Research Article | Vol. 25, No. 25 | 11 Dec 2017 | OPTICS EXPRESS 31827
Optics EXPRESS

Quantum non-demolition measurement of photon number with atom-light interferometers

S. Y. CHEN,[1] L. Q. CHEN,[1,2,*] Z. Y. OU,[1,3] AND WEIPING ZHANG[2,4]

[1]*Quantum Institute for Light and Atoms, Department of Physics, East China Normal University, Shanghai 200062, China*
[2]*Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, China*
[3]*Department of Physics, Indiana University-Purdue University Indianapolis, 402 North Blackford Street, Indianapolis, Indiana 46202, USA*
[4]*Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*
[*]lqchen@phy.ecnu.edu.cn

Abstract: When atoms are illuminated by an off-resonant field, the AC Stark effect will lead to phase shifts in atomic states. The phase shifts are proportional to the photon number of the off-resonant illuminating field. By measuring the atomic phase with newly developed atom-light hybrid interferometers, we can achieve quantum non-demolition measurement of the photon number of the optical field. In this paper, we analyze theoretically the performance of this QND measurement scheme by using the QND measurement criteria established by Holland et al [Phys. Rev. A **42**, 2995 (1990)]. We find the quality of the QND measurement depends on the phase resolution of the atom-light hybrid interferometers. We apply this QND measurement scheme to a twin-photon state from parametric amplifier to verify the photon correlation in the twin beams. Furthermore, a sequential QND measurement procedure is analyzed for verifying the projection property of quantum measurement and for the quantum information tapping. Finally, we discuss the possibility for single-photon-number-resolving detection via QND measurement.

OCIS codes: (270.0270) Quantum optics; (030.5260) Photon counting; (120.3180) Interferometry; (190.4410) Nonlinear optics, parametric processes.

References and links

1. V. B. Braginsky, and F. Y. Khalili, "Quantum nondemolition measurements: the route from toys to tools," Rev. Mod. Phys. **68**(1), 1 (1996).
2. P. Grangier, J. A. Levenson, and J. P. Poizat, "Quantum non-demolition measurements in optics," Nature **396**(6711), 537-542 (1998).
3. B. V. Braginsky, Y. I. Vorontsov, and K. S. Thorne, "Quantum nondemolition measurements," Science **209**(4456), 547–557 (1980).
4. A. Kuzmich , L. Mandel, J. Janis, Y. E. Young, R. Ejnisman, and N.P. Bigelow,"Quantum nondemolition measurements of collective atomic spin," Phys. Rev. A **60**(3), 2346 (1999).
5. W. J. Munro, K. Nemoto, R. G. Beausoleil, and T. P. Spiller, "High-efficiency quantum-nondemolition single-photon-number-resolving detector," Phys. Rev. A **71**(3), 033819 (2005).
6. N. Imoto, H. A. Haus, Y. Yamamoto, "Quantum nondemolition measurement of the photon number via the optical Kerr effect," Phys. Rev. A **32**(4), 2287 (1985).
7. M. D. Levenson, R. M. Shelby, M. Reid, and D. F. Walls, "Quantum nondemolition detection of optical quadrature amplitudes," Phys. Rev. Lett. **57**(20), 2473 (1986).
8. S. R. Friberg, S. Machida, Y. Yamamoto, "Quantum-nondemolition measurement of the photon number of an optical soliton," Phys. Rev. Lett. **69**(22), 3165 (1992).
9. P. Kok, H. Lee, J. P. Dowling, "Single-photon quantum-nondemolition detectors constructed with linear optics and projective measurements," Phys. Rev. A **66**(6), 063814(2002).
10. G. J. Pryde, J. L. OâĂŹBrien, A. G. White, S. D. Bartlett, and T. C. Ralph, "Measuring a photonic qubit without destroying it," Phys. Rev. Lett. **92**(19), 190402 (2004).
11. G. J. Milburn, "Quantum optical Fredkin gate," Phys. Rev. Lett. **62**(18), 2124 (1989).
12. B. Fan, A. F. Kockum, J. Combes, G. Johansson, I.C. Hoi, C.M. Wilson, P. Delsing, G.J. Milburn, and T.M. Stace, "Breakdown of the cross-Kerr scheme for photon counting," Phys. Rev. Lett. **110**(5), 053601 (2013).
13. A. L. Porta, R. E. Slusher, B. Yurke, "Back-action evading measurements of an optical field using parametric down conversion," Phys. Rev. Lett. **62**(1), 28 (1989).
14. J. P. Poizat, P. Grangier, "Experimental realization of a quantum optical tap," Phys. Rev. Lett. **70**(3), 271 (1993).

#303285 https://doi.org/10.1364/OE.25.031827
Journal © 2018 Received 7 Aug 2017; revised 17 Nov 2017; accepted 27 Nov 2017; published 7 Dec 2017

