Indian J. Phys. 81 (12), 1247-1256 (2007)

Theoretical analysis on the performance of a circular sector microstrip antenna under re-entry conditions

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Received 25 May 2007, accepted 5 September 2007

Abstract : Expressions of the field intensities for electromagnetic and electroacoustic modes for a circular sector microstrip patch antenna mounted on a re-entry vehicle are derived by applying linearized hydrodynamic theory coupled with vector function technique. The contribution of both electrons and ions is considered during the theoretical analysis of antenna is plasma medium. Radiation resistance, directivity, bandwidth, quality factor and radiation efficiency of antenna are obtained for different plasma parameter (ζ) values. A comparison between the radiation intensity of antenna under different plasma conditions (different ζ values) indicates that it decreases drastically on decreasing plasma parameter value. The overall performance of antenna is very poor when plasma parameter (ζ) approaches below 0.2 or in other words when plasma frequency is much higher than source frequency.

Keywords : Microstrip antenna, ion electron plasma, radiation parameters, re-entry plasma

PACS Nos. : 52.40.Fd; 84.40. Ba; 75 50 Gg

1. Introduction

The conformal microstrip antennas received much attention in recent times due to their small size, lightweight and low production cost on mass production. Many workers have reported the performance and application of these antennas in free space [1-2]. In addition to application in mobile handsets and communication appliances, these antennas may also be mounted on the surface of a space vehicle due to their conformal planar structure and hence these may be proved useful for high-speed space vehicles and missiles.

The effect of ionosphere plasma medium on the radiation properties of an Alford loop antenna onboard Aerial 3 satellite was observed by Gregory [3] and found quite significant

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effect of plasma medium when operating frequency of antenna was in MHz range. As soon as operating frequency of antenna approaches in GHz range (as in the case of patch geometries), the effect of ionospheric plasma on its radiation properties start loosing its significance since oscillation frequency of ionospheric plasma is also quite small (ω_p ~ 10^7 Hz) in comparison to the resonance frequency of antenna geometry. However during re-entry of space vehicle in earth atmosphere (80 – 100 Km. From earth surface), a high-density plasma sheath (~10^{13} per cc) completely masks the radiator mounted on a space vehicle. The corresponding plasma frequency of such sheath comprising of electrons and ions can be a few GHz, which is of the order of resonance frequency of antenna mounted on space vehicle. This high-density plasma sheath causes serious effect on the performance of a radiating structure. At a particular plasma frequency, even no communication between the satellite and ground station takes place for a smaller duration or in other words, a communication blackout for a smaller duration takes place.

Radiation properties of antenna geometries in electron plasma medium are of great interest for the workers and enough theoretical work on different geometries has been reported [4-6]. In the present communication, the theoretical analysis has been carried out in ion electron plasma medium ($T_e >> T_i$) by adopting the model used by Talekar and Rawat [7] for linear resonant antenna operating in ionospheric plasma medium *i.e.* presence of both electrons and ions in plasma medium are considered simultaneously during the analysis. The condition regarding charge densities of plasma medium is considered similar to that present under re-entry conditions. In actual situation, the plasma sheath under re-entry condition is inhomogeneous in all directions but for simplification of our problem, plasma is considered homogenous. The performance of considered antenna geometry in electron plasma model is compared with that in ion electron plasma model for better understanding.

2. Basic equations and assumptions regarding plasma

In the present communication, in comparison to earlier studies [4-6] a more realistic model of analysis in plasma medium is considered. In free space, the theoretical analysis has been carried out by applying cavity model based model expansion technique, which involves mode dependent study of patch antenna geometry. In plasma medium, the analysis has been carried out by applying hydrodynamic theory in conjunction with vector wave technique. Plasma medium is considered comprising of both electrons and ions with charge densities and collision frequencies similar to that available under re-entry conditions. The plasma medium is considered as warm, homogeneous, isotropic, loss-less, non-drifting continuum of electrons and singly charged ions. Presence of earth's magnetic field is disregarded and interaction between aperture portion of antenna and the plasma medium is considered during present analysis. Under these simplifying assumptions, the electromagnetic and plasma fields are uncoupled and are separated in two independent modes namely electromagnetic (EM) and electroacoustic (EA) modes.

