \beta \log \rho^E \\
= \ & (1+\beta) \log m_t + \beta \log \rho^E - \log(1+\beta) - \beta \log(1+\beta^{-1}) \\
= \ & (1+\beta) \log \psi(1-\tau)(k_t^E)^\alpha (A_t \chi n_t^E)^{1-\alpha} + \beta \log \rho^E - \log(1+\beta) - \beta \log(1+\beta^{-1}) \\
= \ & (1+\beta) \log \psi(1-\tau)^{\frac{1}{\alpha}} [(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}} (\frac{R}{\alpha}) k_t^E + \beta \log[(1-\tau)(1-\psi)]^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R \\
& - \log(1+\beta) - \beta \log(1+\beta^{-1})
\end{aligned}
$$

For entrepreneurs born in the post-transition stage, but before reaching the steady state, the lifetime utility function is:

$$
\begin{aligned}
& \log(m_t - s_t^E) + \beta \log \rho_{t+1}^E s_t^E \\
= \ & \log(\frac{\psi(1-\tau)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}}{(1+\beta)}) + \beta \log \frac{\alpha(1-\tau)(1-\psi)(k_{t+1}^E)^\alpha (A_{t+1}\chi)^{1-\alpha}(1+v)}{\phi_t^E} \\
= \ & \log(\frac{\psi(1-\tau)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}}{(1+\beta)}) + \beta \log \frac{\alpha(1-\tau)(1-\psi)[\frac{\phi_t^E \rho_t^E \psi k_t^E}{(1-\psi)\alpha(1+\beta^{-1})(1+v)}]^\alpha (A_t(1+z)\chi)^{1-\alpha}(1+v)}{\phi_t^E} \\
= \ & \log(\frac{\psi(1-\tau)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}}{(1+\beta)}) \\
& +\beta \log \frac{\alpha(1-\tau)(1-\psi)[\frac{\phi_t^E \psi(1-\tau)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}}{(1+\beta^{-1})(1+v)}]^\alpha (A_t(1+z)\chi)^{1-\alpha}(1+v)}{\phi_t^E} \\
= \ & \log(\frac{\psi(1-\tau)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}}{(1+\beta)}) \\
& +\beta \log \frac{\alpha(1-\tau)^{1+\alpha}(1-\psi)\psi^\alpha (\phi_t^E)^{\alpha-1}(k_t^E)^{\alpha^2}(A_t \chi)^{\alpha-\alpha^2}(A_t(1+z)\chi)^{1-\alpha}(1+v)}{[(1+\beta^{-1})(1+v)]^\alpha} \\
= \ & \log(\frac{\psi(1-\tau)(k_t^E)^\alpha (A_t \chi)^{1-\alpha}}{(1+\beta)}) + \beta \log \frac{\alpha(1-\tau)^{1+\alpha}(1-\psi)\psi^\alpha (\phi_t^E)^{\alpha-1}(k_t^E)^{\alpha^2}(A_t \chi)^{1-\alpha^2}(1+z)^{1-\alpha}}{(1+\beta^{-1})^\alpha(1+v)^{\alpha-1}} \\
= \ & (\alpha + \alpha^2 \beta) \log k_t^E - (1-\alpha)\beta \log \phi_t^E + (1-\alpha)(1+\beta+\alpha\beta) \log A_t \chi \\
& + \log \frac{\psi(1-\tau)}{(1+\beta)} + \beta \log \frac{\alpha(1-\tau)^{1+\alpha}(1-\psi)\psi^\alpha(1+z)^{1-\alpha}}{(1+\beta^{-1})^\alpha(1+v)^{\alpha-1}}
\end{aligned}
$$

