

plasmas. The study of these effects has been paid a great deal of interest in recent years. Dust charge variations play an interesting role in the non-linear propagation characteristics of dust acoustic wave. In case of non-adiabatic dust charge variation, it produces an anomalous dissipation that causes shock wave in dusty plasmas [4,5]; also for weak dissipation, the non-linear waves become damped due to the dust charge variations [6–8].

DAWs have been studied in dusty plasmas with non-thermal ions and constant dust charge [9]. Also, these waves in hot dust plasmas have been investigated in [10]. Wang *et al* have studied effect of negative ions on the formation of solitary waves in dusty plasmas by using the Sagdeev potential [11,12]. Cylindrical KP equation in warm dusty plasmas with two ions has been studied too [13]. Solitary waves of the KdV equation have been investigated in dusty plasmas with variable dust charge in [14,15]. Gill *et al* have also analyzed solitons of KP equation for these plasmas with two temperature ions [16]. This paper deals with the 2D non-linear structures of dust acoustic waves incorporating dust charge variations and non-thermal ions. The problem is interesting from astrophysical point of view, as the observations of PHOBOS and NOZOMI satellites confirm the presence of non-thermal ions on the upper ionosphere of Mars and in the vicinity of Moon. Also in truly dusty plasma the charge on dust grains is an extra dynamical variable that varies in time.

We study the non-linear waves in dusty plasmas with variable dust charge; Boltzmann distributed electrons and the non-thermal ions. In Section 2 the basic set of equations is introduced. We will derive the KP equation by using the reductive perturbation method in Section 3. In Section 4 the modified KP equation is derived at the critical density. Finally, conclusions are given in Section 5.

2. Basic equations

As before mentioned, we consider the propagation of dust acoustic waves in a collisionless, unmagnetized dusty plasma consisting of high negatively charged dust grains, variable dust charges, non-thermal ions and Boltzmann distributed electrons. Total charge neutrality at equilibrium requires that $Z_{0d}n_{0d} + n_{0e} = n_{0i}$, where n_{0i} , n_{0e} and n_{0d} are the equilibrium values of ions, electrons and dust number densities respectively. Z_{0d} is the unperturbed number of charges on the dust particles. The following set of normalized two dimensional equations of motion describe the dynamics of dust acoustic wave in the variable dust charge plasmas :

$$\begin{aligned} \frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) + \frac{\partial}{\partial y}(n_d \nu_d) &= 0, & \frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \nu_d \frac{\partial u_d}{\partial y} &= Z_d \frac{\partial \phi}{\partial x} \\ \frac{\partial \nu_d}{\partial t} + u_d \frac{\partial \nu_d}{\partial x} + \nu_d \frac{\partial \nu_d}{\partial y} &= Z_d \frac{\partial \phi}{\partial x}, & \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} &= Z_d n_d + n_e - n_i \end{aligned} \quad (1)$$

in which u_d and ν_d are velocity components of the dust particles in x and y -directions

and normalized by the dust acoustic speed $c_d = \sqrt{Z_{0d}T_i/m_d}$, where T_i is the temperature of ions, m_d is the mass of dust particles. n_d and ϕ are the dust number density and electrostatic potential that have been normalized by n_{0d} and T_i/e and e is the magnitude of the electron charge, respectively. n_e and n_i are the electron and ion number densities which are normalized by n_{0e} and n_{0i} , respectively. The space and time variables are normalized by the Debye length $\lambda_D = \sqrt{T_i/4\pi n_{0d}Z_{0d}e^2}$ and the inverse of dust plasma frequency $\omega_{pd}^{-1} = \sqrt{m_d/4\pi n_{0d}Z_{0d}^2e^2}$, respectively. Normalized number densities for Boltzmann distributed electrons and non-thermal distributed ions are

$$n_e = (\mu/1 - \mu) e^{\sigma_i\phi}, \quad n_i = (1/1 - \mu) \left[1 + \beta (\phi + \phi^2) \right] e^{-\phi} \quad (2)$$

