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Elongated shape isomers in the ^{36}Ar nucleus

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A recent analysis of the ^{12}C + ^{24}Mg scattering [W. Sciani *et al.*, Phys. Rev. C **80**, 034319 (2009)] suggests the existence of a hyperdeformed band in the ^{36}Ar nucleus, completely in line with the predictions of α [W. D. M. Rae and A. C. Merchant, Phys. Lett. **B279**, 207 (1992)] and binary cluster calculations [J. Cseh *et al.*, Phys. Rev. C **70**, 034311 (2004)]. Here we review the structural understanding of the superdeformed and the hyperdeformed states of ^{36}Ar and present new results on the shape isomers as well. Special attention is paid to the clusterization of these states, which indicates the appropriate reaction channels for their formation.

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I. INTRODUCTION

The investigation of extreme, e.g., superdeformed (SD) and hyperdeformed (HD) nuclear shapes is one of the exciting topics in present day nuclear structure research. In particular, the superdeformed bands of $N = Z$ nuclei were observed during the last decade. These nuclei are known to have a rich cluster structure as well. The possible clusterizations of the shape isomers are important not only for better understanding of their structure but also for finding the reaction channels in which they can be populated.

The superdeformed band of ^{36}Ar was observed for the first time in Ref. [1], in a ^{24}Mg(^{20}Ne,2α)^{36}Ar reaction, by multiple (γ-ray and charged-particle) coincidence techniques. Following the experimental observation, considerable theoretical effort was concentrated on this state. We review this effort in the next section.

For the hyperdeformed band the first prediction was obtained from a Bloch-Brink α-cluster calculation [2]. In Ref. [3] the possible binary clusterizations of the ground, superdeformed, and predicted hyperdeformed states were investigated systematically. The aim of that study was partly to shed some light on the deformation dependence of the cluster configuration and partly to give a hint of the favorable reaction channels for populating the shape isomers. It turned out that the best reactions for reaching the hyperdeformed band are ^{24}Mg + ^{12}C and ^{20}Ne + ^{16}O. The recent careful analysis of the ^{24}Mg + ^{12}C scattering seems to justify this prediction. In particular, it revealed the existence of resonances, which together with previous ^{20}Ne + ^{16}O states [4,5] determine a rotational band, corresponding to the hyperdeformed state of the ^{36}Ar nucleus. It is in very good agreement with the prediction of the α-cluster model. In the next section we also show that a self-consistent-shape calculation, based on the Nilsson model and the quasidynamical SU(3) symmetry, predicts the same HD state, like the α-cluster model, completely in line with the experimental finding.

From the shell-model viewpoint the appearance of shape isomers is a consequence of the stable shell structure. A simple harmonic oscillator shell model shows that the shell structure is especially stable in the case of the ratio of 2:1:1 or 3:1:1 of the main axes of an ellipsoid (or in general when it is expressed as a ratio of small integer numbers). When this is the case then the problem of a single particle in a deformed oscillator potential has an exact SU(3) symmetry [6]. (The states of other deformations in light nuclei can also be characterized by an SU(3) symmetry [7], but this is not an exact symmetry, rather it is a dynamically broken one. In such a case the states have good SU(3) quantum numbers, but the interactions are not invariant.) This finding in the single-particle problem indicates that the appearance of a symmetry and the stability of the shell structure are intimately related to each other. In real nuclei, however, the residual interactions are very important; thus the nucleons do not sit in harmonic oscillator states. Then the question is, what remains from the nice order of the single-particle model in the complex reality? In the next section we illustrate that in terms of the quasidynamical (or effective) SU(3), the symmetry considerations can still be useful in searching for stable shapes of a nucleus. The quasidynamical symmetry is probably the most general symmetry concept of quantum mechanics, describing a situation when neither the operator (i.e., it is not a scalar) nor its eigenvectors (i.e., they do not transform according to an irreducible representation) are symmetric, yet the symmetry is present in some sense and has physical consequences. [8,9].

In Sec. III we pay special attention to the question of the possible clusterization of the shape isomers and extend the previous systematic investigations of Ref. [3] when needed.

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The clusters are not supposed to be spherical here, rather they can be prolate, oblate, or triaxial as well, like the nuclear states in general. The fact that the microscopic connections between the shell-model state and the cluster ones are known in the present treatment allows us to extend some considerations on the classically calculated moments of inertia typical in phenomenological cluster models. This is done in Sec. IV. Finally a summary is given in Sec. V.