1

For entrepreneurs born after reaching the steady state, the lifetime utility function is (after being detrended):

$$\log(\hat{m}^* - \hat{s}^{E*}) + \beta \log \rho^{E*}\hat{s}^{E*}$$

$$= \log(\frac{\psi\rho^{E*}\hat{k}^{E*}}{(1+\beta)\alpha(1-\psi)}) + \beta \log \frac{\rho^{E*}\hat{k}^{E*}(1+\upsilon)}{\phi^{E*}}$$

$$= (1+\beta)\log \rho^{E*}\hat{k}^{E*} - \beta \log \phi^{E*} + \log(\frac{\psi}{(1+\beta)\alpha(1-\psi)}) + \beta \log(1+\upsilon)$$

$$= (1+\beta)\log \alpha\frac{(1-\psi)}{\psi}\frac{(1+\beta^{-1})}{\phi^{E*}}(1+z)(1+\upsilon)[\frac{\psi\phi^{E*}(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\upsilon)}]^{\frac{1}{1-\alpha}} - \beta \log \phi^{E*}$$

$$+ \log(\frac{\psi}{(1+\beta)\alpha(1-\psi)}) + \beta \log(1+\upsilon)$$

$$= [\frac{\alpha(1+\beta)}{1-\alpha} - \beta]\log \phi^{E*} + (1+\beta)\log \alpha\frac{(1-\psi)(1+\beta^{-1})}{\psi}(1+z)(1+\upsilon)[\frac{\psi(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\upsilon)}]^{\frac{1}{1-\alpha}}$$

$$+ \log(\frac{\psi}{(1+\beta)\alpha(1-\psi)}) + \beta \log(1+\upsilon)$$

Proof of Lemma 2

For workers born during the transition, the lifetime utility function is:

$$\log(w_t - s_t^w + T_t) + \beta \log Rs_t^w$$

$$= \log(w_t - s_t^w + \tau(k_t^E)^\alpha(A_t\chi n_t^E)^{1-\alpha}) + \beta \log Rs_t^w$$

$$= \log(w_t - \zeta^w(w_t + \tau(k_t^E)^\alpha(A_t\chi n_t^E)^{1-\alpha}) + \tau[(1-\tau)(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}}(\frac{R}{\alpha})k_t^E)$$

$$+ \beta \log R\zeta^w(w_t + \tau(k_t^E)^\alpha(A_t\chi n_t^E)^{1-\alpha})$$

$$= \log[\frac{1}{1+\beta}(1-\alpha)A_t(\frac{\alpha}{R})^{\frac{\alpha}{1-\alpha}} + \frac{1}{1+\beta}\tau[(1-\tau)(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}}(\frac{R}{\alpha})k_t^E]$$

$$+ \beta \log \frac{R}{1+\beta^{-1}}[(1-\alpha)A_t(\frac{\alpha}{R})^{\frac{\alpha}{1-\alpha}} + \tau[(1-\tau)(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}}(\frac{R}{\alpha})k_t^E]$$

$$= (1+\beta)\log[(1-\alpha)A_t(\frac{\alpha}{R})^{\frac{\alpha}{1-\alpha}} + \tau[(1-\tau)(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}}(\frac{R}{\alpha})k_t^E]$$

$$+ \log \frac{1}{1+\beta} + \beta \log \frac{R}{1+\beta^{-1}}$$

For workers born in the post-transition stage, but before reaching the steady state, the lifetime utility function is:

$$\log(w_t - s_t^w + T_t) + \beta \log R s_t^w$$

$$= \log(w_t - s_t^w + \tau(k_t^E)^\alpha (A_t\chi)^{1-\alpha}) + \beta \log R s_t^w$$

$$= \log(\frac{1}{1+\beta} w_t + \frac{1}{1+\beta}\tau(k_t^E)^\alpha(A_t\chi)^{1-\alpha}) + \beta \log \frac{R}{1+\beta^{-1}}(w_t + \tau(k_t^E)^\alpha(A_t\chi)^{1-\alpha})$$

$$= \log[\frac{1}{1+\beta}A_t(1-\tau)(1-\alpha)(1-\psi)(k_t^E)^\alpha(A_t\chi)^{-\alpha} + \frac{1}{1+\beta}\tau(k_t^E)^\alpha(A_t\chi)^{1-\alpha}]$$