The geometry and coordinate system of a planar circular sector microstrip antenna used for the present theoretical analysis has sector angle 'α' and patch radius 'a'. It is considered designed on a thin dielectric substrate of thickness h, relative substrate permittivity ε_r and loss tangent tan (δ) and is shown in Figure 1. The proposed geometry in free space is analyzed by applying cavity model based modal expansion technique because in comparison to other analysis techniques, it is simple in application and reliable

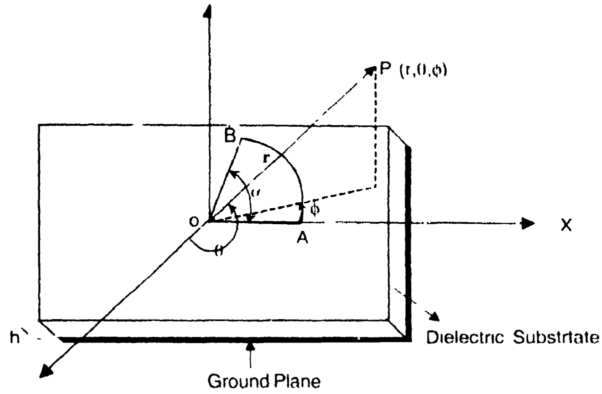

Figure 1. Geometry and co-ordinate system of circular sector microstrip antenna

results may be obtained. For this purpose, the solution of wave equation is obtained by finding its eigen function under specified boundary conditions. If its eigen function under specified conditions be ψ_{nm} and eigen value of K be K_{nm} then assuming eigen functions to be orthogonal, the solution of wave equation for E_z in the cavity with excitation current in z direction will be [2]

$$E_z = j\omega\mu \sum_m \sum_\nu \frac{1}{k^2 - k_{\nu m}^2} \frac{\langle J_z \psi_{\nu m}\rangle}{\langle \psi_{\nu m}\psi_{\nu m}\rangle} \psi_{\nu m} \tag{1}$$

Since the substrate thickness $h << \lambda_0$, the fields within the substrate do not vary along to the z-direction and component of current normal to the edge of antenna approaches to zero at the edges, present structure supports only TM_{nm} modes. With this assumption, antenna is considered as a circular resonator with magnetic sidewalls bounded at its top and bottom by electric walls. Many modal waves may get excited when a cable feeds such an antenna. A uniform current of effective width '$2w$' centered on the feed axis at a distance 'd' from the center of the patch is considered flowing from the ground plane to the patch. If $\psi_{\nu m}$ is the orthogonal eigen functions of the homogeneous wave equation given by

$$\psi_{\nu m} = J_\nu (k_{\nu m}\, \rho)\cos(\phi') \tag{2}$$

and $k_{\nu m}$ is the eigen values of wave number k in the dielectric substrate given by

$$k = k_0 \sqrt{\varepsilon_r (1 - j\tan(\delta))} \tag{3}$$

then following [7], the solution of wave equation for E_z in the cavity with excitation current J_z in z direction satisfying the boundary conditions will be

$$E_z = -j\omega\mu \sum_{\nu}\sum_{m} \frac{2J\frac{\sin(\nu w)}{\nu}\cos(\nu\phi'')J_{\nu}(k_{\nu m}d)J_{\nu}\left(k_{\nu m}\rho\right)\cos(\nu\phi')}{(k^2-k_{\nu m}^2)\frac{1}{4}\left[J_{\nu}\left(k_{\nu m}a\right)\right]^2\left(a^2-\frac{\nu^2}{k_{\nu m}^2}\right)\left(\alpha+\frac{\sin(2\nu\alpha)}{2\nu}\right)} \tag{4}$$

here $J_{\nu}\left(k_{\nu m}\rho\right)$ is the cylindrical function of first kind of Bessel's function of order ν and

$$\nu = \frac{n\pi}{\alpha} \tag{5}$$

Here k_{nm} is the m^{th} zero of the derivative of the Bessel function of order n and a_e is the effective radius of patch radiator [8] given by

$$a_e = \left[a^2+\left(\frac{2ha}{\pi\varepsilon_r}\right)\left(\ln\left(\frac{a}{2h}\right)+(1.41\varepsilon_r+1.77)+\frac{h}{a}(0.268\varepsilon_r+1.65)\right)\right]^{1/2} \tag{6}$$

