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AVAILABILITY ASSESSMENT BASED ON STOCHASTIC MAINTENANCE PROCESS MODELING⊗

RENDELKEZÉSREÁLLÁS KARBANTARTÁSI FOLYAMAT SZTOCHASZTIKUS MODELLEZÉSSEL TÖRTÉNŐ BECSLÉSE

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Abstract: *The operation of technical systems is a stochastic process based upon the equipment, equipment operation and maintenance, equipment preparation and repairs, and also the personnel carrying out repairs, as well as the regulations for operations. From the mathematical point of view, the operation of technical systems and equipment is a discrete state space stochastic process without after-effects, so it can be approximated with a Markov-chain. After setting up the transition probability matrix, matrix-algebraic tools can be used for investigating these processes with systems approach analysis. This paper is aimed to demonstrate the possibilities of the use of Markov matrix in case of stationary maintenance processes. In this paper a well-algorithmizable method developed by the author for mathematical modeling of stationary stochastic maintenance process is presented. The presented modeling method can be used for the assessment of availability, reliability and maintenance cost of a technical system.*

Keywords: *Maintenance; Markov-chain; Stochastic modeling*

Kivonat: *Technikai rendszerek üzemeltetése egy a berendezésekre, azok üzemeltetését, karbantartását, előkészítését és javításukat végző személyekre és berendezésekre, illetve annak irányítására szolgáló utasításokra épülő sztochasztikus folyamat. Matematikai szempontból technikai rendszerek és berendezések üzemeltetése egy diszkrét állapotterű, utóhatásmentes sztochasztikus folyamat, így azt Markov-lánccal lehet matematikailag leírni. Az átmeneti valószínűségi mátrix felállítása után, mátrix-algebrai eszközök segítségével tudjuk a vizsgált folyamatot rendszerszemléletű megközelítéssel elemezni. A tanulmány célja bemutatni a stacionárius karbantartás folyamatok Markov-mátrix felhasználásának történő elemzési lehetőségeit. A cikk a Szerző által kidolgozott, jól algoritmizálható stacioner sztochasztikus modellmegoldási eljárást mutat, mely segítségével prognosztizálható a gyártóberendezések megbízhatósága, rendelkezésre állása, valamint karbantartási költségei.*

Kulcsszavak: *Üzemeltetés; Markov-lánc; Sztochasztikus modellezés*

1 INTRODUCTION

In case of a production line operation, equipment's repairing time could be decreased, but it would increase the repair cost. The task the author undertook was to forecast the changes in system availability and total repair cost if new repairing technology were installed.

The aims were to solve the task above and, in addition, to develop a well-algorithmizable mathematical modeling method for this and similar problems. The solution and this paper have been inspired mainly by book of Rohács and Simon [12].

The operation of a manufacturing system is a stochastic process based upon the equipment, its maintenance, its preparation, and also the personnel carrying out repair, and the regulations for the

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whole process. This process can be considered as a mathematically continuous time, discrete state space Markov-process that can be approximated by a Markov-chain.

There are several books and papers that present the theory of Markov and Semi-Markov processes and their application to model and investigate operational systems of technical equipment. The mathematical basis of Markov-processes is discussed in the following: Bharucha-Reid [2], Helstrom, [6], Karlin and Taylor [8], Korn end Korn, [9].

The paper of Jalali Naini et al. [7] studies a maintenance policy for a system composed of two components, which are subject to continuous deterioration and consequential stochastic failure. Deterioration of these components is inspected periodically and the degrees of deterioration are monitored. The components can be maintained using different maintenance actions with different costs. For using stochastic regenerative properties of the system, a stochastic model has been developed in order to analyze the deterioration process and a novel approach is presented. It simultaneously determines the time between two successive inspection periods and the appropriate maintenance action for each component based on the observed degrees of deterioration. The proposed approach considers different criteria like system reliability and long-run expected cost of the system.

Shahanaghi et al. emphasized that reliability is one of the most important issues in the assessment of industrial equipment or products. Their paper focused on a continuous deterioration of two-unit series equipment whose failure can not be measured by the cost criterion. For these types of systems avoiding failure during the actual operation of the system is extremely important. Shahanaghi determined inspection periods and maintenance policy in a way that failure probability is limited to a pre-specified value and then optimum policy and inspection period are applied to minimize long-run cost per time unit [13].

The paper of El-Dancese deals with a system of *n*-independent repairable units that can be described by a homogeneous, continuous-time, discrete-state Markov process [5]. Transition probability matrix of the system in the form of a modified Kronecker sum of transition-rate matrices of its units was investigated in the paper.

