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Master's Thesis

Pricing and Revenue Sharing between ISPs under Content Sharing

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Pricing and Revenue Sharing between ISPs under Content Sharing

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Abstract

As sponsored data with subsidized access cost gains popularity in industry, it is essential to understand its impact on the Internet service market. We investigate the interplay among Internet Service Providers (ISPs), Content Provider (CP) and End User (EU), where each player is selfish and wants to maximize its own profit. In particular, we consider multi-ISP scenarios, in which the network connectivity between the CP and the EU is jointly provided by multiple ISPs. We first model non-cooperative interaction between the players as a four-stage Stackelberg game, and derive the optimal behaviors of each player in equilibrium. Taking into account the transit price at intermediate ISP, we provide in-depth understanding on the sponsoring strategies of CP. We then study the effect of cooperation between the ISPs to the pricing structure and the traffic demand, and analyze their implications to the players. We further build our revenue sharing model based on Shapley value mechanism, and show that the collaboration of the ISPs can improve their total payoff with a higher social welfare.

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I Introduction

As demand for mobile data increases, Internet service providers (ISPs) are turning to new types of smart data pricing to bring in additional revenue and to expand the capacity of their current network [1]. One way to keep up funding such investment is content sponsorship. Content providers (CPs) split the cost of transferring mobile data traffic, and sponsor the user's access to the content by making direct payment to the ISPs. For example, GS Shop, a Korea TV home shopping company, has partnered with SK Telecom to sponsor data incurred from its application, so consumers are incentivized to continue browsing and making purchases from their mobile devices without ringing up data charges [2]. Content sponsoring may benefit all players in the market: the ISPs can generate more revenue with CP's subsidies, and users can enjoy free or low-cost access to certain services, which in turn increases the demand and attracts more traffic, resulting in higher revenue of the CP.

There are several studies on content sponsoring despite of a short history. Most of the works either focus on a simple model with a single ISP and a single CP interacting in a game theoretic setting, or considers Quality-of-Service (QoS) prioritization and its implications for net neutrality [3, 4, 5, 6]. In a two-sided market with a single ISP providing connection between CPs and EUs, profit maximization of the players under sponsoring mobile data has been studied in [7, 8]. In [7], single monopolistic ISP determines optimal price to charge the CPs and the EUs, while the authors in [8] study the contractual relationship between the CPs and the ISP under a similar model. Nevertheless, none of them consider the interaction between multiple ISPs. Although the authors in [9] proposes a model with a transit ISP and a user-facing ISP, their understanding of the interaction between these non-cooperative ISPs are limited to the environments without content sponsoring. Other works, e.g. [10, 11], have analyzed content sponsorship from the economic point of view. They examine the implications of sponsored data on the CPs and the EUs, and identify how sponsored data influence the CP inequality.

In many Internet markets, there are multiple ISPs that cooperate to provide end-to-end connectivity service between the CPs and the EUs, in which case the assumption of a single representative ISP no longer holds. Since each ISP aims to maximize its own profit, the establishment of interconnection among multiple ISPs is a thorough process that depends on specific profit sharing/inter-charging arrangements.

As the most commercial traffic originates from the CPs and terminates at the EUs, some ISPs positioned on the middle of the traffic delivery chain will have more power and request a transit-price. An ISP serving a large population of users might have a dominant influence in determining the transit price paid by other relatively weak ISPs for traffic delivery. For an example, a large entertainment company Netflix directly uses the service provided by ISPs such as Level 3, which is connected with residential broadband ISPs like Comcast to get access to the customers [12]. Level 3 charges Netflix and Comcast charges the users. Netflix may partially or fully sponsor its traffic, which is likely to increase the amount of traffic through both ISPs. Due to high traffic volume, the access ISP (Comcast) may require additional transit price for traffic delivery, which will impact on the pricing decision at Level 3 and subsequently on the sponsoring decision at Netflix. In this work, we are interested in the dynamics between the play-

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ers with focus on content sponsoring and transit pricing. To this end, we study the interplay among two Internet Service Providers (ISPs), Content Provider (CP), and End User (EU), where each player selfishly maximizes its own profit. We model this non-cooperative interaction between ISP_1, ISP_2, CP, and EU as a four-stage Stackelberg game. Specifically, in our model, we assume that the EU-facing ISP has a dominant power and can be considered as the game leader who decides the transit cost preceding the choice of the follower ISP. We aim to understand the behaviors of the players in non-cooperative equilibrium and their decisions to maximize their own utility. Also we investigate the responses of the players when the ISPs cooperate with each other. We show that, under collaboration with appropriate revenue sharing, each ISP can achieve a higher revenue while improving the social welfare.

The rest of the paper is organized as follows. We present the basic system model in Section II, and investigate the strategies of the CP, the EU, and the ISPs to maximize their utility in Section III. We also study the effect of collaboration and build our revenue sharing model based on Shapley value mechanism in Section IV and V, respectively. Numerical results are presented in Section VI, followed by the conclusion and future work in Section VII.

II Two-ISP Pricing Model

We consider an Internet market model with one CP and two ISPs as shown in Figure 1. Two interconnected ISPs have their own cost structures and each provides connectivity to either the CP or the EU. The CP-facing ISP (ISP_1) obtains its profits by directly charging the CP (CP) by p_{cp} per unit traffic while the EU-facing ISP (ISP_2) charges the EU (EU) by p_{eu} per unit traffic. Further ISP_2 charges ISP_1 with transit-price p_{tr} for traffic delivery. CP can sponsor the cost of EU by $s \cdot p_{eu}$ per unit traffic with $s \in [0,1]$. We assume that the sponsored amount is paid to ISP_1 and then indirectly delivered to ISP_2 through the transit price, which allows both ISPs to benefit from the sponsoring. Let m_1 and m_2 denote the marginal costs of traffic delivery for ISP_1 and ISP_2, respectively. We denote x as the traffic amount of flow between CP and EU.

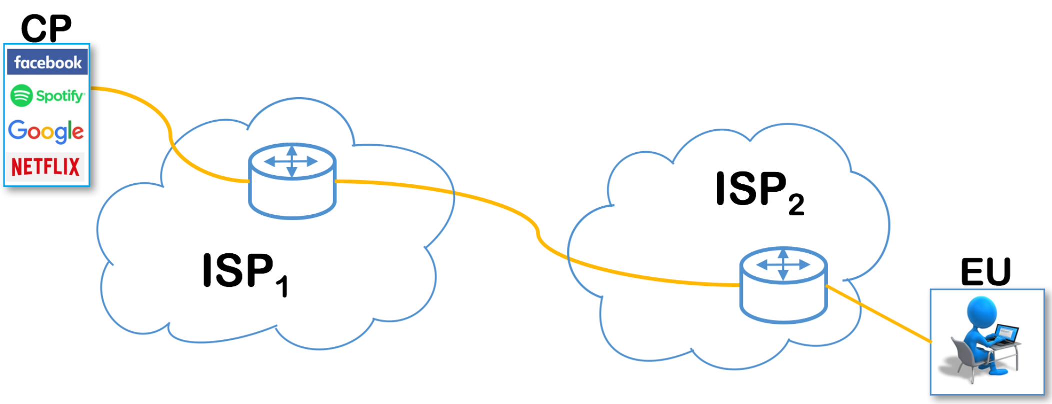

Figure 1: Two-sided Internet market.

We assume that the players in this non-cooperative game make decisions in four stages as follows:

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1. ISP_2 sets prices p_{eu} and p_{tr} to charge EU and ISP_1, respectively.

2. ISP_1 determines the optimal value of p_{cp} to charge CP.

3. CP decides how much content to sponsor, i.e., the value of s.

4. The traffic volume is decided by both EU and CP.

Each player selfishly maximizes its own profit subject to the others' decisions. We model this non-cooperative interaction as a four-stage Stackelberg game and use the backward induction method to find optimal strategy of each player.

Let us define the utility of EU by the multiplication of a scaling factor $\sigma_{eu} \geq 0$ and a utility-level function. The utility represents user's desire to obtain traffic. We assume a concave and non-decreasing function $u_{eu}(x)$ with decreasing marginal satisfaction, i.e., $u_{eu}(x) = \frac{x^{1-\alpha_{eu}}}{1-\alpha_{eu}}$ with parameter $\alpha_{eu} \in (0,1)$. Given unit price p_{eu} that ISP_2 charges user, EU will maximize its utility minus the payment by solving

$$\begin{aligned} (\boldsymbol{EU-P}) \quad & \max_{x} \quad \sigma_{eu} \cdot u_{eu}(x) - (1-s) \cdot x \cdot p_{eu}, \\ & s.t. \quad x \geq 0, \end{aligned} \tag{1}$$

where $s \in [0,1]$ denotes the sponsored percentage, and $(1-s) \cdot x \cdot p_{eu}$ denotes the payment of EU to ISP_2. The solution x^*_{eu} to (1) can be obtained as $x^*_{eu}(s, p_{eu}) = (\frac{\sigma_{eu}}{(1-s)p_{eu}})^{\frac{1}{\alpha_{eu}}}$.

