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STOPPED NUCLEONS IN CONFIGURATION SPACE*

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In this note, using the simplified colour string model, we study the configuration space distribution of stopped nucleons in heavy-ion collisions. Given this model, we find that the stopped nucleons from the target and the projectile end up separated from each other by a distance which increases with the collision energy. In consequence, for the center-of-mass energies larger than 6 or 10 GeV (depending on the details of the model), it appears that the stopped nucleons are not necessarily in thermal and chemical equilibrium, and the net-baryon density reached is likely not much higher than that already present in the colliding nuclei.

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1. Introduction

Searching for structures in the QCD phase diagram has been at the centre of the research on the strong interactions for many years. Experimentally, dense and hot QCD matter is created by colliding heavy ions at ultra-relativistic energies, and experiments at RHIC and the LHC have found that this matter has remarkable properties, such as a small shear-viscosity to entropy ratio as well as an unexpected opaqueness for jets [1–3]. At the same time, Lattice QCD methods have improved to the point that continuum extrapolated results for the equation of state [4, 5] and also for the nature of the phase transition at vanishing baryon density [6] have been obtained. The latter was found to be an analytic cross-over transition. Model calculations, on the other hand, predict (see *e.g.* [7, 8]) a first order transition at vanishing temperature and large baryon densities. If this were the case, this first order phase co-existence will have to end at a critical point, the location of which is not well-constrained by neither model nor lattice calculations.

The existence of a critical point and first order phase co-existence region in the QCD phase diagram has sparked a dedicated experimental program, the so-called RHIC beam energy scan (BES). The basic idea behind this program is that by lowering the beam energy, one can create systems at higher average net-baryon density, and indeed experiments at the CERN SPS and results from the first phase of the BES show that the observed net-baryon number at mid rapidity increases with decreasing beam energy (see *e.g.* [9, 10]). Since no net-baryon number is being produced in the collision, the only way to increase the average net-baryon density is to transport baryons from projectile and target to the mid-rapidity region. Or, in other words, the baryons need to be stopped in the centre-of-mass frame. However, stopping the baryons is not enough. In order to have matter at high baryon density, the stopped baryons *from both nuclei* also need to be located within the same (and small enough) region in *configuration* space. That this is not so easy to achieve will be the central point of this note.

This paper is organised as follows: In the next section, we will set up the formalism based on a simple string picture. Next, we will show some results before we conclude.

2. Configuration space distribution of stopped nucleons

As already pointed out in the introduction, studies of the new states of matter created in collisions of heavy ions must involve the discussion of the particle distributions in configuration space. As these distributions cannot be directly measured, one has to rely on indirect methods or on models of particle production. Here, we study the distribution along the beam (z) direction of nucleons which, after the collision of two heavy nuclei, lost a large part of their momentum and are located in the mid-rapidity region, *i.e.* close to the c.m. rapidity, $y = 0$.

Denoting by σ the energy loss per unit length, one has

$$\mathrm{d}E_z = -\sigma \mathrm{d}z \,, \tag{1}$$

where E_z is the nucleon energy (depending on z) and z is its position along the beam axis. Assuming σ to be a constant, as is the case of the Lund model [11] and in the bremsstrahlung mechanism [12], one obtains

$$E_z = E_{\rm i} - \sigma(z - z_{\rm c})\,, \tag{2}$$

where $E_{\rm i}$ is the initial energy and $z_{\rm c}$ is the collision point in configuration space.

In the c.m. frame, the two nucleons which are to collide, move right and left with velocity V, defined by the initial energy $E_{\rm i} = \frac{1}{2}\sqrt{s} = \sqrt{P^2 + M^2}$. We denote their positions at $t = 0$ by $\zeta_{\rm L}$ and $\zeta_{\rm R}$.