II. SUPERDEFORMED AND HYPERDEFORMED STATES

Nilsson-model calculations are very illuminating in searching for stable deformations and shape isomers, both in general and in the specific case of the ^{36}Ar nucleus [10]. In addition to the oblate ground state they gave a largely deformed prolate state ($\epsilon = 0.74$, $\gamma = 7$ deg, with eight excitation quanta) and two very deformed states close to the oblate shape.

In relation with the experimental discovery of the superdeformed band in ^{36}Ar more detailed calculations of this type have been carried out that were able to reproduce the observed data to a good approximation, with four nucleon excitations to the next major shell [1,11]. This state is not exactly the same as the first shape isomer of Ref. [10], because of the different nucleon configurations, though the shape is similar. These studies were concentrated on the superdeformed state and no further systematic search was performed.

The superdeformed state was found also in large-scale shell-model calculations [1,11], as well as in cranked Skyrme-Hartree-Fock theory [12] and from the antisymmetrized molecular dynamics [13]. They turned out to be built on four excited particle configurations. As far as the shell model is concerned, more recent studies showed that, for the detailed explanation of its low-energy appearance and its electric transitions, the mixing of many-particle many-hole states is needed [14]. The coexistence of spherical, deformed, and superdeformed states was investigated also in Hartree-Fock BCS calculation in Ref. [15].

Here we present the results of a different kind of calculation, based on the Nilsson model. In particular, it consists in a self-consistency check with respect to the quadrupole deformation. It is done in terms of U(3) symmetries in the following way. The U(3) symmetry, which is an approximately good symmetry of light nuclei [7], is known to be uniquely related to the quadrupole shape [16]. Furthermore, when the real U(3) symmetry breaks down, e.g., with increasing excitation energy (due to some symmetry breaking interactions, like spin-orbit, pairing, etc.), a generalized version of it, called quasidynamical or effective U(3) symmetry, still survives [8]. The quasidynamical U(3) quantum numbers can be obtained from Nilsson calculations [17,18]. Thus, the self-consistency calculation consists in the continuous variation of the quadrupole deformation, as an input for a Nilsson calculation, and determination of the effective U(3) quantum numbers or, from them, the corresponding β_{out} quadrupole deformation. For lighter nuclei, like ^{24}Mg and ^{28}Si, where more detailed comparison could be made, the results of this self-consistency calculation are in very good agreement with those of the energy-minima calculations [19,20]. Thus

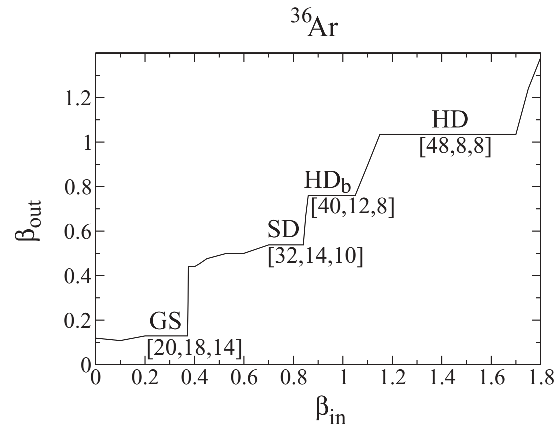

FIG. 1. Quadrupole deformation of the ^{36}Ar nucleus from the Nilsson model with the effective U(3) quantum numbers at the plateaus.

the stability of the quasidynamical U(3) symmetry can also indicate the appearance of shape isomers.

The result for the prolate input deformation of the ^{36}Ar nucleus, which is relevant for clusterization, is shown in Fig. 1. In this figure it is not the minima, but rather the horizontal plateaus, that correspond to the stable shapes. (They are insensitive to the small changes of the input parameter. Furthermore, these deformations fulfill the self-consistency argument, to a reasonable approximation, between the input and output deformation parameters.)

As is seen from Fig. 1, when proceeding toward larger deformation, after the stable region of the ground state, the next one corresponds to the superdeformed shape, with some uncertainty of the shape. This state represents four-nucleon excitation, being very much in line with the more recent Nilsson calculations, as well as with the shell-model calculations [1,10]. The uncertainty of the exact shape in the present Nilsson calculation may be related to what is observed as a many-particle many-hole mixing (to the dominant four-particle excitation) in the large-scale shell-model calculation.