where $\mu = n_{0e}/n_{0i}$, $\sigma_i = T_i/T_e$ and $\beta = 4\alpha/(1 + 3\alpha)$ in which α arises due to the effects of non-thermal ions. The electron and ion currents for spherical dust grains with radius r are [17]

$$\begin{aligned} I_e &= -e\pi r^2 \sqrt{8T_e/\pi m_e} \left(\frac{\mu}{1 - \mu} \right) e^{\sigma_i(\phi - \psi)} \\ I_i &= -e\pi r^2 \sqrt{8T_i/\pi m_i} \left(\frac{1}{1 - \mu} \right) \left(\frac{1}{1 + 3\alpha} \right) \\ &\quad \times \left\{ \left[\left(1 + \frac{24\alpha}{5} \right) + \frac{16\alpha}{3} \phi + 4\alpha\phi^2 \right] - \psi \left[\left(1 + \frac{8\alpha}{5} \right) + \frac{8\alpha}{3} \phi + 4\alpha\phi^2 \right] \right\} e^{-\phi} \end{aligned} \quad (3)$$

in which $\psi = ZQ_d$ denotes the dust grain surface potential relative to the plasma potential ϕ and $Q_d = q_d/z_d e$, where q_d is the dust charge. $z_d e$ is the magnitude of the equilibrium dust charge and $Z = z_d e^2/4\pi\epsilon_0 r T_e$ is the non-dimensional dusty plasma parameter. The term $4\pi\epsilon_0 r$ is the capacitance of the spherical dust grain with average radius r . By considering the only electron and ion currents due to collisions with plasma particles, the dust grain charging equation is given by

$$\partial Q_d / \partial t = (I_e + I_i) / z_d e. \quad (4)$$

If the thermal velocities of electrons and ions are larger than their streaming velocities [15,16] the charge-current balance equation reads [17]

$$(I_e + I_i)_{Q_d = -1, \phi = 0} = 0 \quad (5)$$

and we have

$$\sqrt{\sigma_i/\mu_i} \left(\frac{1}{1+3\alpha} \right) \left[\left\{ 1 + \frac{24\alpha}{5} + \frac{16\alpha}{3} \phi + 4\alpha\phi^2 \right\} - \psi \left\{ 1 + \frac{8\alpha}{5} + \frac{8\alpha}{3} \phi + 4\alpha\phi^2 \right\} \right] \times \exp(-\phi)(1-\psi) - \mu \exp[\sigma_i(\phi+\psi)] = 0 \quad (6)$$

where $\mu_i = m_i/m_e \cong 1840$. Z_d is defined as $Z_d = \frac{\psi}{\psi_0}$, where $\psi_0 = \psi(\phi=0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at an infinite region. By substituting $\phi=0$ into (5) we have

$$\sqrt{\sigma_i/\mu_i} \left(\frac{1}{1+3\alpha} \right) \left[1 + \frac{24\alpha}{5} - \psi_0 \left(1 + \frac{8\alpha}{5} \right) \right] (1-\psi_0) - \mu \exp(\sigma_i\psi_0) = 0 \quad (7)$$

Z_d can be expanded respect to ϕ as

$$Z_d = 1 + \gamma_1\phi + \gamma_2\phi^2 + \gamma_3\phi^3 + \dots \quad (8)$$

where $\gamma_1 \equiv \frac{\psi'_0}{\psi_0}$, $\gamma_2 \equiv \frac{\psi''_0}{2\psi_0}$ and $\gamma_3 \equiv \frac{1}{6} \frac{\psi'''_0}{\psi_0}$ come from expanding ψ near ψ_0 so we can write

$$\psi'_0 = \frac{\left[\frac{8\alpha}{15} - 1 - \psi_0 \right] (1-\psi_0) + \sigma_i \left[\left(1 + \frac{24\alpha}{5} \right) - \psi_0 \left(1 + \frac{8\alpha}{5} \right) \right]}{\left[\left(1 + \frac{24\alpha}{5} \right) - \psi_0 \left(1 + \frac{8\alpha}{5} \right) \right] (1-\sigma_i(1-\psi_0)) - (1-\psi_0) \left(1 + \frac{8\alpha}{5} \right)} \quad (9)$$

