Hierarchical and Nesting Approaches for the Facility Layout Problem

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The Facility Layout Problem (FLP) seeks to determine the dimensions, coordinates and arrangement of rectangular departments within a given facility. The goal is to minimize the cost of inter-department flow. It has several real-world applications, including the design of manufacturing and warehousing facilities and electronic chips. Despite being studied for several decades, the FLP is still very difficult to solve for facilities with thirty or more departments. Thus, many heuristic approaches have been developed to solve the problem in a reasonable time. One such approach tackles the problem in two stages. In the first, some decision, usually the relative positioning of the departments, is fixed. In the second, an easier restricted problem is solved.

This thesis explores hierarchical and nesting approaches for the FLP, in an attempt to leverage the fact that smaller instances of the FLP can be solved to optimality relatively quickly. The goal is to find ways in which the FLP can be decomposed into several smaller problems and recombined to form a high-quality solution to the original problem.

Hierarchical approaches use clustering or related methods to generate a tree where the leaves are the original departments and the root is the facility. The intermediate nodes are super-departments within an overall layout. A new hierarchical approach for the FLP is presented which performs layouts down this tree in a manner that controls deadspace and generates high-quality solutions. The approach provides solutions competitive with the best-known solutions on benchmark instances from the literature, with up to 8% improvement.

The success of the hierarchical approach provided the motivation for a new formulation that nests departments within super-departments. The resulting formulation is even more difficult to solve directly than the original FLP; however, it is suitable for a two-stage solution approach. The first stage determines the assignment of departments to super-departments and the relative positioning of the super-departments. In the second stage, the remainder of the formulation is solved. The approach is found to provide better solutions than the hierarchical approach. Solutions are found with up to 14% improvement over the best-known solutions from the literature.

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List of Symbols

M A very large number. 24

S The set of all super-departments. 30–33, 36, 37, 40

U A set of departments/super-departments, in the fixed location FLP formulation, whose coordinates and dimensions are variable. 22–24

α A parameter that penalizes departments which lie outside of the specified boundary in the fixed location FLP formulation. 22

A_i The minimum area required for department i. 4, 5, 30, 32, 33, 37

β_S The maximum aspect ratio for super-departments. 19, 25, 26, 30, 32, 35, 41

β The maximum aspect ratio, or the maximum allowable ratio of width to height and height to width, defined here as ≥ 1. 4, 5, 25–28, 30, 32, 35, 36, 42, 43, 49, 51, 61–63, 65–67

c_{ij} The inter-department cost, or the cost per unit distance of transporting all goods from department i to department j and vice versa 4, 5, 18, 21, 22, 30, 32, 34, 36, 37

$\hat{d_{kl}}$ An approximation of the distance between two departments, where one department is placed in super-department k and the other in super-department l. 37, 38

d_{ij}^x The horizontal component of the rectilinear distance between department i and department j. 4, 5, 22, 30, 32, 33, 36

d_{ij}^y The vertical component of the rectilinear distance between department i and department j. 4, 5, 22, 30, 32, 33, 36

D The set of all departments. Used to distinguish departments from super-departments in the nesting approach. 30, 32, 33, 36, 37

F A set of departments/super-departments, in the fixed location FLP formulation, whose coordinates and dimensions are fixed. 22, 23

γ A trade-off factor which weighs the value of intra-cluster similarity to inter-cluster dissimilarity. 21, 26, 34, 35

$\bar{h}_i$ The value of department i's fixed height, which was determined in a previous layout, in the fixed location FLP formulation. 23

h_f The height of the facility. 4, 5, 30, 32, 33, 36, 37

h_i The height of department i. 4, 5, 22–24, 30, 32, 33

h_k The height of super-department k. 31–33, 36

$h_i^{\max}$ The maximum height of department i. 4, 5

$h_i^{\min}$ The maximum height of department i. 4, 5

m The number of super-departments. 34, 35, 46

$n^{\mathbf{cols}}$ The number of columns of super-departments in the grid-based approach. 49, 65–67

$n^{\max}$ The maximum number of departments per cluster/super-department. 21, 25, 26, 34, 37, 38, 40, 61–63

$n^{\mathbf{rows}}$ The number of rows of super-departments in the grid-based approach. 46, 49, 65–67

n The number of departments to be placed in the facility. 3–5, 22, 24, 30, 34, 40

p_i^{x+} In the fixed location FLP formulation, the distance along the x-axis in the positive direction in which department i has surpassed the boundary of its parent. 22, 23

p_i^{x-} In the fixed location FLP formulation, the distance along the x-axis in the negative direction in which department i has surpassed the boundary of its parent. 22, 23

p_i^{y+} In the fixed location FLP formulation, the distance along the y-axis in the positive direction in which department i has surpassed the boundary of its parent. 22, 23

p_i^{y-} In the fixed location FLP formulation, the distance along the y-axis in the negative direction in which department i has surpassed the boundary of its parent. 22, 23

q_{ik} Assignment variable which takes value 1 if department i is assigned to super-department k and 0 otherwise. 31–34, 36–38

s_{uv} The similarity measure between cluster or super-department u and cluster or super-department v. 21

$T_{\mathbf{search}}$ The time in seconds that the first stage search algorithm took to find its solution. 41, 42, 51, 61–63, 65–67

$T_{\mathbf{solve}}$ The time in seconds that it took to solve the second stage model. 41, 42, 51, 61–63, 65–67

$\bar{w}_i$ The value of department i's fixed width, which was determined in a previous layout, in the fixed location FLP formulation. 23

w_f The width of the facility. 4, 5, 30, 32, 33, 36

w_i The width of department i. 4, 5, 22–24, 30, 32, 33

w_k The width of super-department k. 30, 32, 33, 36

$w_i^{\max}$ The maximum width of department i. 4, 5

$w_i^{\min}$ The minimum width of department i. 4, 5

$\bar{x}_i$ The value of department i's fixed x-coordinate, which was determined in a previous layout, in the fixed location FLP formulation. 23

x_i The x-coordinate of the centroid of department i. 4, 5, 22–24, 30, 32, 33

x_k The x-coordinate of the centroid of super-department k. 31–33, 36

x_s^+ The upper limit of the x-axis, in the fixed location FLP formulation, where unfixed departments may be placed. 22

x_s^- The lower limit of the x-axis, in the fixed location FLP formulation, where unfixed departments may be placed. 22

$\bar{y}_i$ The value of department i's fixed y-coordinate, which was determined in a previous layout, in the fixed location FLP formulation. 23

y_i The y-coordinate of the centroid of department i. 4, 5, 22–24, 30, 32, 33

y_k The y-coordinate of the centroid of super-department k. 31–33, 36

y_s^+ The upper limit of the y-axis, in the fixed location FLP formulation, where unfixed departments may be placed. 22

y_s^- The lower limit of the y-axis, in the fixed location FLP formulation, where unfixed departments may be placed. 22, 23

$\bar{z_{ij}}$ A parameter in the fixed relative positioning formulation that determines in which direction non-overlap should be enforced between department i and department j. 23, 24

$\bar{z_{ij}}^x$ A parameter in the fixed relative positioning formulation that determines whether department i should be placed to the right or the left of department j. 23, 24

z_{kl}^x Horizontal relative positioning variable for departments. Takes value 1 if $x_k > x_l$ and 0 otherwise. 31, 33, 36

z_{ij}^x Horizontal relative positioning variable for departments. Takes value 1 if $x_i > x_j$ and 0 otherwise. 4, 5, 30, 32, 33

$\bar{z_{ij}}^y$ A parameter in the fixed relative positioning formulation that determines whether department i should be placed above or below department j. 23, 24

z_{kl}^y Vertical relative positioning variable for departments. Takes value 1 if $y_k > y_l$ and 0 otherwise. 31, 33, 36

z_{ij}^y Vertical relative positioning variable for departments. Takes value 1 if $y_i > y_j$ and 0 otherwise. 5, 30, 32, 33

List of Abbreviations

Chapter 1

Introduction

The FLP is concerned with determining the layout of non-overlapping departments within a facility such that the cost of inter-department flow is minimized. Given shape and area requirements, the heights, widths, and coordinates of each department must be determined. In addition to inter-department material handling, other costs, such as construction or communication, can be incorporated into the objective function. Given the difficulty of the FLP, the literature offers several exact and heuristic methods.

This thesis provides a literature review which details many of these exact and heuristic methods. The notable trend in FLP research is that many of the heuristic approaches are two or multi-stage approaches which fix some aspect of the problem, most often the relative positioning of the departments, in a first stage, and solve the remaining problem in a second stage.

Relatively small instances of the FLP can be solved very quickly, but once there is a moderate number of departments in the facility, such as thirty, it becomes nearly impossible to solve using an exact approach. Thus, this thesis explores ways in which the FLP can be decomposed into smaller problems, whose solutions can be combined into a solution to the overall problem. A hierarchical approach, which performs exactly this, is presented, tested, and shown to provide improvements over existing approaches.

This idea works well; however, the performance losses associated with solving long sequences of models and the potential of obtaining an infeasible result motivates a mathematical model that incorporates hierarchical aspects. Some approaches along these lines have been presented in the literature, most notably Flexible Bays approaches covered in Meller (1997) and Konak et al. (2006). These approaches, however, make extremely restrictive assumptions, such as the linear ordering of departments within bays and a linear ordering of bays. This thesis presents a new model that incorporates the idea of encapsulating departments within bays, or super-departments as they are referred to in this thesis, but without these restrictions. Such a model is shown to be initially unwieldy. However, by making a few restrictions about only the relative positioning of the super-departments and the department to super-department assignments, the model produces high quality solutions in a reasonable amount of time.

This nesting approach requires a method of determining the relative positioning of the super-departments and the assignments of departments to super-departments. For the relative positioning aspect of the problem, two approaches are developed. The first assumes that super-departments are oriented as columns in the facility. The second approach uses a more general case where super-departments are arranged as a grid. Three metaheuristics were developed to solve the first stage assignment problem for each approach. These two approaches provide improvements over the hierarchical approach. However, the column-based approach provides better results for some instances, while the grid approach provides better results for other instances. Furthermore, there is no clear best metaheuristic for solving the first stage problem. These findings provide motivation for future research.

The remainder of this chapter is organized as follows: first a discussion of the practicality and importance of the FLP is provided. Next, a mathematical formulation of the FLP will be presented, as well the conventions to be used in all subsequent formulations. Finally, the contributions of this thesis are outlined, and the structure of the remainder of the thesis is presented.

1.1 The Facility Layout Problem

The facility layout problem is a well-studied problem, with a rich set of literature developed over a span of several decades. In this section, the reader is provided with a specific mathematical formulation of the FLP which will be the focus of the thesis. Several other formulations for the FLP have been provided, and are discussed in Chapter 2.

1.1.1 Motivation and Context of the FLP

There is a reason why the FLP is such a well-studied problem with research dating back to the 1950s. Tompkins et al. (2010) suggest that “over $300 billion will be spent annually in the United States alone on facilities that will require planning or replanning.” They also claim that material handling costs make up between 20 and 50% of operating costs within manufacturing and that "effective facilities planning can reduce these costs by at least 10 to 30%."

Although determining the orientation and locations of departments within a facility is only one part of "effective facilities planning", it has the most significant impact on material handling costs. Algorithms for solving the FLP help facility designers develop a multiple alternative layout plans to evaluate and select among.

Furthermore, with the rapid development and implementation of robotics and artificial intelligence into the manufacturing setting, it is reasonable to expect some form of real time facility layout to emerge in some facilities in the near future. Such an environment would require high quality solutions very fast, something that is still difficult to achieve even today.

1.1.2 Conventions

Similar to two-dimensional bin-packing, the output of the FLP is a two-dimensional layout, and thus a mathematical formulation requires a coordinate system. Several coordinate conventions have been used in the bin-packing and layout literature, most notably the bottom-left-corner convention and the centroid convention. This thesis will use the centroid convention. This means that the coordinates of a given department will correspond to the centroid of that department. The centroid of the facility will lie at (0,0), and thus the department coordinate variables may take on positive or negative values. This convention is illustrated in Figure 1.1 where the outer rectangle represents the facility, and the inner rectangle represents a department.

Another assumption being made throughout the thesis is that the departments will have a rectangular shape and that the facility will also have a rectangular shape. A point of some confusion for some readers may be the use of the terms width and height. In this thesis, and much of the literature, the length of an entity along the x-axis is referred to as width, and the length of an entity along the y-axis is referred to as height. This can sometimes cause confusion, because the layout itself represents a top-down view of a facility whose actual height would run along orthogonal to the x-y plane. Some argue that the terms width and length should be used; however, this will not be the case in this work.

1.1.3 Mathematical Formulation

This thesis adopts the formulation of Anjos and Vieira (2016). The first formulation of the continuous plane FLP was presented in Montreuil (1991). Given n departments, indexed

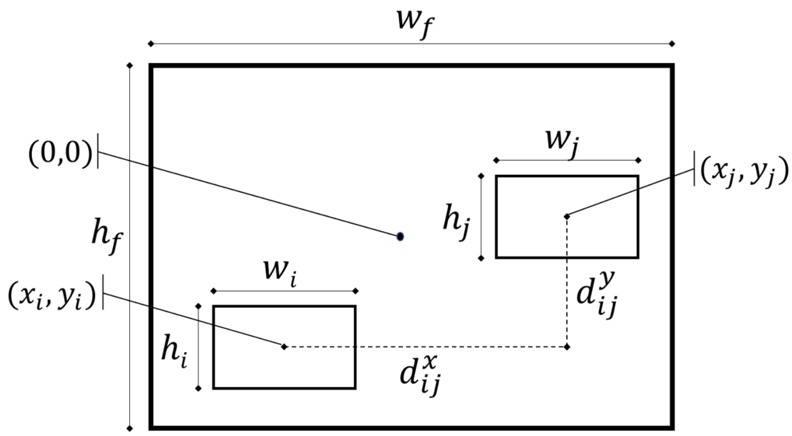

Figure 1.1: Coordinate convention diagram

by i and j, and interdepartmental cost c_{ij}, solving the FLP involves placing all departments within a facility of width, w_f, and height, h_f, such that the overall interdepartmental cost is minimized and no two departments are overlapping. The cost, c_{ij}, can be understood as the product of the expected flow and the cost per unit flow between departments i and j. Each department has a minimum and maximum width, $w_i^{\min}$ and $w_i^{\max}$, and a minimum and maximum height, $h_i^{\min}$ and $h_i^{\max}$ respectively. Departments also have a minimum area, A_i, and a maximum aspect ratio, β.

The decision variables are defined as:

d_{ij}^x: the horizontal component of the rectilinear distance between department i and department j, $i = 1, ..., n$, $j = i + 1, ..., n$.

d_{ij}^y: the vertical component of the rectilinear distance between department i and department j, $i = 1, ..., n$, $j = i + 1, ..., n$.

w_i: the width of department i, $i = 1, ..., n$.

h_i: the height of department i, $i = 1, ..., n$.

x_i: the x-coordinate of the centroid of department i, $i = 1, ..., n$.

y_i: the y-coordinate of the centroid of department i , $i = 1, ..., n$.

z_{ij}^x: $= 1$ if $x_i > x_j$, 0 otherwise, $i = 1, ..., n$, $j = i + 1, ..., n$.

z^y_{ij}**:** $= 1$ if $y_i > y_j$, 0 otherwise, $i = 1, ..., n$, $j = i + 1, ..., n$.

and the continuous FLP is modelled as:

$$
\begin{array}{llrr}
\min & \displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{n} c_{ij}(d^x_{ij} + d^y_{ij}) & & \\
\text{s.t.} & d^x_{ij} \geq x_j - x_i & i = 1, ..., n, j = i + 1, ..., n & (1.1) \\
& d^x_{ij} \geq x_i - x_j & i = 1, ..., n, j = i + 1, ..., n & (1.2) \\
& d^y_{ij} \geq y_j - y_i & i = 1, ..., n, j = i + 1, ..., n & (1.3) \\
& d^y_{ij} \geq y_i - y_j & i = 1, ..., n, j = i + 1, ..., n & (1.4) \\
& \beta w_i - h_i \geq 0 & i = 1, ..., n & (1.5) \\
& \beta h_i - w_i \geq 0 & i = 1, ..., n & (1.6) \\
& w_i^{\min} \leq w_i \leq w_i^{\max} & i = 1, ..., n & (1.7) \\
& h_i^{\min} \leq h_i \leq h_i^{\max} & i = 1, ..., n & (1.8) \\
& z^x_{ij} + z^x_{ji} + z^y_{ij} + z^y_{ji} \geq 1 & i = 1, ..., n, j = i + 1, ..., n & (1.9) \\
& x_j + \dfrac{w_j}{2} \leq x_i - \dfrac{w_i}{2} + (1 - z^x_{ij})w_f & i = 1, ..., n, j = 1, ..., n, i \neq j & (1.10) \\
& y_j + \dfrac{h_j}{2} \leq y_i - \dfrac{h_i}{2} + (1 - z^y_{ij})h_f & i = 1, ..., n, j = 1, ..., n, i \neq j & (1.11) \\
& A_i \leq w_i h_i & i = 1, ..., n & (1.12) \\
& x_i + \dfrac{w_i}{2} \leq \dfrac{w_f}{2} & i = 1, ..., n & (1.13) \\
& x_i - \dfrac{w_i}{2} \geq -\dfrac{w_f}{2} & i = 1, ..., n & (1.14) \\
& y_i + \dfrac{h_i}{2} \leq \dfrac{h_f}{2} & i = 1, ..., n & (1.15) \\
& y_i - \dfrac{h_i}{2} \geq -\dfrac{h_f}{2} & i = 1, ..., n & (1.16) \\
& x_i, y_i \in \mathbb{R} & i = 1, ..., n & \\
& w_i, h_i \geq 0 & i = 1, ..., n & \\
& d^x_{ij}, d^y_{ij} \geq 0 & i = 1, ..., n, j = i + 1, ..., n & \\
& z^x_{ij}, z^y_{ij} \in \{0, 1\} & i = 1, ..., n, j = 1, ..., n, i \neq j &
\end{array}
$$

The objective function is the sum-product of the rectilinear distance between each pair of departments and the cost per unit distance. Constraints 1.1-1.4 determine the rectilinear distance. The maximum aspect ratio is enforced by constraints 1.5 and 1.6. Constraints 1.7 and 1.8 provide lower and upper limits for the width and height of each department. Constraints 1.9-1.11 prevent departments from overlapping. Constraints 1.12 ensure that each department has the required area. Constraints 1.13-1.16 ensure that the departments reside within the boundary of the facility.