The collision space-time point $(z_{\rm c}, t_{\rm c})$ satisfies

$$z_{\rm c} = \zeta_{\rm R} + Vt_{\rm c}\,; \qquad z_{\rm c} = \zeta_{\rm L} - Vt_{\rm c} \tag{3}$$

giving

$$Vt_{\rm c} = \frac{\zeta_{\rm L} - \zeta_{\rm R}}{2}\,; \qquad z_{\rm c} = \frac{\zeta_{\rm L} + \zeta_{\rm R}}{2}\,. \tag{4}$$

It is straightforward to relate $z_{\rm c}$ and $t_{\rm c}$ to the position of the nucleon z, its rapidity y, and the time at which it arrives at z. From (2), we have

$$z - z_{\rm c} = \pm\frac{E_{\rm i} - E_z}{\sigma} = \pm\frac{E_{\rm i} - M_\perp \cosh y}{\sigma}\,, \tag{5}$$

where the sign depends on the direction of the nucleon (left or right) and $M_\perp^2 = M^2 + P_\perp^2$. The time when the nucleon arrives at the point z can be obtained using the equation of motion $\mathrm{d}P/\mathrm{d}t = -\sigma$ giving

$$t - t_{\rm c} = \frac{P_{\rm i} - P_z}{\sigma} = \frac{P_{\rm i} - M_\perp \sinh y}{\sigma}\,. \tag{6}$$

Now that we have an expression for the space and time distance from the collision space-time point for the right and left moving particles in a nucleon–nucleon collision, we turn to nucleus–nucleus collisions. To this end, we first derive the distribution of collision points, given the distributions of nucleons in the colliding nuclei.

The distributions of the nucleon positions at $t = 0$, given by the nuclear shapes, depend on $(\zeta_{\rm L} - \zeta_{\rm L0})$ and $(\zeta_{\rm R} - \zeta_{\rm R0})$, where the subscript 0 denotes the position of the centre of the nucleus (at $t = 0$). Using (4), one obtains the distribution of collision points

$$\begin{aligned} F_{\rm c}(z_{\rm c}) &= \int \mathrm{d}\zeta_{\rm L}\mathrm{d}\zeta_{\rm R} G_{\rm L}[\gamma(\zeta_{\rm L} - \zeta_{\rm L0})]\; G_{\rm R}[\gamma(\zeta_{\rm R} - \zeta_{\rm R0})]\delta\left(\frac{\zeta_{\rm L} + \zeta_{\rm R}}{2} - z_{\rm c}\right) \\ &= 2\int \mathrm{d}\zeta_- G_{\rm L}[\gamma(z_{\rm c} - \zeta_- - \zeta_{\rm L0})]\; G_{\rm R}[\gamma(z_{\rm c} + \zeta_- - \zeta_{\rm R0})]\,, \end{aligned} \tag{7}$$

where $\zeta_{\pm} = (\zeta_{\rm R} \pm \zeta_{\rm L})/2$ and γ is the Lorentz factor. Here, $G_{\rm L}$ and $G_{\rm R}$ are the distributions of the left- and right-going nuclei, respectively. It should be realised that this formula treats the multiple collisions of a nucleon as independent collection of nucleon–nucleon collisions. Although this is a crude approximation, we feel that it is sufficient for our semi-quantitative study.

The distribution of the positions z, where the particles are decelerated to rapidity y for the right-moving, $P_{\rm R}$, and the left-moving nucleons, $P_{\rm L}$, are then readily obtained from the collision point distribution, Eq. (7), by using Eq. (5)

$$\begin{aligned} P_{\rm R}(z;y) &= F_{\rm c}\left(z - \frac{E_{\rm i} - M_{\perp}\cosh y}{\sigma}\right) , \\ P_{\rm L}(z;y) &= F_{\rm c}\left(z + \frac{E_{\rm i} - M_{\perp}\cosh y}{\sigma}\right) , \end{aligned} \tag{8}$$

and

$$P(z;y) = P_{\rm R}(z;y) + P_{\rm L}(z;y) \tag{9}$$

is then the distribution of the space points where the incident nucleons from the projectile and the target nuclei are decelerated to rapidity y (see the last section for further discussion).