Orcesi and Cremona proposed a new methodology in their paper [10] to help the bridge owner in scheduling its bridge maintenance strategies at the scale of the transportation network. The originality of shown approach is in determining performance indicators through the use of Markov chains which, on the other hand, makes it possible to determine an event-tree decision at each inspection time.

The aim of Zhao et al.'s paper [16] is to discuss the problem of modeling and optimizing condition-based maintenance policies for a deteriorating system in the presence of covariates. The expected average of maintenance cost per time unit is calculated by the authors, and the optimal inspection/replacement policies are derived from the different maintenance unit costs. The different policies proposed by the authors were compared and the limits of each one are pointed out.

Dijoux presents a new reliability model for complex repairable systems, which combines a bathtub shaped ageing and imperfect maintenance. A bathtub shaped initial intensity function allows taking into account the burn-in period, the useful life and wearing out of the systems. Repair effect is expressed by the reduction of system virtual age, which depends on the ageing of the system. The main characteristics of the model are derived. The most important is that the maintenance efficiency allows an extension of the system's useful life duration. A statistical analysis of the model and an application to real failure data are presented by [4].

Smith discusses and evaluates preventive maintenance practices. It is discussed in his book [14] how the reliability-centered maintenance method can provide for an extremely cost-effective manufacturing.

By Zheng and Liu Markov chains are stochastic processes that can be parameterized by empirically estimating transition probabilities between discrete states in the observed systems. The main property of the Markov chain is that, given the present state, future states are independent of the past states [15].

A new methodology for failure rate evaluation with influencing factors was proposed by Brissaud et al. [3]. This proposed methodology combines a quantitative part to integrate available data, with a qualitative analysis to compensate for a potential lack of feedback knowledge.

Bertolini et al. defined that the management of failure analysis has a strategic importance within an oil refinery from the organizational, engineering and economic points of view [1].

The theoretical bases of aircraft maintenance management can be studied in the book of Rohács and Simon [12]. Pokorádi describes stochastic modeling methods to investigate aircraft operations system with Markov matrix [11].

This paper is aimed to show the possibilities of the use of Markov matrix in the case of stationary maintenance processes. A well-algorithmizable method for mathematical modeling of stationary stochastic industrial process will be presented. This modeling method can be used to estimate maintenance cost and the time of availability of the investigated manufacturing equipment.

The structure of this paper is as follows: Section 1 contains the applied literatures and the main goals of investigation. Section 2 presents the Markov-processes. Section 3 contains the maintenance processes and their stationary modeling. Section 4 shows a case study for maintenance management decision. A summation, conclusion and the extension of the topic in the future will be given in Section 5.

2. MARKOV PROCESSES

A stochastic process $\eta(t)$ whose development in the future is influenced by its development in the past only through its development in the present, that is, a stochastic process without after- effects, is called Markov-process.

The mathematically described random process $\eta(t)$ is called a Markov-one if the equation of conditional probabilities

$$P\left(\eta(t_{n+1}) = X_{n+1} \middle| \eta(t_1) = X_1 \ldots \eta(t_n) = X_n\right) = P\left(\eta(t_{n+1}) = X_{n+1} \middle| \eta(t_n) = X_n\right) \tag{1}$$

proves to be true with the probability 1 for each $t_1 < \ldots < t_n < t_{n+1}$ and $X_1 ;X_2; \ldots ;X_i ;X_{i+1}$ real number [8].

If process $\eta(t)$ during the study period can have an X value at any moment, it is called a continuous-time process. If $\eta(t)$ can only have some value at certain moments, the process is called a discrete-time one. A random process is considered to be a discrete state space one, if the possible values of variance $\eta(t)$ constitute a finite set or a count non-finite set.

A Markov-process can be characterized unambiguously by supplying the transition probabilities, and the distributions of leaving different states. If distribution of leaving different states is not of the exponential character, such a stochastic process is called a Semi-Markov one [2].

Finite or count non-finite stochastic processes, that is, the discrete state space ones with no after-effects, are called Markov-chain [2]. In this case, the value established in the equation (1) is called the transition probability:

$$P_{ij}^{n,n+1} = P\left(\eta(t_{n+1}) = X_{n+1} \middle| \eta(t_n) = X_n\right) \quad . \tag{2}$$

The transition probability expresses that $\eta(t_{n+1}) = X_j$, supposing that $\eta(t_n) = X_i$.