Similarly, we model the behavior of CP. The utility of CP is given by $\sigma_{cp} u_{cp}(x)$, where $\sigma_{cp} \geq 0$ is a scaling factor (e.g., the popularity of the content) and $u_{cp}(x)$ is a concave utility-level function $u_{cp}(x) = \frac{x^{1-\alpha_{cp}}}{1-\alpha_{cp}}$ with parameter $\alpha_{cp} \in (0,1)$. CP will maximize its payoff by solving

$$\begin{aligned} (\boldsymbol{CP-P}) \quad & \max_{x,s} \quad \sigma_{cp} \cdot u_{cp}(x) - s \cdot x \cdot p_{eu} - x \cdot p_{cp}, \\ & s.t. \quad x \geq 0 \quad and \quad 0 \leq s \leq 1. \end{aligned} \tag{2}$$

In the objective, the first term denotes its utility, the second term denotes the cost due to sponsorship, and the third term is from the network usage cost to ISP_1. Given s, p_{cp}, and p_{eu}, it can be easily shown that the optimal amount of traffic for CP is $x^*_{cp}(s, p_{cp}, p_{eu}) = (\frac{\sigma_{cp}}{s p_{eu} + p_{cp}})^{\frac{1}{\alpha_{cp}}}$.

Since ISP_1 obtains its revenue from charging CP, it decides the optimal value of p_{cp} to maximize its total profit as

$$\begin{aligned} (\boldsymbol{ISP1-P}) \quad & \max_{p_{cp}} \quad (p_{cp} + s^* \cdot p_{eu} - p_{tr} - m_1) \cdot x^*(p_{cp}, p_{eu}), \\ & s.t. \qquad p_{cp} \geq 0, \end{aligned} \tag{3}$$

where m_1 is the marginal cost for traffic delivery and thus $p_{cp} + s^* \cdot p_{eu} - p_{tr} - m_1$ is the net-gain of ISP_1 per unit traffic.

ISP_2 obtains its revenue from charging ISP_1 with transit-price p_{tr} and charging EU with traffic-price p_{eu}. Therefore, in order to maximize its total profit, it will solve

$$\begin{aligned} (\boldsymbol{ISP2-P}) \quad & \max_{p_{eu}, p_{tr}} \quad ((1-s^*) \cdot p_{eu} + p_{tr} - m_2) \cdot x^*(p_{cp}, p_{eu}), \\ & s.t. \quad p_{eu} \geq 0 \quad and \quad p_{tr} \geq 0, \end{aligned} \tag{4}$$

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where m_2 is the marginal cost for traffic delivery.

Through the sequential decision, we investigate the interactions of the players described in (1), (2), (3), (4), and find the optimal strategies for pricing and sponsoring.

III Strategies for Utility Maximization

In this section, we sequentially find the optimal strategies of CP, ISP_1, and ISP_2 by exploiting the backward induction.

3.1 Sponsoring of Content Provider (CP)

Note that each solution to (1) and (2) results in user-side traffic demand x_{eu}^* and CP-side traffic amount x_{cp}^*, respectively, and the actual traffic amount x^* between CP and EU will be determined by their minimum, i.e., $x^* = min\{x_{cp}^*, x_{eu}^*\}$. In general $x_{eu}^* \neq x_{cp}^*$. For instance, a certain website may restrict the number of simultaneous on-line clients, which implies $x_{cp}^* \leq x_{eu}^*$.

Suppose that p_{eu} and p_{cp} are given. The actual traffic $x^*(s)$ will be determined by the sponsoring rate s, and CP will decide its optimal sponsored percentage s^* by solving the following problem:

$$\begin{aligned} (\boldsymbol{CP-P}) \quad & \max_{s} \ \sigma_{cp} \cdot u_{cp}(x^*(s)) - s \cdot x^*(s) \cdot p_{eu} - x^*(s) \cdot p_{cp}, \\ & s.t. \quad 0 \leq s \leq 1. \end{aligned} \tag{5}$$

We assume $\alpha_{eu} = \alpha_{cp} = \alpha \in (0,1)$, i.e., EU and CP utility components have the same utility shape. This assumption is reasonable in the scenarios where CP makes its pricing decision according to the user response. On the other hand, the scaling factors σ_{eu} and σ_{cp} of EU and CP can be quite different. The sponsoring behavior will be affected by whether the traffic volume is constrained by EU or CP. If $x_{eu}^* \leq x_{cp}^*$, we have $s \leq \frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}$ and $x^* = x_{eu}^*$. Similarly, if $x_{eu}^* \geq x_{cp}^*$, we have $s \geq max(\frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}, 0)$ and $x^* = x_{cp}^*$. We consider each case.

Case i) When $x^* = x_{cp}^*$. The profit of the CP can be written as

$$V(s) = \sigma_{cp} \cdot u_{cp}(x_{cp}^*(s)) - s \cdot x_{cp}^*(s) \cdot p_{eu} - x_{cp}^*(s) \cdot p_{cp}. \tag{6}$$

By substituting $x_{cp}^*(s, p_{cp}, p_{eu}) = (\frac{\sigma_{cp}}{s p_{eu} + p_{cp}})^{\frac{1}{\alpha}}$ into (6), it can be easily shown that $V(s)$ is a decreasing function of s, and we have the optimal value $s^* = max(\frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}, 0)$. Thus, the traffic amount and the sponsoring rate will be

$$(x_{cp}^*, s^*) = \begin{cases} ((\frac{\sigma_{cp}}{p_{cp}})^{\frac{1}{\alpha}}, \quad 0), & if \ \frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}, \\ ((\frac{\sigma_{cp}+\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}, \quad \frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}), & if \ \frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}. \end{cases} \tag{7}$$

The maximum profit of CP is given as

$$V^*(x_{cp}^*, s^*) = \begin{cases} \frac{\alpha (\sigma_{cp})^{\frac{1}{\alpha}}}{1-\alpha} (p_{cp})^{1-\frac{1}{\alpha}}, & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}, \\ \frac{\alpha \sigma_{cp}}{1-\alpha} (\frac{p_{eu}+p_{cp}}{\sigma_{eu}+\sigma_{cp}})^{1-\frac{1}{\alpha}}, & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}. \end{cases} \tag{8}$$

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Case ii) When $x^* = x^*_{eu}$. In this case, we have $s \leq \frac{\sigma_{cp}p_{eu}-\sigma_{eu}p_{cp}}{(\sigma_{eu}+\sigma_{cp})p_{eu}}$, $x^*_{eu}(s,p_{eu}) = (\frac{\sigma_{eu}}{(1-s)p_{eu}})^{\frac{1}{\alpha}}$ and $\frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}$. *CP* will optimize its sponsorship percentage by solving

$$\begin{aligned} max \quad & \frac{\sigma_{cp}(\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}-1}}{1-\alpha}(1-s)^{1-\frac{1}{\alpha}} - \frac{(sp_{eu}+p_{cp})(\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}}{(1-s)^{\frac{1}{\alpha}}}, \\ s.t. \quad & 0 \leq s \leq \frac{\sigma_{cp}p_{eu}-\sigma_{eu}p_{cp}}{(\sigma_{eu}+\sigma_{cp})p_{eu}}, \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}. \end{aligned} \tag{9}$$

From the first order condition, the optimal data rate x^* and the optimal sponsoring rate s^* can be obtained as

$$(x^*_{eu}, s^*) = \begin{cases} ((\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}, \quad 0), & if \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}, \\ ((\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}, \quad \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}), & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, \end{cases} \tag{10}$$

and the maximum profit of CP is

$$V^*(x^*_{eu}, s^*) = \begin{cases} (\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}[\frac{\sigma_{cp}p_{eu}}{(1-\alpha)\sigma_{eu}} - p_{cp}] & if \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}, \\ \\ \frac{\alpha(p_{cp}+p_{eu})}{1-\alpha}(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}} & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}. \end{cases} \tag{11}$$

To summarize, we have

(i) If $\frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}$,
then $(x^*, s^*) = (x^*_{cp}, 0)$ and $V^*(x^*_{cp}, s^*) = \frac{\alpha(\sigma_{cp})^{\frac{1}{\alpha}}}{1-\alpha}(p_{cp})^{1-\frac{1}{\alpha}}$.