1.2 Contributions of the Thesis

This first contribution of this thesis is the presentation of a new hierarchical approach which addresses the limitations of existing hierarchical approaches in the literature. This new approach provides a framework which controls deadspace in the layout and is used to solve instances which have no deadspace available. Furthermore, the approach provides improvements over the best-known solutions to benchmark instances.

The second contribution is a second approach which incorporates hierarchical aspects into the mathematical model. The model, which encapsulates departments within super-departments, is too difficult to solve exactly, so a two-stage approach was developed. In the first stage, an orientation of the super-departments is assumed, either column or grid-based, and the assignment of departments to super-departments is determined. This first stage was implemented using three different metaheuristics for each orientation. The two-stage approach provides improvements which are better than those of the hierarchical approach.

Finally, this thesis provides the motivation for two future directions of research. The first is to explore hybrid column-grid approaches for solving larger instances of the FLP. The second is to use a data-driven approach to predict the relative performance of different approaches to solving the FLP.

1.3 Structure of the Thesis

The remainder of the thesis is organized into three three chapters: a literature review, hierarchical approaches to the FLP, and a nesting approach for the FLP. The literature review provides an extensive survey of the FLP literature, covering formulations, heuristics, metaheuristics, multi-stage approaches, special cases, and more complex solution approaches. The chapter on hierarchical approaches to the FLP examines more closely this important approach to solving FLPs, providing a new approach, and demonstrating improvements over the best-known solutions in the literature. Next, a new model is presented which is inspired by hierarchical approaches. This model, which is cumbersome to work with, is explored. Through some simple restrictions, this model provides significant improvements over both the best-known solutions of benchmark instances, as well as the hierarchical approach presented in the preceding chapter.

Chapter 2

Literature Review

The literature on the FLP spans several decades. Multiple formulations have been developed for generating two-dimensional layouts. Most of these formulations are extremely difficult to solve, so the literature presents a variety of heuristic methods. Many of these heuristic methods can be classified as two or multi-stage approaches. The approaches presented in this thesis can also be placed in this category. This chapter aims to provide an overview of the extensive literature on the FLP, giving the reader an understanding of where the contributions of this thesis lie.

In terms of the overall facilities planning literature, the FLP, as presented, is concerned with determining the “block layout”, which specifies the location and dimensions of each department. In contrast, some work focuses on the “detailed layout” which specifies the location of equipment, machinery, and the like, within each department. This distinction has not always been clear in the literature, with some works producing block layouts that incorporate some aspects of detailed layouts. The focus of this thesis will be on determining the block layout.

2.1 Surveys

We begin by acknowledging a number of review papers. Kusiak and Heragu (1987) provide an early survey of various models and algorithms for solving the facility layout problem. They focus on the Quadratic Assignment Problem (QAP) formulation first presented in Koopmans and Beckmann (1957), and the graph theoretic formulation (Foulds, 2012). They discuss optimal algorithms to solve the QAP categorized into either branch and bound algorithms or cutting plane algorithms. They note that optimal solutions tended to be found earlier in the branching process, but a large number of alternative solutions had to be enumerated in order to prove optimality. They also discuss historical approaches to facility layout, as well as suboptimal algorithms categorized as: construction algorithms, improvement algorithms, hybrid algorithms, or graph theoretic algorithms. The hybrid algorithms have characteristics of optimal and suboptimal algorithms or algorithms which use both construction and improvement techniques. They provide examples of algorithms which generate an initial solution by terminating a branch and bound process early, and then use improvement techniques to generate better solutions.

Meller and Gau (1996) provide a background on facility layout, including alternative objective functions, distance metrics, and problem parameters. They cover QAP and graph theoretic approaches similar to those presented in Kusiak and Heragu (1987). They also discuss a Mixed-Integer Program (MIP) approach presented by Montreuil (1991) which treats the location of the departments as continuous, rather than discrete as in the approaches discussed above. They mention that, at the time of writing, this approach could only be solved optimally for less than six departments, so a heuristic is used where the binary variables are fixed, and the resulting linear program is solved. Furthermore, they describe extensions, such as dynamic layouts, stochastic layouts, multi-criteria extensions, and special cases, such as integrating manufacturing system designs, which were not a focus of Kusiak and Heragu (1987).

Mavridou and Pardalos (1997) give a survey of the use of simulated annealing and genetic algorithms for the FLP. The authors note that given the increased interest in parallel computing, genetic algorithms and simulated annealing are becoming more attractive.

Drira et al. (2007) provide a more recent survey of FLP literature. They focus on the QAP and MIP formulations described above. They also present fuzzy formulations and multi-objective formulations. Because exact methods are difficult to solve for realistic problem sizes, most of the research presented in Drira et al. (2007) focuses on approximation approaches using heuristics, metaheuristics and hybrid approaches. Several future areas of research are identified, including: incorporating complex and realistic characteristics in the model, such as pickup-drop off points, multiple floors, dynamic modelling, and the use of heuristics to solve larger problems.

Anjos and Vieira (2017) provide a very recent overview of mathematical optimization approaches to row FLPs, unequal areas FLPs, and multi-floor FLPs. They discuss several two stage approaches to the unequal areas FLP, as well as the flexible bay approach. They present research on symmetry breaking constraints and valid inequalities, and discuss directions for future research.

2.2 Modeling

As indicated above, there are three main formulations of the FLP. The first is the QAP formulation presented in Koopmans and Beckmann (1957). Their paper begins by discussing the problem of assigning discrete plants to discrete locations, while ignoring inter-plant costs, in such a way that maximizes the profit. This problem can be solved as a linear assignment problem. They then incorporate the cost of transportation between the plants being placed. The resulting problem is a quadratic assignment problem.

Montreuil (1991) provides a continuous formulation of the facility layout problem as a MIP. It uses binary variables to represent overlap, and the coordinates, widths, heights must be determined. A perimeter-based area approximation is also presented. A number of improvements to this model have been presented (Meller et al., 1998), (Sherali et al., 2003), (Meller et al., 2007).

The graph theory approach to facility layout is concerned with maximizing departmental adjacency scores. The input is a relationship chart which describes the desirability of placing each pair of departments adjacent to one another. Hassan and Hogg (1987) provide a review of graph theory approaches to the facility layout problem. They describe the graph theoretic approach as a three step procedure: The first step is to generate a Maximal Planar Weighted Graph (MPWG) from the relationship chart. The second is to construct the dual of the MPWG, and the third is to convert the dual graph into a block layout.

2.3 Special Cases

Several extensions to the static, deterministic FLP exist in the litaeerature. Rosenblatt (1986) presents a multi-period layout problem called the Dynamic Plant Layout Problem (DPLP) using the QAP layout formulation. Montreuil and Venkatadri (1991) analyze dynamic facility layout in the expansion phase of a manufacturing system. Lacksonen and Enscore Jr (1993) extend the QAP formulation of the layout problem to include discrete times. Rosenblatt and Kropp (1992) present an approach for the single period stochastic layout problem where the product mix, represented as a flow matrix, is non-deterministic. Kouvelis and Kiran (1991) analyze two cases of the layout problem with multiple periods and non-deterministic product mixes: one where the product mix, once realized, remains constant during the future periods and rearrangements after the design phase are infeasible, and the other where the product mix may change between periods and rearrangements may be made at any period. Rosenblatt and Lee (1987) outline a robustness approach for the QAP formulation of the single period layout problem.

Heragu (2008) provides an overview of group technology and cellular manufacturing. Different clustering algorithms are examined. These algorithms are applied to the part-machine matrix, in order to define machine cells. Different design approaches are discussed; for example, a three stage sequential approach consisting of defining the contents of each cell, finding the intra-cell layout for each cell, and finding the layout of the cells.

Hassan (1994) describes the process of developing a group technology layout exactly as the three stage approach discussed above. It is argued that there are characteristics particular to the machine layout problem that distinguish it significantly from a block layout problem. It is also noted that the third stage of the sequential approach is equivalent to the block layout problem. Barbosa-Povoa et al. (2001) attempt to combine block layouts and detailed layouts into one process. They provide two MILP approaches, one of which incorporates the idea of equipment being assigned to production zones. They note that, while their solution quality degrades when using the production zones due to the additional constraints, the run time is improved.

Meller (1997) defines and provides a two-stage solution methodology for the multi-bay FLP. In this problem, the facility is organized into parallel bays where the cost to move material between bays is much higher than the cost to move material within a given bay. Meller (1997) assumes a linear arrangement of departments within each bay. The first stage of the approach assigns departments to bays, and the second stage generates the layout for each bay.

Konak et al. (2006) presents a formulation for the flexible bay structure FLP. In this case, the parallel bays have variable widths and have straight aisles on both sides. The solution approach simultaneously assigns departments to bays and generates the layout of each bay. Again, a linear ordering of departments is used for the intra-bay layout.

2.4 Heuristic Approaches

Because the FLP is very difficult to solve optimally, many heuristics have been developed to provide good solutions in a reasonable amount of time. Traditionally, FLP heuristics were categorized either as construction or improvement heuristics. Construction heuristics generate a layout from scratch; whereas, improvement heuristics begin with a layout and improve it.

Foulds and Robinson (1978), Montreuil et al. (1987), and Goetschalckx (1992) provide heuristics for the graph theoretic FLP. Armour and Buffa (1963) provide a heavily referenced improvement heuristic called CRAFT for the discrete layout representation. The algorithm works by examining pair-wise exchanges of locations, choosing the one which reduces the objective function the most, and repeating. Bozer et al. (1994) apply spacefilling curves to the QAP formulation of the FLP. They improve the exchange procedure in CRAFT and apply the extended algorithm, named MULTIPLE, to multi-floor facilities. The spacefilling curves ensure that no departments are split, or equivalently, each department is a contiguous collection of grid squares.

2.5 Clustering Approaches to Facility Layout

The idea of grouping similar departments, or clustering them, is not a new idea in facility layout. These approaches can provide insight for designers, or reduce the solution space of the FLP.

O'brien and Abdel Barr (1980) present interactive facility layout software based on a construction procedure INLAYT and an improvement procedure S-ZAKY. INLAYT suggests groups of departments that should be placed together based on the cost of the interdepartment flow and a flow factor provided by the user. The procedure ignores the areas of the departments and is concerned only with determining the relative positioning. The improvement heuristic works by interchanging the locations of three departments at each iteration, until the solution can not improvement by another exchange.

Scriabin and Vergin (1985) present the FLAC (Facility Layout by Analysis of Clusters) algorithm. It uses cluster analysis to reduce the QAP formulation of the layout problem to an assignment problem.

Tam and Li (1991) develop a hierarchical approach to solve a continuous formulation of the FLP. They discuss various parameters of the continual plane layout problem including alternative department shapes, geometric characteristics of each department, the required constraints of the problem, and alternative objective functions. This work, which sets the basis for hierarchical solution approaches to the FLP is described in further detail in Chapter 3.

2.6 Metaheuristic Approaches

Tam (1992b) introduces a simulated annealing based algorithm for developing a layout in a cellular manufacturing environment. The algorithm uses a continuous space definition and a slicing tree representation for the layout. A discussion is presented on how clustering can be used to create a dendrogram for the departments which takes the same form as a slicing tree. The min cut approach from the VLSI literature is also considered, but, because it requires more computational effort and the resulting trees are similar, clustering is preferred. The problem is modeled with an objective function minimizing the sum of the flow-distance, and there are two constraints: one on the aspect ratios of the partitions, and one limiting the amount of deadspace in any partition. The constraints are penalized in the objective function, and simulated annealing is used to solve the problem.

Tam (1992a) apply genetic algorithms to solve the FLP using the same slicing tree facility representation in Tam (1992b). Trees are constructed using dendrograms, and the optimization model is formulated just as in Tam (1992b).

2.7 Multi-Stage Approaches

Due to the FLP's complexity and disjunctive constraints, many multi-stage approaches have been developed to solve the problem. Most aim to determine the relative positioning in a relaxed first stage. Then they fix these values and solve an exact second stage.

Lacksonen (1994) provides a two stage framework for solving both the static and the dynamic layout problem. Stage one is the QAP-based algorithm provided in Lacksonen and Enscore Jr (1993). Stage two incorporates unequal areas among departments by solving a MILP. This stage two model uses a piecewise linearization of the area constraint of Montreuil (1991).

Meller and Bozer (1997) compare a single-stage approach to a two-stage approach to the multi-floor FLP where, in stage one, departments are assigned to a floor, and, in stage two, the layout is generated for each floor. Stage one assigns departments to floors such that the vertical flow cost is minimized. It is modelled using a linear objective and solved using CPLEX. In stage two, SABLE (Meller and Bozer, 1996) is adapted so that departments remain in their assigned floor. They find that the two-stage algorithm outperforms the single stage algorithm when the vertical flow costs were greater than the horizontal and the algorithms were given equal run time.

Gau and Meller (1999) use an iterative algorithm that combines genetic algorithms and mixed-integer programming to solve the facility layout problem. First a genetic algorithm is used to generate a good solution. Some of the relative positions from the initial solution are then used to solve an MIP formulation of the FLP. The solved problem can then be used as an initial solution for the genetic algorithm, and so on. This use of a pre-processing heuristic to determine the binary variables in the MIP is used frequently. However, the iterative procedure provides marginal improvements and increased confidence in the heuristic solutions.

Other multi-stage approaches include Chen et al. (2002),Lee and Lee (2002), and Kulturel-Konak and Konak (2013).

2.8 Dispersion-Concentration Inspired Approaches

Many multi-stage approaches are inspired by the dispersion-concentration approach from Drezner (1980). The idea is that departments should be placed very close and overlapping or very far from each other in a stage one model. The later stages then enforce non-overlap or push the departments closer in an attempt to converge on a near optimal solution.

Drezner (1980) presents the DISCON (dispersion-concentration) procedure for facility layout. The departments are modelled as circles, and the algorithm uses euclidean distance. In the dispersion phase, the centers of the departments very close to the origin such that they are overlapping, and the attractive forces (the cost matrix) is scaled down so that the departments are not directly adjacent in the solution. In the concentration phase, the positions of the departments from the dispersion phase are used as a starting point for the LDG method, and the problem is solved using the original costs.

Van Camp et al. (1992) present the NLT (Nonlinear Layout Technique) algorithm. Their model attempts to minimize the inter-department cost, as well as the cost of flow between departments and the outer wall. This approach was able to layout unequal area departments without forcing them to be composed of a number of equally sized squares, as well as model the departments as squares rather than circles as in DISCON (Drezner, 1980). The primary model of the work is highly sensitive to initial positions, so two other models are presented which are used to construct an initial solution to input into the primary model. The resulting three stage procedure was tested on equal area problems from the literature, as well as a small number of unequal area problems that existed in the literature at the time. The procedure was found to produce good solutions using few computational resources.

Imam and Mir (1993) present a technique for solving a continuous formulation of the FLP. It is a two stage process where departments are placed far apart in stage one, and converge together using a univariate search in stage two. It uses "envelop blocks" which departments are placed in. Initially, the envelop block is much larger than the given department, and it's size is reduced gradually creating a "controlled convergence" in which the resulting layout is less dependent on the initial spacing of the departments.

Mir and Imam (2001) present a hybrid optimization approach for a continuous formulation of the FLP. It is similar to the process in Imam and Mir (1993), except it uses simulated annealing to generate the initial placement of the departments, and it replaces the univariate search in stage two with a steepest decent approach.

Anjos and Vannelli (2002) present an Attractor-Repeller approach to facility layout based on DISCON (Drezner, 1980) and NLT (Van Camp et al., 1992). They replace stages one and two of NLT with a mathematical model that finds an initial solution for stage three of NLT. They remove the non-overlap constraints which make the problem nonconvex and add a "repeller" term to the objective function to enforce the non-overlap constraints. They name this model AR. They then modify the formulation in order to yield a convex problem, which they name CoAR. They find that the non-convex formulation, AR, performed well in their testing.

Anjos and Vannelli (2006) modify the CoAR model from Anjos and Vannelli (2002) into a new stage one model, ModCoAR, which is non-conex, but easier to solve. They develop a formulation of the FLP using equilibrium constraints for the second stage. They then penalize the difficult constraints in the objective function, resulting in the model they name the bilinear penalty layout model (BLP).

Jankovits et al. (2011) provide a two stage convex optimization based framework for finding near-optimal solutions. The first model is based on stage one of Anjos and Vannelli (2006). Improvements to this model are applied, and a systematic approach for setting parameter values is presented. The second model fixes the relative positioning from stage one, and solves the resulting problem using semidefinite programming. They also provide five new instances for benchmarking.

Anjos and Vieira (2016) present a two stage solution approach for the facility layout problem. While, in Jankovits et al. (2011), the first stage models departments as circles, here they are modelled as rectangles, so aspect ratio constraints can be incorporated in stage one. The second stage is a convex optimization formulation that uses the relative positions from stage one in order to solve the problem. They test on several instances including those presented in Jankovits et al. (2011), and they present four new, larger instances.

2.9 VLSI Floorplanning Literature

VLSI floorplanning is a problem related to the FLP, but one which usually incorporates domain specific concerns such as wiring. The physical design step of VLSI design usually involves partitioning the circuit into a hierarchy of blocks which are then placed on the chip in the floorplanning step (Sherwani, 2012).

Adya and Markov (2002) present a floorplanning algorithm that generates an initial placement, clusters cells based on their placement, performs a layout of the clusters, and then replaces the cells within their cluster boundary. Adya and Markov (2003) evaluate a similar hierarchical approach where a top-level layout is created first using clusters with an area equal to 115% of the area of the sub blocks. The blocks are then placed inside the boundary provided by the top-level layout. Adya et al. (2004) present a hierarchical floorplacement algorithm which integrates the partitioning step of VLSI design. It is based on min-cut partitioning where the placement area is recursively refined into a more detailed layout. Once the area is refined enough, the blocks are clustered based on connectivity and laid out in modules using a fixed-outline floorplanner. The floorplanning is not guaranteed to succeed, and if it does not, the previous partition is undone, and the algorithm continues with the larger bin.

Chang et al. (2003) develop a multi-level optimization algorithm, mPG-MS, to solve the floorplacement problem where the size ratio of the objects to be placed can be very large. The algorithm works with a coarsening phase and a refinement phase. In the coarsening phase, the objects are clustered into a hierarchy. The top level modules in the hierarchy are laid out, and then there is a refinement phase, where the floor plan is refined at each iteration. Simulated annealing is used for the floorplanning optimization.

Luo et al. (2008) present a two stage methodology for solving the fixed-outline floorplanning problem. The first stage is a relaxation of the problem which provides relative positions for the second stage.

