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2008 J. Phys.: Conf. Ser. 111 012043

(http://iopscience.iop.org/1742-6596/111/1/012043)

# Clusters and the quasi-dynamical symmetry

**J Cseh[1], P O Hess[2], J Darai[3], A Algora[1,4] and H Yepez-Martinez[5]**
[1] Institute for Nuclear Research, Hung. Acad. Sci., Debrecen, POB 51, Hungary-4001
[2] Instituto de Ciencias Nucleares, UNAM, A.P. 70-543, Mexico City, 04510 D.F. Mexico
[3] Inst. of Experimental Phys., Univ. of Debrecen, Debrecen, POB 105, Hungary-4010
[4] IFIC (CSIC-Univ. of Valencia), Valencia, Spain
[5] Univ. Aut. Ciudad de Mexico, Prolong. San Isidro 151, Mexico City, 09790 D.F. Mexico
E-mail: [1] `cseh@atomki.hu`

**Abstract.** The possible role of the quasi-dynamical symmetry in nuclear clusterization is discussed. Two particular examples are considered: i) the phases and phase-transitions of some algebraic cluster models, and ii) the clusterization in heavy nuclei. The interrelation of exotic (superdeformed, hyperdeformed) nuclear shapes and cluster-configurations are also investigated both for light, and for heavy nuclei, based on the dynamical and quasi-dynamical SU(3) symmetries, respectively.

## 1. Introduction

Clusterization is a rich phenomenon, which shows up among different circumstances in atomic nuclei. In the present contribution we also consider various aspects of it, therefore we start here by recalling (one of) its basic definition(s). It seems most natural to approach the problem from the side of the experimental observation, thus (similarly to other simple states of atomic nuclei) we call a state cluster state, if its wavefunction largely overlaps with that of an observation channel. [1]. Several other definitions are possible, and are in use in the literature, including more theoretical ones. Their interrelation is an interesting topic, but it obviously goes much beyond the limits of this short paper.

Another important feature of clusterization is that it is rather complex, therefore its exact theoretical description is very complicated, or in many cases impossible. The completely microscopic methods incorporate all the relevant aspects of the problem, but their applicability is limited to simple systems, e.g. with small nucleon numbers. Some phenomenological models are widely applied, but the price we have to pay for it is the application of strongly simplifying assumptions, which are not always well-understood. The semimicroscopic approaches, including most of the discussion of the present contribution, try to incorporate the most important consequences of the microscopic structure, i.e. the Pauli-principle in a well-controlled way, when still considerable simplification of the many-body-problem is done.

The structure of this paper is as follows. First we review very briefly the concepts of the well-known dynamical symmetry, and its extension to quasi-dynamical symmetry. Then we recall some applications of the dynamical symmetry in nuclear structure studies in general, and in cluster physics in particular. The main part deals with the role of the quasi-dynamical symmetry in clusterization. Two particular examples are shown: the question of phases and phase-transitions of cluster states, and the problem of the exotic clusterization of heavy nuclei, with an attempt to incorporate the consequences of the exclusion principle on an approximate level. Then the exciting question of the interrelation of the exotic nuclear deformation and clusterization is addressed both in light and in heavy nuclei, by applying selection rules, based on real and quasi-dynamical symmetries, respectively. Finally some conclusions are drawn.

## 2. Dynamical and quasi-dynamical symmetry

A quantum mechanical system governed by a time-independent Hamiltonian $H$ is said to have an exact dynamical symmetry described by the Lie-algebra $L$ if the basis vectors of $L$ commute with $H$ [2]. E.g. the three dimensional harmonic oscillator problem has $U(3)$ as an exact dynamical symmetry. If $H = T + V$, and the elements of $L$ commute not only with $H$, but with $T$ and $V$ as well, the symmetry is called geometrical one. E.g. the oscillator has $O(3)$ as a geometrical symmetry. When exact symmetry holds then not only the operator is symmetric, but so are its eigenvectors, too (i.e. they transform according to an irreducible representation) [3]. In such a case the Hamiltonian can contain the elements of the symmetry Lie-algebra only via its invariant operators.