$$\psi''_0 = \frac{\left\{ \sigma_i(1+\psi_0)^2 \left[1 + \frac{24\alpha}{15} - \psi_0 \left(1 + \frac{8\alpha}{5} \right) \right] - \left[1 + \frac{32\alpha}{15} + 2 \left(1 + \frac{64\alpha}{15} \right) \psi'_0 - \left(1 + \frac{44\alpha}{15} \right) \psi_0 \right] \right\} (1-\psi_0)}{\left[\left(1 + \frac{24\alpha}{5} \right) - \psi_0 \left(1 + \frac{8\alpha}{5} \right) \right] (1-\sigma_i(1-\psi_0)) - (1-\psi_0) \left(1 + \frac{8\alpha}{5} \right)}$$

and

$$\psi'''_0 = \frac{A}{B},$$

where A and B are

$$A = \left\{ 2\sigma_i(1+\psi_0)^2 + 2\psi''_0 \left[1 + \sigma_i(1+\psi'_0) \right] \right\} \times$$

$$\left[1 + \frac{24\alpha}{5} - \psi_0 \left(1 + \frac{8\alpha}{5} \right) \right] (1-\psi_0) \sigma_i - \psi'''_0 \left[\frac{8\alpha}{5} - 1 + \psi_0 \left(1 - \frac{16\alpha}{5} \right) - \psi'_0 \left(1 + \frac{8\alpha}{5} \right) \right]$$

$$\begin{aligned}
& + (1 - \psi_0) \left\{ - \left(3 + \frac{372\alpha}{15} \right) + \left(1 + \frac{88\alpha}{5} \right) \psi_0 - \left(3 + \frac{104\alpha}{5} \right) \psi_0' + \left(3 - \frac{8\alpha}{15} \right) \psi_0'' \right\} \\
& - \psi_0' \left[\psi_0'' \left(1 + \frac{8\alpha}{5} \right) + 2\psi_0' \left(1 - \frac{16\alpha}{3} \right) + \psi_0 \left(1 + \frac{104\alpha}{5} \right) \right], \\
B = & \left[\left(1 + \frac{24\alpha}{5} \right) - \psi_0 \left(1 + \frac{8\alpha}{5} \right) \right] (1 - \sigma_i (1 - \psi_0)) - (1 - \psi_0) \left(1 + \frac{8\alpha}{5} \right)
\end{aligned} \tag{11}$$

3. Derivation of the KP equation

Let us study the behaviour of small amplitude dust-acoustic waves. The KP equation is obtained by using the reductive perturbation method. The stretched coordinates are defined by

$$\xi = \varepsilon(x - \lambda t), \quad \eta = \varepsilon^2 y \quad \text{and} \quad \tau = \varepsilon^2 t \tag{12}$$

where λ is the phase velocity of waves and ε is a small parameter which is characterizes the strength of the nonlinearity. Dependent variables are expanded as follows

$$\begin{aligned}
n_d &= 1 + \varepsilon^2 n_{1d} + \varepsilon^4 n_{2d} + \dots \\
u_d &= \varepsilon^2 u_{1d} + \varepsilon^4 u_{2d} + \dots \\
v_d &= \varepsilon^3 v_{1d} + \varepsilon^5 v_{2d} + \dots \\
\phi &= \varepsilon^2 \phi_1 + \varepsilon^4 \phi_2 + \dots \\
Z_d &= 1 + \varepsilon^2 Z_{1d} + \varepsilon^4 Z_{2d} + \varepsilon^6 Z_{3d} + \dots
\end{aligned} \tag{13}$$