Chapter 3

Hierarchical Approaches to the FLP

In this chapter, hierarchical approaches to the FLP are explored. Hierarchical approach partition the FLP into a series of smaller subproblems which can be combined, through a hierarchy, in order to solve the overall problem. This type of approach is extremely useful for the FLP, because small problems are easily solved, but moderately sized problems become extremely difficult to solve exactly.

The hierarchy refers to a tree generated by clustering, where the leaves of the tree are the original departments from the problem, and the nodes in the tree represent subproblems. When these subproblems are solved and combined effectively, high quality solutions can be obtained to the original problem.

This chapter is organized as follows. First, an existing hierarchical approach is explained. The drawbacks and limitations of this approach are highlighted. Then an improved hierarchical approach is presented, which addresses the drawbacks of those approaches existing in the literature. The improved approach is tested on benchmark instances from the literature, and the results are compared with the best known solutions for those instances.

3.1 Existing Approaches

Hierarchical approaches to the FLP and related problems were first proposed in the literature decades ago. This section will present in detail the approach presented in Tam and Li (1991), which, along with the related approaches in Tam (1992a) and Tam (1992b), remains the only work focused on hierarchical approaches specific to the FLP. There is significantly more interest in hierarchical approaches to the related problem of VLSI Floorplanning, probably due to the large-scale nature of that problem. This literature is outlined in Chapter 2.

The approach proposed in Tam and Li (1991) is divided into three phases: a clustering phase, where the departments are clustered into a hierarchy, an initial layout phase, where the relative positioning of departments is determined, and a layout refinement phase, where the department positions and dimensions are determined. The main idea is that departments are first laid out within their cluster, and then each cluster is treated as a large department in a merge step to determine the overall layout. The second and third phases, along with a merge step, may occur more than once if the hierarchy of departments has more than one level. By dividing the layout problem into smaller subproblems, large FLPs become manageable for computers to solve.

The first phase clusters the departments into a hierarchy based on the pairwise flow-costs, c_{ij}. The clustering procedure is based on K-means. A maximum number of departments per cluster is specified, and the algorithm iteratively merges departments based on the mean inter-cluster flow until no more merging can be done.

The relative positioning of each department within their cluster is then determined in the initial placement phase. The departments are modeled as enlarged circles with a constant radius in this phase, because only the relative positioning of the departments will be carried over into the next phase. They use an objective function based on Hooke's law which attempts to minimize the product of the flow and the squared euclidean distance. To enforce the non-overlapping constraint, they add a penalty to the objective function when the centroid-centroid distance is smaller than the sum of the radii. They then solve the unconstrained optimization problem using a quasi-Newton procedure in the NAG package to determine the centers of each circular block. The relative positioning of the centers is then used in the layout refinement phase.

In the layout refinement phase, the departments are modeled as rectangles. They develop a non-linear constrained optimization model to solve layouts using rectangles. They include non-overlap, aspect ratios, orientation, and boundary constraints. The departments are first laid out within their cluster, which may or may not have boundary constraints. Then each cluster is treated as a rigid rectangle in the merge step. These models are solved using a sequential augmented Lagrangian method, and the sub problems are solved using the quasi-Newton procedure in the NAG package. They provide adjustments for incorporating fixed facility features, as well as a penalty for penalizing wasted space in the facility.

They test their approach on facilities of various sizes from 5 departments to 30 departments. They note that the larger instances cannot be solved without clustering. They discuss that the approach works better when there are distinct communities of departments which have little inter-community flow.

Although this approach was not able to take advantages of the improved MIP formulations that have been developed since its publication, it outlines a very useful overarching method of dealing with the complexity of the FLP. A significant limitation of this approach is that, as the merging process occurs from the bottom up, any deadspace that exists within a cluster propagates through each step and remains in the final merged layout. Furthermore, because the layout within each cluster is determined in isolation, the flows from departments in one cluster to a department in another cluster are ignored when generating the intra-cluster layout. The improved approach presented in the next section addresses these issues specifically.

3.2 A New Hierarchical Approach

In this section, a new hierarchical approach is presented which addresses the limitations of the approach from Tam and Li (1991). First, the departments are clustered into a tree based on the flow cost matrix using the same k-means variant described in Tam and Li (1991). This hierarchical clustering is performed so that a manageable layout problem can be solved at each node in the tree. Each node in the tree represents a super-department: an artificial department with area requirements equal to the sum of the areas of its child departments. Aspect ratios for these super-departments, denoted β_S, can be arbitrarily chosen, or they can be based on the position in the cluster hierarchy.

A layout solution is generated for the super-departments at the very top level of the tree. Next, the location and dimensions of the super-departments are fixed. One super-department at a time is removed and replaced with its children, which are laid out in their parent's place in the layout. Overlap is permitted, but is penalized.

The process of replacing super-departments with their children continues until all of the original departments are laid out. At this point a final layout is generated by fixing the relative positioning of the departments and solving the resulting Second-Order Cone Program (SOCP). The first and last three steps of the algorithm are shown in Figure 3.1 for the instance AnVi100, a 100 department problem. The departments were clustered into a three level tree with 57 artificial departments.

By laying out the top level of the hierarchy first and performing the layouts down the hierarchy, it is possible to generate high quality layouts for instances with little-to-no

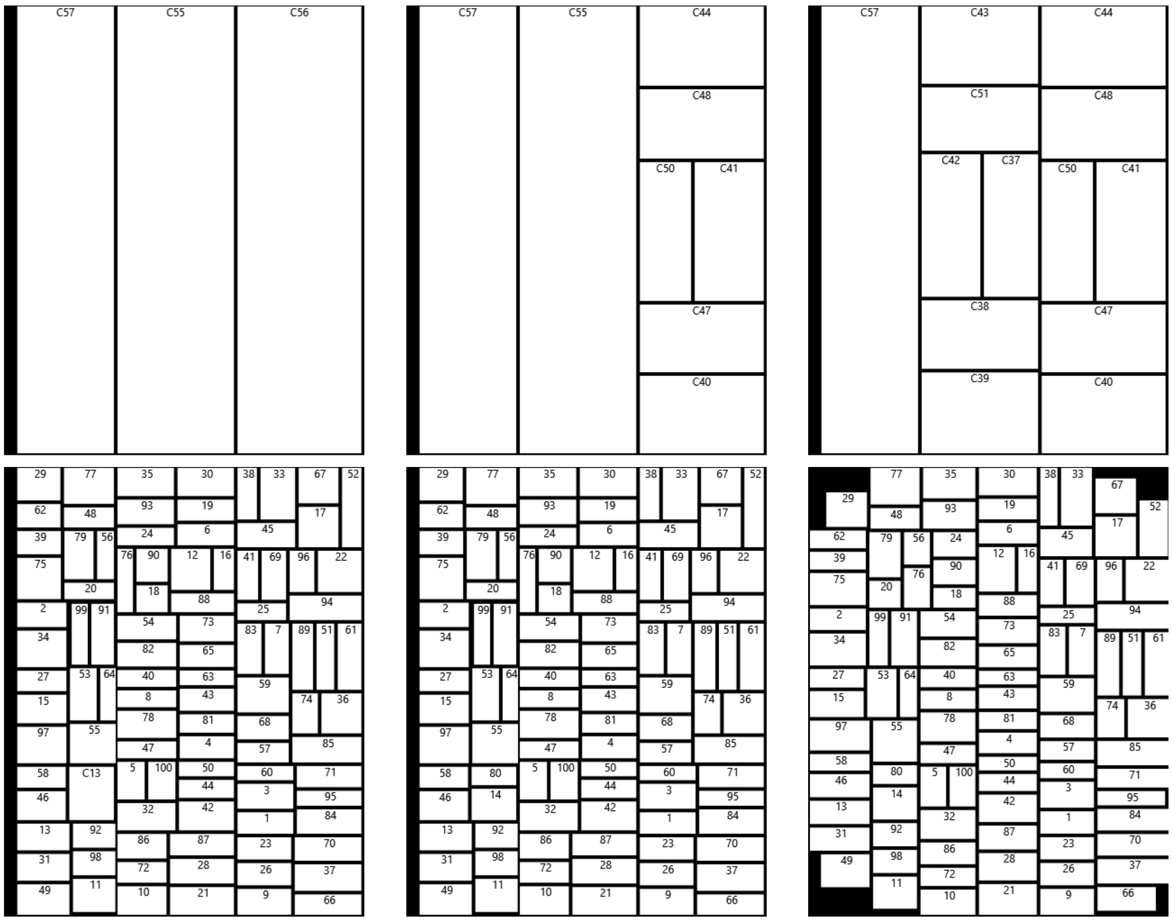

Figure 3.1: The first and last three stages of the hierarchical approach on ANVI100

deadspace. Because the layout is performed down the cluster tree, rather than up, the deadspace within a given artificial department can be controlled. The layout and merge procedure provided in Tam and Li (1991), which begins by laying out the roots and merging layouts, may lead to significant deadspace. Our algorithm is tested on several instances with little-to-no possible deadspace within the facility boundary. Not only does our approach produce feasible solutions, it produces solutions competitive with the best know solutions in the literature. This solution methodology involves clustering the departments and solving variants of the FLP formulation presented in Chapter 1; the clustering algorithm and FLP variants are described in the following sections.

3.2.1 Clustering

The clustering approach presented in Tam and Li (1991) works by merging departments and clusters based on similarity values. They define the similarity between any two clusters u and v, s_{uv}, as the average of the pairwise costs between their constituent departments:

$$s_{uv} = \frac{\sum_{i \in C_u} \sum_{j \in C_v} c_{ij}}{|C_u||C_v|} \tag{3.1}$$

The clustering algorithm begins by treating every department to be clustered as a cluster. A maximum value for the number of departments per cluster, $n^{\max}$, is specified. The algorithm iteratively merges the two clusters with the highest similarity which, when merged, will contain no more than $n^{\max}$ departments. The algorithm terminates when no two of the remaining clusters may be merged.

Alternative Clustering Approach

As an experiment, the clustering technique described above was adapted to incorporate a different similarity metric. It was hypothesized that a clustering technique, which was concerned with not only intra-cluster similarity but also inter-cluster dissimilarity, may perform better. A new similarity measure can be created which penalizes the average similarity to other clusters with a penalty factor γ.

$$s_{uv}^{\text{new}} = s_{uv} - \gamma \frac{\sum_{i \neq u,v} \sum_{\substack{j \neq u,v \\ j<i}} s_{ij}}{m-2} \tag{3.2}$$

3.2.2 FLP Formulation Variants

The improved solution approach makes use of two variants of the FLP formulation: the fixed location formulation, and the fixed relative positioning formulation. This section describes these variants.

Fixed Location Formulation

A fixed location variant of the FLP is described as follows. Let F represent the set of departments and super-department whose location and dimensions will be fixed, and let U represent the set of all departments and super-departments whose coordinates and dimensions must be determined. Let x_s^+, x_s^-, y_s^+, y_s^- represent the boundary of the area where the unfixed departments should be placed. In the context of the overall solution approach, the boundary is the area where a super-department was placed, and the unfixed departments to be placed in the current layout are the children of that super-department.

Define the following additional decision variables for each unfixed department.

p_i^{x+}**:** the distance along the x-axis in the positive direction in which department i has surpassed the boundary of its parent, $i \in U$

p_i^{x-}**:** the distance along the x-axis in the negative direction in which department i has surpassed the boundary of its parent, $i \in U$

p_i^{y+}**:** the distance along the y-axis in the positive direction in which department i has surpassed the boundary of its parent, $i \in U$

p_i^{y-}**:** the distance along the y-axis in the negative direction in which department i has surpassed the boundary of its parent, $i \in U$

The departments lying outside of the given boundary are penalized by modifying the objective function using the penalty parameter α:

$$\min \qquad \sum_{i=1}^{n} \sum_{j=i+1}^{n} c_{ij}(d_{ij}^x + d_{ij}^y) + \alpha \sum_{i \in U} (p_i^{x+} + p_i^{x-} + p_i^{y+} + p_i^{y-})$$

The formulation also requires the addition of the following constraints:

$$p_i^{x+} \geq x_i + \frac{w_i}{2} - x_s^+ \qquad \forall i \in U \tag{3.3}$$

$$p_i^{x-} \geq x_s^- - (x_i - \frac{w_i}{2}) \qquad \forall i \in U \tag{3.4}$$

$$p_i^{y+} \geq y_i + \frac{h_i}{2} - y_s^+ \qquad \forall i \in U \tag{3.5}$$

$$p_i^{y-} \geq y_s^- - (y_i - \frac{h_i}{2}) \qquad \forall i \in U \tag{3.6}$$
$$x_i = \bar{x}_i \qquad \forall i \in F \tag{3.7}$$
$$y_i = \bar{y}_i \qquad \forall i \in F \tag{3.8}$$
$$h_i = \bar{h}_i \qquad \forall i \in F \tag{3.9}$$
$$w_i = \bar{w}_i \qquad \forall i \in F \tag{3.10}$$
$$p_i^{x+}, p_i^{x-}, p_i^{y+}, p_i^{y-} \geq 0 \qquad \forall i \in U$$

where $\bar{x}_i$, $\bar{y}_i$, $\bar{h}_i$, and $\bar{w}_i$ are fixed values determined in previous layout iterations. Constraints 3.3-3.6 define the variables p_i^{x+}, p_i^{x-}, p_i^{y+}, p_i^{y-}, and constraints 3.7-3.10 ensure that departments in the fixed set have fixed coordinates, heights, and widths. Furthermore, in this variant of the FLP, we add the non-overlap constraints, constraints 1.9-1.11, for only unfixed departments; in other words, we allow unfixed departments to overlap with fixed departments to ensure that a feasible solution is generated at every iteration.

Fixed Relative Positioning Formulation

The other variant of the FLP to be presented is the fixed relative positioning variant. In the context of the overall solution approach, this variant is solved only once, yielding the final solution. This formulation takes as inputs the locations of every department in the last solution of the fixed location formulation. The information passing here is similar to the information passed to the second stage of the optimization framework presented in (Anjos and Vieira, 2016). The fixed relative positioning formulation requires the following three parameters:

$$\bar{z_{ij}} = \begin{cases} 1 & \text{if } |x_i - x_j| > |y_i - y_j| \text{ in the previous layout} \\ 0 & \text{otherwise} \end{cases}$$
$$\bar{z_{ij}}^x = \begin{cases} 1 & \text{if } x_i > x_j \text{ in the previous layout} \\ 0 & \text{otherwise} \end{cases}$$
$$\bar{z_{ij}}^y = \begin{cases} 1 & \text{if } y_i > y_j \text{ in the previous layout} \\ 0 & \text{otherwise} \end{cases}$$

The non-overlap constraints, constraints 1.9-1.11, are replaced with the following linear constraints with no binary variables:

$$M(1 - \bar{z_{ij}}(1 - \bar{z_{ij}}^x)) + x_j - x_i \geq \frac{w_i + w_j}{2} \qquad i = 1, ..., n, j = 1, ..., n, i \neq j \tag{3.11}$$

$$M(1 - \bar{z_{ij}}\bar{z_{ij}}^x) + x_i - x_j \geq \frac{w_i + w_j}{2} \qquad i = 1, ..., n, j = 1, ..., n, i \neq j \tag{3.12}$$

$$M(1 - (1 - \bar{z_{ij}})(1 - \bar{z_{ij}}^y)) + y_j - y_i \geq \frac{h_i + h_j}{2} \qquad i = 1, ..., n, j = 1, ..., n, i \neq j \tag{3.13}$$

$$M(1 - (1 - \bar{z_{ij}})\bar{z_{ij}}^y) + y_i - y_j \geq \frac{h_i + h_j}{2} \qquad i = 1, ..., n, j = 1, ..., n, i \neq j \tag{3.14}$$

These constraints use the coordinates in the previous layout to determine in which direction the non-overlap constraints will be enforced. The resulting model contains no integer variables, and can be solved as a SOCP.

3.2.3 Overall Solution Approach

The algorithm begins by solving a typical FLP for the top level of the tree, assigning coordinates, heights and widths to those super-departments. Then each of these super-departments is added to a priority queue. The first department in the queue is removed from the facility and replaced with its child departments from the cluster tree. The fixed location formulation is solved to allocate the new departments to the facility. The set of unfixed departments, U, is comprised of those child departments just added. The rest of the departments are fixed departments. If the child departments just added are also super-departments, they are added to the priority queue.

If a First-in-First-Out (FIFO) queue is used, then the algorithm moves through the tree like a breadth first search: refining each cluster on the current level before moving to the next level of the tree. If a Last-in-First-Out (LIFO) queue is used then the algorithm works like a depth first search: refining a single cluster down to the original departments before moving to the next cluster.

Once all super-departments have been removed from the queue, the fixed relative positioning formulation is solved to generate the final layout.

Algorithm 1 Improved Hierarchical Facility Layout

Input: C, the priority queue containing the super-departments in the top level of the tree

```
Layout(C)
F ← C.Copy()
while C ≠ ∅ do
  U ⇐ ∅
  x ⇐ C.Pop()
  F.Remove(x)
  for all department d in Children(x) do
    U.Add(d)
    if Children(d) ≠ ∅ then
      C.Add(d)
    end if
  end for
  FixedLocationLayout(U, F)
  for all department d in U do
    F.Add(d)
  end for
end while
D ⇐ U ∪ F
FixedRelativePositionLayout(D)
```

3.3 Testing and Results

The improved approach was tested on a workstation with a Xeon E5607 and 48GB of RAM. It was implemented using c# and Gurobi 7.0.2. Several instances of different sizes were retrieved from the unequal areas facility layout section in Anjos (2017). A time limit of 30 minutes was set for each layout optimization. A variety of one level trees were tested with different aspect ratios for the super-departments, shown in Table 3.1. The best result for each problem is shown in Table 3.2. β is the aspect ratio of the original departments, $n^{\max}$ is the maximum number of departments per cluster, and β_S is the aspect ratio of the artificial departments. The results are compared with the best known solutions from the literature (Anjos and Vieira, 2016). The approach of Tam and Li (1991) was not tested on these instances, as it relies on specialized software packages, such as the NAG package.

Table 3.1: The parameters tested for the 30 department instances.

Parameter	Values
$n^{\max}$	4,5,6
β_S	2,3,4,5,6

Table 3.2: Best solutions from hierarchical testing on 30 department instances.