Optics EXPRESS

15. S. F. Pereira, Z. Y. Ou, H. J. Kimble, "Backaction evading measurements for quantum nondemolition detection and quantum optical tapping," Phys. Rev. Lett. **72**(2), 214 (1994).
16. S. Haroche, "Nobel Lecture: Controlling photons in a box and exploring the quantum to classical boundary," Rev. Mod. Phys. **85**(3), 1083 (2013).
17. C. Guerlin, J. Bernu, S. Deleglise, C. Sayrin, S. Gleyzes, S. Kuhr, M. Brune, J.M. Raimond, and S. Haroche, "Progressive field-state collapse and quantum non-demolition photon counting," Nature **448**(7156), 889–893 (2007).
18. S. H. Autler, C. H. Townes, "Stark effect in rapidly varying fields," Phys. Rev. **100**(2), 703 (1955).
19. N. F. Ramsey, "A molecular beam resonance method with separated oscillating fields," Phys. Rev. **78**(6), 695 (1950).
20. S. A. Haine, W. Y. S. Lau, "Generation of atom-light entanglement in an optical cavity for quantum enhanced atom interferometry," Phys. Rev. A **93**(2), 023607(2016).
21. D. Linnemann, H. Strobel, W. Muessel, J. Schulz, R. J. Lewis-Swan, K. V. Kheruntsyan, and M. K. Oberthaler, "Quantum-enhanced sensing based on time reversal of nonlinear dynamics," Phys. Rev. Lett. **117**(1), 013001 (1950).
22. I. Kruse, K. Lange, J. Peise, B. LÃijcke, L. PezzÃÍ, J. Arlt, W. Ertmer, C. Lisdat, L. Santos, A. Smerzi, and C. Klempt, "Stark effect in rapidly varying fields," Phys. Rev. **100**(2), 703 (1955).
23. M. Gabbrielli, L. PezzÃÍ, and A. Smerzi, " Spin-mixing interferometry with Bose-Einstein condensates," Phys. Rev. Lett. **115**(16), 163002(2015).
24. W. J. Munro, K. Nemoto, R. G. Beausoleil, and T. P. Spiller, "High-efficiency quantum-nondemolition single-photon-number-resolving detector," Phys. Rev. A **71**(3), 033819 (2005).
25. B. Yurke, S. L. McCall, J. R. Klauder, "SU (2) and SU (1, 1) interferometers," Phys. Rev. A **33**(6), 4033 (1986).
26. J. Jacobson, G. BjÃűrk, Y. Yamamoto, "Quantum limit for the atom-light interferometer," Appl. Phys. B: Lasers and Optics, **60**(2), 187–191 (1995).
27. G. Campbell, M. Hosseini, B. M. Sparkes, P. K. Lam, and B. C. Buchler, "Time-and frequency-domain polariton interference," New J. Phys. **14**(3), 033022 (2012).
28. B. Chen, C. Qiu, S. Chen, J. Guo, L. Q. Chen, Z. Y. Ou, and W. Zhang, "Atom-light hybrid interferometer," Phys. Rev. Lett. **115**(4), 043602 (2015).
29. C. Qiu, S. Chen, L. Q. Chen, B. Chen, J. Guo, Z. Y. Ou, and W. Zhang, "AtomâĂŞlight superposition oscillation and Ramsey-like atomâĂŞlight interferometer," Optica **3**(7), 775-780 (2016).
30. M. J. Holland, M. J. Collett, D. F. Walls, and M. D. Levenson, "Nonideal quantum nondemolition measurements," Phys. Rev. A **42**(5), 2995 (1990).
31. S. S. Szigeti, R. J. Lewis-Swan, S. A. Haine, "Pumped-up SU (1, 1) interferometry," Phys. Rev. Lett. **118**(15), 150401 (2017).
32. H. Ma, D. Li, C. H. Yuan, L. Q. Chen, Z. Y. Ou, and W. Zhang, "SU (1, 1)-type light-atom-correlated interferometer," Phys. Rev. A **92**(2), 023847 (2015).
33. Z. D. Chen, C. H. Yuan, H. M. Ma, D. Li, L. Q. Chen, Z. Y. Ou, and W. Zhang, "Effects of losses in the atom-light hybrid SU (1, 1) interferometer," Opt. Express **24**(16), 17766–17778(2016).
34. J. Jing, C. Liu, Z. Zhou, Z.Y. Ou, and W. Zhang, "Realization of a nonlinear interferometer with parametric amplifiers," Appl. Phys. Lett. **99**(1), 011110 (2011).
35. Z. Y. Ou, "Enhancement of the phase-measurement sensitivity beyond the standard quantum limit by a nonlinear interferometer," Phys. Rev. A **85**(2), 023815 (2012).
36. F. Hudelist, J. Kong, C. Liu, J. Jing, Z. Y. Ou, and W. Zhang, "Quantum metrology with parametric amplifier-based photon correlation interferometers," Nat. Commun. **5**, 3049 (2014).
37. Z. Y. Ou, "Efficient conversion between photons and between photon and atom by stimulated emission," Phys. Rev. A **78**(2), 023819 (2008).
38. C. M. Caves, "Quantum-mechanical noise in an interferometer," Phys. Rev. D **23**(8), 1693 (1981).
39. M. D. Reid, "Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification," Phys. Rev. A **40**(2), 913 (1989).
40. A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and G. Camy, "Observation of quantum noise reduction on twin laser beams," Phys. Rev. Lett. **59**(22), 2555 (1987).
41. O. AytÃijr, P. Kumar, "Pulsed twin beams of light," Phys. Rev. Lett. **65**(13), 1551 (1990).
42. L. Q. Chen, C. Bian, G. W. Zhang, Z. Y. Ou, and W. Zhang, "Observation of temporal beating in first-and second-order intensity measurement between independent Raman Stokes fields in atomic vapor," Phys. Rev. A**82**(3), 033832 (2010).
43. J. Geng, G. T. Campbell, J. Bernu, D. B. Higginbottom, B. M. Sparkes, S. M. Assad, W. P. Zhang, N.P. Robins, P. K. Lam, and B. C. Buchler, "Electromagnetically induced transparency and four-wave mixing in a cold atomic ensemble with large optical depth," New J. Phys. **16**(11), 113053 (2014).
44. J. F. Roch, K. Vigneron, P. Grelu, A. Sinatra, J. P. Poizat, and P. Grangier, "Quantum Nondemolition Measurements using Cold Trapped Atoms," Phys. Rev. Lett. **78**(78), 634-637 (1997).
45. D. Kruse, M. Ruder, J. Benhelm, C. V. Cube, C. Zimmermann, P. W. Courteille, T. ElsÃďsser, B. Nagorny, and A. Hemmerich, "Cold atoms in a high-Q ring cavity," Phys. Rev. A **67**(5), 051802 (2003).

Optics EXPRESS

1. Introduction

Quantum nondemolition (QND) measurement [1,2] is a special type of measurement of a quantum system in which the physical observable to be measured is not altered in the measurement process. It requires the coupling of the system to be measured to another so-called meter system for easy reading out and in the meantime must evade the back action of the measurement [3,4]. In optical regime, commonly used scheme is the cross phase modulation through Kerr nonlinear interaction between optical fields [5–10]. When working at single-photon level, it was suggested as a control NOT-gate for quantum information processing [11]. However, recent study [12] casts doubt on the validity of Kerr interaction at single-photon level. Other systems were also considered for QND measurement [13–15].

When working in atomic ensemble, QND measurement in microwave regime had tremendous success [16,17] where AC Stark effect [18] was used for coupling photon number to atomic phase shift and Ramsey interferometer [19–23] was used for precise atomic phase measurement. In optical system, AC Stark effect was suggested for QND measurement of photon number by using EIT method for atomic phase shift measurement [24]. However, the on-resonant EIT method can not work for the short and off-resonant optical probe pulse. And the EIT process involves a strong coupling field interacting simultaneously with a weak probe field at few-photon level. Thus it will be difficult to separate the weak probe field from the strong coupling field and measure the weak probe field with low loss and low noise. Recently a new type of hybrid interferometers involving both atomic ensemble and light was demonstrated [25–28], and attracts lots of attention. This type of interferometers is based on Raman interaction between atoms and light, and is sensitive to both optical and atomic phase shift, so can be used to measure the atomic AC Stark phase shift induced by an optical probe field. Compared with the EIT scheme, the QND measurement based on the hybrid interferometers can work for short and off-resonant probe pulse. In this paper, we investigate theoretically the QND measurement scheme using such hybrid interferometers. We will use the QND measurement criteria derived by Holland et al [30] to assess the quality of this measurement scheme. A sequential measurement scheme involving multiple QND measurement processes is considered to verify the projection property of quantum measurement process.