By substituting (13) into (1) and collecting the terms in the different powers of ε we have

$$\begin{aligned}
N_{1d} &= -\frac{\phi_1}{\lambda^2}, \quad u_{1d} = -\frac{\phi_1}{\lambda}, \quad \frac{1}{\lambda^2} = \gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu}, \quad \lambda \frac{\partial v_{1d}}{\partial \xi} = \frac{-\partial \phi_1}{\partial \eta} \\
\frac{\partial^2 \phi_1}{\partial \xi^2} &= Z_2 + n_{2d} + Z_{1d} n_{1d} + \frac{1}{1 - \mu} [\sigma_i \mu + (1 - \beta)] \phi_2 + \frac{1}{2(1 - \mu)} (\mu\sigma_i^2 - 1) \phi_1^2 \\
\frac{\partial n_{1d}}{\partial \tau} - \lambda \frac{\partial n_{2d}}{\partial \xi} + \frac{\partial}{\partial \xi} (n_{1d} u_{1d} + u_{2d}) + \frac{\partial v_{1d}}{\partial \eta} &= 0 \\
\frac{\partial u_{1d}}{\partial \tau} - \lambda \frac{\partial u_{2d}}{\partial \xi} + u_{1d} \frac{\partial u_{1d}}{\partial \xi} &= \frac{\partial \phi_2}{\partial \xi} + Z_{1d} \frac{\partial \phi_1}{\partial \xi}.
\end{aligned} \tag{14}$$

Finally the KP equation is obtained

$$\frac{\partial}{\partial \xi} \left[\frac{\partial \phi_1}{\partial \tau} + a \phi_1 \frac{\partial \phi_1}{\partial \xi} + b \frac{\partial^3 \phi_1}{\partial \xi^3} \right] + c \frac{\partial^2 \phi_1}{\partial \eta^2} = 0. \tag{15}$$

Coefficients of non-linear and dispersion terms are

$$\begin{aligned}
 a = & - \left[\gamma_2 + \frac{\mu\sigma_i^2 - 1}{2(1-\mu)} \right] \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1-\mu} \right]^{-3/2} + 3\gamma_1 \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1-\mu} \right]^{-1/2} \\
 & - \frac{3}{2} \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1-\mu} \right]^{1/2} \\
 b = & \frac{1}{2} \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1-\mu} \right]^{-3/2}, \quad c = \frac{1}{2} \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1-\mu} \right]^{-1/2}
 \end{aligned} \tag{16}$$

Figures 1 show "a" as function of μ , α and σ_i . Figure 1a shows "a" as a function of μ and α with $\sigma_i = 0.3$. In this figure we can see that with a fixed values for μ , "a" increases when the value of non-thermal parameter (α) is decreased. Also "a" gets increased with an increasing μ when α is fixed. In Figure 1b α has been thaken 0.5.

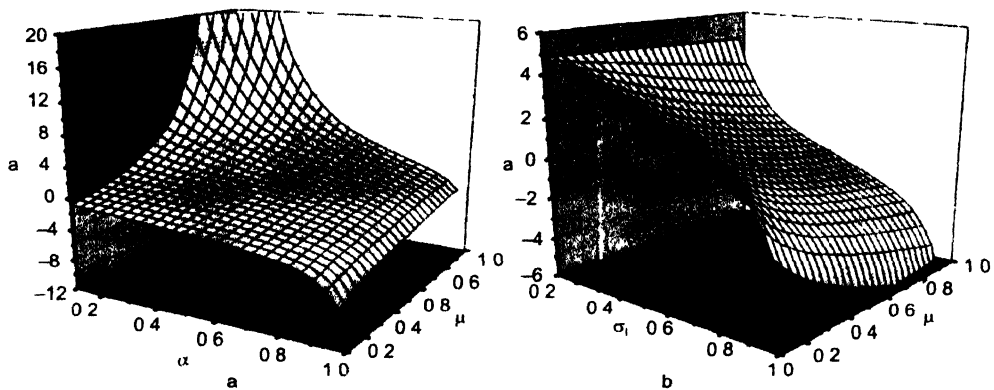


Figure 1. Parameter "a" as a function of α , μ and σ_i , (a) $\sigma_i = 0.3$ and (b) $\alpha = 0.5$.

In this Figure we can see that "a" decreases when σ_i is increased with fixed values for μ . In all of the Figures parameters γ_1 and γ_2 have been taken equal to zero. Figures 1 show that for some values of parameters, "a" is positive and for some other values it is negative. This means that both compressive and rarefactive solitons are available.