γ		0		0.001		0.2	
Problem	β	Best Solution	% Imp.	Best Solution	% Imp.	Best Solution	% Imp.
A	4	8745.95	**0.28**	8860.75	-1.03	8860.75	-1.03
A	5	8745.95	-1.97	8883.63	-3.57	8883.63	-3.57
A	6	8653.56	**0.92**	8884.44	-1.73	8884.44	-1.73
A	7	8713.57	-2.86	8836.91	-4.31	8836.91	-4.31
A	8	8707.96	-3.65	8842.77	-5.25	8842.77	-5.25
A	9	8707.96	-2.32	8837.52	-3.85	8837.52	-3.85
A	10	8696.05	**0.04**	8816.7	-1.34	8816.7	-1.34
B	4	9869.94	-0.26	9844	0	9844	0
B	5	9839.55	**0.92**	9664.1	**2.68**	9664.1	**2.68**
B	6	9784.38	-1.17	9592.1	**0.82**	9592.1	**0.82**
B	7	9771.53	-0.04	9551.2	**2.22**	9551.2	**2.22**
B	8	9602.31	**1.64**	9450.04	**3.2**	9450.04	**3.2**
B	9	9506.3	**2.71**	9551.17	**2.25**	9551.17	**2.25**
B	10	9430.13	**1.15**	9508.33	**0.33**	9508.33	**0.33**
C	4	14117.26	**3.27**	14136.86	**3.13**	14136.86	**3.13**
C	5	14066.74	**4.33**	14317.31	**2.62**	14317.31	**2.62**
C	6	13959.59	**2.68**	14060.78	**1.97**	14060.78	**1.97**
C	7	13608.66	**1.88**	13991.67	-0.88	13991.67	-0.88
C	8	13640.56	**6.35**	13735.94	**5.69**	13735.94	**5.69**
C	9	13640.56	**3.5**	13815.37	**2.26**	13815.37	**2.26**
C	10	13473.64	**4.12**	13684.06	**2.62**	13684.06	**2.62**
D	4	10428.63	-1.4	10603.38	-3.1	10603.38	-3.1
D	5	10503.42	-2.32	10606.31	-3.32	10606.31	-3.32
D	6	10546.01	-5.42	10581.64	-5.77	10581.64	-5.77
D	7	10441.2	-3.76	10563.78	-4.97	10563.78	-4.97
D	8	10346.34	-2.55	10564.82	-4.71	10564.82	-4.71
D	9	10384.81	-2.27	10564.69	-4.04	10564.69	-4.04
D	10	10359.42	-1.4	10453.99	-2.32	10453.99	-2.32

E	4	11586.54	**2.12**	12145.42	-2.6	12145.42	-2.6
E	5	11475.3	**1.5**	11980.52	-2.83	11980.52	-2.83
E	6	11315.16	**3.47**	11888.86	-1.42	11888.86	-1.42
E	7	11315.19	**4.12**	11695.59	**0.9**	11695.59	**0.9**
E	8	11307.98	**3.67**	11647.17	**0.78**	11647.17	**0.78**
E	9	11213.45	**3.6**	11640.5	-0.07	11640.5	-0.07
E	10	10885.41	**8.11**	11606.57	**2.02**	11606.57	**2.02**
Min		-5.42		-5.77		-5.77	
Max		8.11		5.69		5.69	
Average		0.83		-0.67		-0.67	
Number Better		21		15		15	

The objective function is compared with the improved results from Anjos and Vieira (2016). Anjos and Vieira (2016) report that their solution time for the thirty department problems were 30s for 10 choices of their α parameter.

Several larger instances were presented in Anjos and Vieira (2016). Our solution algorithm was tested on the 50 department problem with a variety of two-level cluster trees, shown in Table 3.3, and the best results are shown in Table 3.4. Anjos and Vieira (2016) report their time for these problems to be 70s.

Table 3.3: The parameters used for the 50 department instances.

Parameter	Values
n_1^{max}	2,3,4, 5, 6
β_1^s	2,3,4,6
n_2^{max}	$\frac{50}{3n_1^{max}}$ $\frac{50}{4n_1^{max}}$ $\frac{50}{5n_1^{max}}$ $\frac{50}{6n_1^{max}}$
β_2^s	2,3,4,6

Table 3.4: Best solutions from hierarchical testing on 50 department instances.

Problem	β	n_1^{max}	β_1^s	n_2^{max}	β_2^s	Objective	Time(s)	AnVi Obj.	% Imp.
AnVi50	4	6	3	2	4	17341.03	1891.32	17927.4	3.27
AnVi50	5	6	2	2	2	17141.61	1935.92	17727	3.30
AnVi50	6	6	2	2	2	17070.06	2044.97	17714.2	3.64

The algorithm was tested on the 70, 80, and 100 department problems using three-level cluster trees, and the best results are shown in Table 3.5. Anjos and Vieira (2016) report their solution times for these problems to be 150, 230, and 760s for the 70, 80 and 100 department problems respectively.

Table 3.5: Best solutions from hierarchical testing on 70, 80, and 100 department instances.

Problem	β	Objective	Time(s)	AnVi Obj.	% Imp.
AnVi70	4	42416.31	2041.38	42927.5	1.191
AnVi70	5	42318.65	1851.42	43432.1	2.563
AnVi70	6	42279.77	1856.23	42902.4	1.451
AnVi80	4	63274.61	957.56	64509.2	1.914
AnVi80	5	63151.47	26.91	63744.1	0.930
AnVi80	6	62630.40	1841.17	63717.4	1.706
AnVi100	4	117308.09	5563.31	117253.6	-0.046
AnVi100	5	117740.61	188.93	117791.9	0.044
AnVi100	6	117133.99	227.98	117493.5	0.306

The algorithm is competitive over the the thirty department instances. Our approach has a higher solution time on most instances; however, these solutions times are not unreasonable at the design stage of large facilities.

3.4 Conclusion

In this chapter, hierarchical approaches were explored, and a new hierarchical approach was presented. Hierarchical approaches divide the FLP into a hierarchy of smaller sub problems that are easily solved. Then these solutions are combined in a way that provides a high quality solution to the original problem.

An improved hierarchical approach was presented which addressed the limitations of those approaches existing in the literature. Specifically, it provides a means of controlling deadspace in the layout. This approach was implemented and tested on benchmark instances with little-to-no deadspace. It provided improvements over the best known solutions in the literature. On the thirty department instances, improvement of up to 8% was observed, while improvement between 1-3% persists on even larger instances.

Chapter 4

A Nesting Approach for the FLP

The hierarchical approach presented in the preceding chapter worked by dividing the FLP into a series of smaller subproblems. In this chapter, this idea is incorporated into a mathematical model. This new model encapsulates departments within super-departments. However, because this requires the addition of several new departments, and also assignment variables, it results in a model that is more difficult to solve than the original FLP.

This new model is explored, and is found to produce good solutions under certain restrictions. Namely, if the assignment variables are fixed, and the orientation of the super-departments is known, then a good solution for the original problem may be found. This motivates a two-stage approach where the orientation of the super-departments and assignment variables is determined in the first stage, and the resulting problem is solved in the second stage.

Two approaches are developed and tested. The first assumes that super-departments are oriented as columns within the facility. The first stage solves for the assignment variables by approximating the inter-department distance based on the assignments. This stage is implemented using three different metaheuristics. A second approach assumes that the super-departments are oriented as a grid. The three metaheuristics are modified to solve the first stage for this approach, and both approaches are tested on benchmark instances. The new approaches provide improvements over the hierarchical approach presented in the previous chapter.

4.1 A Nesting Model for the FLP

In this section, a new model for the FLP is presented. This model attempts to incorporate aspects of the hierarchical approaches presented in the previous section that made them successful.

Again, let the facility be centred at $(0,0)$ with fixed width and height, w_f, h_f. If we let D be the set of departments indexed by i and j, c_{ij} is the inter-department cost between departments i and j, and A_i is the area required for department i. Let β be the maximum aspect ratio of the departments.

We still have all of the variables from the FLP to model departments:

d^x_{ij}: the horizontal component of the rectilinear distance between department i and department j, $i = 1, ..., n$, $j = i+1, ..., n$.

d^y_{ij}: the vertical component of the rectilinear distance between department i and department j, $i = 1, ..., n$, $j = i+1, ..., n$.

w_i: the width of department i, $i = 1, ..., n$.

h_i: the height of department i, $i = 1, ..., n$.

x_i: the x-coordinate of the centroid of department i, $i = 1, ..., n$.

y_i: the y-coordinate of the centroid of department i , $i = 1, ..., n$.

z^x_{ij}: $= 1$ if $x_i > x_j$, 0 otherwise, $i = 1, ..., n$, $j = i+1, ..., n$.

z^y_{ij}: $= 1$ if $y_i > y_j$, 0 otherwise, $i = 1, ..., n$, $j = i+1, ..., n$.

Now define a set of super-departments, S, which behave exactly as regular departments, except that a given super-department must encapsulate those regular departments which have been assigned to it. In other words, the super-departments and regular departments are simultaneously placed in the facility. Super-departments must not overlap with each other but may overlap with regular departments. Let β_S be the maximum aspect ratio for super-departments. We introduce location and dimension variables for the super-departments similar to those of regular departments:

w_k: the width of super-department k, $k \in S$.

h_k: the height of super-department k, $k \in S$.

x_k: the x-coordinate of the centroid of super-department k, $k \in S$.

y_k: the y-coordinate of the centroid of super-department k , $k \in S$.

z^x_{kl}: $= 1$ if $x_k > x_l$, 0 otherwise, $k, l \in S$, $k \neq l$.

z^y_{kl}: $= 1$ if $y_k > y_l$, 0 otherwise, $k, l \in S$, $k \neq l$.

Finally, we introduce the variables

$$q_{ik} = \begin{cases} 1, \text{if department } i \text{ is assigned to super-department } k. \\ 0, \text{otherwise} \end{cases}$$

to link departments and super-departments. The new model is formulated as:

$$
\begin{aligned}
\min \quad & \sum_{i \in D} \sum_{j > i \in D} c_{ij}(d^x_{ij} + d^y_{ij}) \\
\text{s.t.} \quad & d^x_{ij} \geq x_i - x_j && i, j \in D, i < j && (4.1) \\
& d^x_{ij} \geq x_j - x_i && i, j \in D, i < j && (4.2) \\
& d^y_{ij} \geq y_i - y_j && i, j \in D, i < j && (4.3) \\
& d^y_{ij} \geq y_j - y_i && i, j \in D, i < j && (4.4) \\
& \beta h_i \geq w_i && i \in D && (4.5) \\
& \beta w_i \geq h_i && i \in D && (4.6) \\
& w_i h_i \geq A_i && i \in D && (4.7) \\
& x_j + \frac{w_j}{2} \leq x_i - \frac{w_i}{2} + (1 - z^x_{ij}) w_f && i, j \in D, i \neq j && (4.8) \\
& y_j + \frac{h_j}{2} \leq y_i - \frac{h_i}{2} + (1 - z^y_{ij}) h_f && i, j \in D, i \neq j && (4.9) \\
& \sum_{k \in S} q_{ik} = 1 && i \in D && (4.10) \\
& z^x_{ij} + z^x_{ji} + z^y_{ij} + z^y_{ji} + (1 - q_{ik}) + (1 - q_{jk}) \geq 1 && i, j \in D, i < j, k \in S && (4.11) \\
& x_i + \frac{w_i}{2} \leq x_k + \frac{w_k}{2} + (1 - q_{ik}) w_f && i \in D, k \in S && (4.12) \\
& x_i - \frac{w_i}{2} \geq x_k - \frac{w_k}{2} - (1 - q_{ik}) w_f && i \in D, k \in S && (4.13) \\
& y_i + \frac{h_i}{2} \leq y_k + \frac{h_k}{2} + (1 - q_{ik}) h_f && i \in D, k \in S && (4.14) \\
& y_i - \frac{h_i}{2} \geq y_k - \frac{h_k}{2} - (1 - q_{ik}) h_f && i \in D, k \in S && (4.15) \\
& x_k + \frac{w_k}{2} \leq \frac{w_f}{2} && k \in S && (4.16) \\
& x_k - \frac{w_k}{2} \geq -\frac{w_f}{2} && k \in S && (4.17) \\
& y_k + \frac{h_k}{2} \leq \frac{h_f}{2} && k \in S && (4.18) \\
& y_k - \frac{h_k}{2} \geq -\frac{h_f}{2} && k \in S && (4.19) \\
& \beta_S w_k \geq h_k && k \in S && (4.20) \\
& \beta_S h_k \geq w_k && k \in S && (4.21)
\end{aligned}
$$

$$w_k h_k \geq \sum_{i \in D} A_i q_{ik}^2 \qquad k \in S \qquad (4.22)$$

$$x_l + \frac{w_l}{2} \leq x_k - \frac{w_k}{2} + (1 - z_{kl}^x) w_f \qquad k, l \in S, k \neq l \qquad (4.23)$$

$$y_l + \frac{h_l}{2} \leq y_k - \frac{h_k}{2} + (1 - z_{kl}^y) h_f \qquad k, l \in S, k \neq l \qquad (4.24)$$

$$z_{kl}^x + z_{lk}^x + z_{kl}^y + z_{lk}^y \geq 1 \qquad k, l \in S, k < l \qquad (4.25)$$

$$x_i, y_i \in \mathbb{R} \qquad i \in D$$

$$w_i, h_i \geq 0 \qquad i \in D$$

$$d_{ij}^x, d_{ij}^y \geq 0 \qquad i, j \in D, i < j$$

$$z_{ij}^x, z_{ij}^y \in \{0, 1\} \qquad i, j \in D, i \neq j$$

$$q_{ik} \in \{0, 1\} \qquad i \in D, k \in S$$

$$x_k, y_k \in \mathbb{R} \qquad k \in S$$

$$w_k, h_k \geq 0 \qquad k \in S$$

$$z_{kl}^x, z_{kl}^y \in \{0, 1\} \qquad k, l \in S, k \neq l$$

Constraints 4.1 - 4.4 define the rectilinear distance for each pair of departments. Constraints 4.5, 4.6, 4.20, and 4.21 enforce the maximum aspect ratio for departments and super-departments. 4.7 and 4.22 enforce the area requirements for departments and super-departments; while 4.22 are not necessary, they are specified for completeness. 4.8 and 4.9 define the non-overlap variables for the x and y directions respectively. 4.10 specify that each department must be assigned to exactly one super-department. 4.11 ensure that departments are separated in at least one direction. Constraints 4.12 - 4.15 ensure that each super-department encapsulates those departments which have been assigned to it. 4.16 - 4.19 specify that each super-department must lie within the facility boundary. Finally, constraints 4.23 and 4.24 define the non-overlap variables for super-departments, and 4.25 specify that each super-department must be separated in at least one direction.

4.2 Model Exploration

One would expect that the nesting approach formulation would be more difficult to solve than the original FLP, due to the additional complexity of assigning departments to super-departments. However, one would also suspect that by restricting the number of super-departments, and finding ways to fix some of the variables in the model, one could find high quality solutions that are also feasible to the original FLP. In this section we explore the model by making it more tractable in various ways and observing whether or not it provides any valuable insight.

4.2.1 Fixing Assignment Variables

One would expect that by fixing the assignment variables, q_{ik}, and limiting the number of super-departments, the new model would be able to produce competitive solutions. This is the case for some instances. When the q_{ik} variables are fixed, high quality solutions are obtained within 30 minutes for some problems. However, for most problems, this approach fails to generate any feasible solutions within a 30 minute cutoff. Furthermore, it is difficult to know how many super-departments should be chosen, and how the assignment variables should be fixed. Two methods for finding assignments were tested, with different values for the number of super-departments.

Clustering

The first method considered for determining the assignment variables is the clustering approach that was used in the improved hierarchical approach of Chapter 3. The clustering algorithm is described in detail in Section 3.2.1.

MIP

A MIP was also formulated to solve the assignment of departments to super-departments. Let m be the number of super-departments. We can formulate the problem of assigning departments to super-departments as the following QAP:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=1}^{m} \left[-\gamma c_{ij} q_{ik} q_{jk} + c_{ij} q_{ik} \sum_{l=1}^{m} q_{jl} \right] \\
\text{s.t.} \quad & \sum_{k=1}^{m} q_{ik} = 1 && i = 1, ..., n \qquad (4.26) \\
& \sum_{i=1}^{n} q_{ik} <= n^{\max} && k = 1, ..., m \qquad (4.27) \\
& q_{ik} \in \{0, 1\} && {\scriptstyle i=1,...,n \atop k=1,...,m}
\end{aligned}
$$

Results

The two-stage approach was tested on all 30 department instances with the following parameter values: $\gamma = 0, 1$, $m = 5, 6, 7$, $\beta_S = 8, 9, 10$, and a time limit of 10 minutes for the first stage and 30 minutes for the second stage. Those tests which successfully generated an incumbent within the time limit are shown in Table 4.1. The best solution found for each instance is summarised in Table 4.2.

Table 4.1: The instances for which a solution was generated within 30 minutes using the fixed assignment approach.

Prob.	β	Obj.	Benchmark	% Imp.	Time	m	β_S	Stage I	γ
A	4	8790.46	8770.2	-0.23	1800.8	5	8	Clustering	-
A	4	8892.53	8770.2	-1.39	2400.7	5	8	MIP	0
A	4	8892.53	8770.2	-1.39	2400.7	5	8	MIP	1
A	4	8782.38	8770.2	-0.14	2400.9	6	8	MIP	0
A	4	8709.44	8770.2	0.69	2400.6	6	9	MIP	0
A	4	8916.20	8770.2	-1.66	1800.6	5	10	Clustering	-
A	4	8755.86	8770.2	0.16	2400.8	5	10	MIP	0
A	4	8755.86	8770.2	0.16	2400.8	5	10	MIP	1
A	4	8666.49	8770.2	1.18	2400.9	6	10	MIP	0
A	4	8717.66	8770.2	0.60	2400.9	6	10	MIP	1
A	5	9299.94	8577.4	-8.42	1800.8	5	8	Clustering	-
A	5	9312.06	8577.4	-8.57	2400.4	6	9	MIP	0
A	5	9312.06	8577.4	-8.57	2400.3	6	9	MIP	1
A	5	8915.91	8577.4	-3.95	2400.3	6	10	MIP	0
A	5	8915.91	8577.4	-3.95	2400.4	6	10	MIP	1
A	6	9383.02	8733.6	-7.44	2400.5	6	9	MIP	0
A	6	9383.02	8733.6	-7.44	2400.6	6	9	MIP	1
A	6	9184.15	8733.6	-5.16	2400.4	6	10	MIP	0
A	6	9184.15	8733.6	-5.16	2400.8	6	10	MIP	1
C	4	14254.95	14594	2.32	2400.5	5	10	MIP	0
C	4	14254.95	14594	2.32	2400.6	5	10	MIP	1
C	6	14929.21	14343.3	-4.08	2400.3	5	10	MIP	0
C	6	15226.81	14343.3	-6.16	2400.3	5	10	MIP	1
D	4	11198.17	10284.8	-8.88	1800.9	7	10	Clustering	-

Table 4.2: The best solution found for each instance using the fixed assignment approach.