The resonance frequency of antenna is determined by

$$f_r = \frac{\left(k_{\nu m}a\right)c}{2\pi a_e\sqrt{\varepsilon_d}}. \tag{7}$$

The dynamic dielectric constant ε_d is calculated following [8] with little changes as per geometry of structure. It is assumed that only aperture portion of antenna encounters the plasma medium and rest of structure is covered with a protective layer ($d \leq 1$mm). The fractional change in resonance frequency due to such dielectric covering is found negligibly small (~ 1%) at our selected resonance frequency *i.e.* 3.806 GHz, which does not cause any serious change in the performance of antenna.

For the analysis of antenna in EM mode, equivalence principal and image theory [9] are applied to find the components of equivalent magnetic current sources M_ϕ which represents outer M source associated with curved aperture and equivalent source associated linear apertures ($M_{\rho 1}$ and $M_{\rho 2}$) and hence expression of vector potential (F) are obtained. On substituting them in following equations,

$$E_\theta = -i\omega\frac{\eta_0}{\zeta}\left[F_\rho\sin(\phi'-\phi)+F_{\phi'}\cos(\phi'-\phi)\right] \tag{8}$$

$$E_\phi = +i\omega\frac{\eta_0}{\zeta}\left[F_\rho\cos(\theta)\cos(\phi'-\phi)-F_{\phi'}\cos(\theta)\sin(\phi'-\phi)\right] \tag{9}$$

far-field radiation patterns in $E\left(\phi = 0^0\right)$ plane and $H\left(\phi = 90^0\right)$ plane are obtained. Here η_0 is the wave impedance in free space and ζ is the plasma parameter given by:

$$\zeta = \sqrt{1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2}} \tag{10}$$

Here ω_{pe} and ω_{pi} are electron and ion plasma frequencies respectively. The total far field radiation pattern factors of antenna in EM mode are obtained by using the relation

$$R_{thEM}(\theta,\phi) = \quad |E_\theta|^2 + \quad \Big] = \omega^2 \frac{\eta_0^2}{\zeta^2}\left[F_\theta(\theta,\phi) + F_\phi(\theta,\phi)\right]^2 . \tag{11}$$

The free space results are obtained by substituting $\zeta = 1$ in the above expressions. The computed free space radiation patterns are in nice agreement with those obtained for the same geometry experimentally in free space [10].

Two sets of wave equation for electroacoustic modes are obtained by simplifying Maxwell equations, momentum transfer equations for electron and ion and mass transport equations for electron and ion:

$$\nabla^2 p_e + T_{11} p_e + T_{12} p_i \quad \frac{eN}{j\omega\varepsilon_0} \nabla.J \tag{12}$$

$$\nabla^2 p_i + T_{22} p_i + T_{21} p_e = -\frac{eN}{j\omega\varepsilon_0} \nabla.J \tag{13}$$

here

$$T_{11} = \omega^2 \quad 1 - \frac{\omega_{pe}}{\omega^2} \qquad T_{12} = \frac{\omega_{pi}^2}{u_i^2}$$

$$T_{21} = \frac{\omega_{pe}^2}{u_e^2} \qquad T_{22} = \frac{\omega^2}{u_i^2}\left(1 - \frac{\omega_{pi}^2}{\omega^2}\right)$$

The electromagnetic mode is a pure one but mutual coupling exists between two plasma modes *viz.* electron plasma mode and the ion plasma mode. These modes can be decoupled by introducing new pressures p_1 and p_2 defined by,

$$\begin{aligned} p_1 &= p_e + \alpha_1 p_i \\ p_2 &= p_e + \alpha_2 p_i \end{aligned} \tag{14}$$

Here,

$$p_e = \frac{(p_1 + p_2)}{2} - \frac{(\alpha_1 + \alpha_2)(p_1 - p_2)}{2\sqrt{(\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2}} \quad \text{and} \quad p_i = \frac{(p_1 - p_2)}{\sqrt{(\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2}}$$