The paper is organized as follows. In Sect.(2), we first introduce a quantum measurement scheme of photon number via AC Stark effect, and QND measurement of photon number by the atom-light interferometers. Then in Sect.(3), we discuss the case of non-ideal QND measurement and apply the QND criteria to the QND schemes with atom-light interferometers. In Sect.(4), we apply the QND measurement scheme on the photon number correlated state of twin beam. In Sect.(5), we discuss the sequential QND problem and apply it to quantum information tapping. We conclude in Sect.(6) with a summary and a discussion on the possiblility for single-photon-number-resolving detection via this QND measurement.

2. QND measurement of photon number

2.1. Atomic phase shift by AC Stark effect

It is well-known that when atoms are subject to the illumination of an electromagnetic field (denoted here as the probe field), their energy levels will be shifted [18]. When the frequency of the probe field is near an atomic transition but is far off resonance with a large detuning Δ, the interaction between atom and light will lead to an AC Stark shift in atomic level without atomic absorption:

$$\Delta\omega_{AC} = \Omega_R^2/\Delta, \tag{1}$$

Here $\Omega_R = |\mu_{ge}|E_{rms}$ is the Rabi frequency for the interaction between the atom and light with μ_{ge} as the dipole moment of the atomic transition and E_{rms} as the rms value of the electric field of the electromagnetic field. Since the optical field intensity is $I = cn\epsilon_0|E_{rms}|^2$, the AC Stark

Optics EXPRESS

shift is then proportional to the optical field intensity. Furthermore for a time period of ΔT, the atomic state will accumulate a phase of

$$\varphi_{AC} = \Delta\omega_{AC}\Delta T = \frac{|\mu_{ge}|^2 I \Delta T}{cn\epsilon_0 \Delta} = \frac{|\mu_{ge}|^2}{cn\epsilon_0 \Delta}\frac{P\Delta T}{A} = \frac{|\mu_{ge}|^2}{cn\epsilon_0 \Delta}\frac{N_p}{A} \equiv \kappa N_p, \tag{2}$$

with $\kappa \equiv |\mu_{ge}|^2/cn\epsilon_0 A\Delta$ is the AC Stark effect coefficient. Here n is the index of refraction of the atomic medium, P is the power of the electromagnetic field (probe field), A is the cross section of the field, and N_p is the total number of photons illuminating the atoms. So the atomic phase is dependent on the photon number of probe field. If we can measure the atomic phase shift, we will be able to measure the photon number of the probe light. The atomic phase can be measured by a new type of interferometers, which were recently demonstrated [27–29]. This type of interferometers is an atom-light hybrid interferometer that involves both atoms and light in the interference and is sensitive to the phases of both atoms and light. Furthermore, since the probe light field is far off-resonant, the absorption effect on the probe light is negligible. Therefore, the photon number of the probe field is not changed in the process. If the atomic phase can be measured with enough precision, we can achieve a quantum non-demolition measurement (QND) of the photon number of the probe field.

2.2. QND measurement of photon number by atom-light interferometers

Schematic of QND measurement with atom-light interferometers is given in Fig. 1(a). There are two types of atom-light interferometers [28, 29, 32, 33]. The first type is an analog of Mach-Zehnder/Michaelson optical interferometer but with atom-light mixer replacing regular beam splitters for linear superposition of atomic waves and optical waves [27, 29]. We call it linear atom-light interferometer, whose laser frequencies are shown in Fig. 1(b). The second type is the newly developed SU(1,1) nonlinear interferometer [21–23, 25, 28, 29, 31–37]. This type of interferometer utilizes parametric amplifiers to replace regular beam splitters for wave splitting and recombination. We call it nonlinear atom-light interferometer, whose laser frequencies are shown in Fig. 1(c). Both interferometers can be used to measure the phase information. The difference is that the interference intensity of linear interferometer depends on the phase difference and some common phase fluctuations in two interference beams can be canceled, but SU(1,1) interferometer depends on phase sum, common fluctuations can not be canceled. So phase locking on optical beam or magnetic shielding on atomic beam are needed in experiment. However, in theory, SU(1,1) nonlinear interferometers are more sensitive than the traditional linear interferometers with the same phase-sensing intensity [35] due to the noise reduction coming from the quantum correlation between two interference beams. The improvement of $4.1dB$ in signal-to-noise ratio has been demonstrated in atomic vapor under the same phase-sensing intensity with $60uW$ [36]. Since both types can be used for atomic phase measurement, we will analyze them in the following.

QND measurement schemes with two atom-light interferometers are similar. The atomic ensemble by interaction with a strong coherent pump field acts like a beam splitter and mixer for superposition of the atomic spin wave and optical wave. After the first interaction for wave splitting, a superposition between the atomic and optical waves is created and the optical wave travels out of the atomic ensemble but the atomic spin wave stays. After a delay through a fiber, the optical wave returns and mixes with the atomic wave by a second interaction to complete the interferometer, as in Fig. 1(a). During the delay, a probe field illuminates the atoms to induce an atomic phase shift which is proportional to the photon number of the probe field. Measurement of the atomic phase shift by the atom-light interferometers leads to the QND measurement of the photon number of the probe field.

For a linear atom-light interferometer as shown in Fig. 1 (b), we can treat it like a regular Mach-Zehnder interferometer but with one of the optical fields replaced by an atomic spin wave.