Problem	β	Obj.	Benchmark	% Imp.
A	4	8666.49	8770.2	1.18
A	5	8915.91	8577.4	-3.95
A	6	9184.15	8733.6	-5.16
C	4	14254.95	14594	2.32
C	6	14929.21	14343.3	-4.09
D	4	11198.17	10284.8	-8.88

4.2.2 A Super-Department Focused Model

Another way in which the model may be able to be simplified is if we can perform the assignment to super-departments and the layout of the super-departments, but ignore the layout of the original departments in a first stage, and then layout the departments within their super-departments in the second stage. Such a first stage problem can be formulated as the following SOCP:

$$
\begin{aligned}
\min \quad & \sum_{i \in D} \sum_{j < i \in D} c_{ij}(d_{ij}^x + d_{ij}^y) && && (4.28)\\
\text{s.t.} \quad & d_{ij}^x \geq x_k - x_l - (1 - q_{ik}q_{jl})w_f && i < j \in D, k \neq l \in S && (4.29)\\
& d_{ij}^x \geq x_l - x_k - (1 - q_{ik}q_{jl})w_f && i < j \in D, k \neq l \in S && (4.30)\\
& d_{ij}^y \geq y_k - y_l - (1 - q_{ik}q_{jl})h_f && i < j \in D, k \neq l \in S && (4.31)\\
& d_{ij}^y \geq y_l - y_k - (1 - q_{ik}q_{jl})h_f && i < j \in D, k \neq l \in S && (4.32)\\
& x_l + \frac{w_l}{2} \leq x_k - \frac{w_k}{2} + (1 - z_{kl}^x)w_f && k \neq l \in S && (4.33)\\
& y_l + \frac{h_l}{2} \leq y_k - \frac{h_k}{2} + (1 - z_{kl}^y)h_f && k \neq l \in S && (4.34)\\
& z_{kl}^x + z_{lk}^x + z_{kl}^y + z_{lk}^y \geq 1 && k < l \in S && (4.35)\\
& q_{ik} \in \{0, 1\} && i \in D, k \in S &&\\
& x_k, y_k \in \mathbb{R} && k \in S &&\\
& d_{ij}^x, d_{ij}^y \geq 0 && i < j \in D &&
\end{aligned}
$$

This model was implemented and tested; however, it was too difficult to solve due to the large quantity of second order cone constraints.

4.3 A Column-Based Two-Stage Approach

Fixing the assignment variables, q_{ik}, provides competitive solutions for some problems, but fails to find a feasible solution for others. This is because the model is still too difficult, which hints that if one restricts it further in an intelligent manner, one would obtain good solutions to the original problem. To begin, the assumption is made that the super-departments will be oriented as columns in the facility. Each super-department centroid will have the same y-coordinate, and the relative positioning will be fixed.

4.3.1 Metaheuristics for Stage I

This approach provides the motivation to also find a new first stage model that can incorporate more information about the locations or relative positions of the super-departments into the second stage model. In order to incorporate the naive assumption that each super-department forms a column of the facility, the following first stage model is devised:

$$\begin{aligned}
\min \quad & \sum_{k \in S} \left[\sum_{i \in D} \sum_{j < i \in D} c_{ij} q_{ik} q_{jk} + \sum_{i \in D} \sum_{j \neq i \in D} \sum_{l < k \in S} c_{ij} q_{ik} q_{jl} \hat{d_{kl}} \right] \\
\text{s.t.} \quad & \hat{d_{kl}} = \sum_{u=l}^{k} \sum_{i \in D} \frac{A_i q_{iu}}{h_f} && k, l \in S, l < k \qquad (4.36) \\
& \sum_{k \in S} q_{ik} = 1 && i \in D \qquad (4.37) \\
& \sum_{i \in D} q_{ik} \leq n^{\max} && k \in S \qquad (4.38) \\
& q_{ik} \in \{0, 1\} && i \in D, k \in S \\
& \hat{d_{kl}} \geq 0 && k, l \in S, l < k
\end{aligned}$$

The objective function minimizes the the inter-department cost of departments placed in the same super-department, plus the inter-department cost of departments placed in different super-departments, k and l, scaled by an approximation of the distance between those super-departments, $\hat{d_{kl}}$. Constraints 4.36 define the distance between pairs of super-departments. The distance across a single super-department is approximated by the sum of the areas of those departments assigned to that super-department divided by the height of the facility, since each super-department must span the entire height of the facility. Constraints 4.37 ensure that each department is assigned to exactly one super-department, and 4.38 place a limit on how many departments can be assigned to each super-department.

Since this model involves the product of three variables, $q_{ik}q_{jl}\hat{d_{kl}}$, it seems unlikely that it can be solved as a MIP. Therefore, in this section, different metaheuristics are applied to solving this problem.

Metaheuristic Structure

Before detailing the implementation of various metaheuristics to solve this problem, the common components will be described here. Namely, the objective function, solution representation, and the neighborhood. The objective function used to evaluate a given solution for each metaheuristic will be the same as the model presented above. Each solution is represented as a sequence of departments, with the first $n^{\max}$ departments assigned to the first super-department, and so on. The neighborhood of each solution will be explored through swapping pairs of departments. Swapping refers to interchanging the locations of two departments in the ordered list which represents the solution; this procedure is illustrated in Figure 4.1.

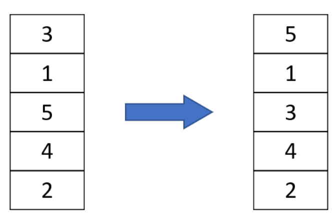

Figure 4.1: Illustration of a swap of departments 3 and 5.

Various metaheuristics have different strengths and weaknesses, and it is difficult to predict beforehand which one is more appropriate for a given problem. Therefore, three well known metaheuristics were implemented to solve the first stage.

Tabu Search

Tabu Search (TS) works similarly to a hill-climbing search, but prevents the search from terminating at a local minima by maintaining a tabu list, a FIFO queue of prohibited transitions with a fixed length. When the tabu search moves from one solution to another, that solution is added to the tabu-list, and the search may not enter that solution again until the tabu-list has been cycled through.

TS was implemented using random initial solutions. At each iteration, every possible pair of swaps in the current solutions is performed, generating a neighbor, and each of these neighbors is evaluated. Then, the search moves to the neighbor solution with the lowest cost which is not on the tabu list. Note that the search may move from a better to a worse solution. The parameters for this simple tabu search are the number of iterations to perform, and the length of the tabu list.

Simulated Annealing

A Simulated Annealing (SA) algorithm was implemented as a first stage solution procedure. SA differs from TS in that it does not perform and evaluate each neighbor of the current solution. Rather, it chooses one neighbor randomly. If the neighbor is better than the current solution, the search moves to that solution. If the neighbor is worse than the current solution, there is probability that the search will move to the neighbor. This probability is typically expressed as

$$e^{\frac{Z_{\text{old}}-Z_{\text{new}}}{T_t}}$$

where Z_{old} is the current solution, Z_{new} is the worse neighbor solution, and T_t is the temperature in stage t of the algorithm. This probability is determined by a temperature schedule, a parameter of SA that specifies how the probability of moving to a worse solution changes over the course of the search. In most cases, this probability is initially large to allow the search to move around the feasible region, and eventually becomes smaller, to obtain a local minimum in the final iterations.

Genetic Algorithms

Finally, a Genetic Algorithm (GA) was implemented to solve the first stage. The first way in which GA differs from TS and SA is that GA maintains multiple solutions at each iteration in what is called the 'current population'. At each iteration, pairs of solutions are selected from the current population to become 'parents'. A 'crossover' method is applied to generate children from parents, and a new population containing some parents and some children is created. Moving from the current population to the new population is analogous to moving from the current solution to a neighbor in TS and GA. During or after the creation of the new population, 'mutation' or 'immigration' is performed to prevent the search from prematurely converging. The intention of these procedures is to allow the search to move away from local minima.

The GA implemented to solve the first stage problem was developed using many ideas presented in Ahuja et al. (2000). Specifically the path-based crossover method, the next generation selection criteria, and the immigration procedure.

In the path crossover approach of Ahuja et al. (2000), the two solutions chosen to be parents, I_1 and I_2, produce multiple children, the best of which, I_3, is chosen to be the single output. The approach to combining the solutions involves creating a path of children between I_1 and I_2 such that each solution along the path is one swap away from both of its neighbors. The first node on the path is I_1 and the final node on the path is I_2. The algorithm begins with I_1 and looks at a random chromosome (indexed department in the solution). If the chromosome (assigned department) is the same in both I_1 and I_2 the algorithm continues to the next node on the path. If they are different, there are two possible swaps that can be made to make that specific chromosome the same, either a swap in I_1 to appear like I_2 or a swap in I_2 to appear like I_1. Both of these swaps are evaluated, and the one with the better objective function value is chosen to be the second node on the path. The algorithm moves to the next chromosome and compares the second node on the path to I_2 as before, and continues so on. Once all chromosomes have been considered, the best node on the path excluding I_1 and I_2 is chosen to be the child, I_3.

Ahuja et al. (2000) discuss choosing the two best among I_1, I_2, and I_3 for the next population, but decide that it leads to premature convergence. For the sake of simplicity, in this work, this was the rule implemented to decide the next generation.

Ahuja et al. (2000) also present the use of an immigration procedure as opposed to a mutation procedure. That is, there is a small probability at each iteration that immigration will take place. This involves replacing some of the worst performing solutions with solutions that are significantly different than the current population. Taking the approach from Ahuja et al. (2000) and applying it here, let Q be an nx$|S|$ matrix. An element (i, k) of the matrix is the number of individuals in the current population (as opposed to all past individuals in Ahuja et al. (2000)), who have department i assigned to super-department k. For each new immigrant, a random sequence of departments is created, and each department in the sequence is assigned to the open super-department (that is, a super-department where the number of assigned departments less than $n^{\max}$) with the lowest value of Q_{ik}.

4.3.2 Testing and Results

The three metaheuristics described above were tested using the following parameters. Each metaheuristic was run 50 times in parallel using different initial solutions(populations). The tabu search iteration limit was set to 5000 and the list length was set to 1000. For the genetic algorithm, the population size was 100, the iteration limit was 10000, the immigration probability was 10% and the number of immigrants per immigration was 1. β_S was set to 15 for all tests. Finally, the temperature schedule for simulated annealing was set according to Table 4.3 where 30000 iterations are performed at each value of t and the temperature at stage t is defined by

$$T_t = z_0 \prod_{i=0}^{t} s_i \tag{4.39}$$

where z_0 is the objective function value for the initial solution.

Table 4.3: Simulated annealing temperature schedule

t	Iterations	s_t
0	30000	0.9
1	30000	0.8
2	30000	0.8
3	30000	0.8
4	30000	0.5
5	30000	0.5
6	30000	0.5
7	30000	0.5
8	30000	0.3
9	30000	0.2

Detailed results for each algorithm are presented in tables A.1, A.2, and A.3 in Appendix A. The best solutions from the column based approach, and the metaheuristics which produced them are shown in Table 4.4, where the T_{search} column displays the time to solve the first stage, and the T_{solve} column shows the time to solve the second stage. Note that there is no clear best metaheuristic as each finds several of the best solutions. The improvement ranges from 0.85-14.18% over the best known solutions in the literature.

Table 4.4: Best solutions using the column-based approach.

Prob	β	Obj	Stage I	% Imp.	T_{search}	T_{solve}
A	4	8356.68	GA	4.72	354.9	1800.4
A	4	8356.68	TS	4.72	351.4	1800.3
A	5	8339.55	SA	2.77	42.1	1800.4
A	6	8360.14	GA	4.28	455.4	1800.4
A	7	8296.63	SA	2.07	38.5	1800.3
A	8	8329.89	SA	0.85	44.3	1800.4
A	9	8204.15	SA	3.60	58.3	1800.4
A	10	8212.82	SA	5.60	55.7	1800.4
B	4	9138.69	TS	7.16	434.5	1800.4
B	5	9021.57	TS	9.15	436.6	1800.4
B	6	9060.04	TS	6.32	427.3	1800.4
B	7	9017.66	TS	7.68	433.4	1800.4
B	8	8923.77	GA	8.59	422.7	1800.4
B	9	8966.99	TS	8.23	424.3	1800.6
B	10	8817.01	TS	7.57	425.3	1800.6
C	4	13775.99	SA	5.61	42.0	1800.4
C	5	13791.66	TS	6.20	351.8	1800.4
C	6	13707.00	TS	4.44	436.5	1800.4
C	7	13714.31	TS	1.12	433.8	1800.4
C	8	13724.71	GA	5.77	362.5	1800.5
C	9	13625.85	TS	3.60	469.2	1800.4
C	10	13625.85	TS	3.04	436.4	1800.4
D	4	9824.94	TS	4.47	421.3	1800.3
D	5	9868.30	TS	3.87	436.6	1800.3
D	6	9823.74	GA	1.80	369.7	1800.8
D	7	9634.92	TS	4.26	431.0	1800.6
D	8	9561.63	SA	5.23	59.5	1800.5
D	9	9318.09	SA	8.24	59.5	1800.4
D	10	9403.44	GA	7.96	420.7	1800.6
E	4	11104.30	TS	6.19	438.1	1800.7
E	5	10746.96	SA	7.75	59.5	1800.4
E	6	10498.00	GA	10.44	417.2	1800.4
E	7	10534.71	TS	10.74	425.5	1800.6
E	8	10836.81	SA	7.69	57.5	1800.6
E	9	10475.60	GA	9.94	434.2	1800.8
E	10	10166.38	SA	14.18	59.2	1800.5

Relationship Between Stages

Because the first stage metaheuristics can be tuned and optimized, it would be interesting to know the relationship between the objective function found in Stage I and the objective function found in Stage II. In other words, it would be useful to know if significant time should be spent in finding an optimal solution to the first stage problem, or if find a good one quickly is a better approach. To examine this relationship, the objective function values of each stage were graphed for each problem. Figure 4.2 shows the second stage objective versus the first stage objective, with every problem instance placed on the same graph. The legend lists the different instances. Note that for a given problem, there appears to be a positive relationship. The relationship for each problem broken down by β is shown in Figures 4.3 - 4.7.

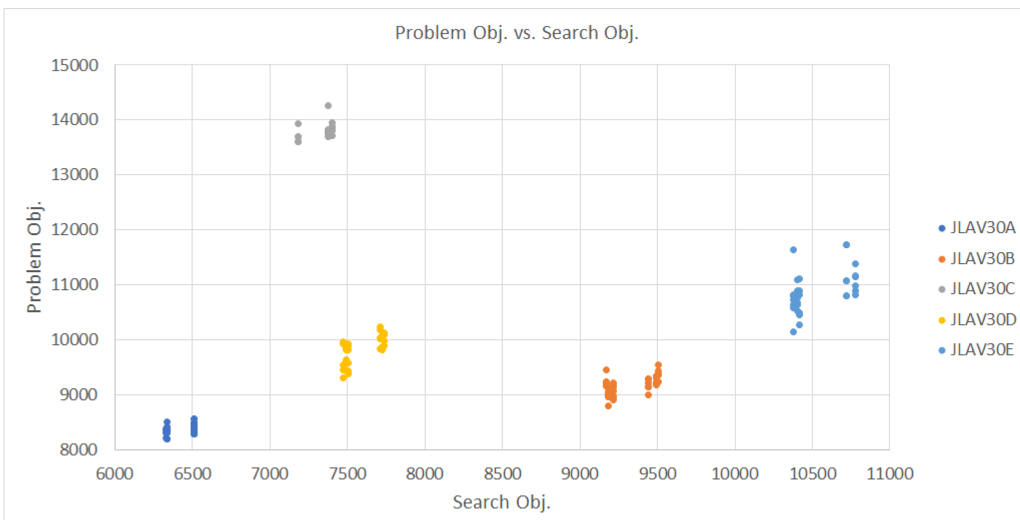

Figure 4.2: Stage II vs. Stage I objectives.

The trend, for the most part, is increasing; however, for some problems it is decreasing, which suggests that the objective function of the first stage problem could be improved.

4.3.3 Limitations

The column-based approach is not very flexible, as it forces the facility to be partitioned into columns. Furthermore, it is nearly identical to the Flexible Bays approach (Meller, 1997) (Konak et al., 2006), except that it does not force departments to be placed in a

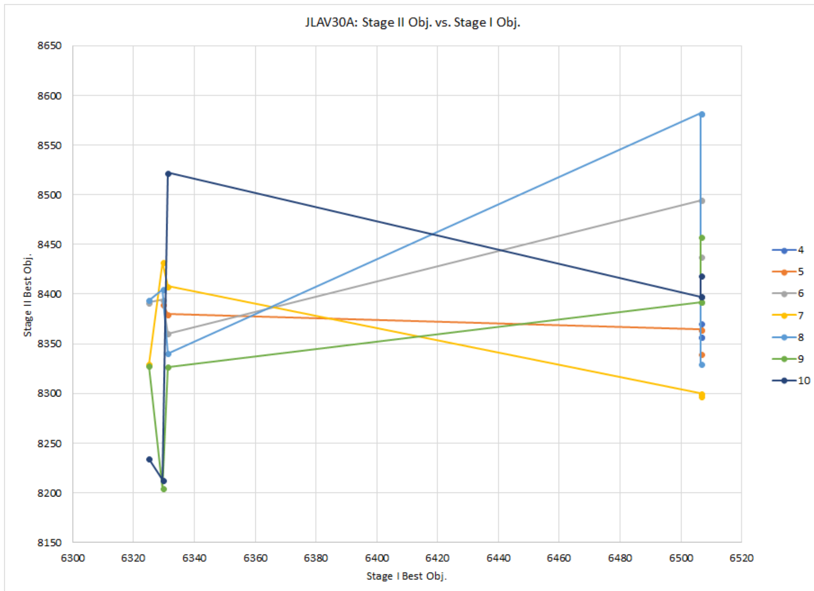

Figure 4.3: Stage II vs Stage I objectives by maximum aspect ratio for JLAV30A.