(ii) If $\frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}$ and $x^* = x^*_{cp}$,
then $(x^*, s^*) = (x^*_{cp}, max(\frac{\sigma_{cp}p_{eu}-\sigma_{eu}p_{cp}}{(\sigma_{eu}+\sigma_{cp})p_{eu}}, 0))$ and $V^*(x^*_{cp}, s^*) = \frac{\alpha\sigma_{cp}}{1-\alpha}(\frac{p_{eu}+p_{cp}}{\sigma_{eu}+\sigma_{cp}})^{1-\frac{1}{\alpha}}$.

(iii) If $\frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}$, $x^* = x^*_{eu}$, and $\frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$,
then $(x^*, s^*) = (x^*_{eu}, 0)$ and $V^*(x^*_{eu}, s^*) = (\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}[\frac{\sigma_{cp}p_{eu}}{(1-\alpha)\sigma_{eu}} - p_{cp}]$.

(iv) If $\frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}$ and $x^* = x^*_{eu}$,
then $(x^*, s^*) = (x^*_{eu}, \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha})$ and $V^*(x^*_{eu}, s^*) = \frac{\alpha(p_{cp}+p_{eu})}{1-\alpha}(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}$.

From the two-case response of *CP*, we can obtain the following Proposition.

Proposition 1 *Given prices p_{cp} and p_{eu}, the optimal sponsorship rate s^* of the CP is*

$$\begin{aligned} &case\ 1) \quad if \quad \frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}, && s^* = 0, \\ &case\ 2) \quad if \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}, && s^* = 0, \\ &case\ 3) \quad if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, && s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}. \end{aligned} \tag{12}$$

Proof For case 1, the maximum available profit of CP can be easily obtained as $V^*(x^*_{cp}, s^*) = \frac{\alpha(\sigma_{cp})^{\frac{1}{\alpha}}}{1-\alpha}(p_{cp})^{1-\frac{1}{\alpha}}$ from (8).

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For $\frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}$, the CP will choose the largest one among available profits of $V^*(x^*_{cp}, s^*)$ and $V^*(x^*_{eu}, s^*)$, given in (8) and (11), respectively. Let $\sigma = \frac{\sigma_{cp}}{\sigma_{eu}}$ and $p = \frac{p_{cp}}{p_{eu}}$. We decompose it into two subcases as below.

1) When $p < \sigma \leq \alpha + p$, each profit function can be written as

$$V^*(x^*_{cp}, s^*) = \frac{(\sigma_{eu})^{\frac{1}{\alpha}}(p_{eu})^{1-\frac{1}{\alpha}}}{(1-\alpha)}\left(\frac{1+p}{1+\sigma}\right)\left(\frac{1+p}{1+\sigma}\right)^{-\frac{1}{\alpha}}\alpha\sigma,$$
$$V^*(x^*_{eu}, s^*) = \frac{(\sigma_{eu})^{\frac{1}{\alpha}}(p_{eu})^{1-\frac{1}{\alpha}}}{(1-\alpha)}(\sigma - (1-\alpha)p).$$

Consider the ratio $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)}$. By using the generalized form of Bernoulli's inequality $(1+x)^r \geq 1 + rx$ for $r \leq 0$ or $r \geq 1$ and $x > -1$, we can obtain

$$\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} \geq \left(\frac{\sigma-(1-\alpha)p}{\alpha\sigma}\right)\left(\frac{1+\sigma}{1+p}\right)\left(1 + \frac{p-\sigma}{(1+\sigma)\alpha}\right) = 1 + \frac{(1-\alpha)(\sigma-p)(p+\alpha-\sigma)}{\sigma\alpha^2(1+p)}.$$

Hence, if $p < \sigma \leq \alpha + p$, we have $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} \geq 1$, implying $x^* = x^*_{eu}$ and $s^* = 0$ from (10)

2) When $\sigma > \alpha + p$, we have

$$V^*(x^*_{cp}, s^*) = \left(\frac{\alpha}{1-\alpha}\right)(p_{eu} + p_{cp})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}(\sigma)(1+\sigma)^{\frac{1}{\alpha}-1},$$
$$V^*(x^*_{eu}, s^*) = \left(\frac{\alpha}{1-\alpha}\right)(p_{eu} + p_{cp})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}(1+\sigma-\alpha)^{\frac{1}{\alpha}}.$$

Again we consider the ratio $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} = \frac{1+\sigma}{\sigma}\left(1 - \frac{\alpha}{1+\sigma}\right)^{\frac{1}{\alpha}}$. Applying the generalized form of Bernoulli's inequality, we have $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} \geq \frac{1+\sigma}{\sigma}\left(1 - \frac{1}{1+\sigma}\right) = 1$, and thus we have $x^* = x^*_{eu}$ and $s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}} + 1 - \alpha}$ from (10).

According to Proposition 1, *CP* has no incentive to invest in sponsored data plan when $\frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$. On the other hand, when $\frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}$, *CP* will invest in sponsoring as in (10). The data rate under sponsoring will be

$$\begin{aligned}
&\textit{case 1) if} \quad \frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}, && x^*(p_{cp}, p_{eu}) = \left(\frac{\sigma_{cp}}{p_{cp}}\right)^{\frac{1}{\alpha}},\\
&\textit{case 2) if} \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}, && x^*(p_{cp}, p_{eu}) = \left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}},\\
&\textit{case 3) if} \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, && x^*(p_{cp}, p_{eu}) = \left(\frac{\sigma_{cp} + (1-\alpha)\sigma_{eu}}{p_{cp} + p_{eu}}\right)^{\frac{1}{\alpha}}.
\end{aligned} \tag{13}$$

3.2 Utility Maximization of ISP_1

ISP_1 also tries to maximize its total profit in each region specified in (13). We obtain the optimal response of ISP_1 in each case.

Case 1) When $x^* = \left(\frac{\sigma_{cp}}{p_{cp}}\right)^{\frac{1}{\alpha}}$ and $s^* = 0$. From (3), ISP_1 maximizes $(p_{cp} - p_{tr} - m_1) \cdot \left(\frac{\sigma_{cp}}{p_{cp}}\right)^{\frac{1}{\alpha}}$ subject to $\frac{\sigma_{cp}}{\sigma_{eu}} \cdot p_{eu} \leq p_{cp}$. The best response p^*_{cp} of ISP_1 can be easily obtained as $p^*_{cp} = \frac{p_{tr}+m_1}{1-\alpha}$. The maximum profit P^*_1 is

$$P^*_1 = \left[\frac{\alpha(m_1+m_2)}{(1-\alpha)} \cdot \frac{\sigma}{1+\sigma(1-\alpha)}\right] \cdot \left(\frac{\sigma_{cp}(1-\alpha)(1+\sigma(1-\alpha))}{\sigma(m_1+m_2)}\right)^{\frac{1}{\alpha}},$$

where $\sigma = \frac{\sigma_{cp}}{\sigma_{eu}}$.

Case 2) When $x^* = (\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}$ and $s^* = 0$. From (3), ISP_1 has the objective of $\max_{p_{cp}\geq 0} \quad (p_{cp} - p_{tr} - m_1)\cdot(\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}$ subject to $\frac{p_{cp}}{p_{eu}} - \frac{\sigma_{cp}}{\sigma_{eu}} \leq 0$ and $\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}}{p_{eu}} \leq 0$. From the constraints, we have $p_{cp} \in [(\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha)p_{eu}, \frac{\sigma_{cp}}{\sigma_{eu}}p_{eu}]$. Note that since the objective is an increasing function of p_{cp}, we set the largest $p_{cp} = \frac{\sigma_{cp}}{\sigma_{eu}}\cdot p_{eu}$ for the optimal solution, which gives us maximum utility $P_1^* = (\frac{\sigma_{cp}}{\sigma_{eu}}\cdot p_{eu} - p_{tr} - m_1)\cdot(\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}$. By differentiating it with respect to p_{eu}, we can find $p_{eu}^* = \frac{\sigma_{eu}}{\sigma_{cp}}\cdot(\frac{p_{tr}+m_1}{1-\alpha})$ that maximizes P_1^*, which results in the optimal $p_{cp}^* = \frac{p_{tr}+m_1}{1-\alpha}$. The maximum profit is

$$P_1^* = [\frac{\alpha(m_1+m_2)}{(1-\alpha)}\cdot\frac{\sigma}{1+\sigma(1-\alpha)}]\cdot(\frac{\sigma_{eu}(1-\alpha)(1+\sigma(1-\alpha))}{(m_1+m_2)})^{\frac{1}{\alpha}}.$$

Case 3) When $x^* = (\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}$ and $s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}} + 1 - \alpha}$. The problem can be rewritten as $\max_{p_{cp}\geq 0}(p_{cp} + s^* p_{eu} - p_{tr} - m_1)\cdot(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}$, subject to $p_{cp} \leq (\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha)p_{eu}$. From the first order condition, we can obtain the optimal price $p_{cp}^* = \frac{(k+1)(p_{tr}+m_1)}{k(1-\alpha)} - p_{eu}$, where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$. The maximum profit is

$$P_1^* = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}\cdot(\sigma_{cp} + (1-\alpha)\sigma_{eu})^{\frac{1}{\alpha}}\cdot(\frac{1+k(1-\alpha)}{1+k})^{\frac{1}{\alpha}}\cdot\frac{k}{1+k(1-\alpha)}.$$

3.3 Utility Maximization of ISP_2

For the behaviors of ISP_2, we also consider the three cases of (13) and find the best strategy of ISP_2 for each case.