$P_{ij}^{n,n+1}$ marking above also shows that the transition probability is not only the function of the i-th beginning state and of the *j*-th next state, but also the function of t_n time. In order to have a simpler marking, the following formula is used:

$$P_{ij}^{n,n+1} = P_{ij}(t_n) = P_{ij}(t) \quad . \tag{3}$$

Having N number of states, P_{ij} transition probabilities can be arranged in matrix. The

$$\mathbf{P}_{NxN}(t) = [P_{ij}(t)] \tag{4}$$

matrix is called the Markov-matrix (or the transition probability matrix) of the process.

Using the Markov-matrix, the change in time of the probability of staying in different states can be determined by

$$\mathbf{p}(t+\Delta t)=\mathbf{P}^{T}(t)\mathbf{p}(t) \quad , \tag{5}$$

where $\mathbf{P}^T$ is the transposed matrix of **P**.

If the one-step transition probabilities are not time-dependent, we call the Markov-process stationary. In this case we can state that

$$P_{ij}=P_{ij}^{n,n+1} \quad , \tag{6}$$

or

$$\mathbf{P}_{NxN}=[P_{ij}] \tag{7}$$

as it does not depend on the value of n, and P_{ij} means that the value of $\eta(t)$ is probably transiting from X_i to X_j during the (t_n ; t_{n+1}) or Δt time interval.

For the sake of further analyses, it is advisable for us to consider the case where after Δt time period the value of $\eta(t)$ will be the beginning one again. So the determination of varieties in the main diagonal of the matrix is carried out as follows:

$$P_{ii}=1-\sum_{\substack{j=1\\ j\neq i}}^{N}P_{ij} \quad , \tag{8}$$

(As the total space means that the object of operation enters into a new state or it remains in the beginning state.)

In case of stationary processes, using the property of unit matrix **E**, you can set up the equation

$$\mathbf{p}(t+\Delta t)=\mathbf{P}^{T}\mathbf{p}(t)=\mathbf{E}\mathbf{p}(t), \tag{9}$$

which can be transformed into the following formula:

$$[\mathbf{P}^{T}-\mathbf{E}]\mathbf{p}=\mathbf{0} \quad , \tag{10}$$

where **0** is the zero vector.

3 MAINTENANCE PROCESSES

The operational process of engineering systems (which is the complex of events that happen to the system from its manufacturing to its discarding) is a random in time and in frequency succession of so-called states of operation. This process can be described with the so called operational chain, that is, a Markov-chain from the mathematical point of view (see Fig. 1).

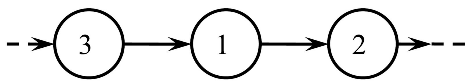

Fig.1 Operational Chain (Example)
1 — Applicability of Equipment; ***2*** — ***Type A*** Failure's Repair;
3 — ***Type B*** Failure's Repair.

When analyzing operational processes with the systems approach, the actual succession of the single states of the equipment is no concern of ours. It is rather complicated to describe the whole operational process with an operational chain. In order to achieve a clearer picture it is advisable to describe the operational process as a directed graph.

Within the graph operation states are represented by the nodes of the graph, and transitions from one state to another are represented by the directed edges of the graph (for example, Fig. 3). Analyzing the operational chain or the operational graph, we assume that states are clearly defined, and transitions occur during zero time.

The staying of the object of the operation in different states can also be characterized by the vector of mean costs **k**, and vector of mean work expenditures **m** of the staying in states of the operation. Knowing the characteristics above we are able to determine expected values of the total operational cost K_{Σ} and work expenditure M_{Σ}.

For characterization of transitions from one state to another, we use their probability transition (failure or repair) rates. The limiting value

$$\beta_{ij} = \lim_{t_{n+1} \to t_n} \frac{P_{ij}^{n,n+1}}{t_{n+1} - t_n} \tag{11}$$

is called transition probability (in this study: λ **-** failure and μ **-** repair) density.