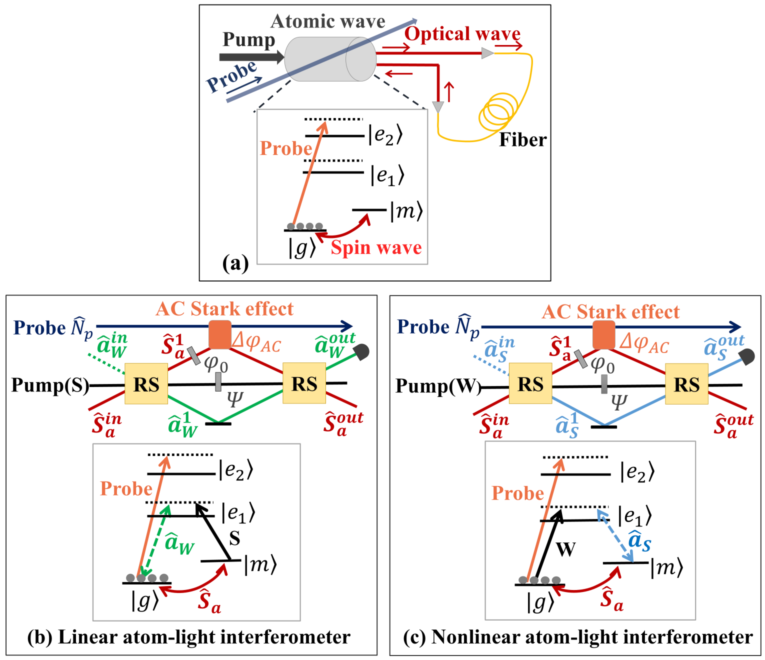

Fig. 1. (a) Schematic of QND measurement with atom-light interferometers. $|g\rangle$ and $|m\rangle$ are two ground state levels; $|e_1\rangle$ and $|e_2\rangle$ are two excited state levels. Atoms are initially prepared in the $|g\rangle$ ground state.The laser frequency of each optical field and corresponding signatures of pump, optical and atomic fields are given in (b) and (c). (b) In the linear interferometer, the strong pump field labeled as S, couples the state $|m\rangle$ and $|e_1\rangle$, and generates an optical signal field $\hat{a}_W^1$ and atomic spin wave $\hat{S}_a^1$, as two beams of the interferometer. The RS is shorted for Raman system. (c) In the nonlinear interferometer, the strong pump field labeled as W, couples $|g\rangle$ and $|e_1\rangle$ and generates an optical signal $\hat{a}_S^1$ and atomic spin wave $\hat{S}_a^1$. Between the two Raman processes in the atom-light interferometers, the atomic spin wave experiences a phase modulatiion $\Delta\varphi_{AC}$ via the AC Stark effect by probe field $\hat{N}_p$.

The input-output relation is given by

$$\begin{aligned} \hat{a}_W^{out} &= \hat{a}_W^{in} \cos\varphi/2 + \hat{S}_a^{in} \sin\varphi/2 \\ \hat{S}_a^{out} &= \hat{S}_a^{in} \cos\varphi/2 - \hat{a}_W^{in} \sin\varphi/2, \end{aligned} \tag{3}$$

where the optical signal field $\hat{a}_W$ and the atomic spin wave $\hat{S}_a$ are mixed via Raman processes [35, 37]. φ is the overall phase shift difference between the two beams of the interferometer. When the atoms are illuminated by another field, denoted as the probe field, an extra phase shift is introduced on atomic spin wave: $\varphi = \varphi_0 + \Delta\varphi_{AC}$ with $\Delta\varphi_{AC} = \kappa\hat{N}_p$ caused by AC Stark effect as described in Sect.(2.1). The input to the atom-light interferometer is chosen to be at the atomic spin wave: $\langle\hat{S}_a^{in}\rangle \neq 0$. We assume it is in a coherent state $|\alpha\rangle$: $\hat{S}_a^{in}|\alpha\rangle = \alpha|\alpha\rangle$. We will measure the photon number $\hat{N}_W$ of the output optical signal field $\hat{a}_W^{out}$ as our meter quantity for the QND measurement of probe field photon number $\hat{N}_p$:

$$\begin{aligned} \hat{N}_W &= \hat{a}_W^{out\dagger}\hat{a}_W^{out} \\ &= \hat{a}_W^{in\dagger}\hat{a}_W^{in}(1+\cos\varphi)/2 + \hat{S}_a^{in\dagger}\hat{S}_a^{in}(1-\cos\varphi)/2 \\ &\quad + \sin\varphi(\hat{a}_W^{in\dagger}\hat{S}_a^{in} + h.c.)/2. \end{aligned} \tag{4}$$

The interferometer is most sensitive to the phase change at $\varphi = \pi/2$. So we set $\varphi_0 = \pi/2$ and assume $\Delta\varphi_{AC} << \pi/2$. Making Taylor expansion around $\varphi_0 = \pi/2$, equation (4) can be approximated as

$$\hat{N}_W \approx \hat{S}_a^{in\dagger}\hat{S}_a^{in}(1+\Delta\varphi_{AC})/2 + (\hat{a}_W^{in\dagger}\hat{S}_a^{in} + h.c.)/2$$
$$= \hat{S}_a^{in\dagger}\hat{S}_a^{in}(1+\kappa\hat{N}_p)/2 + (\hat{a}_W^{in\dagger}\hat{S}_a^{in} + h.c.)/2. \quad (5)$$

Here, since there is no input in $\hat{a}_W^{in}$ field, we can assume it is in vacuum and drop those terms that only involve this field. The above relation describes how the QND observable, which is the photon number $\hat{N}_p$ of the probe field, is coupled to the meter observable, which is the photon number $\hat{N}_W$ of the output optical signal field, for measurement.

The general theoretical analysis for an SU(1,1) nonlinear interferometer is presented in reference [35]. For an atom-light SU(1,1) interferometer, we use Raman amplifiers to replace parametric amplifiers as shown in Fig. 1(c). So, we just need to replace the idler field by the atomic spin field $\hat{S}_a$ and the signal field by the field $\hat{a}_S$, then the input and output relation is given by

$$\hat{a}_S^{out} = (G^2e^{i\varphi}+g^2)\hat{a}_S^{in} + Gg(1+e^{i\varphi})\hat{S}_a^{in\dagger} = G_T(\varphi)\hat{a}_S^{in} + g_T(\varphi)\hat{S}_a^{in\dagger},$$
$$\hat{S}_a^{out} = Gg(1+e^{i\varphi})\hat{a}_S^{in\dagger} + (G^2e^{i\varphi}+g^2)\hat{S}_a^{in} = g_T(\varphi)\hat{a}_S^{in\dagger} + G_T(\varphi)\hat{S}_a^{in}, \quad (6)$$

where we assume that the amplitude gains for the two Raman amplifiers are the same and are labeled as G and $g^2 = G^2 - 1$. The combined phase sensitive gains of the two Raman amplifiers are $G_T(\varphi) = G^2e^{i\varphi} + g^2$ and $g_T(\varphi) = Gg(1+e^{i\varphi})$. Here the phase is $\varphi = \varphi_0 + \Delta\varphi_{AC}$ with φ_0 as the overall phase of the interferometer and $\Delta\varphi_{AC} = \kappa\hat{N}_p$ as the induced phase shift by the probe field.