Case 1) When $x^*(p_{cp}^*, p_{eu}) = (\frac{\sigma_{cp}}{p_{cp}^*})^{\frac{1}{\alpha}}$ and $s^* = 0$. We already have $p_{cp}^* = \frac{p_{tr}+m_1}{1-\alpha}$. From (4) and (13), the ISP_2 determines its prices p_{eu} and p_{tr} by solving $\max_{p_{eu}\geq 0, p_{tr}\geq 0}((1-s^*)\cdot p_{eu} + p_{tr} - m_2)\cdot(\frac{\sigma_{cp}}{p_{cp}^*})^{\frac{1}{\alpha}}$, subject to $\frac{\sigma_{cp}}{\sigma_{eu}} - \frac{p_{cp}^*}{p_{eu}} \leq 0$.

Let P denote the objective function. From the Karush-Kuhn-Tucker (KKT) conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{tr}} = 0$, and $\lambda\cdot[\frac{\sigma_{cp}}{\sigma_{eu}} - \frac{p_{cp}^*}{p_{eu}}] = 0$. By solving these equations, we have the optimal prices

$$p_{eu}^* = \frac{(m_1+m_2)}{(1-\alpha)(1+(k+\alpha)(1-\alpha))} \quad and \quad p_{tr}^* = \frac{(k+\alpha)(m_1+m_2)}{(1+(k+\alpha)(1-\alpha))} - m_1,$$

at which the maximum profit P_2^* is

$$P_2^* = [\frac{\alpha(m_1+m_2)}{(1-\alpha)}](\frac{\sigma_{cp}(1-\alpha)(1+(k+\alpha)(1-\alpha))}{(k+\alpha)(m_1+m_2)})^{\frac{1}{\alpha}},$$

where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$.

Case 2) When $x^*(p_{cp}^*, p_{eu}) = (\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}$ and $s^* = 0$. In this case, we have $p_{cp}^* = \frac{p_{tr}+m_1}{1-\alpha}$. From (4) and (13), the ISP_2 determines its prices by solving $\max_{p_{eu}\geq 0, p_{tr}\geq 0} \quad ((1-s^*)\cdot p_{eu} + p_{tr} - m_2)\cdot(\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}$, subject to $\frac{p_{cp}^*}{p_{eu}} - \frac{\sigma_{cp}}{\sigma_{eu}} \leq 0$ and $\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}^*}{p_{eu}} \leq 0$.

From the KKT conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{tr}} = 0$, $\lambda_1\cdot(\frac{p_{cp}^*}{p_{eu}} - \frac{\sigma_{cp}}{\sigma_{eu}}) = 0$ and $\lambda_2\cdot(\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}^*}{p_{eu}}) = 0$, where $\lambda_i \geq 0$, $p_{cp} \geq 0$, and $p_{eu} \geq 0$. There are three possible subcases: i) $\lambda_1 = 0$, $\lambda_2 \neq 0$, ii) $\lambda_1 \neq 0$, $\lambda_2 = 0$, iii) $\lambda_1 = 0$ and $\lambda_2 = 0$. The solution to each subcase can be obtained as follows.

i) When $\lambda_1 = 0$ and $\lambda_2 \neq 0$, the optimal prices will be

$$p_{eu}^* = \frac{m_1+m_2}{(1-\alpha)(1+k(1-\alpha))} \quad and \quad p_{tr}^* = \frac{k(m_1+m_2)}{1+k(1-\alpha)} - m_1,$$

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Scenario	CP: s^*	ISP 1: p^*_{cp}	ISP 2: p^*_{eu} , p^*_{tr}
$\frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$	$s^* = 0$	$p^*_{cp} = \frac{p_{tr}+m_1}{1-\alpha}$	$p^*_{eu} = \frac{(m_1+m_2)}{(1-\alpha)(1+(k+\alpha)(1-\alpha))}$, $p^*_{tr} = \frac{(k+\alpha)(m_1+m_2)}{(1+(k+\alpha)(1-\alpha))} - m_1$
$\frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}$	$s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}$	$p^*_{cp} = \frac{(k+1)(p_{tr}+m_1)}{k(1-\alpha)} - p_{eu}$	$p^*_{eu} = \frac{(m_1+m_2)}{(1-\alpha)(1+k(1-\alpha))}$, $p^*_{tr} = \frac{k(m_1+m_2)}{(1+k(1-\alpha))} - m_1$

Table 1: Optimal sponsoring rate and prices.

where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$, and we have the maximum profit

$$P^*_{\lambda_1} = [\frac{\alpha(m_1+m_2)}{(1-\alpha)}](\frac{(\sigma_{cp}-\sigma_{eu}\alpha)(1-\alpha)^2+\sigma_{eu}(1-\alpha)}{m_1+m_2})^{\frac{1}{\alpha}}.$$

ii) When $\lambda_1 \neq 0$ and $\lambda_2 = 0$, the optimal prices will be

$$p^*_{eu} = \frac{(m_1+m_2)}{(1-\alpha)(1+(k+\alpha)(1-\alpha))} \ \ and \ \ p^*_{tr} = \frac{(k+\alpha)(m_1+m_2)}{(1+(k+\alpha)(1-\alpha))} - m_1,$$

and the maximum profit

$$P^*_{\lambda_2} = \frac{\alpha(m_1+m_2)}{(1-\alpha)}(\frac{\sigma_{cp}(1-\alpha)^2+\sigma_{eu}(1-\alpha)}{m_1+m_2})^{\frac{1}{\alpha}}.$$

iii) When $\lambda_1 = 0$ and $\lambda_2 = 0$, the two inequality constraints should be an active constraint (i.e., the equalities hold). However, it is not possible to satisfy both equalities, and hence, this case is infeasible.

From $P^*_{\lambda_2} > P^*_{\lambda_1}$, we should have $\lambda_2 = 0$ and the best response of the ISP_2 is that of ii), which also equals the result of Case 1.

Case 3) In this case, we have the optimal sponsoring rate $s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}$ and the traffic demand is $x^*(p^*_{cp}, p_{eu}) = (\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}$. As shown in Section 3.2, the best-response p^*_{cp} of ISP_1 is $\frac{(k+1)(p_{tr}+m_1)}{k(1-\alpha)} - p_{eu}$. From (4) and (13), ISP_2 determines its prices by solving $\max_{p_{eu}\geq 0, p_{tr}\geq 0}((1-s^*)\cdot p_{eu}+p_{tr}-m_2)\cdot(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p^*_{cp}+p_{eu}})^{\frac{1}{\alpha}}$, subject to $\frac{p^*_{cp}}{p_{eu}} + \alpha - \frac{\sigma_{cp}}{\sigma_{eu}} \leq 0$.

From the KKT conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{tr}} = 0$, and $\lambda \cdot [\frac{p^*_{cp}}{p_{eu}} + \alpha - \frac{\sigma_{cp}}{\sigma_{eu}}] = 0$. By solving the equations, we can obtain without difficulty that

$$p^*_{eu} = \frac{(m_1+m_2)}{(1-\alpha)(1+k(1-\alpha))} \ \ and \ \ p^*_{tr} = \frac{k(m_1+m_2)}{(1+k(1-\alpha))} - m_1.$$

The maximum profit P^*_2 will be

$$P^*_2 = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp}+(1-\alpha)\sigma_{eu})^{\frac{1}{\alpha}}(\frac{1+k(1-\alpha)}{1+k})^{\frac{1}{\alpha}}.$$

To sum up, by investigating the structure of the proposed game, we derived the optimal responses of the EU, the CP, and two *ISPs* in a non-cooperative equilibrium. The Table 1 summarizes the best response of each player when they maximize their utility in a greedy manner. We further investigate the players' behaviors when the two ISPs cooperate.