Figures 2 present the parameter "b" respect to μ , α and σ_i . In the b - μ plot $\sigma_i = 0.3$, while in the b - σ_i , $\alpha = 0.3$ and in the b - α plot $\sigma_i = 0.3$ has been selected. The γ_1 and γ_2 are zero in all the cases. Behaviour of the parameter "c" is the same as what we have seen for the parameter "b".

Figures show that dispersive parameter "b" decreases when μ is increased while α and σ_i are fixed. On the other hand "b" increases with an increasing non-thermal parameter α when other parameters ermine unchanged. But for high density situations which $0.6 < \mu < 1$ "b" approximately is independent of α . Thus the width of soliton increases when α increases.

Eq. (15) has solitonic solutions and one-soliton solution for this equation is given by [16]

$$\phi_1 = \phi_m \operatorname{sech} h^2 \frac{\chi}{w} \tag{17}$$

where $\chi = \xi + \eta - u\tau$ and soliton amplitude and width are

$$\phi_m = \frac{3(u-c)}{a} \quad w = 2\sqrt{\frac{b}{u-c}} \tag{18}$$

Figures 3 show the soliton profiles with different values for the parameters. We can see that both compressive and rarefactive kinds can be created.

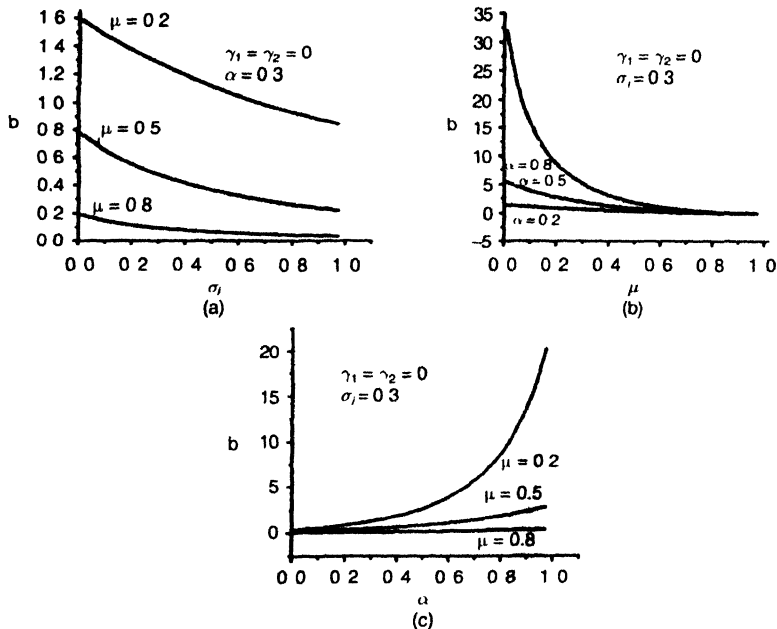


Figure 2. Parameter "b" as a function of μ , α and σ_i .

The change of the type of the soliton from rarefactive to compressive type is in agreement with the change of the parameter "a" with respect to medium parameters (α , σ_i and μ). As an example, the a - α curve in Figure 1 shows that for $\mu = 0.5$ and $0 < \alpha < 0.43$ the parameter "a" is negative. Thus we have rarefactive soliton in this region of parameters. Also for $a > 0.43$ "a" is positive and we expect to have compressive soliton. The ϕ - χ plots with different values of α in Figure 3 are in agreement with the results of Figures 1.