$$+\beta \log \frac{R}{1+\beta^{-1}}[A_t(1-\tau)(1-\alpha)(1-\psi)(k_t^E)^\alpha(A_t\chi)^{-\alpha} + \tau(k_t^E)^\alpha(A_t\chi)^{1-\alpha}]$$

$$= \log \frac{1}{1+\beta}[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi]A_t(k_t^E)^\alpha(A_t\chi)^{-\alpha}$$

$$+\beta\log\frac{R}{1+\beta^{-1}}[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi]A_t(k_t^E)^\alpha(A_t\chi)^{-\alpha}$$

$$= (1+\beta)\log[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi]A_t(k_t^E)^\alpha(A_t\chi)^{-\alpha}$$

$$+\log\frac{1}{1+\beta} + \beta\log\frac{R}{1+\beta^{-1}}$$

For workers born after reaching the steady state, the lifetime utility function is (after being detrended):

$$\log(\hat{w}^* - \hat{s}^{w*} + \hat{T}^*) + \beta \log R\hat{s}^{w*}$$

$$= \log(\hat{w}^* - \hat{s}^{w*} + \tau(\hat{k}^{E*})^\alpha\chi^{1-\alpha}) + \beta \log R\hat{s}^{w*}$$

$$= \log(\frac{1}{1+\beta}\hat{w}^* + \frac{1}{1+\beta}\tau(\hat{k}^{E*})^\alpha\chi^{1-\alpha}) + \beta \log \frac{R}{1+\beta^{-1}}(\hat{w}^* + \tau(\hat{k}^{E*})^\alpha\chi^{1-\alpha})$$

$$= \log \frac{1}{1+\beta}[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi](\hat{k}^{E*})^\alpha\chi^{-\alpha}$$

$$+\beta\log\frac{R}{1+\beta^{-1}}[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi](\hat{k}^{E*})^\alpha\chi^{-\alpha}$$

$$= \log \frac{1}{1+\beta}[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi][\frac{\psi\phi^{E*}(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+v)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha}$$

$$+\beta\log\frac{R}{1+\beta^{-1}}[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi][\frac{\psi\phi^{E*}(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+v)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha}$$

$$= (1+\beta)\log[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi][\frac{\psi\phi^{E*}(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+v)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha}$$

$$+\log\frac{1}{1+\beta} + \beta\log\frac{R}{1+\beta^{-1}}$$

Proof of Lemma 3

The housing bubble arises because high capital returns driven by resource reallocation are not sustainable in the long run. Rational expectations of a

strong future demand for alternative stores of value can thus induce currently productive agents to speculate in the housing market. During both the transition and post-transition stages, the growth rate of housing prices equals the rate of return of capital by no-arbitrage condition, i.e. $\frac{P_{t+1}^H}{P_t} = \rho_{t+1}^E$. Specifically, during the transition stage, ρ^E is constant, $\rho^E = [(1-\tau)(1-\psi)]^{\frac{1}{\alpha}}\chi^{\frac{1-\alpha}{\alpha}}R$; during the post-transition stage, ρ_t^E is a function of two state variables, k_t^E and A_t, $\rho_t^E = \alpha(1-\tau)(1-\psi)(k_t^E)^{\alpha-1}(A_t\chi)^{1-\alpha}$. In either case, the housing growth rate is decreasing with a positive tax rate τ.

Proof of Proposition 1

From Lemma 1, it is obvious that during both the transition and post-transition stages, entrepreneur's welfare is monotonically decreasing with τ. From Lemma 2, for workers born during the transition, the lifetime utility function is:

$$
\begin{aligned}
& \log(w_t - s_t^w + T_t) + \beta \log R s_t^w \\
= \; & (1+\beta)\log[(1-\alpha)A_t(\frac{\alpha}{R})^{\frac{\alpha}{1-\alpha}} + \tau[(1-\tau)(1-\psi)\chi]^{\frac{1-\alpha}{\alpha}}(\frac{R}{\alpha})k_t^E] \\
& + \log\frac{1}{1+\beta} + \beta\log\frac{R}{1+\beta^{-1}}
\end{aligned}
$$