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IV Cooperative Model

In this section, we study the effect of collaboration to the pricing structure and the traffic demand between CP and EU, and analyze their implications for the total payoff of ISPs. When ISP_1 and ISP_2 collaborate to deliver traffic from CP to EU, we can consider them as one ISP who obtains its revenue from charging CP by p_{cp} and EU by p_{eu}. The two ISPs are in peering with no transit-cost: neither party pays the other in association with the exchange of traffic. Instead, they need to fairly redistribute the total revenue according to their marginal contributions. We will use Shapley value mechanism for this purpose.

We first obtain the total revenue of the ISPs. The utility maximization of the ISPs can be written as

$$\begin{aligned} (\boldsymbol{ISP-P}) \quad & \max_{p_{cp},p_{eu}} \quad (p_{cp}+p_{eu}-m_1-m_2)\cdot x^*(p_{cp},p_{eu}), \\ & s.t. \qquad p_{cp}\geq 0 \ \ and \ \ p_{eu}\geq 0. \end{aligned} \tag{14}$$

Given unit price p_{eu} that ISP charges user, EU will maximize its utility minus the payment by solving

$$\begin{aligned} (\boldsymbol{EU-P}) \quad & \max_{x} \quad \sigma_{eu}\cdot u_{eu}(x)-(1-s)\cdot x\cdot p_{eu}, \\ & s.t. \quad x\geq 0, \end{aligned} \tag{15}$$

where $s\in[0,1]$ denotes the sponsored percentage, and $(1-s)\cdot x\cdot p_{eu}$ denotes the payment of EU to ISP. The solution x^*_{eu} to (2) can be obtained as $x^*_{eu}(s,p_{eu})=(\frac{\sigma_{eu}}{(1-s)p_{eu}})^{\frac{1}{\alpha_{eu}}}$.
Similarly, CP will maximize its payoff by solving

$$\begin{aligned} (\boldsymbol{CP-P}) \quad & \max_{x,s} \quad \sigma_{cp}\cdot u_{cp}(x)-s\cdot x\cdot p_{eu}-x\cdot p_{cp}, \\ & s.t. \quad x\geq 0 \quad and \quad 0\leq s\leq 1, \end{aligned} \tag{16}$$

where the first term denotes its utility, the second term denotes the cost due to sponsorship, and the third term is from the network usage cost to ISP. Given s, p_{cp}, and p_{eu}, it can be easily shown that the optimal amount of traffic for CP is $x^*_{cp}(s,p_{cp},p_{eu})=(\frac{\sigma_{cp}}{sp_{eu}+p_{cp}})^{\frac{1}{\alpha_{cp}}}$.

Since the actual traffic amount x^* between CP and EU will be determined by their minimum, i.e., $x^*=min\{x^*_{cp},x^*_{eu}\}$, we can obtain the optimal sponsorship rate s^* and the data rate under sponsoring by considering three cases as before. We omit the detailed derivation and provide the result as

$$\begin{aligned} & case\ 1)\ if \ \ \frac{\sigma_{cp}}{\sigma_{eu}}\leq\frac{p_{cp}}{p_{eu}}, && (x^*,s^*)=((\frac{\sigma_{cp}}{p_{cp}})^{\frac{1}{\alpha}},\ 0), \\ & case\ 2)\ if \ \ \frac{p_{cp}}{p_{eu}}<\frac{\sigma_{cp}}{\sigma_{eu}}\leq\alpha+\frac{p_{cp}}{p_{eu}}, && (x^*,s^*)=((\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}},\ 0), \\ & case\ 3)\ if \ \ \frac{\sigma_{cp}}{\sigma_{eu}}>\alpha+\frac{p_{cp}}{p_{eu}}, && (x^*,s^*)=((\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}},\ \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}). \end{aligned} \tag{17}$$

$ISPs$ cooperate and try to maximize their total profit in each region specified in (17). We obtain the optimal response of $ISPs$ in each case.

Case 1) When $x^*(p_{cp}, p_{eu}) = (\frac{\sigma_{cp}}{p_{cp}})^{\frac{1}{\alpha}}$ and $s^* = 0$. From (14) and (17), the coalition-*ISP* determines its prices p_{eu} and p_{cp} by maximizing $(p_{cp} + p_{eu} - m_1 - m_2) \cdot (\frac{\sigma_{cp}}{p_{cp}})^{\frac{1}{\alpha}}$, subject to $\frac{\sigma_{cp}}{\sigma_{eu}} - \frac{p_{cp}}{p_{eu}} \leq 0$, $p_{eu} \geq 0$, and $p_{cp} \geq 0$.

Let P denote the objective function. From the Karush-Kuhn-Tucker (KKT) conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{cp}} = 0$, and $\lambda \cdot [\frac{\sigma_{cp}}{\sigma_{eu}} - \frac{p_{cp}}{p_{eu}}] = 0$. By solving these equations, it is not difficulty to obtain the optimal prices of

$$p_{eu}^{**} = \frac{\sigma_{eu}(m_1+m_2)}{(1-\alpha)(\sigma_{cp}+\sigma_{eu})} \quad and \quad p_{cp}^{**} = \frac{\sigma_{cp}(m_1+m_2)}{(1-\alpha)(\sigma_{cp}+\sigma_{eu})}, \tag{18}$$

at which the maximum profit P^{**} equals

$$P^{**} = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp} + \sigma_{eu})^{\frac{1}{\alpha}}.$$

Case 2) When $x^*(p_{cp}, p_{eu}) = (\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}$ and $s^* = 0$. From (14) and (17), the coalition-*ISP* determines its prices by solving $\max\limits_{p_{eu}\geq 0, p_{cp}\geq 0}(p_{cp} + p_{eu} - m_1 - m_2) \cdot (\frac{\sigma_{eu}}{p_{eu}})^{\frac{1}{\alpha}}$, subject to $\frac{p_{cp}}{p_{eu}} \leq \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$.

From the KKT conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{cp}} = 0$, $\lambda_1 \cdot (\frac{p_{cp}}{p_{eu}} - \frac{\sigma_{cp}}{\sigma_{eu}}) = 0$ and $\lambda_2 \cdot (\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}}{p_{eu}}) = 0$, where $\lambda_i \geq 0$, $p_{cp} \geq 0$, and $p_{eu} \geq 0$. There are four possible subcases: i) $\lambda_1 = 0, \lambda_2 \neq 0$, ii) $\lambda_1 \neq 0, \lambda_2 = 0$, iii) $\lambda_1 = 0$ and $\lambda_2 = 0$, iv) $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$.

i) When $\lambda_1 = 0$ and $\lambda_2 \neq 0$, the optimal prices will be

$$p_{eu}^{**} = \frac{m_1+m_2}{(1-\alpha)(1+k)} \quad and \quad p_{cp}^{**} = \frac{k(m_1+m_2)}{(1-\alpha)(1+k)},$$

where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$, and we have the maximum profit

$$P_{\lambda_1}^{**} = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp} + (1-\alpha)\sigma_{eu})^{\frac{1}{\alpha}}.$$

ii) When $\lambda_1 \neq 0$ and $\lambda_2 = 0$, the optimal prices will be

$$p_{eu}^{**} = \frac{\sigma_{eu}(m_1+m_2)}{(1-\alpha)(\sigma_{cp}+\sigma_{eu})} \quad and \quad p_{cp}^{**} = \frac{\sigma_{cp}(m_1+m_2)}{(1-\alpha)(\sigma_{cp}+\sigma_{eu})}, \tag{19}$$

and the maximum profit $P_{\lambda_2}^{**} = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp} + \sigma_{eu})^{\frac{1}{\alpha}}$.

iii) When $\lambda_1 = 0$ and $\lambda_2 = 0$, the two inequality constraints of $\frac{p_{cp}}{p_{eu}} \leq \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$ should be an active constraint (i.e., the equalities hold). However, it is not possible to satisfy both equalities, and hence, it is infeasible.

iv) Similarly, when $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$, we cannot find a feasible solution for any $\alpha > 0$.

From $P_{\lambda_2}^{**} > P_{\lambda_1}^{**}$, we should have $\lambda_2 = 0$ and the best response of the *ISPs* is (19), which is exactly the same as in (18).