As shown in reference [35], the best sensitivity for SU(1,1) interferometer occurs at the dark fringe. So we set $\varphi_0 = \pi$ for the overall gain G_T near unity and g_T near zero. From equation(6), we find the information about the probe light in optical signal output $\hat{a}_S^{out}$ and atomic spin wave output $\hat{S}_a^{out}$ are of the same form, so we make observation at the output optical field $\hat{a}_S^{out}$ only. If the induced phase shift $\Delta\varphi_{AC} << \pi$, equation (6) can be approximated as

$$\hat{a}_S^{out} = -(1+iG^2\Delta\varphi_{AC})\hat{a}_S^{in} - iGg\Delta\varphi_{AC}\hat{S}_a^{in\dagger}. \quad (7)$$

As shown in reference [35], in a SU(1,1) interferometer, we measure the quadrature amplitude for precision phase measurement. So, the meter quantity is then

$$\hat{X}_S^{out} = \hat{a}_S^{out} + \hat{a}_S^{out\dagger}$$
$$= -X_S^{in} + G^2\Delta\varphi_{AC}\hat{Y}_S^{in} - Gg\Delta\varphi_{AC}\hat{Y}_a^{in}$$
$$= -X_S^{in} + \kappa G\hat{N}_p(G\hat{Y}_S^{in} - g\hat{Y}_a^{in}), \quad (8)$$

with $X_S^{in} = \hat{a}_S^{in} + \hat{a}_S^{in\dagger}, \hat{Y}_{(S,a)}^{in} \equiv (\hat{a}_{(S,a)}^{in} - \hat{a}_{(S,a)}^{in\dagger})/i$ as the quadrature amplitude and phase.

2.3. Signal-to-noise ratio analysis

Next, let us examine the performance of the QND measurement process by signal-to-noise analysis. For this, we place the probe field in a number state $|n_p\rangle$, which is the eigen-state of the measurement process: $\hat{N}_p|n_p\rangle = n_p|n_p\rangle$, so that there is no uncertainty in the quantity to be measured. We will calculate the signal-to-noise ratio (SNR) of the meter output. This provides some indication of how good the QND measurement is. We start with the linear atom-light interferometer.

2.3.1. Linear interferometer

Assume the input to the linear atom-light interferometer is like that the atomic spin wave beam $\hat{S}_a^{in}$ is in a coherent state and the optical field beam $\hat{a}_W^{in}$ is in vacuum. It is straightforward to find from equation (5) that:

$$\langle\hat{N}_W\rangle = |\alpha|^2(1+\kappa n_p)/2. \quad (9)$$

Optics EXPRESS

So, the signal part extracted from the phase variation is $\langle \hat{N}_W \rangle_s = |\alpha|^2 \kappa n_p/2$, with $\langle \hat{a}_S^{in\dagger} \hat{a}_S^{in} \rangle = |\alpha|^2$ as the intensity of the input spin wave. The noise is

$$\langle \Delta^2 \hat{N}_W \rangle = |\alpha|^2 (1 + \kappa n_p)^2/4 + |\alpha|^2/4 \approx |\alpha|^2/2. \tag{10}$$

The approximation in the last equation is because we usually have $\kappa n_p << 1$. So the SNR is

$$R_L = \langle \hat{N}_W \rangle_s^2 / \langle \Delta^2 \hat{N}_W \rangle = |\alpha|^2 \kappa^2 n_p^2/2 = \kappa^2 n_p^2 I_{ps}, \tag{11}$$

where $I_{ps} \equiv |\alpha|^2/2$ is the phase sensing photon number inside the interferometer. To have single-photon resolution, we need AC effect interaction coefficient $\kappa \sim 1/\sqrt{I_{ps}} = \delta_{SQL}$. Here, δ_{SQL} is the standard quantum limit of phase measurement with a linear interferometer [38].

2.3.2. Nonlinear interferometer

Next, for the SU(1,1) interferometer with the probe field in a number state $\hat{N}_p|n_p\rangle = n_p|n_p\rangle$ and the atomic spin wave input field in a coherent state and the input optical field in vacuum, the meter quantity $\hat{X}_S$ in equation (8) has an expectation value of

$$\langle \hat{X}_S^{out} \rangle = -Gg\kappa n_p \langle \hat{Y}_a^{in} \rangle, \tag{12}$$

and the fluctuation of

$$\langle \Delta^2 \hat{X}_S^{out} \rangle = 1 + G^2(G^2 + g^2)\kappa^2 n_p^2. \tag{13}$$

Here the first term is from the vacuum noise of the input optical field $\hat{a}_S^{in}$ and the second term is from the coupling between atom and light. Usually, κ is very small so that most of the noise is from input vacuum noise. So the signal-to-noise ratio is

$$\begin{aligned} R &= \frac{|\langle \hat{X}_S^{out} \rangle|^2}{\langle \Delta^2 \hat{X}_S^{out} \rangle} = \frac{4G^2 g^2 \kappa^2 n_p^2 |\alpha|^2}{1 + G^2(G^2 + g^2)\kappa^2 n_p^2} \\ &\approx 4G^2 g^2 \kappa^2 n_p^2 |\alpha|^2 = 4G^2 \kappa^2 n_p^2 I_{ps}. \end{aligned} \tag{14}$$

Here, $I_{ps} \equiv g^2|\alpha|^2$ is the phase sensing photon number inside the SU(1,1) interferometer. For single-photon resolution, we need $4G^2\kappa^2 I_{ps} \sim 1$ or $\kappa \sim 1/2G\sqrt{I_{ps}} = \delta_{SQL}/2G$. This value is smaller by a factor of $1/2G$ than the one required with the linear atom-light interferometer. Thus, the SU(1,1) interferometer has the advantage of using lower photon number inside the interferometer to reach the same phase measurement sensitivity [35, 36].