By taking the first-order condition with respect to τ, we find that the optimal tax rate is:

$$\tau^* = \alpha$$

For workers born in the post-transition stage, but before reaching the steady state, the lifetime utility function is:

$$
\begin{aligned}
& \log(w_t - s_t^w + T_t) + \beta \log R s_t^w \\
= \; & (1+\beta)\log[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi]A_t(k_t^E)^\alpha(A_t\chi)^{-\alpha} \\
& + \log\frac{1}{1+\beta} + \beta\log\frac{R}{1+\beta^{-1}}
\end{aligned}
$$

Given the capital stock k_0^E and technology level A_0 when the economy initially enters the post-transition stage, the worker's utility function can be rewritten as:

$$
\begin{aligned}
& \log(w_t - s_t^w + T_t) + \beta \log R s_t^w \\
= \; & (1+\beta)\log[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi] \\
& + (1+\beta)(\frac{\alpha - \alpha^{T+1}}{1-\alpha})\log(1-\tau) + \tilde{C}
\end{aligned}
$$

where $\tilde{C}$ is a function of parameters. By taking the first-order condition with respect to τ, we find that the optimal tax rate is:

$$\tau^* = \frac{\chi(1-\alpha) - (1 - \alpha^{T+1})(1-\alpha)(1-\psi)}{\chi(1 - \alpha^{T+1}) - (1 - \alpha^{T+1})(1-\alpha)(1-\psi)}$$

Proof of Proposition 2

From Lemma 2, for workers born after reaching the steady state, the lifetime utility function is (after being detrended):

$$\log(\hat{w}^* - \hat{s}^{w*} + \hat{T}^*) + \beta \log R \hat{s}^{w*}$$

$$= (1+\beta) \log[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi][\frac{\psi\phi^{E*}(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+v)}]^{\frac{\alpha}{1-\alpha}} \chi^{-\alpha}$$

$$+ \log \frac{1}{1+\beta} + \beta \log \frac{R}{1+\beta^{-1}}$$

By taking the first-order condition with respect to τ, we find the optimal tax rate is:

$$\tau^* = \frac{\chi(1-\alpha) - (1-\alpha)(1-\psi)}{\chi - (1-\alpha)(1-\psi)}$$

The social welfare is:

$$W = u^w + u^E$$

$$= [\log(\hat{w}^* - \hat{s}^{w*} + \hat{T}^*) + \beta \log R \hat{s}^{w*}] + [\log(\hat{m}^* - \hat{s}^{E*}) + \beta \log \rho^{E*} \hat{s}^{E*}]$$

$$= (1+\beta) \log[(1-\tau)(1-\alpha)(1-\psi) + \tau\chi][\frac{\psi\phi^{E*}(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+v)}]^{\frac{\alpha}{1-\alpha}} \chi^{-\alpha}$$

$$+ (1+\beta) \log \alpha \frac{(1-\psi)(1+\beta^{-1})}{\psi}(1+z)(1+v)[\frac{\psi(1-\tau)\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+v)}]^{\frac{1}{1-\alpha}}$$

$$+ \log \frac{1}{1+\beta} + \beta \log \frac{R}{1+\beta^{-1}} + [\frac{\alpha(1+\beta)}{1-\alpha} - \beta] \log \phi^{E*}$$

$$+ \log(\frac{\psi}{(1+\beta)\alpha(1-\psi)}) + \beta \log(1+v)$$

By taking the first-order condition with respect to τ, we find the optimal tax rate is:

$$\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha) - (1-\alpha)(1-\psi)}{\chi - (1-\alpha)(1-\psi)}$$