Case 3) In this case, we have the optimal sponsoring rate $s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}} + 1 - \alpha}$ and the traffic demand is $x^*(p_{cp}, p_{eu}) = (\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}$. From (14) and (17), *ISPs* determine their prices by solving $\max\limits_{p_{eu}\geq 0, p_{cp}\geq 0}(p_{cp} + p_{eu} - m_1 - m_2) \cdot (\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}})^{\frac{1}{\alpha}}$, subject to $\frac{p_{cp}}{p_{eu}} + \alpha - \frac{\sigma_{cp}}{\sigma_{eu}} \leq 0$. From the KKT conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{cp}} = 0$, and $\lambda \cdot [\frac{p_{cp}}{p_{eu}} + \alpha - \frac{\sigma_{cp}}{\sigma_{eu}}] = 0$. By solving the equations, we can obtain without difficulty that

$$p_{eu}^{**} = \frac{(m_1+m_2)}{(1-\alpha)(k+1)} \quad and \quad p_{cp}^{**} = \frac{k(m_1+m_2)}{(1-\alpha)(k+1)},$$

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where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$, with the maximum profit as

$$P^{**} = \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp} + (1-\alpha)\sigma_{eu})^{\frac{1}{\alpha}}.$$

Comparing the results with those in non-cooperative scenarios, we can obtain the following proposition.

Proposition 2 *The ISPs obtain higher total payoff when they collaborate.*

$$P^{**} \geq P_T^*, \tag{20}$$

where P_T^* denotes the total profit in non-cooperative case, i.e., $P_T^* = P_1^* + P_2^*$.

Proof We consider each case as before.

For Case 1. From our previous results for non-cooperative game, we know that the maximum profits of ISP_1 and ISP_2 are $[\frac{\alpha(m_1+m_2)}{(1-\alpha)} \cdot \frac{\sigma}{1+\sigma(1-\alpha)}](\frac{\sigma_{cp}(1-\alpha)(1+\sigma(1-\alpha))}{\sigma(m_1+m_2)})^{\frac{1}{\alpha}}$ and $[\frac{\alpha(m_1+m_2)}{(1-\alpha)}] \cdot (\frac{\sigma_{cp}(1-\alpha)(1+\sigma(1-\alpha))}{\sigma(m_1+m_2)})^{\frac{1}{\alpha}}$, respectively, where $\sigma = \frac{\sigma_{cp}}{\sigma_{eu}}$. Hence, the total profit is

$$P_T^* = [\tfrac{\alpha(m_1+m_2)}{(1-\alpha)} \cdot \tfrac{1+\sigma+\sigma(1-\alpha)}{1+\sigma(1-\alpha)}](\tfrac{\sigma_{cp}(1-\alpha)(1+\sigma(1-\alpha))}{\sigma(m_1+m_2)})^{\frac{1}{\alpha}}.$$

We can rewrite the total profits of *ISPs* for each non-cooperative and cooperative case as

$$\begin{aligned} P_T^* &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}(1+\sigma(1-\alpha))^{\frac{1}{\alpha}}(\tfrac{1+\sigma+\sigma(1-\alpha)}{1+\sigma(1-\alpha)}), \\ P^{**} &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}(1+\sigma)^{\frac{1}{\alpha}}. \end{aligned}$$

Considering the ratio $\frac{P^{**}}{P_T^*} = (\frac{1+\sigma}{1+\sigma-\alpha\sigma})^{\frac{1}{\alpha}}(\frac{1+\sigma-\alpha\sigma}{1+\sigma+\sigma-\alpha\sigma})$ and applying the generalized form of Bernoulli's inequality, we obtain

$$\tfrac{P^{**}}{P_T^*} \geq (1 + \tfrac{\sigma}{1+\sigma-\alpha\sigma})(\tfrac{1+\sigma-\alpha\sigma}{1+\sigma+\sigma-\alpha\sigma}) = 1,$$

which immediately implies $P^{**} \geq P_T^*$.

For Case 2, we have the same total profits P_T^* and P^{**} as in Case 1. Thus, we have $P^{**} \geq P_T^*$.

For Case 3, from Section 3.2 and 3.3, The total profits in non-cooperative and cooperative cases can be written as

$$\begin{aligned} P_T^* &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp} + (1-\alpha)\sigma_{eu})^{\frac{1}{\alpha}}(\tfrac{1+k(1-\alpha)}{1+k})^{\frac{1}{\alpha}}\tfrac{k+1+k(1-\alpha)}{1+k(1-\alpha)}, \\ P^{**} &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp} + (1-\alpha)\sigma_{eu})^{\frac{1}{\alpha}}, \end{aligned}$$

where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$ and $\sigma = \frac{\sigma_{cp}}{\sigma_{eu}}$. Again we apply Bernoulli's inequality to $\frac{P^{**}}{P_T^*} = (\frac{1+k}{1+k(1-\alpha)})^{\frac{1}{\alpha}} \frac{1+k(1-\alpha)}{k+1+k(1-\alpha)}$, and obtain

$$\tfrac{P^{**}}{P_T^*} \geq \left(1 + \tfrac{k}{1+k(1-\alpha)}\right) \cdot \tfrac{1+k(1-\alpha)}{k+1+k(1-\alpha)} = 1.$$

This completes the proof, and in all three cases, ISPs obtain a higher total payoff when they collaborate.

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V Shapley Revenue Distribution

One remaining task under the collaboration is how to distribute the payoff P^{**} to each ISP. To this end, we apply Shapley value mechanism.

Suppose that a network consists of a set of *ISPs* denoted as $\mathbb{N}$ with $N = |\mathbb{N}|$. Any nonempty subset $\mathbb{S} \subseteq \mathbb{N}$ is a coalition of ISPs. For any coalition $\mathbb{S}, P(\mathbb{S})$ denotes the profit (i.e., revenue minus cost) generated by the sub-network formed by the set of ISPs $\mathbb{S}$. We define the marginal contribution of ISP_i to a coalition $\mathbb{S} \subseteq \mathbb{N} \setminus \{i\}$ as $\triangle_i(\mathbb{S}) = P(\mathbb{S} \cup \{i\}) - P(\mathbb{S})$. The Shapley value ϕ is defined by

$$\phi_i = \frac{1}{N!} \sum_{\pi \in \Pi} \triangle_i(\mathbb{S}(\pi, i)) \quad \forall i \in N, \tag{21}$$

where Π is the set of all $N!$ orderings of $\mathbb{N}$ and $\mathbb{S}(\pi, i)$ is the set of players preceding i in the ordering π [13, 14]. The Shapley value depends only on the values $\{P(\mathbb{S}) : \mathbb{S} \subseteq \mathbb{N}\}$ and satisfies desirable efficiency and fairness properties [15]. Revenue sharing model based on the Shapley value belongs to a cooperation-based game theory, and the mechanism has a capacity to divide the revenue *fairly* between the involved parties [16, 17].

In our model, we have $\mathbb{N} = \{1, 2\}$, and ISP_1 and ISP_2 receive their Shapley value, which can be obtained as

$$\begin{aligned} \phi_1 &= \tfrac{1}{2}P(\{1\}) + \tfrac{1}{2}[P(\{1,2\}) - P(\{2\})], \\ \phi_2 &= \tfrac{1}{2}P(\{2\}) + \tfrac{1}{2}[P(\{1,2\}) - P(\{1\})], \end{aligned} \tag{22}$$

where $P(\{1,2\}) = \phi_1 + \phi_2$ is the total profit under collaboration, and $P(\{1\}) = P_1^*$ and $P(\{2\}) = P_2^*$ are the profit of ISP_1 and ISP_2 in non-cooperative case, respectively.