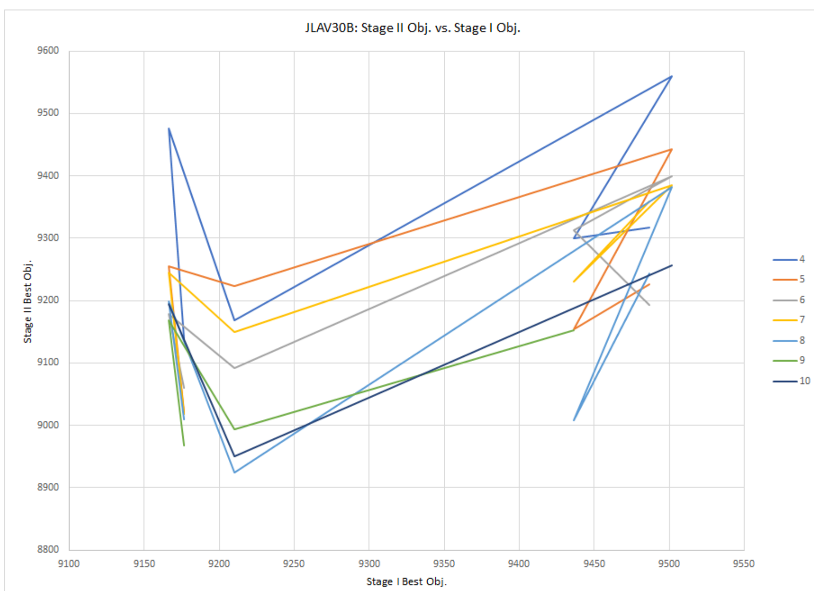

Figure 4.4: Stage II vs Stage I objectives by maximum aspect ratio for JLAV30B.

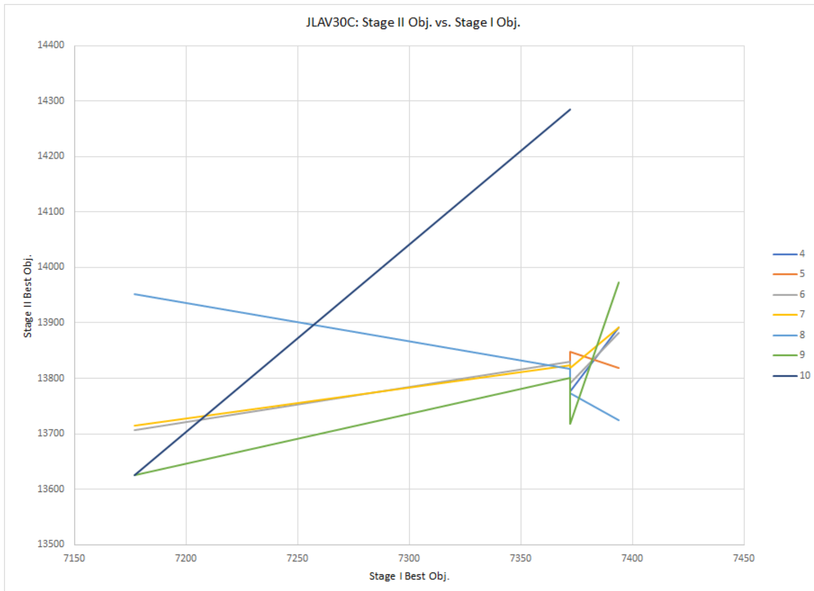

Figure 4.5: Stage II vs Stage I objectives by maximum aspect ratio for JLAV30C.

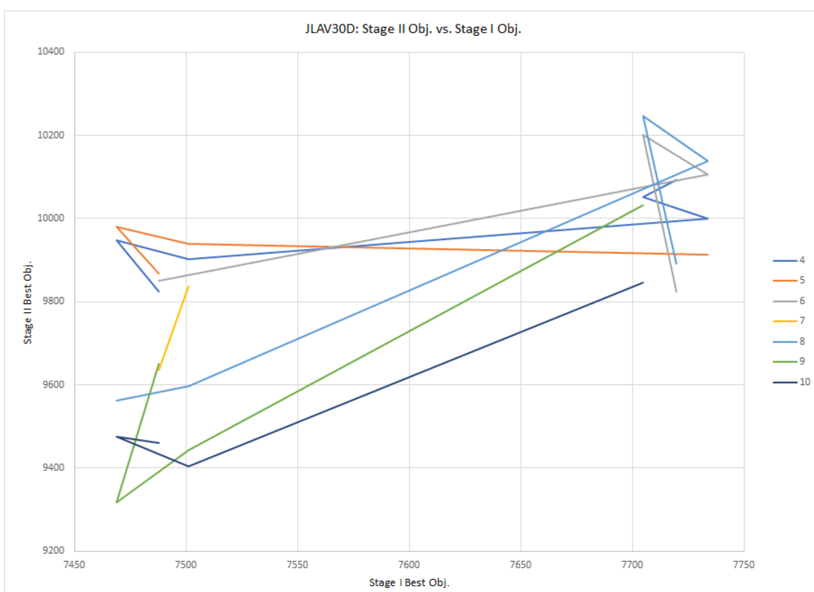

Figure 4.6: Stage II vs Stage I objectives by maximum aspect ratio for JLAV30D.

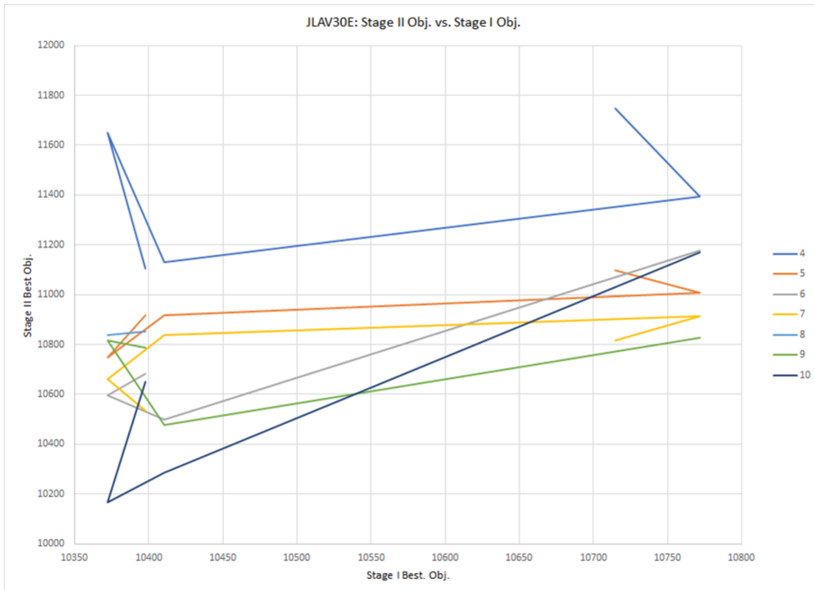

Figure 4.7: Stage II vs Stage I objectives by maximum aspect ratio for JLAV30E.

linear ordering within their column. In other words, the departments being placed within a given super-department still require non-overlap variables for both dimensions and may be placed in any orientation. In the next section, a more sophisticated approach is developed in an attempt to address these limitations.

4.4 A Grid-Based Two-Stage Approach

In this section, a new two-stage approach is developed to address the limitations of the column-based approach. In the grid-based approach, m super-departments are oriented in a pre-specified grid, with n^{rows} rows. This introduces additional complexity, as the non-overlap conditions are not known before solving, as was the case in the column-based approach. The following sections discuss the changes made in order to adapt the column-based approach to a grid-based approach.

4.4.1 Adapting to a Grid-Based Approach

One aspect of the column-based approach that made it easier to work with was that the vertical non-overlap variables could be removed. Each super-department represented an entire slice of the facility. This is illustrated in Figures 4.8 and 4.9, which show the solution to JLAV30A and JLAV30B using a column-based approach with six and seven super-departments, respectively.

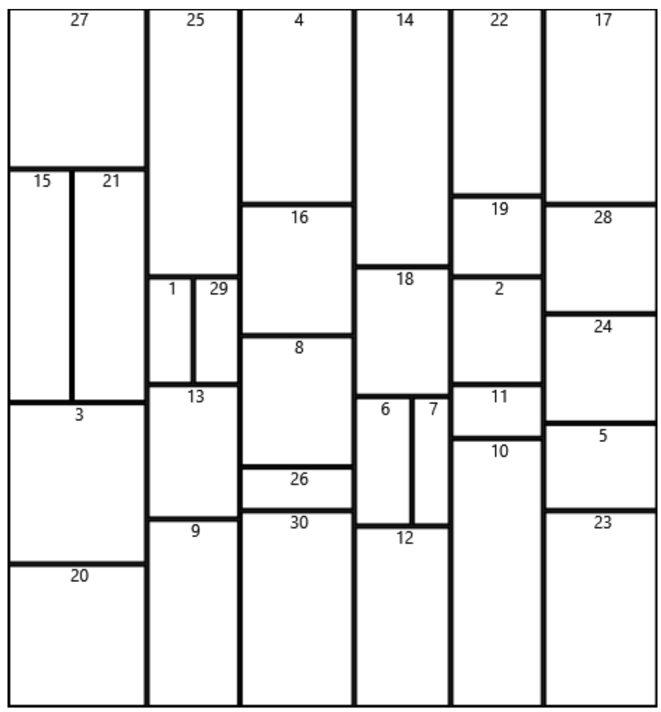

Figure 4.8: The final solution to JLAV30A using a column-based approach.

When adding back the vertical non-overlap conditions, the non-overlap conditions may not be fixed between adjacent rows and columns; otherwise, a rigid grid will be created where the facility will have cuts that extend the length of the entire facility in both the vertical and horizontal direction.

Thus, the problem becomes harder to solve, because not all of the non-overlap variables for super-departments may be fixed. In order to keep the problem solvable in a reasonable amount of time, another restriction was introduced for the grid-based approach. Within a given super-department, departments will be arranged as a series of rows or columns, with their order and orientation determined in the first stage model.

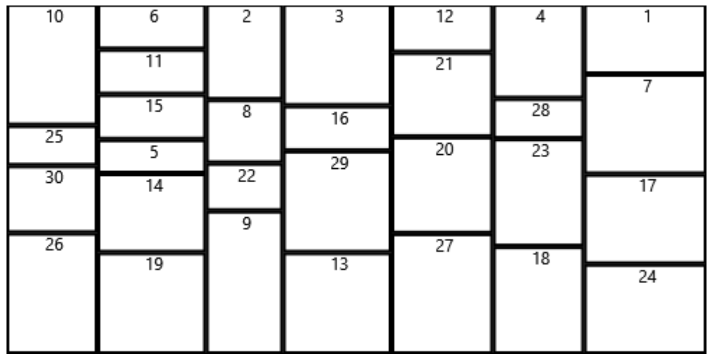

Figure 4.9: The final solution to JLAV30B using a column-based approach.

A variable was added to the solution representation in the first stage metaheuristics. This variable represents the orientation of the departments within the super-department and may take on four possible values. Each value corresponds with the departments being ordered as: columns, rows, reverse ordered columns, and reverse ordered rows. In other words, the ordering of the departments within their super-department now affects their orientation.

The neighbor selection methods for each metaheuristic were adapted to now consider both swaps and rotations of super-departments in the candidate solutions. The metaheuristics move to the neighbor which resulted in a better objective function, regardless of whether it was a swap or a rotation.

The first stage model was adapted to incorporate an approximation of the distance between departments in different super-departments. Let A_r be the sum of the areas of all super-departments assigned to row r, and A_c be the sum of the areas of all super-departments assigned to column c, then the width of a column c in the grid is approximated by $\frac{A_c}{h_f}$ and the height of a row r in the grid is approximated by $\frac{A_r}{w_f}$. Then the vertical distance between the boundary of two rows r_1 and r_2 where $r_1 < r_2$ is $\sum_{r=r_1+1}^{r_2-1} \frac{A_r}{w_f}$. Similarly, the horizontal distance between the boundaries of two columns c_1 and c_2 where $c_1 < c_2$ is $\sum_{c=c_1+1}^{c_2-1} \frac{A_c}{h_f}$.

Since the orientation of the departments within their grid square is also determined in the first stage, the distance from the boundary of a grid square to the department of interest can be approximated in a similar way, instead by taking the ratio of the department area to the approximated width or height of its grid square.

4.4.2 Testing and Results

The grid-based approach was implemented and tested on several benchmark instances. Each instance was attempted with the grids specified in Table 4.5. Detailed results are provided in Section A.2. Table 4.6 shows the best solution for each instance for both the grid and column-based approaches. It also specifies which first stage metaheuristic generated the best solution for each approach.

Table 4.5: Parameters for grid-based approach tests.

n^{rows}	n^{cols}
2	4
3	3
3	4

Solutions for JLAV30A and JLAV30B are shown in Figures 4.10 and 4.11. These figures show several changes from the column-based approach. The first is that the solution no longer requires divisions between super-departments to extend the entire length of the facility. The second is that the linear ordering within a given super-department is clearly observable.

Although this approach outperforms the column-based approach on only a few instances, it provides advantages in terms of flexibility. It would be easier to construct an approach using the grid-approach to solve much larger FLPs, because a pure column-based approach would require several long strips across a facility, which would be difficult to implement in real life. Exploring hybrid hierarchical column-grid approach for solving very large instances would be a promising direction for future research. Such an approach could have a first level which would be a grid approach, and a second level which encapsulates individual grids within different columns.

4.5 Motivation for a Data-Driven Approach

It was shown throughout this section that there is no single best approach for solving the FLP. Furthermore, there is no clear best metaheuristic within any given approach. The performance of different approaches and metaheuristics seems to depend on the instance data and the restrictiveness of the parameter β. Since solving even moderately sized FLPs using the approaches presented thus far takes a significant amount of time, it would be desirable to have a tool which could predict the relative performance of different approaches

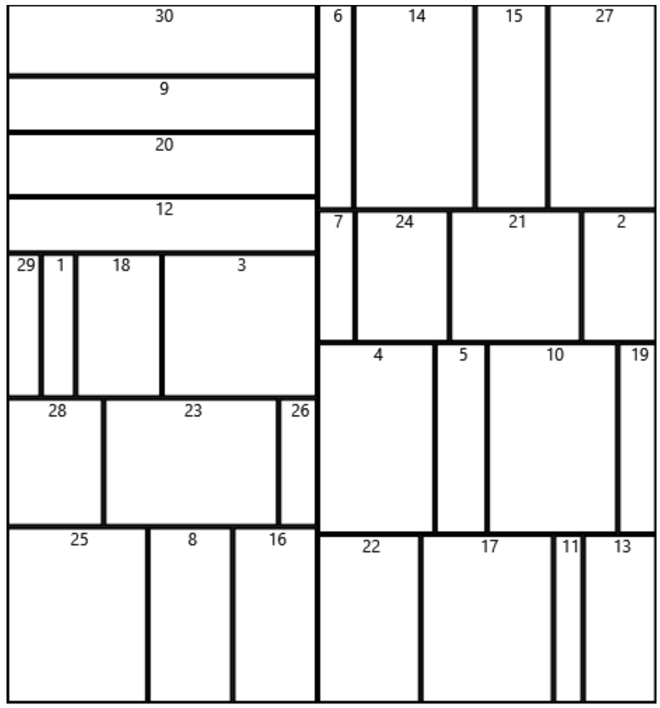

Figure 4.10: The final solution to JLAV30A using a grid-based approach.

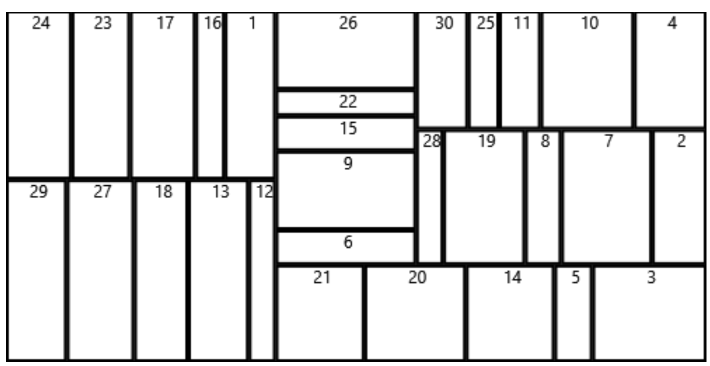

Figure 4.11: The final solution to JLAV30B using a grid-based approach.

by examining the performance of those approaches on similar instances. Research involving the incorporation of machine learning tools into solving problems such as the FLP has become a very active area, so this is a promising direction of future research.

Table 4.6: Best solutions for grid and column-based approaches

		Grid Approach					Column Approach	
Prob.	β	Obj.	T_{search}	T_{solve}	Stage I	% Imp.	Stage I	% Imp.
A	4	NA	NA	NA	NA	NA	GA,TS	4.72
A	5	NA	NA	NA	NA	NA	SA	2.77
A	6	9503.25	274.5	1158.1	GA	-8.81	GA	4.28
A	7	9424.45	493.2	779.8	SA	-11.25	SA	2.07
A	8	9424.45	545.3	1392.6	SA	-12.18	SA	0.85
A	9	8990.97	530.6	1800.2	SA	-5.65	SA	3.60
A	10	8979.51	489.3	1800.3	SA	-3.22	SA	5.60
B	4	10036.75	534.9	636.4	SA	-1.96	TS	7.16
B	5	9872.03	1539.2	1800.1	TS	0.59	TS	9.15
B	6	9814.73	250.4	1800.2	GA	-1.49	TS	6.32
B	7	9577.60	567.8	1800.3	SA	1.95	TS	7.68
B	8	9355.92	532.5	1800.2	SA	4.16	GA	8.59
B	9	9080.51	543.5	1800.3	SA	7.07	TS	8.23
B	10	8980.99	540.3	1800.2	SA	5.86	TS	7.57
C	4	14382.86	1666.3	1800.2	TS	1.45	SA	5.61
C	5	13889.25	204.9	382.5	GA	5.54	TS	6.20
C	6	13687.37	1498.5	1231.8	TS	4.57	TS	4.44
C	7	13663.75	1410.9	1800.2	TS	1.48	TS	1.12
C	8	13663.75	1507.7	1800.2	TS	6.19	GA	5.77
C	9	13636.48	1524.9	1800.2	TS	3.53	TS	3.60
C	10	13461.19	1528.0	1800.2	TS	4.21	TS	3.04
D	4	10003.32	534.7	1800.3	SA	2.74	TS	4.47
D	5	9802.09	504.9	1800.3	SA	4.51	TS	3.87
D	6	9776.51	1635.7	1800.3	TS	2.27	GA	1.80
D	7	9652.83	547.0	1800.3	SA	4.08	TS	4.26
D	8	9453.57	580.3	1800.2	SA	6.30	SA	5.23
D	9	9376.45	494.6	1800.2	SA	7.66	SA	8.24
D	10	9326.71	1504.1	1800.3	TS	8.71	GA	7.96
E	4	11067.22	519.8	1800.3	SA	6.51	TS	6.19
E	5	10652.20	1450.9	1800.2	TS	8.57	SA	7.75
E	6	10459.51	552.8	1800.2	SA	10.77	GA	10.44
E	7	10394.68	555.1	1800.3	SA	11.92	TS	10.74
E	8	10379.94	515.5	1800.3	SA	11.58	SA	7.69
E	9	10297.77	533.8	1800.3	SA	11.47	GA	9.94
E	10	10265.49	1516.8	1800.3	TS	13.34	SA	14.18

Chapter 5

Conclusion

The FLP is well-known and well-studied optimization problem with many important applications in industry. Since the first formulations dating back to Koopmans and Beckmann (1957), it has been recognized that the FLP is difficult to solve. Thus, as explored in Chapter 2, various exact and heuristic approaches have been devised to solve the problem. Many of the heuristic approaches fall into a category of two or multi-stage solution approaches where some aspect of the problem, such as the relative positioning variables, are determined in a first stage, and the remaining problem is solved in a second stage. In this theis, two solution approaches to the FLP have been presented, explored, tested, and shown to provide significant improvements over existing approaches from the literature.