With increasing β values two further stable plateaus appear, a less-pronounced one around 0.8 (β_{out}) and a very big one at $\sim$1.1. The first one coincides with the prolate state of Ref. [10] ($\epsilon \approx 0.8$, $\gamma \approx 7$ deg). They can be the candidates for the hyperdeformed band. Cluster studies can help us to select between them, as discussed in the next section.

III. CLUSTERIZATION

Bloch-Brink α-cluster model calculations have been performed to search for a hyperdeformed state of ^{36}Ar in Ref. [2] and have been performed systematically in Ref. [21]. As a result two triaxial excited states (b and c) and a hyperdeformed state, with a ratio of main axes of 3:1, were identified.

The lower-lying triaxial state, denoted by (b) in Ref. [21] corresponds to the SD state because it has $4\hbar\omega$ excitations, just like the superdeformed states of the recent Nilsson calculations (both for energy and for self-consistency), and it is largely prolate with a small triaxiality, with U(3) symmetry [32,16,8], close to the effective U(3) quantum numbers [32,14,10] of our Nilsson model calculations for the SD state.

The α-cluster calculation and our present shell-model study give exactly the same HD state of [48,8,8] symmetry.

It is an open question if the other stable α-cluster configuration (state (c) in Ref. [21]) corresponds to the less deformed shape isomer of Fig. 1 between the superdeformation and the hyperdeformation.

The allowed binary clusterizations of the superdeformed and hyperdeformed states in ^{36}Ar (together with those of the ground state) were investigated systematically in Ref. [3]. The clusters were supposed to be in their intrinsic ground state, with spherical or deformed (even triaxial) shapes, with no simplifying assumption for their relative orientation, and all the stable isotopes were considered as possible clusters. The problem was investigated both from the viewpoint of the microscopic structure and from the angle of energetic preference, calculated in terms of the criterion of maximum stability (combined with the no-dipole constraint) [22]. The structural aspect was treated by the combined application of the U(3) selection rule and Harvey's prescription (for a general discussion, see Ref. [20]).

In Ref. [3] the superdeformed and hyperdeformed states were defined in two different ways: by their quadrupole deformation (from the experiment for the superdeformed state and by the usual $\beta \approx 0.86$ value for the hyperdeformed state, noted there as SD(b) and HD(b) states, respectively) or by corresponding simple shell-model configurations noted as SD(a) and HD(a). The comparison with our present calculations (Fig. 1) gives further support for the superdeformed states of Ref. [3]. The hyperdeformed state obtained by the quadrupole deformation in Ref. [3] corresponds to the plateau at [40,12,8], and the one determined by simple shell-model configuration is the state [48,8,8].

The same study also revealed that the ground state allows more asymmetric cluster configurations, whereas the hyperdeformed state allows more symmetric cluster configurations, and the superdeformed state shows a picture in between. It also turned out that the ^{24}Mg + ^{12}C clusterization is allowed in each of these three states; the difference lies in the relative orientations of the deformed clusters.

To complete the binary cluster study we have looked at the allowed configurations of the [32,16,8] Ar(b) state of the α-cluster calculation in Ref. [21]. It turned out that core plus lighter clusters ($A = 4$–8) are allowed in this slightly triaxial state (whereas they are forbidden in the cylindrically symmetric one with U(3) symmetry [32,12,12]).

The role of the ^{32}S + α configuration can actually be important in the SD state, which is indicated by the fact that the spectroscopic details of the superdeformed band could be quantitatively reproduced by supposing such a configuration in Ref. [23].

As far as the hyperdeformed state is concerned, in addition to the ^{24}Mg + ^{12}C binary configuration, the ^{20}Ne + ^{16}O turned out to be allowed from among the α-like cluster-configurations, which are energetically preferred. This result was obtained as a prediction, based on the microscopic structure, without any use of experimental information, like the ones presented here. Therefore, the observation that the highly deformed molecular band is populated in the ^{24}Mg + ^{12}C and ^{20}Ne + ^{16}O reactions seems to be a nice experimental confirmation of a theoretical prediction.

A similar study concerning the possible ternary clusterizations of the ground, superdeformed, and hyperdeformed states is described in Ref. [24].