3. Nonideal QND measurement criteria

As we have seen in the previous section, even with the probe field in the eigen-state (number state) of the measurement process, the meter output has fluctuation and does not yield a definite value. This leads to a non-ideal QND measurement. For this kind of measurement, Holland et al [30] derived a set of three criteria to test how close the measurement scheme is to an ideal QND measurement. The first one is the non-demolition criterion: how much the measurement scheme degrades the signal to be measured. Because we work with far off-resonant probe field, there is no loss for the photon number of the probe field and thus the criterion for non-demolition is satisfied: $\hat{N}_p^{out} = \hat{N}_p^{in}$. The second one is about how good the measurement scheme is a measurement device. For this, we examine the correlation between the input signal S_{in} and meter output M

$$C_{S_{in},M} \equiv \frac{|\langle \Delta S_{in} \Delta M \rangle|}{\sqrt{\langle \Delta^2 S_{in} \rangle \langle \Delta^2 M \rangle}}. \tag{15}$$

Optics EXPRESS

For a good QND measurement, $C_{S_{in},M} \to 1$. The third one is about how good the measurement scheme is as a state preparation device. For this, we examine the conditional variance of the signal output upon the meter outcome $Var(S_{out}|M)$. For Gaussian process, we have

$$Var(S_{out}|M) = Var(S_{out})(1 - C^2_{S_{out},M}). \tag{16}$$

Here $C_{S_{out},M}$ is the correlation coefficient between the signal output and the meter output. For $C_{S_{out},M} \to 1$, we have $Var(S_{out}|M) \to 0$ and this corresponds to a perfect state preparation. Furthermore, we have [39]

$$Var(S_{out})(1 - C^2_{S_{out},M}) = \langle(\Delta S_{out} - \lambda\Delta M)^2\rangle_m, \tag{17}$$

where the subscript "m" means the minimum value for an optimum value of parameter λ.

In our scheme, the signal quantity is the photon number of the probe field: $S = N_p$. Since we have the non-demolition condition: $N_p^{in} = N_p^{out}$, the second and the third criteria become the same: $C_{S_{in},M} = C_{S_{out},M}$. We will evaluate this quantity next.

3.1. QND measurement criteria applied to the linear atom-light interferometer

For simplicity of calculation, let us place the input probe field in a coherent state $|\alpha_p\rangle$. With both the probe field and the input atomic spin wave field in coherent states and the input optical signal field in vacuum, we can evaluate by using equation (4) the correlation coefficient

$$C_{N_W N_p} \equiv \frac{|\langle\Delta\hat{N}_W\Delta\hat{N}_p\rangle|}{\sqrt{\langle\Delta^2\hat{N}_W\rangle\langle\Delta^2\hat{N}_p\rangle}}. \tag{18}$$

It is straightforward to find

$$\langle\Delta\hat{N}_W\Delta\hat{N}_p\rangle = \kappa|\alpha|^2|\alpha_p|^2/2. \tag{19}$$

$$\langle\Delta^2\hat{N}_W\rangle = \frac{|\alpha|^2}{2}(1 + \kappa|\alpha_p|^2) + \frac{\kappa^2|\alpha|^2|\alpha_p|^2}{4}(|\alpha|^2 + |\alpha_p|^2 + 1). \tag{20}$$

$$\langle\Delta^2\hat{N}_p\rangle = |\alpha_p|^2. \tag{21}$$

So, the correlation coefficient is

$$C_{N_W N_p} = \frac{1}{\sqrt{1 + \frac{1+|\alpha_p|^2}{2I_{ps}} + \frac{1+\kappa|\alpha_p|^2}{\kappa^2 I_{ps}|\alpha_p|^2}}}, \tag{22}$$

where $I_{ps} \equiv |\alpha|^2/2$ is the number of the atomic spin wave to sense the atomic phase shift inside the linear interferometer. For $C_{N_W N_p} \to 1$, we will need $I_{ps} >> |\alpha_p|^2$ and $\kappa^2 I_{ps}|\alpha_p|^2 >> 1$. For the probe field at single-photon level ($|\alpha_p|^2 \approx 1$), the first one is easy to satisfy but the second one becomes $\kappa >> 1/\sqrt{I_{ps}} = \delta_{SQL}$. This simply implies that the interferometer, with a phase resolution of the standard quantum limit δ_{SQL}, is able to resolve the single-photon phase shift κ.

3.2. QND measurement criteria applied to the SU(1,1) nonlinear atom-light interferometer

With both the probe field and the input atomic spin wave field in the coherent states and the input optical field in vacuum, we can find the QND measurement quantity

$$C_{N_p X_S} \equiv \frac{|\langle\Delta\hat{N}_p\Delta\hat{X}_S\rangle|}{\sqrt{\langle\Delta^2\hat{N}_p\rangle\langle\Delta^2\hat{X}_S\rangle}}. \tag{23}$$

Optics EXPRESS

We first calculate each term in equation (23). The results are

$$\langle \Delta \hat{X}_S \Delta \hat{N}_p \rangle = 2Gg|\alpha|\kappa|\alpha_p|^2. \tag{24}$$

$$\langle \Delta^2 \hat{X}_S \rangle = 1 + G^2 g^2 \kappa^2 |\alpha_p|^2 (4|\alpha|^2 + |\alpha_p|^2 + 1) + G^4 \kappa^2 |\alpha_p|^2 (|\alpha_p|^2 + 1). \tag{25}$$

With $\langle \Delta^2 \hat{N}_p \rangle$ in equation (21), the correlation coefficient is

$$C_{N_p X_S} = \frac{1}{\sqrt{1 + \frac{(2g^2+1)(1+|\alpha_p|^2)}{4I_{ps}} + \frac{1}{4G^2 I_{ps} \kappa^2 |\alpha_p|^2}}}. \tag{26}$$

Here $I_{ps} \equiv g^2|\alpha|^2$ is the intensity of phase sensing spin wave inside the interferometer. Assuming $g^2 \gg 1$, the condition for a perfect correlation $C_{N_p X_S} \approx 1$ is $I_{ps} \gg g^2(1 + |\alpha_p|^2)$ and $I_{ps}\kappa^2|\alpha_p|^2 \gg 1/4G^2$. The right hand side of the inequality $I_{ps}\kappa^2|\alpha_p|^2 \gg 1/4G^2$ is reduced by a factor of $1/4G^2$ compared to the one discussed in equation (22) for linear atom-light QND measurement. This is due to sensitivity enhancement of the SU(1,1) interferometer.

Since the SU(1,1) interferometer has better sensitivity than the linear interferometer, we will only examine this measurement scheme in the following applications.

4. Application to twin beams

For practical application of the QND measurement scheme, we can use it to confirm the photon number correlation of the twin beams from an optical parametric amplifier (OPA, Fig. 2), which has been demonstrated by direct photo-detection [40, 41]. It is well-known that the output state of a non-degenerate optical parametric amplifier is a correlated thermal state of the form

$$|\mathrm{TwBm}\rangle = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} |n\rangle_s |n\rangle_i, \tag{27}$$

where $\bar{n}$ is the average photon number and “s, i" denote the signal and idler fields of the two outputs, respectively. So, the photon numbers of the two fields have perfect correlation coefficient: $C_{N_s N_i} = 1$. That is why the fields in this state are called “twin beams".