Proof of Corollary 1

$$\tau^{**} - \tau^* = \frac{\frac{1}{2}\chi(1-\alpha) - (1-\alpha)(1-\psi)}{\chi - (1-\alpha)(1-\psi)} - \frac{\chi(1-\alpha) - (1-\alpha)(1-\psi)}{\chi - (1-\alpha)(1-\psi)}$$

$$= \frac{-\frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)} < 0$$

Proof of Proposition 3

As $\tau^{**} > 0$, it is obvious that the tax policy improves worker's welfare while it reduces entrepreneur's welfare. Following the proof of Proposition 2, worker's welfare with an optimal tax rate τ^{**} is:

$$
\begin{aligned}
u_1^w &= (1+\beta)\log[(1-\tau^{**})(1-\alpha)(1-\psi)+\tau^{**}\chi][\frac{\psi\phi^{E*}(1-\tau^{**})\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\upsilon)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha} \\
&\quad + \log\frac{1}{1+\beta} + \beta\log\frac{R}{1+\beta^{-1}}
\end{aligned}
$$

Worker's welfare without the tax policy is:

$$
\begin{aligned}
u_2^w &= (1+\beta)\log[(1-\alpha)(1-\psi)][\frac{\psi\phi^{E*}\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\upsilon)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha} \\
&\quad + \log\frac{1}{1+\beta} + \beta\log\frac{R}{1+\beta^{-1}}
\end{aligned}
$$

Thus, given that $\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$,

$$
\begin{aligned}
u_1^w - u_2^w &= (1+\beta)\{\frac{\alpha}{1-\alpha}\log(1-\tau^{**})+\log[(1-\tau^{**})(1-\alpha)(1-\psi)+\tau^{**}\chi] \\
&\quad - \log(1-\alpha)(1-\psi)\} \\
&= (1+\beta)\{\frac{\alpha}{1-\alpha}\log\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)}+\log\frac{1}{2}(1-\alpha)\chi-\log(1-\alpha)(1-\psi)\} \\
&= (1+\beta)\log(\frac{\chi-\frac{1}{2}\chi(1-\alpha)}{\chi-(1-\alpha)(1-\psi)})^{\frac{\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)}
\end{aligned}
$$

Similarly, entrepreneur's welfare with an optimal tax rate τ^{**} is:

$$
\begin{aligned}
u_1^E &= (1+\beta)\log\alpha\frac{(1-\psi)(1+\beta^{-1})}{\psi}(1+z)(1+\upsilon)[\frac{\psi(1-\tau^{**})\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\upsilon)}]^{\frac{1}{1-\alpha}} \\
&\quad + [\frac{\alpha(1+\beta)}{1-\alpha}-\beta]\log\phi^{E*}+\log(\frac{\psi}{(1+\beta)\alpha(1-\psi)})+\beta\log(1+\upsilon)
\end{aligned}
$$

Entrepreneur's welfare without the tax policy is:

$$
\begin{aligned}
u_2^E &= (1+\beta)\log\alpha\frac{(1-\psi)(1+\beta^{-1})}{\psi}(1+z)(1+\upsilon)[\frac{\psi\chi^{1-\alpha}}{(1+\beta^{-1})(1+z)(1+\upsilon)}]^{\frac{1}{1-\alpha}} \\
&\quad + [\frac{\alpha(1+\beta)}{1-\alpha}-\beta]\log\phi^{E*}+\log(\frac{\psi}{(1+\beta)\alpha(1-\psi)})+\beta\log(1+\upsilon)
\end{aligned}
$$

Given that $\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$,

6

$$u_1^E - u_2^E = (1 + \beta) \frac{1}{1 - \alpha} \log(1 - \tau^{**})$$

$$= (1 + \beta) \log(\frac{\chi - \frac{1}{2}\chi(1 - \alpha)}{\chi - (1 - \alpha)(1 - \psi)})^{\frac{1}{1-\alpha}}$$