4. The modified KP equation

The strength of the non-linear term in KP equation depends on the value of parameter

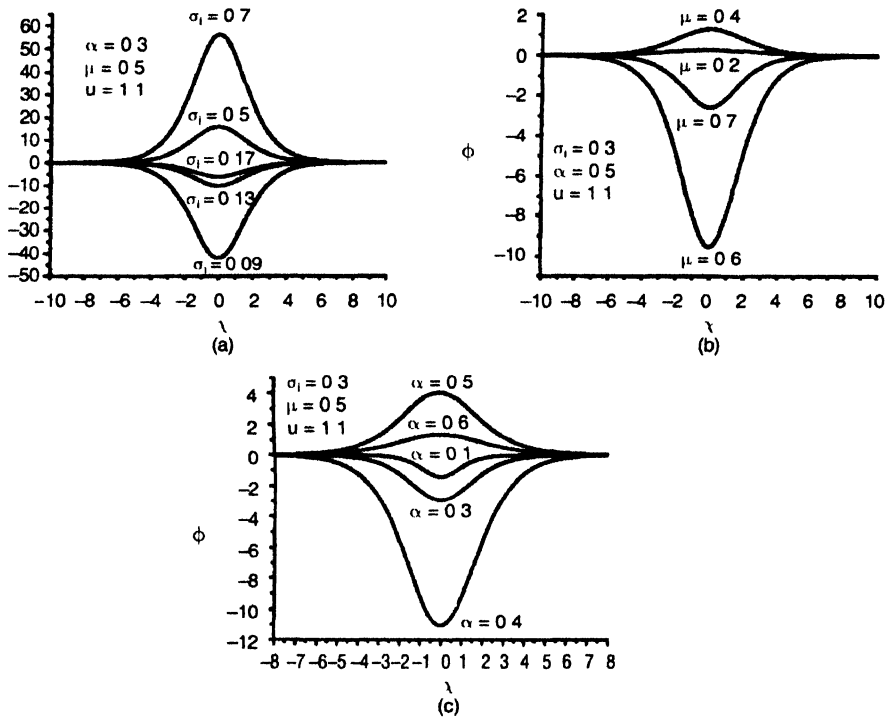


Figure 3. The soliton profiles of the KP equation for different values of parameter γ_1 and γ_2 are zero in all plots

“a” which is a function of $\mu, \beta, \sigma_i, \gamma_1$ and γ_2 . The dependency of “a” can be studied by plotting this quantity as a function of other parameters. By taking a specific value for the density (which is called critical density) it is possible that “a” becomes zero and thus ϕ_m increases to infinity. With the $\gamma_1 = \gamma_2 = 0$ the critical density is

$$\mu_c = \frac{6\sigma_i(1-\beta) + 1 + \sigma_i^2}{4\sigma_i'} \pm \left\{ \left[\frac{6\sigma_i(1-\beta) + 1 + \sigma_i^2}{4\sigma_i'} \right]^2 - 8\sigma_i^2(2 + 3\beta^2 - 6\beta) \right\}^{1/2} \quad (19)$$

In this case the stretching coordinate transformation is not valid. But we can save the equations by using a new set of parameters as follows

$$n_d = 1 + \varepsilon n_{1d} + \varepsilon^2 n_{2d} + \varepsilon^3 n_{3d} + \dots$$

$$u_d = \varepsilon u_{1d} + \varepsilon^2 u_{2d} + \varepsilon^3 u_{3d} + \dots$$

$$v_d = \varepsilon^2 v_{1d} + \varepsilon^3 v_{2d} + \varepsilon^4 v_{3d} + \dots$$

$$\phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \dots$$

$$Z_d = 1 + \varepsilon^2 \gamma_1 \phi_1 + \varepsilon^4 (\gamma_1 \phi_2 + \gamma_2 \phi_1) + \varepsilon^6 (\gamma_1 \phi_3 + 2\gamma_2 \phi_1 \phi_2 + \gamma_3 \phi_1^3) \quad (20)$$