If the Hamiltonian is expressed in terms of the invariant operators of a nested chain of subalgebras (rather than of a single algebra as before), we speak about a broken dynamical symmetry. In such a case the Hamiltonian is not symmetric (scalar) any more, but its eigenvectors are symmetric [4]. E.g. in the Elliott-model (see next section) the Hamiltonian is written in terms of the Casimir invariants of the $U(3) \supset SU(3) \supset SO(3)$ algebra-chain, therefore $U(3)$ and $SU(3)$ are broken dynamical symmetries, while $SO(3)$ is an exact (geometrical) one. In this case the original degeneracy corresponding to the $U(3)$ splits up. The eigenvalue-problem of a Hamiltonian with broken dynamical symmetry still has an analytical solution (similarly to that of the exact symmetry). There are many useful examples of this kind of broken dynamical symmetries in nuclear physics. (In many papers these dynamical symmetries are called exact ones.)

When the symmetry-breaking interaction is even stronger such that it not only splits up different basis states but even mixes them with each other, and yet the symmetry survives (for some of the states) we speak about quasi-dynamical symmetry [5]. In this case neither the operator, nor its eigenvectors are symmetric [4]. The symmetry-breaking interaction which leads to such a situation is still not quite arbitrary, of course.

## 3. Dynamical symmetry and clusterization

The basic assumption of the cluster models is that the relevant degrees of freedom of the atomic nucleus are classified into two categories: some of them describe the internal structure of the clusters (e.g. in terms of a shell model), while others account for their relative motion.

The Elliott-model [6] is an algebraic shell model with a $U(3)$ dynamical symmetry belonging to the spatial degrees of freedom. The spin-isospin sector is accounted for by Wigner's $U^{ST}(4)$ group [7]. The antisymmetrization in this scheme is done exactly.

The relative motion of two clusters can be described by the vibron model [8], which is an algebraic model of the dipole collective motion. It has an $U(4)$ algebraic structure with two limiting cases corresponding to the algebra-chains: $U(4) \supset U(3) \supset SU(3) \supset SO(3)$, $U(4) \supset O(4) \supset SO(4) \supset SO(3)$.

By combining the vibron model and the Elliott-model (or some other algebraic model for the description to the internal cluster structure) one can construct an algebraic cluster model, and it has been done both on the phenomenological and on the semimicroscopical level. The distinction is made by the fact whether or not the Pauli-forbidden states are excluded from the model space. This complication arises in spite of the fact that the Elliott-model has antisymmetrised wavefunctions, because the antisymmetrization is not carried out with respect to the interchange of nucleons from different clusters. When the two models are coupled on the $U(3)$ level, the problem can be solved in a relatively simple way, due to the relation of the unitary and permutational groups [9]. In this way one can obtain a semimicroscopic algebraic cluster model (SACM) [10], in which the model space is microscopic, i.e. it is free from the Pauli-forbidden states, but the physical operators (expressed in terms of group-generators) are treated phenomenologically, i.e. they contain parameters, which are fitted to experimental data. The spin-isospin degrees of freedom are described by the $U^{ST}(4)$ group. When, however, only a single sector of the isospin is considered, its generators do not play any role in the physical operators, and in this sense we can say that e.g. a binary cluster system can be characterized by the chain:

$$U_{C_1}(3) \otimes U_R(4) \otimes U_{C_2}(3) \supset U_C(3) \otimes U_R(3) \supset U(3) \supset SU(3) \supset SO(3), \tag{1}$$

where $C$ and $R$ stand for cluster, and relative motion, respectively.

The SACM proved to be successful in describing the detailed spectra of some cluster systems [11]. It is also worth mentioning that the unified description of different cluster systems can also be carried out in this framework by the extensions of the dynamical symmetry (1). Different cluster-configurations of the same nucleus can be treated on an equal footing by applying the multichannel dynamical symmetry [12], while similar clusterizations (e.g. core-plus-alpha-particle) of different nuclei can be described by the supersymmetric model [13]. By applying large multiplet-structures of the microscopic model space, and unified physical operators, these schemes handle the problem with serious constraints, and consequently they have strong predictive power, too. (It is remarkable that currently another cluster supersymmetry-scheme has been established on the phenomenological level [14].)