The first solution approach, a hierarchical approach, works by decomposing the facility into clusters to be solved as subproblems. Although there are existing approaches in the literature that also take advantage of this type of clustering, they have several drawbacks and fail to control in layouts. The approach presented in this thesis addresses these drawbacks by performing the layout down, rather than up, the hierarchy generated through clustering. The approach was implemented and tested on benchmark instances from the literature. For many instances, the approach provides improvements over the best known solutions. This success motivated another approach, which incorporates hierarchical aspects into the mathematical modelling.

Such a model was presented in Chapter 4. This model encapsulates departments within different super-departments. However, the problem is more difficult to solve than the original FLP, so different experiments were performed to understand the conditions under which the model produced good solutions. It was determined that, if the assignment variables and orientations of the super-departments were determined in a first stage and fixed, high quality solutions could be obtained. Two solution approaches were developed. The first involved orienting the super-departments as a series of columns with variable widths within the facility. Three metaheuristics were implemented to solve the first stage problem. The first stage involved determining the assignment variables by approximating the interdepartment distances. This approach provided even more improvement than the hierarchical approach on the benchmark instances.

Partitioning the facility into a set of columns is sometimes neither desirable nor feasible. Thus, a more sophisticated approach was developed, where the super-departments were arranged in a grid. The three metaheuristics were adapted to approximate the interdepartment distances in the grid setting. This approach also provided improvements, but the column-based approach provided better solutions for some instances. Furthermore, in both approaches, there was no clear best metaheuristic for solving the first stage problem.

These issues provide the motivation for two future directions of research. The first is the development of a hybrid column-grid approach, which would involve a two level hierarchy suitable for solving larger instances. Ideally, this would combine the best of both approaches. The second is for a data-driven approach to determine the relative performance of each approach using each metaheuristic. This area of research is currently very popular, due to the recent advances in machine learning.

Thus far, this thesis has focused on the improvements in terms of computational performance; however, there are possible contributions for practical modellers to take advantage of. Tompkins et al. (2010) recommend developing and considering a pool of possible facility layouts in the overall design process. The approaches presented in this thesis develop a "low resolution" layout first, and increase the granularity in successive stages. These low resolution layouts may provide insight to practitioners when developing their pool of potential layouts.

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APPENDICES

Appendix A

Detailed Results for Two-Stage Nesting Approaches

A.1 Detailed Column-Based Results

Table A.1: Genetic Algorithm results for column-based approach.

$n^{\max}$			6					7				
Problem	β	ANVI Obj.	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}
JLAV30A	4	8770.2	6506.52	8356.68	4.72	354.86	1800.40	6331.15	INF	INF	430.34	4.50
JLAV30A	5	8577.4	6506.52	8364.17	2.49	350.57	1800.35	6331.15	8379.64	2.31	415.87	1800.32
JLAV30A	6	8733.6	6506.52	8494.45	2.74	350.96	1800.41	6331.15	8360.14	4.28	455.42	1800.37
JLAV30A	7	8471.7	6506.52	8300.18	2.02	346.29	1800.37	6331.15	8407.91	0.75	429.38	1800.44
JLAV30A	8	8401.4	6506.52	8581.83	-2.15	350.29	1800.32	6331.15	8340.30	0.73	459.93	1800.36
JLAV30A	9	8510.1	6506.52	8392.25	1.38	358.15	1800.34	6331.15	8326.93	2.15	421.18	1800.41
JLAV30A	10	8699.8	6506.52	8397.44	3.48	348.66	1800.31	6331.15	8522.37	2.04	427.38	1800.41
JLAV30B	4	9843.9	9486.61	9316.95	5.35	337.92	1800.39	9210.27	9167.90	6.87	426.91	1800.72
JLAV30B	5	9930.5	9486.61	9225.94	7.09	342.30	1800.42	9210.27	9223.72	7.12	417.93	1800.40
JLAV30B	6	9671	9486.61	9193.57	4.94	346.84	1800.43	9210.27	9092.10	5.99	413.64	1800.34
JLAV30B	7	9768.1	9486.61	9358.78	4.19	340.22	1800.48	9210.27	9150.03	6.33	415.84	1800.40
JLAV30B	8	9762.5	9486.61	9242.99	5.32	336.33	1800.92	9210.27	8923.77	8.59	422.69	1800.38
JLAV30B	9	9771.2	9486.61	NI	NI	351.45	1801.10	9210.27	8993.51	7.96	424.89	1800.72
JLAV30B	10	9539.6	9486.61	NI	NI	339.77	1801.32	9210.27	8950.85	6.17	420.99	1800.56
JLAV30C	4	14594	7393.66	13891.02	4.82	363.58	1800.36	7176.39	INF	INF	410.25	3.51
JLAV30C	5	14703.2	7393.66	13817.92	6.02	367.73	1800.36	7176.39	INF	INF	416.96	4.51
JLAV30C	6	14343.3	7393.66	13882.41	3.21	360.48	1800.39	7176.39	INF	INF	415.43	4.47
JLAV30C	7	13869.3	7393.66	13891.02	-0.16	361.37	1800.42	7176.39	INF	INF	411.01	4.87
JLAV30C	8	14564.7	7393.66	13724.71	5.77	362.47	1800.46	7176.39	INF	INF	418.06	4.51
JLAV30C	9	14135.1	7393.66	13973.67	1.14	365.73	1800.45	7176.39	INF	INF	411.93	4.10
JLAV30C	10	14052.6	7393.66	NI	NI	364.27	1801.10	7176.39	INF	INF	405.28	4.41
JLAV30D	4	10284.8	7719.68	10092.18	1.87	357.22	1800.58	7501.17	9902.33	3.72	415.56	1800.32
JLAV30D	5	10265.4	7719.68	NI	NI	355.71	1800.75	7501.17	9939.57	3.17	425.02	1800.34
JLAV30D	6	10004	7719.68	9823.74	1.80	369.67	1800.79	7501.17	NI	NI	420.57	1800.79
JLAV30D	7	10063.3	7719.68	NI	NI	360.37	1800.80	7501.17	9837.21	2.25	416.81	1800.56
JLAV30D	8	10089.2	7719.68	9890.61	1.97	354.84	1800.65	7501.17	9595.78	4.89	429.04	1800.46
JLAV30D	9	10154.6	7719.68	NI	NI	365.07	1800.96	7501.17	9442.68	7.01	411.33	1800.63
JLAV30D	10	10216.5	7719.68	NI	NI	365.88	1801.13	7501.17	9403.44	7.96	420.69	1800.59
JLAV30E	4	11837.3	10714.68	11748.10	0.75	345.14	1801.08	10410.36	11132.25	5.96	422.09	1800.65
JLAV30E	5	11650.3	10714.68	11096.28	4.76	357.72	1800.52	10410.36	10916.18	6.30	413.82	1800.34
JLAV30E	6	11722.3	10714.68	NI	NI	350.90	1801.03	10410.36	10498.00	10.44	417.25	1800.40
JLAV30E	7	11802	10714.68	10817.75	8.34	346.53	1800.59	10410.36	10839.71	8.15	423.23	1800.46
JLAV30E	8	11739.1	10714.68	NI	NI	358.69	1801.07	10410.36	NI	NI	421.26	1800.93
JLAV30E	9	11632.3	10714.68	NI	NI	345.08	1801.08	10410.36	10475.60	9.94	434.21	1800.77
JLAV30E	10	11846	10714.68	NI	NI	364.65	1801.00	10410.36	10284.11	13.18	418.57	1800.41

Table A.2: Simulated Annealing results for column-based approach.

$n^{\max}$			6					7				
Problem	β	ANVI Obj.	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}
JLAV30A	4	8770.2	6506.52	8370.25	4.56	41.28	1800.31	6329.62	INF	INF	57.02	5.91
JLAV30A	5	8577.4	6506.52	8339.55	2.77	42.05	1800.41	6329.62	8389.99	2.18	55.53	1800.30
JLAV30A	6	8733.6	6506.52	8437.82	3.39	43.21	1800.46	6329.62	8394.50	3.88	56.70	1800.36
JLAV30A	7	8471.7	6506.52	8296.63	2.07	38.51	1800.32	6329.62	8431.98	0.47	55.31	1800.39
JLAV30A	8	8401.4	6506.52	8329.89	0.85	44.35	1800.38	6329.62	8404.67	-0.04	58.17	1800.47
JLAV30A	9	8510.1	6506.52	8457.99	0.61	44.51	1800.40	6329.62	8204.15	3.60	58.34	1800.41
JLAV30A	10	8699.8	6506.52	8418.47	3.23	39.65	1800.46	6329.62	8212.82	5.60	55.65	1800.39
JLAV30B	4	9843.9	9436.26	9299.70	5.53	42.73	1800.37	9166.43	9475.42	3.74	56.77	1800.40
JLAV30B	5	9930.5	9436.26	9154.23	7.82	42.64	1800.38	9166.43	9255.68	6.80	57.77	1800.35
JLAV30B	6	9671	9436.26	9312.55	3.71	40.77	1800.47	9166.43	9178.89	5.09	56.35	1800.37
JLAV30B	7	9768.1	9436.26	9230.91	5.50	43.82	1800.35	9166.43	9244.78	5.36	56.07	1800.37
JLAV30B	8	9762.5	9436.26	9008.07	7.73	43.54	1800.54	9166.43	9198.09	5.78	56.50	1800.34
JLAV30B	9	9771.2	9436.26	9152.17	6.34	42.44	1800.52	9166.43	9168.73	6.17	56.70	1800.49
JLAV30B	10	9539.6	9436.26	NI	NI	42.50	1801.34	9166.43	9193.80	3.62	53.40	1800.73
JLAV30C	4	14594	7371.82	13775.99	5.61	42.03	1800.37	7147.52	INF	INF	54.03	3.94
JLAV30C	5	14703.2	7371.82	13848.13	5.82	41.47	1800.40	7147.52	INF	INF	61.60	3.99
JLAV30C	6	14343.3	7371.82	13791.66	3.85	41.71	1800.41	7147.52	INF	INF	60.78	10.15
JLAV30C	7	13869.3	7371.82	13818.99	0.36	41.31	1800.46	7147.52	INF	INF	57.05	10.95
JLAV30C	8	14564.7	7371.82	13772.97	5.44	42.40	1800.42	7147.52	INF	INF	59.69	4.67
JLAV30C	9	14135.1	7371.82	13718.34	2.95	42.01	1800.44	7147.52	INF	INF	63.22	4.85
JLAV30C	10	14052.6	7371.82	14284.83	-1.65	46.80	1800.96	7147.52	INF	INF	59.10	4.61
JLAV30D	4	10284.8	7704.64	10050.82	2.28	39.08	1800.38	7468.93	9947.30	3.28	52.62	1800.31
JLAV30D	5	10265.4	7704.64	NI	NI	43.88	1800.86	7468.93	9979.67	2.78	59.92	1800.36
JLAV30D	6	10004	7704.64	10201.30	-1.97	41.33	1800.87	7468.93	NI	NI	58.36	1800.86
JLAV30D	7	10063.3	7704.64	NI	NI	42.91	1800.88	7468.93	NI	NI	57.34	1800.76
JLAV30D	8	10089.2	7704.64	10245.97	-1.55	42.60	1800.77	7468.93	9561.63	5.23	59.46	1800.52
JLAV30D	9	10154.6	7704.64	10033.07	1.20	41.29	1800.66	7468.93	9318.09	8.24	59.54	1800.38
JLAV30D	10	10216.5	7704.64	9845.20	3.63	41.99	1801.12	7468.93	9475.32	7.25	59.57	1800.50
JLAV30E	4	11837.3	10714.68	11748.10	0.75	43.98	1801.12	10372.03	11651.08	1.57	56.26	1801.09
JLAV30E	5	11650.3	10714.68	11096.28	4.76	44.12	1800.41	10372.03	10746.96	7.75	59.53	1800.42
JLAV30E	6	11722.3	10714.68	NI	NI	41.54	1801.09	10372.03	10596.79	9.60	57.93	1800.37
JLAV30E	7	11802	10714.68	10817.75	8.34	42.62	1800.60	10372.03	10659.36	9.68	59.82	1800.56
JLAV30E	8	11739.1	10714.68	NI	NI	42.49	1801.07	10372.03	10836.81	7.69	57.45	1800.64
JLAV30E	9	11632.3	10714.68	NI	NI	41.54	1801.14	10372.03	10817.82	7.00	52.84	1800.70
JLAV30E	10	11846	10714.68	NI	NI	47.64	1801.03	10372.03	10166.38	14.18	59.23	1800.55

Table A.3: Tabu Search results for column-based approach.

$n^{\max}$			6					7				
Problem	β	ANVI Obj.	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}
JLAV30A	4	8770.2	6506.52	8356.68	4.72	351.36	1800.30	6325.03	INF	INF	438.50	4.37
JLAV30A	5	8577.4	6506.52	8364.17	2.49	350.00	1800.31	6325.03	INF	INF	422.93	4.13
JLAV30A	6	8733.6	6506.52	8494.45	2.74	353.63	1800.43	6325.03	8391.39	3.92	427.04	1800.38
JLAV30A	7	8471.7	6506.52	8300.18	2.02	359.17	1800.36	6325.03	8329.44	1.68	437.37	1800.40
JLAV30A	8	8401.4	6506.52	8581.83	-2.15	354.93	1800.33	6325.03	8393.89	0.09	432.94	1800.41
JLAV30A	9	8510.1	6506.52	8392.25	1.38	347.94	1800.36	6325.03	8327.60	2.14	427.77	1800.44
JLAV30A	10	8699.8	6506.52	8397.44	3.48	350.96	1800.38	6325.03	8234.29	5.35	430.38	1800.33
JLAV30B	4	9843.9	9501.88	9560.43	2.88	354.37	1800.36	9176.42	9138.69	7.16	434.48	1800.43
JLAV30B	5	9930.5	9501.88	9442.71	4.91	346.37	1800.34	9176.42	9021.57	9.15	436.56	1800.36
JLAV30B	6	9671	9501.88	9399.13	2.81	353.91	1800.41	9176.42	9060.04	6.32	427.34	1800.38
JLAV30B	7	9768.1	9501.88	9385.12	3.92	353.43	1800.47	9176.42	9017.66	7.68	433.38	1800.38
JLAV30B	8	9762.5	9501.88	9382.87	3.89	348.09	1800.60	9176.42	9010.12	7.71	431.87	1800.42
JLAV30B	9	9771.2	9501.88	NI	NI	346.68	1801.14	9176.42	8966.99	8.23	424.26	1800.58
JLAV30B	10	9539.6	9501.88	9256.07	2.97	345.57	1800.83	9176.42	8817.01	7.57	425.29	1800.56
JLAV30C	4	14594	7371.82	13798.74	5.45	352.54	1800.46	7176.71	INF	INF	428.36	4.07
JLAV30C	5	14703.2	7371.82	13791.66	6.20	351.80	1800.37	7176.71	INF	INF	428.94	4.02
JLAV30C	6	14343.3	7371.82	13830.06	3.58	350.31	1800.36	7176.71	13707.00	4.44	436.51	1800.37
JLAV30C	7	13869.3	7371.82	13823.36	0.33	357.66	1800.46	7176.71	13714.31	1.12	433.83	1800.36
JLAV30C	8	14564.7	7371.82	13817.13	5.13	347.51	1800.45	7176.71	13951.96	4.21	431.98	1800.42
JLAV30C	9	14135.1	7371.82	13800.96	2.36	357.47	1800.46	7176.71	13625.85	3.60	469.16	1800.40
JLAV30C	10	14052.6	7371.82	NI	NI	351.18	1801.01	7176.71	13625.85	3.04	436.39	1800.40
JLAV30D	4	10284.8	7733.71	10000.72	2.76	348.70	1800.45	7487.86	9824.94	4.47	421.32	1800.29
JLAV30D	5	10265.4	7733.71	9912.38	3.44	345.96	1800.71	7487.86	9868.30	3.87	436.58	1800.29
JLAV30D	6	10004	7733.71	10105.79	-1.02	349.98	1800.50	7487.86	9850.42	1.54	428.65	1800.43
JLAV30D	7	10063.3	7733.71	NI	NI	355.96	1800.82	7487.86	9634.92	4.26	431.00	1800.57
JLAV30D	8	10089.2	7733.71	10139.25	-0.50	353.64	1800.93	7487.86	NI	NI	431.32	1800.90
JLAV30D	9	10154.6	7733.71	NI	NI	346.39	1800.98	7487.86	9650.63	4.96	430.68	1800.52
JLAV30D	10	10216.5	7733.71	NI	NI	358.96	1800.99	7487.86	9459.82	7.41	427.71	1800.45
JLAV30E	4	11837.3	10771.75	11395.28	3.73	361.99	1800.92	10397.68	11104.30	6.19	438.07	1800.71
JLAV30E	5	11650.3	10771.75	11006.50	5.53	348.62	1800.46	10397.68	10916.27	6.30	419.39	1800.57
JLAV30E	6	11722.3	10771.75	11178.82	4.64	343.31	1801.35	10397.68	10682.30	8.87	425.83	1800.53
JLAV30E	7	11802	10771.75	10914.44	7.52	347.37	1800.86	10397.68	10534.71	10.74	425.46	1800.61
JLAV30E	8	11739.1	10771.75	NI	NI	345.22	1801.12	10397.68	10852.30	7.55	416.27	1800.48
JLAV30E	9	11632.3	10771.75	10825.75	6.93	359.23	1801.03	10397.68	10788.82	7.25	450.60	1800.98
JLAV30E	10	11846	10771.75	11170.93	5.70	359.18	1801.09	10397.68	10651.92	10.08	423.33	1800.89