Again by using (20) in the main eqs. (1) and collecting terms with the same powers of expanding parameters ε we find eq. (14) for the lowest order. But for higher orders

of ε we will have

$$n_{2d} = \frac{1}{2\lambda^2} \left(\frac{3}{\lambda^2} - \gamma_1 \right) \phi_1^2, \quad u_{2d} = \frac{1}{2\lambda} \left(\frac{1}{\lambda^2} - \gamma_1 \right) \phi_1^2 - \frac{\phi_2}{\lambda}$$

$$\frac{1}{\lambda^2} = \gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \quad (21)$$

$$\frac{\partial n_{1d}}{\partial \tau} - \lambda \frac{\partial n_{3d}}{\partial \xi} + \frac{\partial}{\partial \xi} (n_{1d}n_{2d} + n_{2d}u_{1d} + u_{3d}) \frac{\partial v_{1d}}{\partial \eta} = 0$$

$$\frac{\partial u_{1d}}{\partial \tau} + \frac{\partial}{\partial \xi} (u_{1d}u_{2d}) - \lambda \frac{\partial u_{3d}}{\partial \xi} = \frac{\partial \phi_3}{\partial \xi} + \gamma_1 \frac{\partial}{\partial \xi} (\phi_1\phi_2) + \frac{\gamma_2}{3} \frac{\partial}{\partial \xi} (\phi_1^3)$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = n_1 Z_2 + n_2 Z_1 + n_3 + Z_3 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \phi_3 + \frac{1}{6} \left(\frac{\mu\sigma_i^3 + 1 - 3\beta}{1 - \mu} \right) \phi_1^3 + \frac{\mu\sigma_i^2 - 1}{2(1 - \mu)} \phi_1\phi_2$$

and

$$-\lambda \frac{\partial v_1}{\partial \xi} = \frac{\partial \phi_1}{\partial \eta}.$$

Finally, we have the following equation

$$\frac{\partial}{\partial \xi} \left[\frac{\partial \phi_1}{\partial \tau} + A\phi_1^2 \frac{\partial \phi_1}{\partial \xi} + E \frac{\partial}{\partial \xi} (\phi_1\phi_2) + B \frac{\partial^3 \phi_1}{\partial \xi^3} \right] + C \frac{\partial^2 \phi_1}{\partial \eta^2} = 0 \quad (22)$$

where A , E , B and C are

$$A = \frac{1}{2} \left(\frac{4}{3} \gamma_2 + \frac{\gamma_1^2}{2} \right) \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{-1/2} - \frac{1}{2} \left[\gamma_3 + \frac{\mu\sigma_i^3 + 1 - 3\beta}{2(1 - \mu)} \right] \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{-3/2}$$

$$+ \frac{15}{4} \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{3/2} - \gamma_1 \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{1/2},$$

$$B = \frac{1}{2} \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{-3/2}, \quad C = \frac{1}{2} \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{-1/2}$$

$$E = - \left[\gamma_2 + \frac{\mu\sigma_i^2 - 1}{2(1 - \mu)} \right] \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{-3/2}$$

$$+ 3\gamma_1 \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{-1/2} - \frac{3}{2} \left[\gamma_1 + \frac{\mu\sigma_i + 1 - \beta}{1 - \mu} \right]^{1/2}. \quad (23)$$

For critical density (μ_c) "E" becomes zero and in this situation (22) reduces to the modified KP equation,

$$\frac{\partial}{\partial \xi} \frac{\partial \phi_1}{\partial \tau} + A \phi_1^2 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} + C \frac{\partial^2 \phi_1}{\partial \eta^2} = 0. \tag{24}$$

This equation has solitonic solutions. One soliton solution for this equation is [11,18]

$$\phi_1 = \pm \phi_m \operatorname{sech} \left[(\xi + \eta - u\tau) / W \right] \tag{25}$$

where u , $\phi_m = \sqrt{6(u-C)/A}$ and $W = \sqrt{B/(u-C)}$ are velocity, amplitude and width of solitary wave respectively. The above results for one dimensional propagation with $\gamma_1 =$