In our semi-quantitative studies, we assume Gaussians for $G_{\rm L}$ and $G_{\rm R}$ with central positions $\zeta_{\rm L0}$, $\zeta_{\rm R0}$ and widths $R_{\rm L}$ and $R_{\rm R}$

$$G_{\rm L}(\zeta_{\rm L}) \sim e^{-(\zeta_{\rm L}-\zeta_{\rm L0})^2/R_{\rm L}^2} ; \qquad G_{\rm R}(\zeta_{\rm R}) \sim e^{-(\zeta_{\rm R}-\zeta_{\rm R0})^2/R_{\rm R}^2} . \tag{10}$$

The collision point distribution is consequently given by

$$F_{\rm c}(z_{\rm c}) \sim e^{-4\gamma^2[z_{\rm c}-\Delta_0]^2/\left(R_{\rm L}^2+R_{\rm R}^2\right)} ; \qquad \Delta_0 = \frac{\zeta_{\rm R0}+\zeta_{\rm L0}}{2} , \tag{11}$$

and in the following, we will take $\Delta_0 = 0$.

3. Results

The distributions of z, evaluated for the flat rapidity distribution in the region $|y| < 1$[1], assuming baryon density per unit of rapidity equal to unity, are shown in Fig. 1 ($\sigma = 1$ GeV/fm for the wounded nucleon model [13]) and Fig. 2 ($\sigma = 3$ GeV/fm for the wounded quark model [14]). Here, $R_{\rm L} = R_{\rm R} = 6.5$ fm, see Eq. (10), and the transverse momentum, $P_{\perp}$, of the final nucleons was taken to be 1 GeV/c.

[1] Our results change weakly when narrowing the rapidity bin.

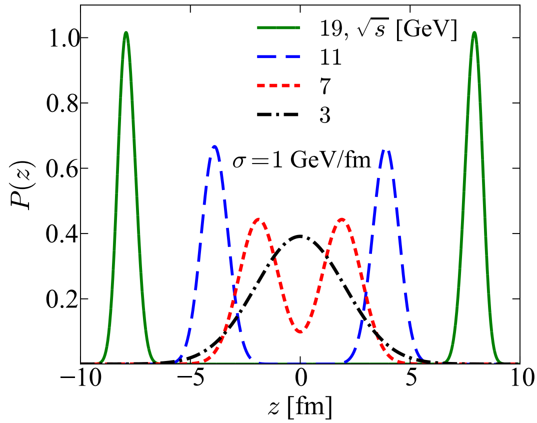

Fig. 1. The z distribution of stopped nucleons at mid-rapidity for $\sigma = 1$ GeV/fm (wounded nucleon model) and various c.m. energies, $\sqrt{s} = 3, 7, 11$ and 19 GeV/c.

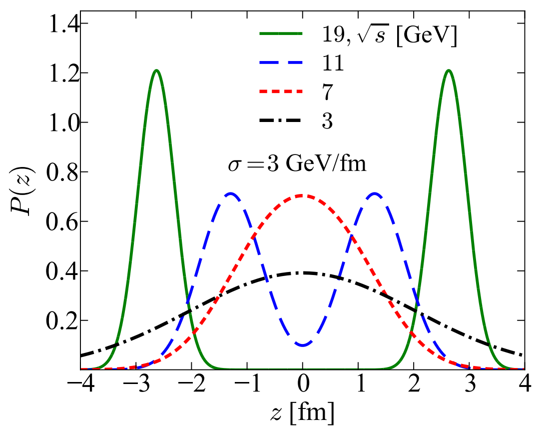

Fig. 2. The same as in Fig. 1 but $\sigma = 3$ GeV/fm (wounded quark model).

With increasing energy of the collision, the separation between the positions of the left- and the right-movers is increasing, while the width of the two peaks is decreasing. They are clearly separated at energies above $\sqrt{s} \simeq 10$ GeV for the wounded nucleon model and for $\sqrt{s} \simeq 20$ GeV for the wounded quark model.

At the same time, one sees that the density in the two separated peaks increases rapidly with the incident energy. Thus, at these higher energies, there are two regions in configuration space where the baryon density may reach substantial values.

One should keep in mind, however, that the distributions shown in Figs. 1 and 2 are normalized to a fixed number (the integrals of P_{R} and P_{L} over z equal 1). Consequently, to obtain the predicted baryon density in the configuration space, they must be multiplied by the measured baryon density in the central rapidity region, $|y| \leq 1$. With increasing energy, the net-baryon density in this region decreases and this may substantially reduce the effect.