Careful experiments have been done concerning the core-plus-α-particle structure of highly excited states of the ^{36}Ar nucleus (see Refs. [25,26]). In particular, in Ref. [25] the (^{6}Li,d) transfer reaction was measured, and states with spin-parities of 5^- to 8^+ were observed in the energy range of 10–25 MeV, whereas in Ref. [26] the ^{32}S + α elastic scattering was measured with good resolution, and the R-matrix analysis resulted in 40 resonances in the ^{36}Ar excitation energy range of 12–16 MeV. The resonances observed in both reactions are situated below the bandhead of the possible hyperdeformed band we are referring to. They are situated in the SD band energy region and it is a challenge for the cluster studies to describe these states along with the shape isomers of ^{36}Ar.

IV. MOMENTS OF INERTIA

We have calculated classical rigid body moments of inertia both for the superdeformed and hyperdeformed states and for the cluster configurations. The nuclei were considered to be ellipsoids and the ratio of their main axes is given by the U(3) symmetry quantum numbers of the states, based on the self-consistency argument [27].

The volume was taken to be the same as the usual value of a sphere with radius $r_0 A^{\frac{1}{3}}$, $r_0 = 1.2$ fm. The moments of inertia are listed in Table I. The second column shows the experimental data (obtained from the linear fit to the SD and HD bands in Fig. 7 of Ref. [30]). The third column gives the values that correspond to the U(3) shell-model configurations. (When there is no axial symmetry, two values are indicated.)

As can be seen the ground-state band cannot be considered a rigid rotor (as usual), but the SD and HD bands give moments of inertia very close to those of the corresponding rigid bodies.

The experimental data definitely indicate that the molecular band corresponds to the HD state of larger deformation of Fig. 1, and not to the other one.

We have determined the moments of inertia also for some relevant cluster configurations. Table I contains the results for the ^{24}Mg + ^{12}C configuration, which is allowed in each of the shape isomers of Fig. 1. When doing these calculations we treated the clusters as ellipsoids whose relative orientations with respect to the molecular axis, as shown in Fig. 2, were obtained from the shell-model picture. We recall here that the ^{24}Mg is considered to be a triaxial cluster (like the ground state of the corresponding nucleus), having a long, a middle, and a short major axis. ^{12}C is oblate, with a short axis and two long major axes. The four different shape isomers correspond to cluster configurations, in which different major axes of the clusters are parallel with the molecular axis.

When supposing a touching configuration of the two rigid clusters, this picture overestimates the moment of inertia. This is a well-known fact, and in cluster studies it is usually corrected by applying a reduction factor (r) smaller than 1 [28]:

TABLE I. Moments of inertia (I) of the ground, superdeformed, and hyperdeformed states of the ^{36}Ar nucleus in 10^5 fm^2 MeV/c^2 units. The subscripts ex, sm, t, and c indicate experimental, shell model (i.e., rigid-body value corresponding to the shape of the shell-model state), touching cluster configuration, and compressed cluster configuration, respectively. The U(3) symmetry of the state (in italics for the quasidynamical symmetry) and the corresponding ratio of the major axes of the ellipsoidal shape are also listed. r shows the reduction factor of the cluster moment of inertia to that of the shell model $rI_t = I_{\rm sm}$, while c gives the reduction of the intercluster distance. (I_t and I_c have two values, when the cluster configuration is not axially symmetric.) For more explanation see the text.

State	$I_{\rm ex}$	$I_{\rm sm}$	U(3)	$a:b:c$	^{24}Mg + ^{12}C			
					I_t	r	I_c	c
		2.00	[20,20,12]	1.3:1.3:1	1.94	1	1.94	0.74
GS	0.92				2.72	0.74	2.00	
		1.97	*[20,18,14]*	1.2:1.1:1				
		2.09						
		2.83	[32,12,12]	1.7:1:1				
		2.75	*[32,14,10]*	1.8:1.1:1				
SD	2.97	2.95						
		2.67	[32,16,8]	1.9:1.3:1	4.52	0.59	2.67	0.66
		3.08			5.03	0.61	3.18	
HD$_b$		3.35	*[40,12,8]*	2.2:1.2:1	4.88	0.69		
		3.54			5.05	0.70		
		4.21	[48,8,8]	2.5:1:1	6.44	0.65	4.21	0.72
HD	4.4				6.47	0.65	4.24	
		4.21	*[48,8,8]*	2.5:1:1	6.44	0.65	4.21	0.72
					6.47	0.65	4.24	

$I = r(I_1 + I_2 + I_R)$, where I_1 and I_2 stand for the moment of inertia of cluster numbers 1 and 2, while I_R belongs to their relative motion. Usually spherical clusters are considered. The reduction of the moment of inertia is explained by the empirical finding that the nuclear moment of inertia is in between those of the rigid body and the irrotational flow ($r = 0.85$) [29].