α_1 and α_2 are chosen to be the roots of the quadratic equation

$$T_2\,\alpha_j^2 + (T_{11} - T_{22})\alpha_j - T_{12} = 0 \tag{15}$$

The decoupled wave equations then are

$$\left(\nabla^2 + \beta_{pj}^2\right)p_j = \frac{eN}{j\omega\varepsilon_0}\left(1 - \alpha_j\right)\nabla J \tag{16}$$

here j = 1,2 The wave numbers β_1 and β_2 are given by the roots of the quadratic equation

$$\beta_{pj}^4 - (T_{11} + T_{22})\beta_{pj}^2 + (T_{11}T_{22} - T_{12}T_{21}) = 0 \tag{17}$$

and are related to α_j by

$$\beta_{pj}^2 = T_{11} + T_{21}\alpha_j \tag{18}$$

The far field radiation pattern factor of antenna in electroacoustic mode (EA mode) are obtained following [11] *i e*

$$S_{thEA} \quad \frac{I}{Ne\zeta^2}\,\frac{\omega_{pe}^2}{\omega^2}\nabla p_e - \frac{\omega_{pi}^2}{\omega^2}\nabla p_i \tag{19}$$

By simplifying above wave equations in free space and plasma medium, expressions of radiation intensity, radiation conductance, bandwidth, directivity, quality factor and radiation efficiency of circular sector microstrip antenna geometry as a function of plasma parameter (ζ) are derived and computed separately in EM and EA modes

The EM mode far field patterns of a circular sector antenna in dominant TM_{11} mode of excitation are obtained in the *E* plane and *H* plane as a function of angle θ and are shown in Figures 2 and 3 respectively for different ζ values The relative variations in *H* – plane radiation intensities of this antenna in free space (ζ = 1 0) and in plasma medium (ζ = 0.2, 0 5) are shown in Figure 4. The normalized *H* plane EA mode radiation patterns are obtained by applying electron plasma model as well as electron ion plasma model and are shown in Figures 5a and 5b respectively for ζ = 0 5 in a reduced range of angle θ from 50 degree to 60 degree with 0 1° variation in θ values The power radiated by antenna in free space as well as in plasma medium is obtained by integrating the poynting vector over the hemisphere and is applied to obtain radiation conductance of antenna in free space as well as in plasma medium. Results are shown in Figure 6. The radiation efficiencies of antenna in EM mode and in EA mode are also obtained by applying electron plasma model as well as electron ion plasma model and are shown in figure 7. The total quality factor, directivity and bandwidth of this antenna are computed in EM mode only and are shown in Table 1.

3. Results and discussions

The normalized radiation patterns of a circular sector microstrip antenna are obtained in EM and EA modes for dominant mode of excitation of antenna for different ζ values and are shown in Figures 2 and 3 respectively. Both *E* and *H*-plane radiation patterns indicate a systematic variation in radiation intensity of antenna with angle θ in free space (ζ = 1) as well as in plasma medium. In both *E* and *H* plane radiation patterns, the radiation intensity in free space as well as in plasma medium is maximum normal to the patch of the geometry *i.e.* in θ = 0° direction. The *E* – plane patterns indicate that 3dB beam width of antenna in free space (ζ = 1.0) is 90^0 degree while in plasma medium it approaches to 100^0 for ζ = 0.5 and 110^0 degree for ζ = 0.2. The 3dB beam width in *H*-plane patterns in free space (ζ = 1) is 100^0 degree while in the plasma medium (both ζ = 0.5 and 0.2), these patterns are almost omni-directional. A comparison between H-plane radiation intensity of antenna in free space and in plasma medium is shown in Figure 4 which indicates that in comparison to free space, the radiation intensity of antenna in plasma medium (ζ = 0.5) is around 6dB down and around 14dB down with ζ = 0.2. H plane patterns of antenna drawn in EA mode are shown in