## 4. Quasi-dynamical symmetry and clusterization

### *4.1. Phases and phase-transitions*

Phases and phase-transitions are usually investigated in systems with very large numbers of degrees of freedom. More recently, however, much interest has been concentrated on the phase-transitions in finite quantum systems e.g. atomic nuclei [15, 16]. Algebraic models seem to be especially useful in this kind of studies, and they were investigated thoroughly concerning the quadrupole collective motion. Here we investigate the problem from the viewpoint of clusterization (i.e. dipole collectivity) based on the results of [17].

In algebraic models one usually considers finite number ($N$) of particles, but it is possible to go to the large $N$ limit, where real phase-transitions can take place. For finite $N$ it can be investigated, whether or not some less robust changes survive. As a control parameter one has the relative weight of the Hamiltonians belonging to the different dynamical symmetries (analytically solvable limits). The energy-minimum is investigated as a function of the control parameter, and the degree of its derivative showing discontinuity (in the large $N$ limit) defines the order of the phase-transition.

In [17] we concentrated on the relative motion of some binary cluster systems. There are two relevant algebra-chains: (1) above, and

$$U_{C_1}(3) \otimes U_R(4) \otimes U_{C_2}(3) \supset U_C(3) \otimes O_R(4) \supset SO_C(3) \otimes SO_R(3) \supset SO(3). \tag{2}$$

Their physical content are the following. From the collective motion viewpoint (1) corresponds to a soft vibrator with spherical equilibrium shape, while (2) describes a rigid rotor with permanent dipole deformation. From the microscopic viewpoint (1) corresponds to shell-model-like clusters, while (2) describes localized clusters.

We have investigated binary cluster systems (with zero, one and two open-shell clusters) both in a phenomenological and in a semimicroscopical model (in order to study the influence of the Pauli-principle on the question of phase-transition). It turned out that first order phase transition takes place at a critical point both in the phenomenological and in the semimicroscopical model in the large $N$ limit. For finite systems the transition is smoothed out somewhat, but still observable. (The larger the model space the more abrupt the transition is.)

Another interesting finding was that in both models the quasi-dynamical $U(3)$ symmetry proved to be valid between the endpoint of the real dynamical symmetry and the critical point; i.e. throughout the whole phase. This observation, combined with some similar results in relation with the quadrupole model [18] indicates that the existence of the quasi-dynamical symmetry could be considered as the general definition of the phase in finite quantum systems. The situation resembles to that of the phase-transitions in Landau's theory, where the different phases are characterized by different symmetries [19].

Let us note here, that the localized (rigid rotor, $O(4)$) and the shell-like (soft vibrator, $U(3)$) cluster phases show very remarkable similarities to the "solid" and "liquid" phases discussed in other context at this conference [16, 20], despite the differences in the applied theoretical methods. In this respect it is an interesting open question, whether or not the third possible limit of the SACM

$$U_{C_1}(3) \otimes U_R(4) \otimes U_{C_2}(3) \supset U_C(3) \otimes U_R(3) \supset SO_C(3) \otimes SO_R(3) \supset SO(3), \tag{3}$$

which corresponds to the weak coupling between the relative motion and internal degrees of freedom, has any features of the "gas-like" phase [21].

Another interesting question (in which the literature is not unicvocal), if it is worth speaking about shell-like clusters at all, or we just could call clusters the localized ones, and consider the others as shell model states. It seems to us, however, more consequent to distinguish between localized and shell-like clusters because these states with approximately good (real) $U(3)$ symmetries may have very large cluster spectroscopic factors for some clusterization [22], and negligible for others. Thus their relation to clusterization is very structured, which may very well have differences in experimental observation, too.

### *4.2. Clusterization in heavy nuclei*

In case of light nuclei one can formulate a selection rule based on the approximate (real) $U(3)$ symmetry. It says that a binary cluster-configuration ($C_1 + C_2$) is allowed in a state characterized by the $[n_1, n_2, n_3]$ quantum numbers, if this set appears in the direct product: $[n_1^{C_1}, n_2^{C_1}, n_3^{C_1}] \otimes [n_1^{C_2}, n_2^{C_2}, n_3^{C_2}] \otimes [n_1^R, 0, 0]$. Otherwise it is forbidden (up to the approximation the $U(3)$ symmetry holds). It can be very useful in investigating the similarity between quadrupole deformation and clusterization, or for taking into account the Pauli-principle without carrying out the antisymmetrization (by checking if the cluster state can be found among the antisymmetric shell model states).