Proof of Proposition 4

Following the proof of Proposition 2, the social welfare with an optimal tax rate τ^{**} is:

$$W_1 = (1 + \beta) \log[(1 - \tau^{**})(1 - \alpha)(1 - \psi) + \tau^{**}\chi][\frac{\psi\phi^{E*}(1 - \tau^{**})\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + v)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha}$$

$$+ (1 + \beta) \log \alpha \frac{(1 - \psi)(1 + \beta^{-1})}{\psi}(1 + z)(1 + v)[\frac{\psi(1 - \tau^{**})\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + v)}]^{\frac{1}{1-\alpha}}$$

$$+ \log \frac{1}{1 + \beta} + \beta \log \frac{R}{1 + \beta^{-1}} + [\frac{\alpha(1 + \beta)}{1 - \alpha} - \beta] \log \phi^{E*}$$

$$+ \log(\frac{\psi}{(1 + \beta)\alpha(1 - \psi)}) + \beta \log(1 + v)$$

The social welfare without the tax policy is:

$$W_2 = (1 + \beta) \log[(1 - \alpha)(1 - \psi)][\frac{\psi\phi^{E*}\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + v)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha}$$

$$+ (1 + \beta) \log \alpha \frac{(1 - \psi)(1 + \beta^{-1})}{\psi}(1 + z)(1 + v)[\frac{\psi\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + v)}]^{\frac{1}{1-\alpha}}$$

$$+ \log \frac{1}{1 + \beta} + \beta \log \frac{R}{1 + \beta^{-1}} + [\frac{\alpha(1 + \beta)}{1 - \alpha} - \beta] \log \phi^{E*}$$

$$+ \log(\frac{\psi}{(1 + \beta)\alpha(1 - \psi)}) + \beta \log(1 + v)$$

The social welfare without housing bubbles is (i.e. $\phi^E = 1$):

$$W_3 = (1 + \beta) \log[(1 - \alpha)(1 - \psi)][\frac{\psi\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + v)}]^{\frac{\alpha}{1-\alpha}}\chi^{-\alpha}$$

$$+ (1 + \beta) \log \alpha \frac{(1 - \psi)(1 + \beta^{-1})}{\psi}(1 + z)(1 + v)[\frac{\psi\chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + v)}]^{\frac{1}{1-\alpha}}$$

$$+ \log \frac{1}{1 + \beta} + \beta \log \frac{R}{1 + \beta^{-1}} + \log(\frac{\psi}{(1 + \beta)\alpha(1 - \psi)}) + \beta \log(1 + v)$$

Thus, given that $\tau^{**} = \frac{\frac{1}{2}\chi(1-\alpha)-(1-\alpha)(1-\psi)}{\chi-(1-\alpha)(1-\psi)}$,

$$W_1 - W_2 = (1+\beta)\{\frac{1+\alpha}{1-\alpha}\log(1-\tau^{**}) + \log[(1-\tau^{**})(1-\alpha)(1-\psi) + \tau^{**}\chi]$$
$$- \log(1-\alpha)(1-\psi)\}$$
$$= (1+\beta)\{\frac{1+\alpha}{1-\alpha}\log\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)} + \log\frac{1}{2}(1-\alpha)\chi - \log(1-\alpha)(1-\psi)\}$$
$$= (1+\beta)\log(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)})^{\frac{1+\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)}$$

If $(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)})^{\frac{1+\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)} > 1$, then $W_1 > W_2$.

$$W_2 - W_3 = [\frac{\alpha(1+\beta)}{1-\alpha} - \beta]\log\phi^{E*} + \frac{\alpha(1+\beta)}{1-\alpha}\log\phi^{E*}$$
$$= [\frac{2\alpha(1+\beta)}{1-\alpha} - \beta]\log\phi^{E*} = (1+\beta)[\frac{2\alpha}{1-\alpha} - \frac{\beta}{1+\beta}]\log\phi^{E*}$$

Thus,

$$W_1 - W_3 = (W_1 - W_2) + (W_2 - W_3)$$
$$= (1+\beta)\log(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)})^{\frac{1+\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)}(\phi^{E*})^{\frac{2\alpha}{1-\alpha} - \frac{\beta}{1+\beta}}$$

If $(\frac{\chi - \frac{1}{2}\chi(1-\alpha)}{\chi - (1-\alpha)(1-\psi)})^{\frac{1+\alpha}{1-\alpha}}\frac{\frac{1}{2}\chi}{(1-\psi)}(\phi^{E*})^{\frac{2\alpha}{1-\alpha} - \frac{\beta}{1+\beta}} > 1$, then $W_1 > W_3$.