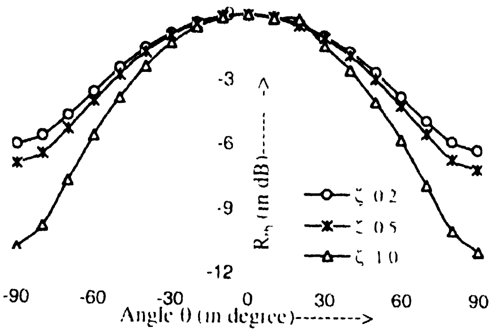

Figure 2. E- Plane radiation pattern for different values of ζ

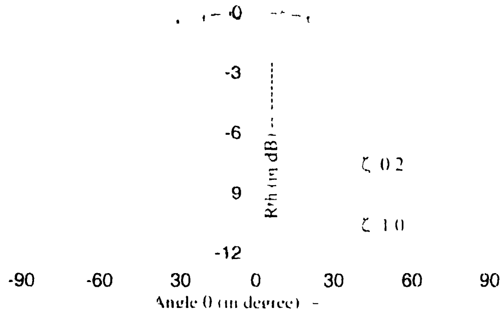

Figure 3. H- Plane radiation pattern for different values of ζ

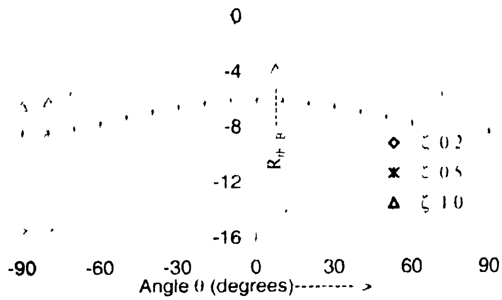

Figure 4. Variation of radiation intensity for different plasma factors in H-plane.

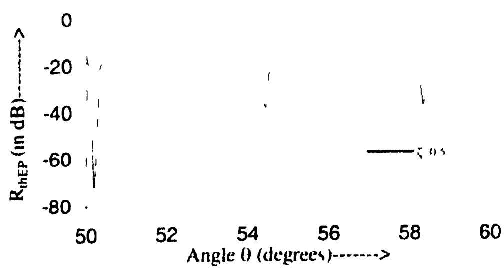

Figure 5(a). EA mode radiation pattern for H-plane at ζ = 0.5 by applying electron plasma model.

Table 1. Radiation parameters of antenna under different plasma conditions.

Plasma Parameter ζ	$_e/\omega$		Quality Factor Q_t	Band-width (%)	Directive Gain (in dB)
0.1	0.99	0.023	827.37	0.09	4.79
0.2	0.98	0.023	687.8	0.10	4.86
0.3	0.95	0.022	476.93	0.15	4.97
0.4	0.91	0.021	304.38	0.23	5.02
0.5	0.86	0.020	195.77	0.36	5.33
0.6	0.80	0.019	131.65	0.54	5.57
0.7	0.71	0.017	93.37	0.76	5.86
0.8	0.60	0.014	69.66	1.02	6.19
0.9	0.43	0.010	54.37	1.3	6.56
1.0	0	0	44.12	1.6	6.97

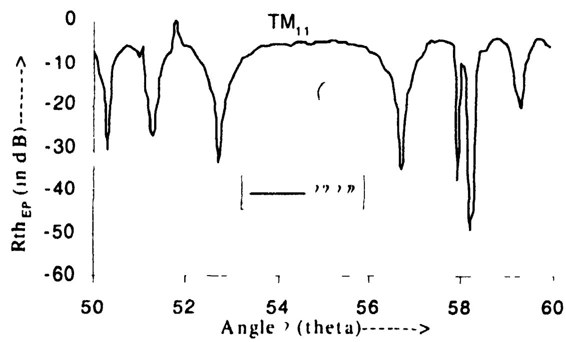

Figure 5(b). EA Mode radiation pattern for H-plane at = 0.5 by applying electron ion plasma model.

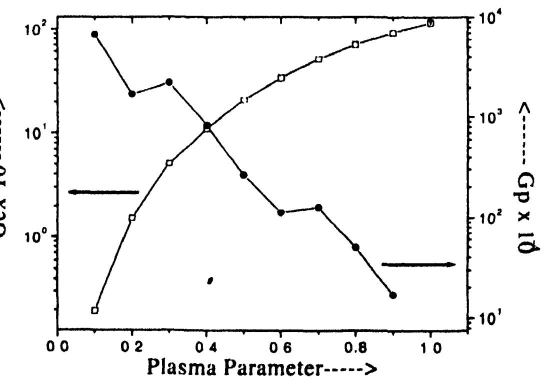

Figure 6. Variation of radiation conductance in EM and EA modes.