Recall that letting $A = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}$, $\sigma = \frac{\sigma_{cp}}{\sigma_{eu}}$ and $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$, we have

$$\begin{array}{lll} \textit{if} \;\; \frac{\sigma_{cp}}{\sigma_{eu}} \le \alpha + \frac{p_{cp}}{p_{eu}}, & P_1^* = A(1+\sigma(1-\alpha))^{\frac{1}{\alpha}}(\frac{\sigma}{1+\sigma(1-\alpha)}), \\ & P_2^* = A(1+\sigma(1-\alpha))^{\frac{1}{\alpha}}, \\ \textit{if} \;\; \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, & P_1^* = A(1+k(1-\alpha))^{\frac{1}{\alpha}}(\frac{k}{1+k(1-\alpha)}), \\ & P_2^* = A(1+k(1-\alpha))^{\frac{1}{\alpha}}. \end{array} \tag{23}$$

From the results in Section IV, we can also obtain the total payoff $P(\{1,2\})$ under cooperation as

$$\begin{array}{ll} \textit{if} & \frac{\sigma_{cp}}{\sigma_{eu}} \le \alpha + \frac{p_{cp}}{p_{eu}}, \quad \textit{(Cases 1 \& 2)} \\ & P(\{1,2\}) = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}(1+\sigma)^{\frac{1}{\alpha}}, \\ \textit{if} & \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, \quad \textit{(Case 3)} \\ & P(\{1,2\}) = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{cp} + (1-\alpha)\sigma_{eu})^{\frac{1}{\alpha}}. \end{array} \tag{24}$$

From (22), we obtain the Shapley value for ISP_1 and ISP_2 as

$$\begin{aligned}
&if \quad \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}, \quad (Cases\ 1\ \&\ 2) \\
&\phi_1 = A[(1+\sigma)^{\frac{1}{\alpha}} - (1+\sigma(1-\alpha))^{\frac{1}{\alpha}} \tfrac{1-\alpha\sigma}{1+\sigma(1-\alpha)}], \\
&\phi_2 = A[(1+\sigma)^{\frac{1}{\alpha}} + (1+\sigma(1-\alpha))^{\frac{1}{\alpha}} \tfrac{1-\alpha\sigma}{1+\sigma(1-\alpha)}], \\
&if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, \quad (Case\ 3) \\
&\phi_1 = A[(\sigma+1-\alpha)^{\frac{1}{\alpha}} - (1+k(1-\alpha))^{\frac{1}{\alpha}} \tfrac{1-\alpha k}{1+k(1-\alpha)}], \\
&\phi_2 = A[(\sigma+1-\alpha)^{\frac{1}{\alpha}} + (1+k(1-\alpha))^{\frac{1}{\alpha}} \tfrac{1-\alpha k}{1+k(1-\alpha)}].
\end{aligned} \tag{25}$$

The following proposition shows that the collaboration with revenue sharing of Shapley mechanism improves the profit of each ISP.

Proposition 3 *The revenue sharing mechanism assures that an ISP_i's revenue portion at least equals to the revenue gained without collaboration, i.e.*

$$\phi_i \geq P(\{i\}) \tag{26}$$

Proof For case 1, according to a Shapley value based revenue sharing scheme defined in Equation (25), ISP_1 receives $\phi_1 = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(1+\sigma)^{\frac{1}{\alpha}} - (1+\sigma(1-\alpha))^{\frac{1}{\alpha}}\frac{1-\alpha\sigma}{1+\sigma(1-\alpha)}]$ portion of the total revenue and from Equation (23) we know the maximum profit of ISP_1 obtained without collaboration is $P(\{1\}) = \alpha(\frac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(1+\sigma(1-\alpha))^{\frac{1}{\alpha}}(\frac{\sigma}{1+\sigma(1-\alpha)})]$. We consider the ratio $\frac{\phi_1}{P(\{1\})} = \frac{1}{2}[\frac{(1+\sigma)^{\frac{1}{\alpha}}(1+\sigma(1-\alpha))}{\sigma(1+\sigma(1-\alpha))^{\frac{1}{\alpha}}} - \frac{1-\alpha\sigma}{\sigma}] = \frac{1}{2}[(\frac{1+\sigma-\alpha\sigma+\alpha\sigma}{1+\sigma-\alpha\sigma})^{\frac{1}{\alpha}}\frac{(1+\sigma(1-\alpha))}{\sigma} - \frac{1-\alpha\sigma}{\sigma}]$. By using the generalized form of Bernoulli's inequality $(1+x)^r \geq 1+rx$ for $r \leq 0$ or $r \geq 1$ and $x > -1$, we can obtain

$$\frac{\phi_1}{P(\{1\})} \geq \frac{1}{2}[(1+\frac{\sigma}{1+\sigma-\alpha\sigma})(\frac{1+\sigma(1-\alpha)}{\sigma}) - \frac{1-\alpha\sigma}{\sigma}] = \frac{1}{2}[\frac{2\sigma}{\sigma}] = 1.$$

Similarly, for case 1, ISP_2's profits in cooperative and non-cooperative cases, defined in (25) and (23), respectively, are as follows

$$\begin{aligned}
\phi_2 \quad &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(1+\sigma)^{\frac{1}{\alpha}} + (1+\sigma(1-\alpha))^{\frac{1}{\alpha}}\tfrac{1-\alpha\sigma}{1+\sigma(1-\alpha)}], \\
P(\{2\}) &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(1+\sigma(1-\alpha))^{\frac{1}{\alpha}}].
\end{aligned}$$

Again we consider the ratio $\frac{\phi_2}{P(\{2\})} = \frac{1}{2}[(\frac{1+\sigma}{1+\sigma(1-\alpha)})^{\frac{1}{\alpha}} + \frac{1-\alpha\sigma}{1+\sigma(1-\alpha)}] = \frac{1}{2}[(\frac{1+\sigma-\alpha\sigma+\alpha\sigma}{1+\sigma-\alpha\sigma})^{\frac{1}{\alpha}} + \frac{1-\alpha\sigma}{1+\sigma(1-\alpha)}]$. Applying the generalized form of Bernoulli's inequality, we have $\frac{\phi_2}{P(\{2\})} \geq \frac{1}{2}[(1+\frac{\sigma}{1+\sigma-\alpha\sigma}) + \frac{1-\alpha\sigma}{1+\sigma(1-\alpha)}] = 1$.

Hence, when $\frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}$, we have $\phi_1 \geq P(\{1\})$ and $\phi_2 \geq P(\{2\})$, implying both *ISPs* gain higher revenue when they cooperate.

For case 2, $\frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$, we have the same profits ϕ_i and $P(\{i\})$ as in Case 1. Hence, using the above method we can easily prove that $\phi_1 \geq P(\{1\})$ and $\phi_2 \geq P(\{2\})$.

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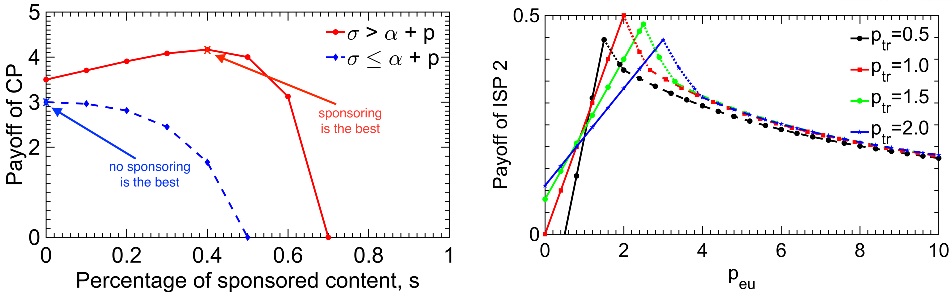

(a) CP

(b) ISP_2 with $\sigma_{cp} = 2, \sigma_{eu} = 1$

Figure 2: Payoff changes of CP and ISP_2 when $\alpha = 0.5$.

For case 3, $\frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}$, ISP_1's profits in cooperative and non-cooperative models, defined in (25) and (23), respectively, are as follows

$$\begin{aligned} \phi_1 &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(\sigma+1-\alpha)^{\frac{1}{\alpha}} - (1+k(1-\alpha))^{\frac{1}{\alpha}}\tfrac{1-\alpha k}{1+k(1-\alpha)}], \\ P(\{1\}) &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(1+k(1-\alpha))^{\frac{1}{\alpha}}\tfrac{k}{1+k(1-\alpha)}]. \end{aligned}$$

Hence, the ratio is $\frac{\phi_1}{P(\{1\})} = \frac{1}{2}[\frac{1+k(1-\alpha)}{k}(\frac{1+k}{1+k(1-\alpha)})^{\frac{1}{\alpha}} - \frac{1-\alpha k}{k}]$. Applying the generalized form of Bernoulli's inequality, we can obtain $\frac{\phi_1}{P(\{1\})} \geq \frac{1}{2}[\frac{1+k(1-\alpha)}{k}(1+\frac{k}{1+k(1-\alpha)}) - \frac{1-\alpha k}{k}] = \frac{2k}{2k} = 1$.
Similarly, for ISP_2 we have

$$\begin{aligned} \phi_2 &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(\sigma+1-\alpha)^{\frac{1}{\alpha}} + (1+k(1-\alpha))^{\frac{1}{\alpha}}\tfrac{1-\alpha k}{1+k(1-\alpha)}], \\ P(\{2\}) &= \alpha(\tfrac{m_1+m_2}{1-\alpha})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}[(1+k(1-\alpha))^{\frac{1}{\alpha}}]. \end{aligned}$$

Hence, the ratio is $\frac{\phi_2}{P(\{2\})} = \frac{1}{2}[(\frac{\sigma+1-\alpha}{1+k(1-\alpha)})^{\frac{1}{\alpha}} + \frac{1-\alpha k}{1+k(1-\alpha)}] = \frac{1}{2}[(\frac{1+k}{1+k-\alpha k})^{\frac{1}{\alpha}} + \frac{1-\alpha k}{1+k(1-\alpha)}]$. Applying the generalized form of Bernoulli's inequality, we can obtain $\frac{\phi_2}{P(\{2\})} \geq \frac{1}{2}[(1+\frac{k}{1+k(1-\alpha)}) + \frac{1-\alpha k}{1+k(1-\alpha)}] = \frac{2(1+k(1-\alpha))}{2(1+k(1-\alpha))} = 1$.
Thus, for all regions specified in (17) we have $\phi_i \geq P(\{i\})$, i.e., an ISP_i's revenue portion in cooperative model is at least equals to the revenue gained in non-cooperative model.