A.2 Detailed Grid-Based Results

Table A.4: Genetic Algorithm results for grid-based approach.

n^{rows}			2					3					3				
n^{cols}			4					3					4				
Prob.	β	ANVI Obj.	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}
30A	4	8770.2	14612.8	INF	INF	276	112	9981.3	INF	INF	231	231	11802.9	NI	NI	342	1800
30A	5	8577.4	14612.8	INF	INF	286	202	9981.3	INF	INF	249	1202	11802.9	NI	NI	326	1800
30A	6	8733.6	14612.8	9503.3	-8.81	274	1158	9981.3	NI	NI	214	1800	11802.9	NI	NI	336	1800
30A	7	8471.7	14612.8	9503.3	-12.18	267	1520	9981.3	NI	NI	220	1800	11802.9	NI	NI	331	1800
30A	8	8401.4	14612.8	9503.3	-13.12	276	1800	9981.3	NI	NI	243	1800	11802.9	NI	NI	324	1800
30A	9	8510.1	14612.8	9531.6	-12.00	275	1800	9981.3	NI	NI	244	1800	11802.9	NI	NI	329	1800
30A	10	8699.8	14612.8	9503.3	-9.24	289	1800	9981.3	NI	NI	242	1800	11802.9	NI	NI	332	1800
30B	4	9843.9	15667.5	INF	INF	256	243	10348.8	INF	INF	336	43	12651.6	NI	NI	274	1800
30B	5	9930.5	15667.5	INF	INF	252	665	10348.8	INF	INF	336	120	12651.6	NI	NI	276	1800
30B	6	9671.0	15667.5	9814.7	-1.49	250	1800	10348.8	INF	INF	338	291	12651.6	NI	NI	270	1800
30B	7	9768.1	15667.5	9684.4	0.86	242	1800	10348.8	INF	INF	335	478	12651.6	NI	NI	289	1800
30B	8	9762.5	15667.5	9545.1	2.23	246	1800	10348.8	9900.8	-1.42	340	1800	12651.6	NI	NI	281	1800
30B	9	9771.2	15667.5	9569.1	2.07	260	1800	10348.8	9900.8	-1.33	336	1800	12651.6	NI	NI	271	1800
30B	10	9539.6	15667.5	9064.3	4.98	248	1800	10348.8	9858.5	-3.34	338	1800	12651.6	NI	NI	283	1800
30C	4	14594.0	21781.6	INF	INF	238	4	15089.5	INF	INF	206	72	17509.3	NI	NI	257	1800
30C	5	14703.2	21781.6	INF	INF	225	35	15089.5	13889.2	5.54	205	383	17509.3	NI	NI	257	1800
30C	6	14343.3	21781.6	14943.0	-4.18	225	69	15089.5	13852.9	3.42	211	1800	17509.3	NI	NI	270	1800
30C	7	13869.3	21781.6	14404.2	-3.86	235	156	15089.5	13802.3	0.48	213	1800	17509.3	14434.0	-4.07	260	1800
30C	8	14564.7	21781.6	14404.2	1.10	219	241	15089.5	13765.7	5.49	208	1800	17509.3	NI	NI	327	1800
30C	9	14135.1	21781.6	14404.2	-1.90	219	414	15089.5	13765.7	2.61	215	1800	17509.3	NI	NI	255	1800
30C	10	14052.6	21781.6	14404.2	-2.50	229	454	15089.5	13787.8	1.88	206	1800	17509.3	NI	NI	258	1800
30D	4	10284.8	16757.2	10675.8	-3.80	291	292	11640.8	10185.0	0.97	231	1800	13784.2	10377.6	-0.90	247	1800
30D	5	10265.4	16757.2	10394.7	-1.26	252	1302	11640.8	10143.1	1.19	231	1800	13784.2	10305.0	-0.39	254	1800
30D	6	10004.0	16757.2	10268.6	-2.64	289	1800	11640.8	10031.3	-0.27	227	1800	13784.2	10247.4	-2.43	280	1800
30D	7	10063.3	16757.2	10184.4	-1.20	292	1800	11640.8	9989.2	0.74	254	1800	13784.2	10575.8	-5.09	310	1801
30D	8	10089.2	16757.2	9586.4	4.98	326	1800	11640.8	9988.7	1.00	245	1800	13784.2	10151.7	-0.62	266	1800
30D	9	10154.6	16757.2	9486.9	6.58	297	1800	11640.8	10082.9	0.71	255	1800	13784.2	9932.3	2.19	240	1800
30D	10	10216.5	16757.2	9433.1	7.67	258	1800	11640.8	9722.7	4.83	230	1800	13784.2	9960.4	2.51	272	1800
30E	4	11837.3	18759.1	11554.4	2.39	233	1387	12912.5	11909.6	-0.61	220	1198	15235.6	11464.8	3.15	301	1800
30E	5	11650.3	18759.1	11338.6	2.68	236	1800	12912.5	11735.7	-0.73	215	1800	15235.6	11428.2	1.91	274	1800
30E	6	11722.3	18759.1	11260.1	3.94	246	1800	12912.5	11487.4	2.00	211	1800	15235.6	11386.4	2.87	266	1800
30E	7	11802.0	18759.1	11191.9	5.17	229	1800	12912.5	11429.4	3.16	213	1800	15235.6	12126.0	-2.74	270	1800
30E	8	11739.1	18759.1	11135.8	5.14	255	1800	12912.5	11394.8	2.93	222	1800	15235.6	11316.2	3.60	276	1800
30E	9	11632.3	18759.1	11112.1	4.47	247	1800	12912.5	11390.0	2.08	220	1800	15235.6	11456.9	1.51	265	1800
30E	10	11846.0	18759.1	10915.4	7.86	245	1800	12912.5	11254.7	4.99	222	1800	15235.6	12047.3	-1.70	269	1800

Table A.5: Simulated Annealing results for grid-based approach.

n^{rows}			2					3					3				
n^{cols}			4					3					4				
Prob.	β	ANVI Obj.	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}
30A	4	8770.2	14089.9	INF	INF	526	11	9604.0	INF	INF	538	1009	11293.4	NI	NI	575	1800
30A	5	8577.4	14089.9	INF	INF	562	48	9604.0	NI	NI	551	1800	11293.4	NI	NI	551	1800
30A	6	8733.6	14089.9	INF	INF	506	138	9604.0	9649.5	-10.49	510	1800	11293.4	NI	NI	550	1800
30A	7	8471.7	14089.9	9424.5	-11.25	493	780	9604.0	NI	NI	555	1800	11293.4	NI	NI	546	1800
30A	8	8401.4	14089.9	9424.4	-12.18	545	1393	9604.0	NI	NI	490	1800	11293.4	NI	NI	580	1800
30A	9	8510.1	14089.9	9424.4	-10.74	524	1800	9604.0	8991.0	-5.65	531	1800	11293.4	NI	NI	559	1800
30A	10	8699.8	14089.9	9424.5	-8.33	551	1800	9604.0	8979.5	-3.22	489	1800	11293.4	NI	NI	572	1800
30B	4	9843.9	14890.1	10036.7	-1.96	535	636	9971.7	INF	INF	548	35	12000.0	NI	NI	537	1800
30B	5	9930.5	14890.1	9872.4	0.59	546	1800	9971.7	INF	INF	551	127	12000.0	NI	NI	570	1800
30B	6	9671.0	14890.1	9872.4	-2.08	558	1800	9971.7	INF	INF	527	421	12000.0	NI	NI	572	1800
30B	7	9768.1	14890.1	9841.7	-0.75	528	1800	9971.7	INF	INF	539	712	12000.0	9577.6	1.95	568	1800
30B	8	9762.5	14890.1	9355.9	4.16	532	1800	9971.7	10025.2	-2.69	509	1800	12000.0	NI	NI	518	1800
30B	9	9771.2	14890.1	9080.5	7.07	543	1800	9971.7	9745.1	0.27	546	1800	12000.0	NI	NI	520	1800
30B	10	9539.6	14890.1	8981.0	5.86	540	1800	9971.7	9616.0	-0.80	547	1800	12000.0	NI	NI	678	1800
30C	4	14594.0	21067.5	INF	INF	502	2	14304.2	INF	INF	544	39	16575.2	NI	NI	581	1800
30C	5	14703.2	21067.5	INF	INF	550	21	14304.2	INF	INF	521	222	16575.2	14053.8		582	1800
30C	6	14343.3	21067.5	INF	INF	541	20	14304.2	14079.8	1.84	528	1800	16575.2	13985.0	2.50	514	1800
30C	7	13869.3	21067.5	INF	INF	517	69	14304.2	13901.9	-0.24	516	1800	16575.2	13911.2	-0.30	593	1800
30C	8	14564.7	21067.5	INF	INF	495	94	14304.2	13901.9	4.55	510	1800	16575.2	13904.7	4.53	573	1800
30C	9	14135.1	21067.5	14362.5	-1.61	558	365	14304.2	13869.9	1.88	521	1800	16575.2	NI	NI	572	1800
30C	10	14052.6	21067.5	14362.5	-2.21	557	312	14304.2	13785.3	1.90	556	1800	16575.2	NI	NI	538	1800
30D	4	10284.8	16012.8	10524.3	-2.33	544	262	11133.0	10288.0	-0.03	548	1800	13053.0	10003.3	2.74	535	1800
30D	5	10265.4	16012.8	10211.5	0.53	538	750	11133.0	9955.0	3.02	556	1800	13053.0	9802.1	4.51	505	1800
30D	6	10004.0	16012.8	10054.2	-0.50	544	1800	11133.0	9866.2	1.38	489	1800	13053.0	9925.1	0.79	569	1800
30D	7	10063.3	16012.8	9652.8	4.08	547	1800	11133.0	9804.7	2.57	603	1800	13053.0	NI	NI	610	1800
30D	8	10089.2	16012.8	9453.6	6.30	580	1800	11133.0	9697.6	3.88	606	1800	13053.0	9528.1	5.56	611	1800
30D	9	10154.6	16012.8	9376.4	7.66	495	1800	11133.0	9650.0	4.97	559	1800	13053.0	9472.4	6.72	508	1800
30D	10	10216.5	16012.8	9333.0	8.65	538	1800	11133.0	9477.6	7.23	536	1800	13053.0	9580.4	6.23	537	1800
30E	4	11837.3	18043.6	11262.4	4.86	550	1639	12192.9	11694.6	1.21	489	1757	14713.6	11067.2	6.51	520	1800
30E	5	11650.3	18043.6	10719.8	7.99	539	1800	12192.9	11503.0	1.26	502	1800	14713.6	10915.0	6.31	573	1801
30E	6	11722.3	18043.6	10459.5	10.77	553	1800	12192.9	11131.0	5.04	532	1800	14713.6	10822.7	7.67	536	1800
30E	7	11802.0	18043.6	10394.7	11.92	555	1800	12192.9	10978.5	6.98	581	1800	14713.6	10744.9	8.96	537	1800
30E	8	11739.1	18043.6	10379.9	11.58	515	1800	12192.9	10948.0	6.74	548	1800	14713.6	10573.1	9.93	576	1800
30E	9	11632.3	18043.6	10297.8	11.47	534	1800	12192.9	10931.4	6.03	530	1800	14713.6	10462.6	10.06	579	1800
30E	10	11846.0	18043.6	10337.2	12.74	503	1800	12192.9	10855.8	8.36	496	1800	14713.6	10382.5	12.35	565	1800

Table A.6: Tabu Search results for grid-based approach.

n^{rows}			2					3					3				
n^{cols}			4					3					4				
Prob.	β	ANVI Obj.	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}	S Obj.	Obj.	% Imp.	T_{search}	T_{solve}
30A	4	8770.2	14272.4	INF	INF	1458	17	9684.5	INF	INF	1575	1342	11304.1	NI	NI	1684	1800
30A	5	8577.4	14272.4	INF	INF	1580	50	9684.5	NI	NI	1503	1800	11304.1	NI	NI	1730	1800
30A	6	8733.6	14272.4	INF	INF	1560	128	9684.5	NI	NI	1530	1800	11304.1	NI	NI	1596	1800
30A	7	8471.7	14272.4	INF	INF	1790	381	9684.5	NI	NI	1569	1800	11304.1	NI	NI	1533	1800
30A	8	8401.4	14272.4	9780.2	-16.41	1530	842	9684.5	NI	NI	1506	1800	11304.1	9787.6	-16.50	1540	1800
30A	9	8510.1	14272.4	9780.2	-14.92	1540	1738	9684.5	9075.9	-6.65	1471	1800	11304.1	9080.1	-6.70	1684	1800
30A	10	8699.8	14272.4	9713.8	-11.66	1586	1800	9684.5	NI	NI	1534	1800	11304.1	NI	NI	1815	1800
30B	4	9843.9	14940.6	10046.6	-2.06	1585	562	10030.3	INF	INF	1441	58	12173.6	NI	NI	1717	1800
30B	5	9930.5	14940.6	9872.0	0.59	1539	1800	10030.3	INF	INF	1578	187	12173.6	NI	NI	1594	1800
30B	6	9671.0	14940.6	9892.7	-2.29	1598	1800	10030.3	INF	INF	1541	378	12173.6	NI	NI	1658	1800
30B	7	9768.1	14940.6	9925.4	-1.61	1551	1800	10030.3	10634.1	-8.87	1436	1466	12173.6	NI	NI	1771	1800
30B	8	9762.5	14940.6	9747.3	0.16	1549	1800	10030.3	10634.1	-8.93	1593	1800	12173.6	NI	NI	1739	1800
30B	9	9771.2	14940.6	9422.2	3.57	1523	1800	10030.3	9986.7	-2.21	1474	1800	12173.6	9641.8	1.32	1743	1800
30B	10	9539.6	14940.6	9085.6	4.76	1506	1800	10030.3	9887.9	-3.65	1592	1800	12173.6	NI	NI	1582	1800
30C	4	14594.0	21174.0	INF	INF	1475	2	14408.8	INF	INF	1410	2	16678.1	14382.9	1.45	1666	1800
30C	5	14703.2	21174.0	INF	INF	1498	18	14408.8	INF	INF	1497	207	16678.1	14292.6	2.79	1711	1800
30C	6	14343.3	21174.0	INF	INF	1685	37	14408.8	13687.4	4.57	1498	1232	16678.1	14139.2	1.42	1742	1800
30C	7	13869.3	21174.0	INF	INF	1440	72	14408.8	13663.7	1.48	1411	1800	16678.1	NI	NI	1729	1800
30C	8	14564.7	21174.0	14179.7	2.64	1570	268	14408.8	13663.7	6.19	1508	1800	16678.1	NI	NI	1747	1800
30C	9	14135.1	21174.0	14179.7	-0.32	1604	163	14408.8	13636.5	3.53	1525	1800	16678.1	NI	NI	1676	1800
30C	10	14052.6	21174.0	14179.7	-0.90	1494	297	14408.8	13461.2	4.21	1528	1800	16678.1	NI	NI	1662	1800
30D	4	10284.8	16339.8	10937.4	-6.34	1564	401	11177.8	10545.1	-2.53	1471	1800	13259.3	10195.0	0.87	1694	1800
30D	5	10265.4	16339.8	10473.4	-2.03	1539	725	11177.8	10129.7	1.32	1517	1800	13259.3	9925.1	3.32	1680	1800
30D	6	10004.0	16339.8	10339.1	-3.35	1488	1484	11177.8	10056.4	-0.52	1572	1800	13259.3	9776.5	2.27	1636	1800
30D	7	10063.3	16339.8	10120.8	-0.57	1746	1800	11177.8	9965.2	0.97	1686	1800	13259.3	9751.2	3.10	1845	1800
30D	8	10089.2	16339.8	9491.1	5.93	1660	1800	11177.8	9796.6	2.90	1713	1800	13259.3	9637.4	4.48	1852	1800
30D	9	10154.6	16339.8	9382.9	7.60	1557	1800	11177.8	9721.8	4.26	1541	1800	13259.3	9659.3	4.88	1597	1800
30D	10	10216.5	16339.8	9326.7	8.71	1504	1800	11177.8	9656.7	5.48	1532	1800	13259.3	9538.7	6.63	1647	1800
30E	4	11837.3	18082.8	11409.7	3.61	1481	1800	12386.7	11480.2	3.02	1438	445	14696.1	11305.7	4.49	1561	1800
30E	5	11650.3	18082.8	10652.2	8.57	1451	1800	12386.7	11354.8	2.54	1445	1169	14696.1	11238.9	3.53	1701	1800
30E	6	11722.3	18082.8	10486.4	10.54	1545	1800	12386.7	11006.8	6.10	1572	1628	14696.1	11136.4	5.00	1713	1800
30E	7	11802.0	18082.8	10439.6	11.54	1459	1800	12386.7	10879.8	7.81	1502	1800	14696.1	11023.7	6.59	1695	1800
30E	8	11739.1	18082.8	10393.9	11.46	1594	1800	12386.7	10843.3	7.63	1569	1800	14696.1	10952.0	6.70	1676	1800
30E	9	11632.3	18082.8	10306.9	11.39	1579	1800	12386.7	10841.1	6.80	1515	1800	14696.1	NI	NI	1721	1800
30E	10	11846.0	18082.8	10265.5	13.34	1517	1800	12386.7	10751.6	9.24	1557	1800	14696.1	10635.3	10.22	1635	1800

Glossary

Block Layout A two-dimensional layout which specifies the location and dimensions of the departments within a facility. 8, 10, 11

Deadspace Area within a facility or super-department which is not occupied by a department; wasted area. iii, 6, 13, 19, 20, 28

Detailed Layout A two-dimensional layout which specifies the location of equipment and machinery within each department. 8, 11

Hierarchical Approach A solution approach which clusters the departments into a hierarchy, solves subproblems from the hierarchy, and combines the subproblems into a solution for the original problem. iii, ix, 1, 2, 6, 7, 12, 16–20, 28–30, 34, 52, 53

Metaheuristic A general structure for the design of heuristics which applies across a variety of specific applications. vi, 2, 6, 9, 13, 29, 37, 38, 41, 43, 48, 49, 53

Nesting Approach A solution approach which encapsulates several departments into super-departments and performs the layout of both simultaneously. iii, 2, 6, 33

Super-Department A group of departments aggregated into a single entity for modeling purposes iii, 2, 6, 19, 22, 24, 25, 29–31, 33, 34, 36–38, 40, 46–49, 52, 53