Figures 5a and 5b for electron plasma and electron ion plasma models respectively. Both these patterns present multi lobe pattern as shown in Figures 5a and 5b for a fixed ζ (= 0.5). It may be observed from H plane pattern obtained by applying electron ion model is relatively more discrete than that obtained by applying electron model for plasma medium.

The radiation conductance (G) and radiation efficiency (η) of circular sector antenna are computed in both EM and EA mode in dominant mode of excitation for different ζ values and are shown in Figures 6 and 7 respectively. As shown in Figure 6, EM mode radiation conductance is maximum in free space and decreases with decrease in plasma parameters. However reverse behaviour is found in EA mode. The normalized radiation efficiency of antenna, which is defined as the useful power in the EM mode to the total power radiated by an antenna is found to decrease with decrease in plasma parameter. The total power radiated by antenna includes both EM and EA mode powers The power radiated in EM mode is the useful power while the power in EA mode is in the form of noise and it increases with the increase in plasma parameter This increase in unwanted noise power decreases the radiation efficiency of antenna in plasma medium Figure 7 includes two curves drawn separately by considering electron plasma model and electron ion plasma model The radiation efficiency by considering electron ion model is little less than that obtained with electron plasma model In two-component plasma model, the effect of electron ion collision is considered which is ignored in electron plasma model It may be pointed out here that in both models, the performance of antenna in plasma medium becomes extremely poor as soon as ζ reduces below 0 2 This ζ value may be considered as the cutoff value for communication or a value at which communication blackout will take place under re-entry condition To avoid or minimize communication blackout under re-entry conditions, the source frequency must be sufficiently higher than the plasma frequency

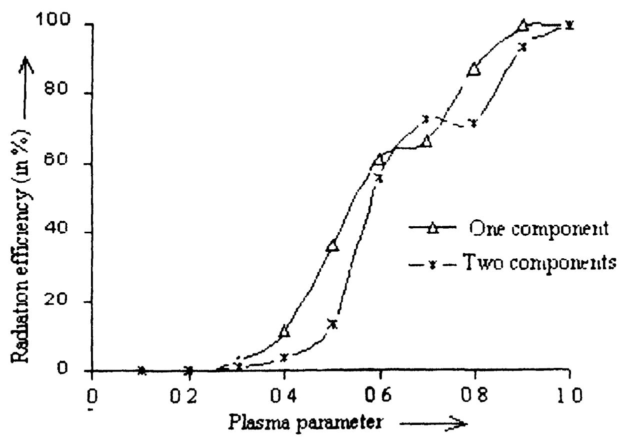

Figure 7. Variation of radiation efficiency with plasma parameter

The directive gain, quality factor and bandwidth of antenna in EM mode are presented in Table 1 for different ζ values of antenna and are found a sensitive function of plasma parameter ζ. Directive gain and bandwidth of antenna increases with increase in the value of plasma parameter and becomes maximum in free space Quality factor that decides the ability of antenna to radiate must be small In free space (ζ = 1) it is sufficiently small but approaches to a significantly higher value as plasma parameter approaches to 0.2.

4. Conclusions

The work reported in this paper indicates that the presence of plasma medium affects the performance of radiating antenna to a great extent The performance of antenna becomes poorer on decreasing the value of plasma parameter ζ and on approaching to a particular plasma condition (ζ = 0 2), the plasma oscillation frequency increases upto an extent that the charge density present around the antenna completely cutoff the communication between the radiator and the receiver. Antenna efficiency under this condition becomes extremely poor and quality factor attains a large value. Gain and bandwidth values of antenna also reduces significantly on decreasing the value of ζ. In the present communication, a more realistic approach regarding plasma conditions is considered i.e presence of both electrons and ions are taken into account for obtaining different theoretical results.

The work reported in this paper is of theoretical nature and requires experimental verification. An experimental setup showing conditions similar to that available under re-entry conditions (with charge density 10^{12} particles/cc) is underway at this center with the support of ISRO Bangalore and DST New Delhi. Antenna analyzed theoretically in this communication has been recently fabricated and tested in free space [10]. These antennas under re-entry conditions will be tested in due course of time.

Acknowledgments

Authors are thankful to ISRO, Banglore for the research project sanctioned to carry out the present work and to IPR Ahmedabad for the technical support. One of us (AK) is thankful to ISRO, Banglore for awarding research fellowship to carry out this work.

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