VI Numerical Simulations

We verify our analytical results through numerical simulations. We consider one CP, one EU, and two ISPs as shown in Figure 1, and assume that CP and the EU share the same utility-level function $\alpha_{eu} = \alpha_{cp} = \alpha \in (0,1)$. Figure 2$a$ shows that, if $\sigma(=\frac{\sigma_{cp}}{\sigma_{eu}}) > \alpha + p(=\frac{p_{cp}}{p_{eu}})$, CP has the maximum profit at $\sigma = 0.4$ and thus has incentive to invest in sponsored data plan. It implies that when CP has a higher utility level than EU (or similarly, when the price charged to CP is relatively lower than the price charged to EU), CP is willing to provide a higher sponsorship rate. In contrast, when $\sigma \leq \alpha + p$, the maximum payoff is achieved at $s^* = 0$, i.e., the best strategy of CP is not sponsoring.

Next we observe the payoff of ISP_2 as we change the price per unit traffic p_{eu} that charges to EU. Figure 2b illustrates the results and show that the payoff of ISP_2 linearly rises till some point, and then declines exponentially, which is due to the fact that the demand of users is inversely proportional to

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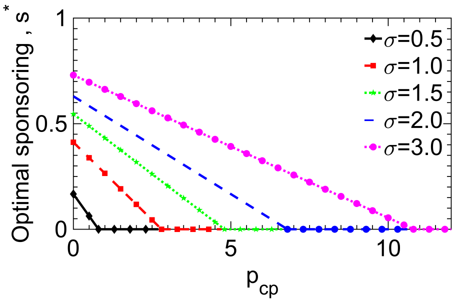
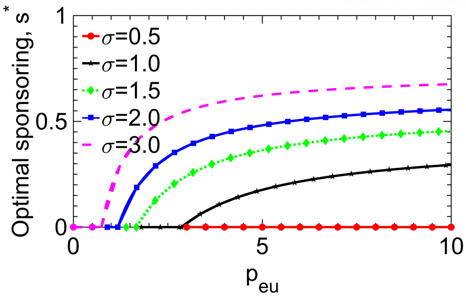
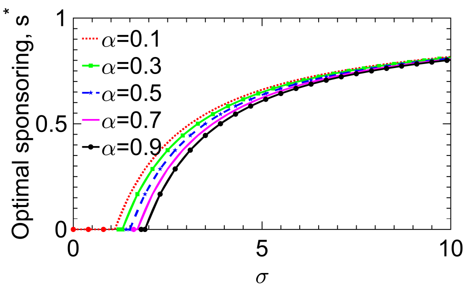


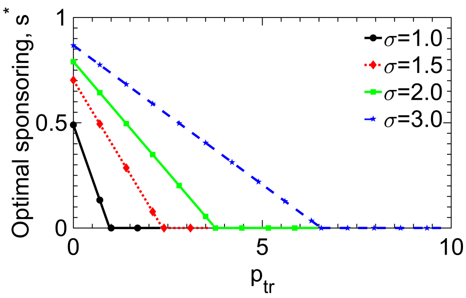
(a) $p_{eu} = 4$, $\alpha = 0.3$



(b) $p_{cp} = 2$, $\alpha = 0.3$



(c) $p_{cp} = 2$, $p_{eu} = 2$



(d) $p_{eu} = 4$, $\alpha = 0.3$

Figure 3: The optimal sponsoring rate with respect to p_{cp}, p_{eu}, σ, and p_{tr}.

p_{eu}. Although ISP_2 obtains its revenue from charging ISP_1 with transit-price p_{tr}, the results show that increasing the p_{tr} does not necessarily increase the payoff of ISP_2. As the transit price becomes higher, CP is forced to increase p_{cp} which in turn results in a decline of the traffic demand. Hence, the maximum point is achieved at $p_{tr} = 1$ and $p_{eu} = 2$.

We examine the impact of ISP prices (p_{cp}, p_{eu}, and p_{tr}) and σ on the optimal sponsoring rate with different parameter sets. Figure 3a shows that as p_{cp} increases, the sponsoring rate drops sharply. The decreasing rate can be mitigated with higher σ. Figure 3b shows that with the increase of p_{eu}, the marginal increase of the sponsoring rate is decreasing. Moreover, a larger σ value indicates a higher and rapidly growing sponsorship rate. Figure 3c demonstrates the change of the optimal sponsoring rate with respect to σ under different α values. The sponsorship rate logarithmically increases as σ increases. It can be explained from the fact that CP with higher revenue level can afford more investment on the sponsoring content. We can also observe that the variation in α has a little impact on the traffic demand. Figure 3d will help us to understand the effect of the transit cost p_{tr} to the optimal sponsoring rate s^*. We can observe that the increase of the transit cost results in a sharp drop of s^*. The rise of transit cost will incur significant loss in ISP_1's revenue, which forces ISP_1 to increase its charge to CP, resulting in a rapid drop of the sponsoring rate.

We now observe the total payoff of ISPs in cooperative and non-cooperative cases. Figure 4a illustrates the results and show that the ISPs obtain higher total payoff when they collaborate. We also examine the impact of collaboration on the individual payoff of ISP_1 and ISP_2. Figure 4b and 4c shows that each ISP's revenue portion in cooperative case increases sharply and highly exceeds the revenue gained without collaboration.

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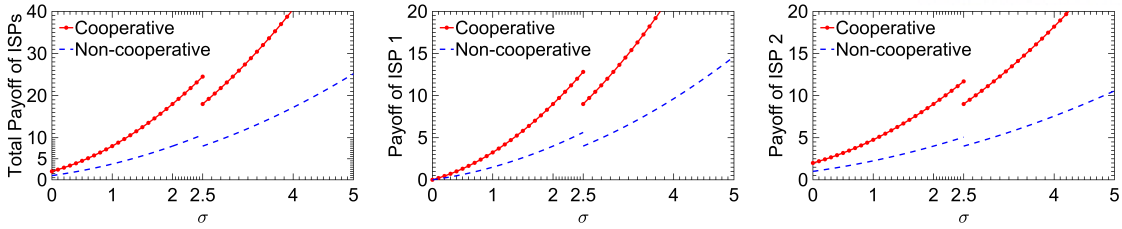

(a) Total payoff of *ISPs* (b) ISP_1 (c) ISP_2

Figure 4: Payoff changes of ISP_1 and ISP_2 when $\alpha = 0.5$.

VII Conclusion

In this work, we studied the inter-pricing among ISPs that jointly deliver the *sponsored data* from CP to EU. We derived the best response of the EU, the CP, and the ISPs, and analyzed their implications for the sponsoring strategy of the CP. We investigate the interactions between strategic EU, CP, and two interconnected ISPs through a sequential Stackelberg game, and verify our results through numerical simulations. Our results clarify the high impact of the transit price of intermediate ISP on the sponsoring strategies of the CP, and demonstrate in what scenarios sponsoring helps. The proposed model assists CPs to make decision on offering content sponsoring services and ISPs to make appropriate pricing scheme. We then study the effect of cooperation between the ISPs and show that the collaboration can improve the total payoff of the ISPs and leads to a higher social welfare. Based on the Shapley value mechanism, we further show that each ISP's revenue portion in cooperative case exceeds the revenue gained without collaboration. In our future work, we will consider the network with multiple ISPs for the service to the EU or the CP which may result in competition between the ISPs and change the system dynamics.

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