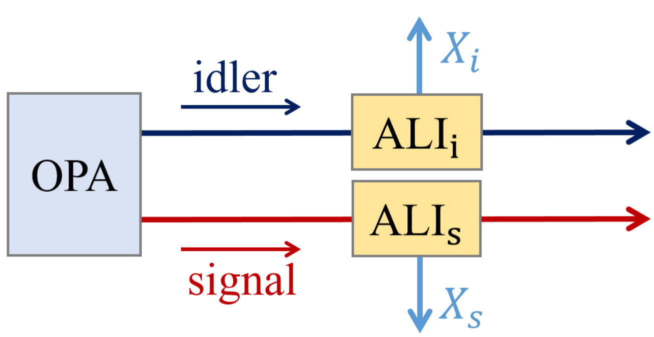

Fig. 2. QND measurement on the twin beams with two atom-light interferometers (ALI). The subscriptions s and i mean signal and idler. X_s and X_i are the output fields of the atom-light interferometers for signal and idler beams.

We now make QND measurements on both fields of the twin beams simultaneously as shown in Fig. 2. The two meter outputs $X_S^{(s)}, X_S^{(i)}$ will show similar correlation depending on how well the QND measurements are. Using equation (8) for the meter output and assuming the two QND measurement schemes same parameters: G, g, κ, α, it is straightforward to show that for the input state in equation (27), the correlation coefficient $C_{X_S^{(s)}X_S^{(i)}}$ is given by

$$C_{X_S^{(s)}X_S^{(i)}} = \frac{1}{1 + \frac{2\bar{n}+1}{\bar{n}+1}\frac{G^2+g^2}{4I_{ps}} + \frac{1}{4G^2\kappa^2 I_{ps}\bar{n}(\bar{n}+1)}}. \quad (28)$$

This result is similar to that in equation (26) with α_p-dependent terms replaced by $\bar{n}$-dependent terms. Thus, the condition to obtain a perfect correlation of $C_{X_S^{(s)}X_S^{(i)}} \approx 1$ for the twin beams is similar: $I_{ps} >> G^2$ and $I_{ps}\kappa^2\bar{n}^2 >> 1/4G^2$ for $\bar{n} >> 1$.

The correlation coefficient can be measured experimentally from the variance of the difference:

$$\langle(\Delta\hat{X}_S^{(s)} - \lambda\Delta\hat{X}_S^{(i)})^2\rangle = \langle\Delta^2\hat{X}_S^{(s)}\rangle(1 - C^2_{X_S^{(s)}X_S^{(i)}}). \quad (29)$$

Here again, λ is an adjustable parameter for the lowest variance.

5. Sequential QND measurement and quantum information tapping

Another way to verify the properties of a QND measurement and make use of it for quantum information distribution is the sequential measurement. Since a QND measurement is non-destructive, the probe field after the first measurement should be the same as that before the measurement. So we can make another measurement on the probe field and see if this measurement is correlated with the previous one.

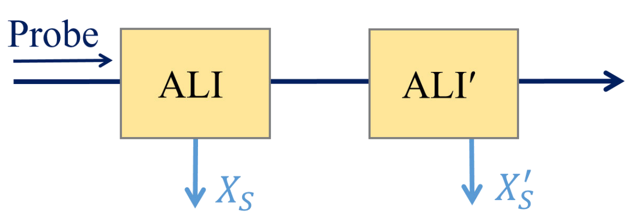

Fig. 3. Sequential QND measurement with atom-light interferometers(ALI). X_S and X_S' are final quadrature outputs of ALI and ALI'.

The schematic is shown in Fig. 3. We denote the meter quantity of the second measurement as X_S'. We can then find the correlation coefficient $C_{X_S X_S'}$ of the two meter quantities X_S, X_S' by measuring the variance in the difference: $\langle(\Delta X_S - \beta\Delta X_S')^2$, which is related to the correlation coefficient $C_{X_S X_S'}$:

$$C_{X_S X_S'} \equiv \frac{|\langle\Delta\hat{X}_S\Delta\hat{X}_S'\rangle|}{\sqrt{\langle\Delta^2\hat{X}_S\rangle\langle\Delta^2\hat{X}_S'\rangle}}. \quad (30)$$

Assuming the same coherent state and vacuum inputs for the two interferometers and a coherent state $|\alpha_p\rangle$ at the probe field, we can use equation (8) for both $\hat{X}_S$ and $\hat{X}_S'$ but with different $\hat{Y}_a^{in}$ and $\hat{X}_S^{in}$ and find the correlation coefficient $C_{X_S X_S'} = C^2_{N_p X_S}$. Thus, a good QND measurement

Optics EXPRESS

in the each of the two measurements ensures good correlation between the two outputs of the sequential measurement.

A very important property of quantum measurement is the state projection: the state of the system after the measurement is projected to the eigen-state of the measurement process. Subsequent measurement will not change this state because it is the eigen-state. For non-ideal QND measurement process, the third criterion concerns the measurement for only single time. For repeated measurements in a sequential QND measurement, we should expect better state preparation. Thus, another quantity to confirm the quantum projection property of a sequential QND measurement is the three-quantity correlation:

$$\langle(\Delta N_p - \lambda\Delta X_S - \lambda'\Delta X'_S)^2\rangle = Var(N_p|X_S, X'_S), \tag{31}$$

which is the conditional variance of the measured quantity upon the outcomes of the two measurements. Here, parameters λ, λ' are optimized to minimize the three-quantity correlation function. By the projection property of quantum measurement theory, this quantity should be smaller than $Var(N_p|X_S)$ because, in non-ideal QND measurements, each additional measurement will narrow down the uncertainty in the measured quantity.

With the parameters λ, λ' optimized, it is straightforward to show that

$$\begin{aligned} & Var(N_p|X_S, X'_S) \\ = \ & \langle\Delta^2 N_p\rangle\left(1 - \frac{C^2_{N_pX_S} + C^2_{N_pX'_S} - 2C_{N_pX_S}C_{N_pX'_S}C_{X_SX'_S}}{1 - C^2_{X_SX'_S}}\right) \\ = \ & \langle\Delta^2 N_p\rangle\frac{1 - C^2_{N_pX_S}}{1 + C^2_{N_pX_S}} = \frac{Var(N_p|X_S)}{1 + C^2_{N_pX_S}}. \end{aligned} \tag{32}$$

Here in the last line, we use $C_{X_SX'_S} = C^2_{N_pX_S} = C^2_{N_pX'_S}$. Since $C^2_{N_pX_S} > 0$, we therefore have $Var(N_p|X_S, X'_S) < Var(N_p|X_S)$, confirming the quantum projection property of QND measurement. We may of course ask further what happens if we perform the QND measurement M times. Following the same procedure above, we can show the conditional variance $Var(N_p|X_S, X'_S, ..., X^M_S) = Var(N_p|X_S)/(1 + MC^2_{N_pX_S})$. So, each additional QND measurement will reduce the conditional variance further.