Naturally, these transition probability densities β_{ij} can be arranged in matrix **B** analogously to equation (7)

$$\mathbf{B}_{NxN} = [\beta_{ij}] \quad , \tag{12}$$

then equation (10) can be modified as:

$$[\mathbf{B}^T - \mathbf{E}]\mathbf{p} = \mathbf{0} \; . \tag{13}$$

Introducing the matrix

$$[\mathbf{B}^T - \mathbf{E}] = \mathbf{M} \; , \tag{14}$$

the equation (13) can be modified as:

$$\mathbf{Mp} = \mathbf{0} \; . \tag{15}$$

During the solution of the system of equations (15) there is a problem, that the numerical algorithms provide (or can provide) the **p** = **0** trivial solution. It is obvious that the concern of investigation is to achieve a solution different from the trivial one. Because the aim of the author was to develop a well-algorithmizable method for this problem, the system of equations with *N* unknowns (15) is transformed into a system of equations with *N+1* unknowns by equation

$$P_{\Sigma} = \sum_{i=1}^{4} P_i = 1 \tag{16}$$

of probability of total event space. Thus the system of equations has changed to

$$\begin{bmatrix} & & & \mathrm{M} & 1 \\ & \mathbf{M} & & \mathrm{M} & \mathrm{M} \\ & & & \mathrm{M} & 1 \\ \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\ 1 & \mathrm{K} & 1 & \mathrm{M} & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ \mathrm{M} \\ P_N \\ \Lambda \\ P_\Sigma \end{bmatrix} = \begin{bmatrix} 1 \\ \mathrm{M} \\ \mathrm{M} \\ \mathrm{M} \\ 1 \end{bmatrix} . \qquad (17)$$

The solution of the system of linear equations is reached by any numerical algorithm will result in a solution of the system of equations (15) different from the trivial one.
At this point a question emerges: when can an operational process be approximated as a stationary one?

The bathtub curves (see Fig. 2) are often used to present the failure rate in the function of time. The curve has three stages [4]:

I. The first stage shows decreasing failure rate, known as the early-life failures, where the system is adapting to the new situation.
II. The second stage is characterized by constant hazard, where the system performs its functions in optimal conditions. The initial intensity is constant during this stage.
III. During the third stage, also called the wear-out phase of the system, the intensity is increasing.

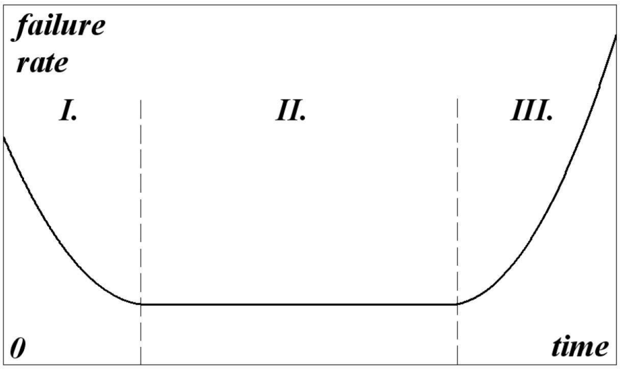

Fig. 2. Bathtub Curve

During the second stage, the bathtub curve shows constant failure rate, therefore at this stage the operational–maintenance process can be approximated as a stationary one. We must admit that a few authors — for example, Smith in reference [14] — have criticized the bathtub curve. But their age-reliability patterns contain linear, basically constant failure rate stages; in this case the failure and maintenance processes can be modeled as non-transient ones. Our investigation will model and study only these stationary operational stages.

4 CASE STUDY

During the operation of investigated manufacturing equipment four main types of failures can be experienced. These (**Type *A***; ***B***; ***D*** and ***E***) failures occur more then 94 % of equipment outages. (The other ones will be modeled by ***Type C*** failures.) When ***Type B*** failures where repaired, the servicemen detect frequently that ***Type A*** failure will occur shortly, then ***Type A*** failure repair is carried out as well. During the repair of ***Type D*** and ***Type E*** failures a similar situation can occur. Maybe in these cases the other failure should be repaired too. Table 1 shows the main failure and repair data of investigated production line equipment (where MTBF means Mean Time Between Failures and MTR means Mean Time to Repair).

Modifying the repairing technology, MRT of ***Type A*** failure will be decreased from 7.08 hours approximately to 5.5 hours, and Mean Work Expenditure will decrease from 14.16 to ab. 11 man hours. At the same time the repairing cost of ***Type A*** failure is increased from 150.2 to around 190 Euros. The task undertaken by the author was to forecast the total repair cost, work expenditure and the potential gains of the application of the new technology.