Although the nucleons in the two separated peaks of Figs. 1 and 2 are close enough in space (and time) to interact, it remains an open and interesting question if they can eventually form an equilibrated state at high baryon density. Even if this is not the case, however, they may contribute to the elliptical flow v_2 of protons measured by the STAR Collaboration [15]. Indeed, in the transverse space, they are apparently correlated with the produced particles, since both groups likely reveal a similar "almond" shape. However, to translate this transverse space correlation into the transverse momentum distribution is not easily achieved as it demands a certain number of collisions to happen before the nucleons fly apart. This question may perhaps be answered by studying various event generators [16, 17].

4. Discussion

As the results shown in the previous section are based on a rather simplified model, it is essential to discuss the arguments supporting our approach, as well as its limitations. They are listed below.

(i) First of all, our main observation is rather straightforward. It is well-established that *e.g.*, at $\sqrt{s} = 19$ GeV, we have a substantial number of protons at $y \approx 0$ originating mostly from stopping (*e.g.*, protons from the initial nuclei). Clearly, it takes time and space to stop protons[2], thus it is quite natural that the stopped protons from the projectile and the target might end up separated in the coordinate z axis, and that this separation may be surprisingly large. This is the main message of our paper.

(ii) While there might be different and more refined microscopic pictures for the deceleration of the nucleons, it is obvious that having the nucleons stop at small z (close to $z = 0$) requires near infinite deceleration, which seems rather unphysical and if confirmed in experiment would constitute an interesting discovery.

(iii) We note that a different picture of baryon stopping is usually assumed in transport models, such as UrQMD, where strings are immediately

[2] Except for very small energies where elastic scattering can stop protons almost instantaneously.

decayed with a formation time, see *e.g.* [18, 19]. It is not clear to us if a picture with essentially an instant stopping is closer to reality than our proposition. To our knowledge, there is no experimental evidence that would support an instant deceleration and we believe that the studies of alternative mechanisms are justified.

(iv) In our approach, we assume that the stopping process can be described by a constant string tension σ [see Eq. (1)], acting during a certain time, needed for neutralisation of the colour field (represented by the colour string). After this "neutralisation" time, particle emission ends and the proton does not loose energy any more. This means that between the collision point where the string is formed and the final string "neutralisation" point, the stopping process is given by a constant deceleration. Such a simplified picture may, naturally, need some refinements. Since we, however, discuss only single particle distributions, such refinements are not expected to change our results qualitatively.

(v) By adequate selection of the distribution of the "neutralisation" times (which is not restricted *a priori*), the colour string model can obviously account for the observed distribution of the proton momentum and/or rapidity (see *e.g.* [20]). This also means, however, that the model cannot *predict* the rapidity distribution of the final protons. Fortunately, this information is not needed for the questions we discuss. Actually, we use the energy of the stopped protons as the input to estimate the "neutralisation" time and thus the distance the nucleon must have travelled before it stopped losing energy. In other words, having rapidity and transverse momentum of the stopped nucleon, we can estimate the time and space needed to reduce the initial beam energy into the final one.

(vi) It is well-known that in collisions of the nucleon with the target nucleus, the distribution of final nucleons depends on the centrality of the collision [21, 22]. This effect is described in the colour string model by observation that the string tension σ increases with the number of collisions inside the nucleus[3]. That is why we consider two values of σ, $\sigma = 1$ GeV/fm and $\sigma = 3$ GeV/fm, ranging from peripheral to central collisions. This point of view can be justified by the success of the wounded quark (quark–diquark) model, see, *e.g.*, [23–26], in describing soft particle production in $p + A$ and $A + A$ across a broad range of energies. In this model, the number of produced soft particles and thus the energy lost by the proton depends on the number of its collisions. In $p + p$ or peripheral $p + A/A + A$ collisions, roughly one

[3] This is equivalent to adding more strings.

quark per nucleon interacts, while in central $p+A/A+A$, practically all three quarks are involved, resulting in stronger stopping. Thus, $\sigma = 1-3$ GeV/fm should cover all various scenarios encountered in $p+A/A+A$ interactions[4].

(vii) Clearly, our arguments are questionable at low energies (below, say, $\sqrt{s} = 10$ GeV) where resonances and elastic scattering should be included. We think, however, that it is interesting to discuss the z distribution of the stopped nucleons even with a not fully adequate model in order to pave the way for future more sophisticated analysis.