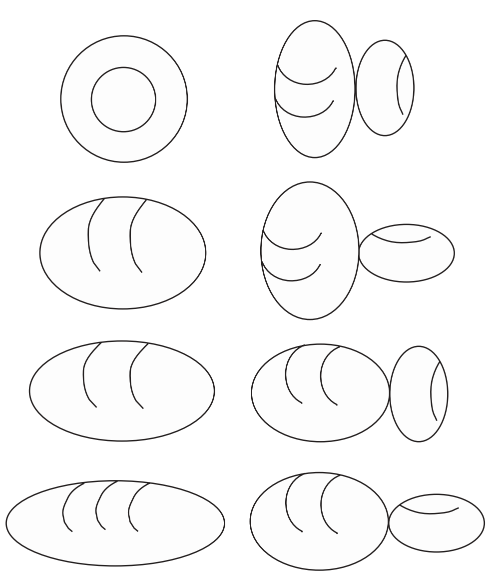

FIG. 2. Shape isomers of Fig. 1 (GS, [20,20,12]; SD, [32,16,8]; HD$_b$, [40,12,8]; HD, [48,8,8]) with their corresponding ^{24}Mg + ^{12}C cluster configurations.

By applying the U(3) selection rule and Harvey's prescription we have the single-nucleon configurations of cluster states as well as the shell-model state obtained from their amalgamation. As mentioned above, the nuclear shape is considered to be ellipsoidal, without any constraint, and the relative orientations were obtained from the nucleon distribution. Thus we could compare directly the moments of inertia of the shell-model and cluster states to determine the factor r. It is shown by Table I to be in the region of 0.6–0.75. Similar values are obtained for the ^{20}Ne + ^{16}O clusterization in the HD state, 0.69 (in this configuration the symmetry axis of the prolate ^{20}Ne is parallel with the molecular axis), and for the ^{32}S + ^{4}He configuration in the SD state, 0.72 or 0.75 (when the triaxial ^{32}S lines its longest axis with the molecular one). It is a bit smaller than what is usually applied in the picture of the spherical clusters in the phenomenological cluster models.

This procedure, however, is unable to reflect any geometrical dependence of the overlap of the clusters, which is expected to be more important in between the two clusters than in other parts of the dinuclear configuration. Therefore, we applied another way of reducing the moment of inertia, which was the reduction of the distance between the centers of gravity of the two clusters (by a factor of c). As can be seen the numerical values are close to those of r for the ^{24}Mg + ^{12}C configuration, and the result is in better agreement with the shell-model value (see the axially symmetric ground state for example). For the ^{20}Ne + ^{16}O clusterization a close-lying value is obtained (0.76), but for the ^{32}S + ^{4}He configuration it is smaller, 0.51, indicating that the light α-cluster penetrates deeper into the core.

V. SUMMARY

For the microscopic structure of the experimentally known superdeformed band of ^{36}Ar many different calculations give $4\hbar\omega$ excitation. The exact shape is a bit uncertain. The quasidynamical U(3) symmetry obtained in this work involves a slight triaxiality. There is actually a close-lying cylindrically symmetric shape; however, the joined conclusion of the experimental observation and several theoretical studies prefers the triaxial shape.

The hyperdeformed shape was first predicted from α-cluster calculations [2] concentrating especially on this deformation. Our present systematic symmetry consideration gives two further prolate shape isomers beyond the SD state. One of them is an $8\hbar\omega$ state with $\beta \approx 0.8$, the other one corresponds to $12\hbar\omega$ and more deformed $\beta \approx 1.1$. This latter one corresponds to the prediction of the α-cluster model for the HD state and is in line with experimental observations (of the moment of inertia). Furthermore the preferred clusterizations of this state are ^{12}C + ^{24}Mg and ^{16}O + ^{20}Ne, as predicted in Ref. [3], completely in line with the molecular resonances, which build up the rotational band [30]. These circumstances suggest that in addition to its recently observed superdeformed band [1] the ^{36}Ar nucleus has a hyperdeformed state as well, observed as resonance states in the ^{12}C + ^{24}Mg and ^{16}O + ^{20}Ne reactions [30].

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