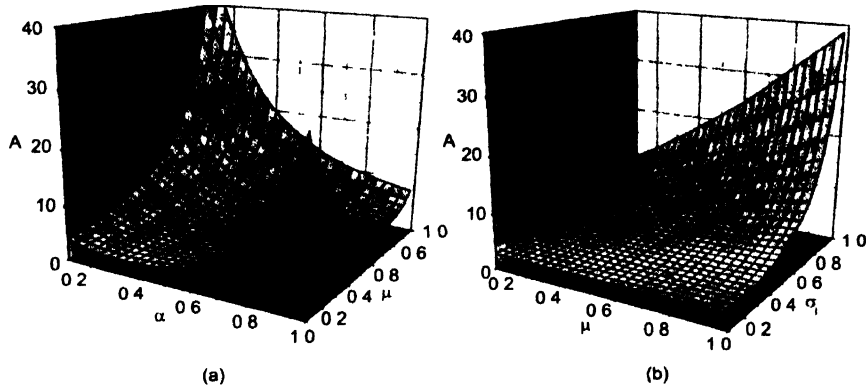


Figure 4. Parameter "A" as a function of μ , α and σ_1 , (a) $\sigma_1 = 0.3$ and (b) $\alpha = 0.3$. In Figures γ_1 , γ_2 and γ_3 are zero

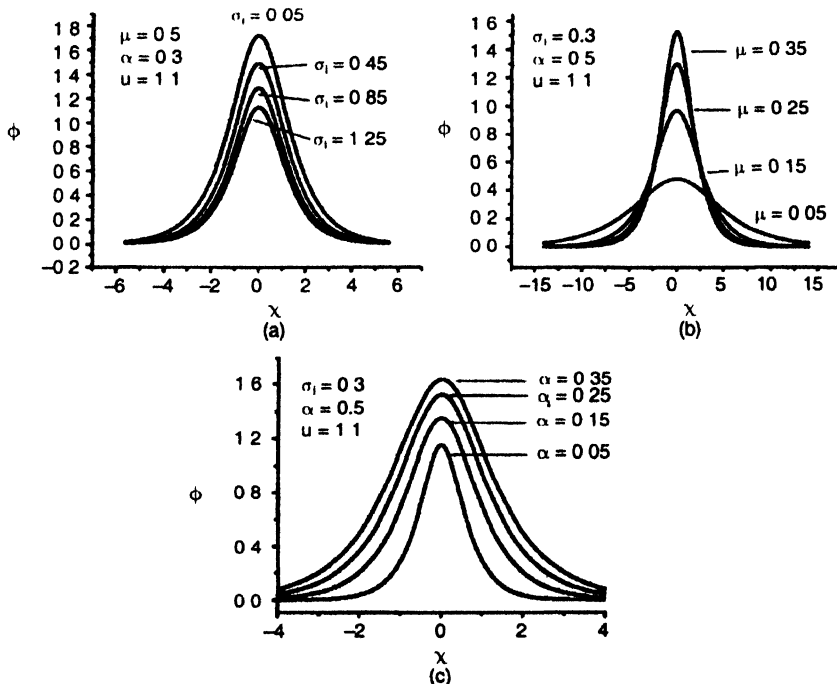


Figure 5. Soliton profiles of the mKP equation for different values of parameters. $\gamma_1 = \gamma_2 = \gamma_3 = 0$, in all the cases

$\gamma_2 = \gamma_3 = 0$ can be compared with results of reference [9]. Figures 4 show the coefficient "A" as functions of μ , α and σ_i . The γ_1 , γ_2 and γ_3 are all equal to zero in these Figures. It is clear that "A" is always positive.

Figures 5 show the soliton profiles with different values of the plasma parameters.

5. Conclusion

We studied small amplitude dust acoustic waves in dusty plasmas. We found the KP equation for a system of dusty plasma which contains Boltzmann distributed electrons and negative and variable charge of dust particles. The results which are presented in this section can be compared with what has appeared in [9] for the same system but with fixed charge for the dust particles and the results of [15] for warm plasma with the external static magnetic field. Also, the mentioned equations agree with those in reference [16].

Varying the amount of dust charge changes the strength of non-linear (parameter "a") and dispersive (parameter "b") terms [10,16,14,15]. A solitonic solution for the KP equation can not be found when the parameter "a" becomes zero. But in this case the KP equation changes to modified type of the KP equation. The mKP equation has stable solitonic solutions while the KP equation does not have any such solution for $a = 0$. The effects of non-thermal ions, density (μ) and temperature (σ_i) on the behaviour of the solitons have been discussed by numerical simulations.

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