Finally, let us remark that we are discussing *soft* collisions which cannot be easily treated in the framework of perturbative QCD. Thus, the recent extensive discussion of the "jet quenching", see *e.g.* [28, 29], is not directly applicable to our problem.

5. Concluding remarks and comments

Based on the colour string fragmentation picture, we have studied the distribution of "stopped" nucleons in the longitudinal configuration space (z). We find that there are two regions where one may hope to create a baryonic system with net-baryon density higher than nuclear density.

In the central region, $z \approx 0$, where the stopped nucleons from the two colliding nuclei overlap, a relatively high baryon density can be achieved at low energies. The actual value of this limiting energy varies from $\sqrt{s} \sim 10$ GeV to $\sqrt{s} \sim 6$ GeV, depending on details of the model.

In the regions far off $z = 0$, Lorentz contraction of nucleons from one of the colliding nuclei may, at high enough energy, also produce relatively large baryon density, provided the number of baryons stopped at $y \sim 0$ is large enough. Since this number is rapidly decreasing with increasing energy, however, it seems unlikely that these regions play an important role, except in events of very high multiplicity.

Clearly, a longitudinal configuration for higher energies, as seen in Figs. 1 and 2, does not resemble a system in thermal and chemical equilibrium, where the baryons should be uniformly distributed in configuration space. Therefore, if our considerations are correct, it may be difficult to extract information about the properties of dense matter such as it is determined, for example, in Lattice QCD.

[4] We have verified that the evaluation of the proton inelasticity in the dual-parton model [27] also falls in this range.

Several comments are in order.

(i) Equation (8) describes the distribution of the z points where the incident nucleons from the projectile and the target nuclei are decelerated to rapidity y. What we neglect in our discussion is the fact that nucleons stop loosing energy (and reach their rapidity y) at different times, not only because they come from different collision points, but also their final rapidities are different (when we consider particles in some finite rapidity bin). Ideally, we should study the time evolution (time snapshots) of all decelerating nucleons, however, this is a rather nontrivial problem to tackle analytically.

(ii) Although at high energies the process of particle emission is practically the only effective way to transport the colliding nucleons to the central rapidity region, at relatively low energies we are considering, other mechanisms may be important and cannot be neglected. The most effective one seems to be the excitation of nucleon resonances[5]. This may give substantially larger baryon density than that expected from the Lund model.

(iii) When the nuclei pass through each other, the baryon density may, of course, reach values which exceed the nuclear density. However, even at the energy as low as $\sqrt{s} = 6$ GeV, the rapidities of the incident nucleons exceed 1, and thus they cannot be observed in the present RHIC experiments. Moreover, it is difficult to see how these nucleons may form an equilibrated system without important inelastic interactions. Investigation of this question (as well as the one mentioned above) using event generators [16, 17] may help to clear up this problem.

(iv) It remains an open question if, at high energy, the nucleons from one nucleus, arriving to the small region of z (see, *e.g.*, the curves for $\sqrt{s} = 19$ GeV in Figs. 1 and 2) may actually form an equilibrated baryon system. This is an interesting problem to study using various event generators.

(v) One may worry that the relative distance between the nucleons which, according to the Lund model, stop at a specific position in the z space, is affected by their interaction through the nuclear forces before they reach the final destination. We expect this effect to be small because the time needed for the nuclear forces to operate (~ 1 fm) is longer than the time needed for nucleons (in their rest frame) to travel from the collision point to the point when they stop to radiate.

[5] *E.g.* production and decay of a baryon resonance of mass M^* in the process $p + p \to M^* + M^*$ gives the approximate shift of the baryon rapidity which varies from $\log(M/2M^*)$ (at threshold) to $\log(M/M^*)$ (at high energy, $E \gg M^*$).

(vi) Recent progress in femtoscopy opens the possibility of experimental verification of the ideas presented in this work[6]. We feel that such measurements could be of real help in determining if at low and medium energies (which are of main interest in investigation of the strongly interacting matter at high density), the colour string model is indeed the dominant mechanism of bringing baryons at rest in the c.m. frame.

In our view, the present study, although admittedly based on a simple model and fairly crude approximations, calls for a detailed investigation of the phase-space density evolution of baryons in heavy-ion collisions.

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