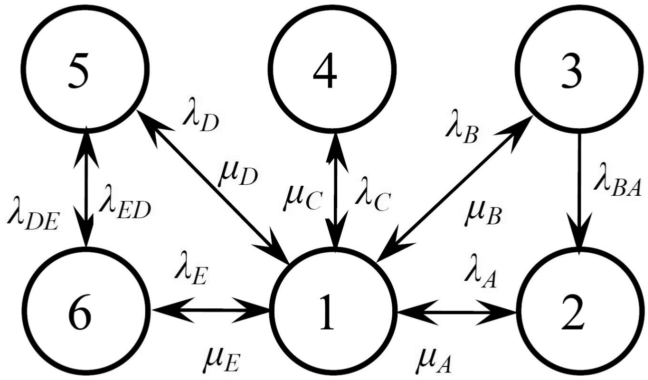

Fig. 3 Graph of the Investigated Maintenance Process
1 — Equipment is Applicable; ***2*** — ***Type A*** Failure's Repair;
3 — ***Type B*** Failure's Repair; ***4*** — ***Type C*** Failures' Repair;
5 — ***Type D*** Failure's Repair; ***6*** — ***Type E*** Failure's Repair

Failures	***A*-type**	***B*-type**	**C-type**	***D*-type**	***E*-type**
MTBF τ [hour]	1316.3	892.8	1339.4	1410.1	1396.4
Failure rate λ [hour^{-1}]	7.60 10^{-4}	1.12 10^{-3}	7.47 10^{-4}	7.91 10^{-4}	7.61 10^{-4}
MTR τ [hour]	7.08	9.63	2.14	8.21	7.62
Repair rate μ [hour^{-1}]	0.141	0.104	0.467	0.122	0.131
Mean Repairing Cost [€]	150.2	115.4	98.7	210.8	352.4
Mean Work Expenditure [man-hour]	14.16	14.45	5.35	24.63	17.5
λ_{ij} [hour^{-1}]	—	0.427	—	0.613	0.524

Table 1 Main Data of Statistical Analysis

The statistical analysis of data has showed that the failure and repair rates are independent on the operating time of the investigated equipment. The data analysis suggests that the process is stationary and its Markov model can be described by matrix equation (15), matrix **M** is the following:

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & 0 & 0 & 0 \\ m_{31} & 0 & m_{33} & 0 & 0 & 0 \\ m_{41} & 0 & 0 & m_{44} & 0 & 0 \\ m_{51} & 0 & 0 & 0 & m_{55} & m_{56} \\ m_{61} & 0 & 0 & 0 & m_{65} & m_{66} \end{bmatrix}, \qquad (18)$$

where elements of the matrix are determined by Table 2.

To get a non-trivial solution of the system of equations (15), the method presented in Section 3 was used.

Knowing the cost of different repairs, the expected value of the total repairing cost is:

$$K_\Sigma = T\sum_{i=2}^{6}\frac{k_i P_i}{MTR_i} \quad , \tag{19}$$

and the expected value of the work expenditure is:

$$M_\Sigma = T\sum_{i=2}^{6}\frac{m_i P_i}{MTR_i} \,, \tag{20}$$

where:
T — investigated time-interval;
MTR_i — Mean Time to *i*-th Repair
k_i — mean cost of the *i*-th repair;
m_i — mean work expenditure of the *i*-th repair.

$m_{11} = -(\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E)$	$m_{12} = \mu_A$
$m_{13} = \mu_B$	$m_{14} = \mu_C$
$m_{15} = \mu_D$	$m_{16} = \mu_E$
$m_{21} = \lambda_A$	$m_{22} = -\mu_A$
$m_{23} = \lambda_{BA}$	$m_{31} = \lambda_B$
$m_{33} = -(\mu_B + \lambda_{BA})$	$m_{41} = \lambda_C$
$m_{44} = -\mu_C$	$m_{51} = \lambda_D$
$m_{55} = -(\mu_D + \lambda_{DE})$	$m_{56} = \lambda_{ED}$
$m_{61} = \lambda_E$	$m_{65} = \lambda_{DE}$
$m_{66} = -(\mu_E + \lambda_{ED})$	

Table 2 Coefficients of Matrix **M**

The non-vanishing solution of equation (17) by data of Table 1 is following probabilities of the staying in states:

P_1 = 0.973994 ; P_2 = 0.011453 ;
P_3 = 0.002054 ; P_4 = 0.001556 ;
P_5 = 0.005101 ; P_6 = 0.005844 .

The expected total repairing cost of the present maintenance system for 10000 hours is:

51996.0 Euros,

and expected work expenditure is:

4252.43 man hours

For forecasting of the total repair cost of the new technology the above calculation is completed by the following modification:

$MTR_B = 5.5$ *hours* ; $k_B = 190$ € and $m_b = 11$ *man hours*.