In medium and heavy nuclei, however, the $U(3)$ symmetry is not valid in its original form, due to the importance of the symmetry-breaking interactions, like spin-orbit and pairing. Nevertheless, it was found in [23] that in spite of the strong symmetry-breaking interactions the effective, or quasi-dynamical $U(3)$ symmetry, may survive even for heavy nuclei. In [24] a method was developed for the determination of the effective $U(3)$ quantum numbers, based on the occupation of the asymptotic Nilsson orbits. The procedure, which was originally invented for the large prolate deformation was extended in [25] to the oblate shape and small deformations as well, based on the expansion of single-particle orbitals in terms of asymptotic Nilsson-states.

The concept of effective symmetry is applicable also to light nuclei, and when the simple leading representation approximation is valid, the real and effective $U(3)$ quantum numbers usually coincide [25]. This circumstance gives a straightforward way for the extension of the simple selection rule consideration. Due to the average nature of these quantum numbers, however, the effect of the selection rule is different from that of the real $U(3)$ selection rule. It gives information on the matching, or mismatching of the average nucleon distributions in the cluster-configuration and in the shell-model-state. Therefore, it acts like a self-consistency check of the quadrupole deformation and the clusterization.

## 5. Exotic shapes and clusters

In order to study the deformation-dependence we have investigated the appearance of cluster-configurations in the ground, superdeformed and hyperdeformed states of some nuclei. The first two (at least in some cases) are known experimentally, the hyperdeformed states were predicted theoretically.

The main motivation of these studies was that in addition to taking into account the energetic preferences of different cluster-configurations we tried to incorporate the consequences of the exclusion principle as well. This latter one is done by the application of the real or quasi-dynamical $SU(3)$ symmetry for light and heavy nuclei, respectively, as mentioned above. (Please, note that the $SU(3)$ symmetry is known to recover for the super- and hyperdeformed shapes [26, 27].) In this way the effect of the Pauli-principle is handled only approximately, of course, but in a microscopic and well-controlled way, and its results can be tested by comparing with those of the fully microscopic calculations, where they are available. The forbiddenness of the cluster-configurations are characterized quantitatively. The deformation of the clusters (and parent nuclei) are taken into account, and no constraint is applied for their relative orientation.

The energetic preference of the clusterization is measured by the binding-energy difference (combined with the no-dipole constraint) of [28], on the one side, and in some cases with the more detailed double-folding potential energy of the dinuclear system model [29] on the other side. This latter quantity is determined both for the usual pole-to-pole configuration, and for the one, which is preferred by the selection rule.

We have considered the possible binary configurations for the $^{36}$Ar, $^{40}$Ca and $^{252}$Cf nuclei, [30, 31], and some ternary configurations [32] for the $^{36}$Ar and $^{252}$Cf.

The main conclusion of these calculations can be summarized as follows. The preference of the exclusion principle and the energy-calculation do not necessarily coincide. Therefore, we think that when searching for the most probable cluster-configuration(s), one has to take into account not only the energetic circumstances, but the exclusion principle, too. It also turned out that sometimes the same clusterization can be present both in the ground and in the superdeformed, as well as in the hyperdeformed state. The difference between them is the spatial arrangement of the deformed clusters.

## 6. Conclusion

In cluster studies, just like in many other physical problems, symmetry-considerations can help to find a simple solution to a complex problem. In this contribution we have tried to illustrate how the quasi-dynamical symmetry, which is one of the most general symmetry concept of quantum mechanics, can be applied. The two phenomena we have considered, the phases and phase-transitions of cluster states, and the exotic clusterization in heavy nuclei are of utmost interest. Obviously much work remains to be done until we reach their proper theoretical understanding, and it seems that symmetry-arguments can be fruitful along this line.

## Acknowledgments

This work was supported by the OTKA (Grant No. 46791), DGAPA, CONACyT, and by the MTA-CONACyT, as well as by the MTA-CSIC exchange programmes. One of us (J Cs) acknowledges fruitful discussions with dr N Itagaki on the phases of nuclear states.

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