Its results — probabilities of the staying in states — are

$P_1 = 0.976496$; $P_2 = 0.008920$;
$P_3 = 0.002059$; $P_4 = 0.001560$;
$P_5 = 0.005114$; $P_6 = 0.005859$.

The expected total cost of modified maintenance system is:

51830.7 Euros,

and expected work expenditure is:

3618.65 man hours.

From the results mentioned above, the following conclusions can be drawn (see Fig. 4):

- the probability of staying in ***Type A*** failure's repair state will decrease measurably (by 2.533 ‰);
- the probabilities of staying in other failures' repair state will increase slightly (by 0.004 ~ 0.015 ‰);

These conclusions should be taken into account during the organization of the new maintenance system management:

- 10000 hours-related expected total repair cost will be reduced by 165.3 Euros (relatively: 3.178 ‰**)**;
- 10000 hours-related expected work expenditure will be decreased by 633.78 man hours (relatively: 14.9 %), the follow-up work of the repairing team can become more flexible (it is important to mention that the given repair can be delayed frequently due to other tasks of the servicemen)
- the availability of the equipment will be increased by 5.5 ‰, which generates more profit by the improvement of the reliability and the productivity of the production line.

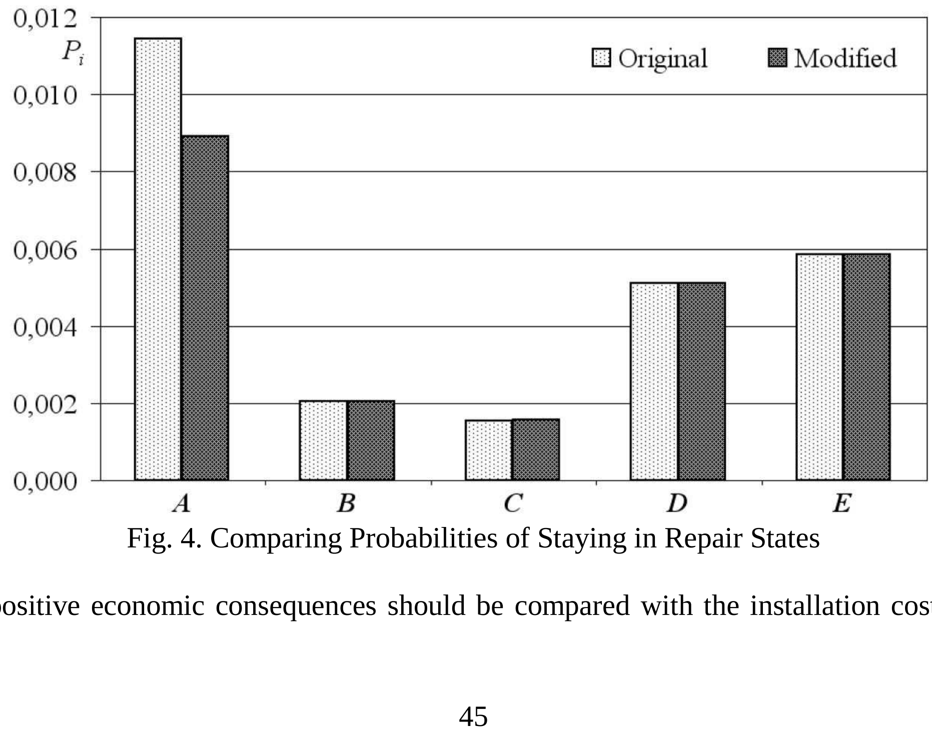

Fig. 4. Comparing Probabilities of Staying in Repair States

These positive economic consequences should be compared with the installation cost of the new repairing technology. It contains charges of new repairing equipment and cost of subsidiary failures and problems occurred during the adaptation to the new maintenance situation. (see Stage 1 of Bathtub Curve – Figure 2). These questions go beyond of this study.

5. CONCLUSIONS

The paper showed a well-algorithmizable method developed by the author for mathematical modeling and investigation of stationary stochastic industrial processes. The presented modeling method can be used to estimate the applicability of equipment, maintenance cost and work expenditures of manufacturing systems.

The general conclusion of this paper is that the stationary Markov model of operational processes can be used to investigate maintenance systems and processes.

During prospective scientific research related to this field of mathematics and the science of engineering management, the author would like to develop

- other models to investigate the maintenance processes of engineering systems;
- methods to analyze the parametrical uncertainties of the model presented above.

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