The above demonstrates the projection property of quantum measurement processes: measurement of a physical quantity on a quantum system will project the state of the system to an eigen-state of the measurement process and subsequent repeated measurement will not change the eigen-state. Here the eigen-state of the measurement is the number state $|n_p\rangle$.

Next, we can use the sequential measurement processes to realize quantum information tapping, that is, we can make multiple copies of the input signal for quantum information distribution. To demonstrate this, we encode a modulation signal on the probe field, which is in a coherent state. We can calculate the signal-to-noise ratios of the input $R^{in}_{N_p}$ and the three outputs: $R^{out}_{N_p}, R_{X_S}, R_{X'_S}$, respectively. The signal outputs at X_S, X'_S are information taps for the input at the measured field. We define the information transfer coefficients

$$T_{N_p} = \frac{R^{out}_{N_p}}{R^{in}_{N_p}},\ \ T_{X_S} = \frac{R^{out}_{X_S}}{R^{in}_{N_p}},\ \ T_{X'_S} = \frac{R^{out}_{X'_S}}{R^{in}_{N_p}}, \tag{33}$$

Since there is no loss in the probe field, we have $T_{N_p} = 1$. For a coherent state input, we have $\langle\Delta^2 N_p\rangle = |\alpha_p|^2$ and $\langle N_p\rangle = |\alpha_p|^2$, so $R^{in}_{N_p} = \langle N_p\rangle^2/\langle\Delta^2 N_p\rangle = |\alpha_p|^2$. From equation (8), we find $\langle X^{out}_S\rangle = 2\kappa Gg|\alpha_p|^2\alpha$. With $\langle\Delta^2 X_S\rangle$ given in equation (25), we find

$$T_{X_S} = \frac{R^{out}_{X_S}}{R^{in}_{N_p}} = C^2_{N_pX_S} = T_{X'_S}. \tag{34}$$

Optics EXPRESS

Therefore

$$T_{N_p} + T_{X_S} = T_{N_p} + T_{X'_S} = 1 + C^2_{N_p X_S} > 1. \quad (35)$$

The quantity one on the right hand side of the inequalities is the classical limit for information tapping of signals at quantum level (quantum-noise limited signals). So, the correlation coefficient is also a measure of how good the information tapping is.

6. Conclusion

In summary, we analyzed a quantum non-demolition measurement scheme of photon number of an optical field, based on the AC Stark effect for photon-induced atomic phase shift and on atom-light hybrid interferometers for precise measurement of the phase shift. We used the QND criteria suggested by Holland et al [30] to access the quality of our measurement scheme. We found that good QND measurement can be achieved when the phase resolution of the atom-light interferometer is on par with the AC Stark phase shift induced by the field to be measured. We applied this QND measurement scheme to a twin photon state and confirmed the photon number correlation in the twin beam. The projection property of quantum measurement is confirmed with sequential multiple QND measurements. Such a scheme can be used for quantum information tapping in order to distribute multiple copies of information.

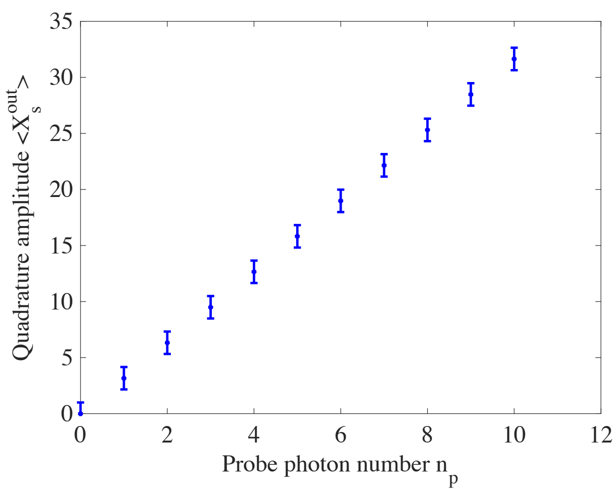

Fig. 4. The quadrature amplitude $\langle \hat{X}_S^{out} \rangle$ as a function of the probe photon number n_p, showing that the meter signal is sensitive to probe photon number. The errorbar is the fluctuation of meter signal quadrature amplitude $\sqrt{\langle \Delta^2 \hat{X}_S^{out} \rangle}$. The data used in this graph is calculated at $\kappa = 5.30 \times 10^{-4} rad$, $G^2 = 10$, and $I_{ps} = 8.91 \times 10^5$.

When the QND measurement scheme achieves single-photon resolution, it can be used as a non-destructive photon counter. From the equation (14) in Sect.(2.3.1), we know that this is achieved when $\kappa > 1/2G\sqrt{I_{ps}}$. We can make an estimation of κ from literature according to the experimental data. In a rubidium atomic cell, we obtain an AC Stark shift of $1.5MHz$ by $0.6mW$ of probe power with $0.45mm$ of the spot size when the probe light is blue detuned $1GHz$ from the transition $|^5S_{1/2}, F = 1\rangle \rightarrow |^5P_{1/2}, F = 2\rangle$ [42]. For the atom-light interferometer demonstrated in reference [28], we have a time duration of $100ns$. At photon frequency detuning $1GHz$ from the transition $|^5S_{1/2}, F = 1\rangle \rightarrow |^5P_{1/2}, F = 2\rangle$, we thus obtain a phase shift of $0.15rad$ for a photon number of 2.35×10^8, and finally the phase shift coefficient can be calculated to be

Optics EXPRESS

$\kappa = 6.38 \times 10^{-10} rad$ per photon. To satisfy the criteria $\kappa > 1/2G\sqrt{I_{ps}}$ under a gian $G^2 = 10$, we need a very large power of phase sensing beam $1.57 \times 10^4 W$. This is difficult to realize experimentally. In reference [29], we know that the magnitude of AC Stark shift is of the same order for atomic vapor and cold atomic ensemble. And the atomic numbers in interaction region for two atomic systems are both $10^9 - 10^{10}$ [28, 29, 43]. To increase the interaction coefficient κ, we suggest to use the cold atomic ensemble in a cavity [44, 45].

With $120MHz$ detuning frequency of probe light, the absorption for probe in cold atomic ensemble is very small and negligible. Using an optical cavity of a modest finesse of 10^5 [45] to increase interaction between the probe field and spin wave, considering the loss of cavity is small, we can obtain an improved interaction coefficient $\kappa = 5.30 \times 10^{-4} rad$ per photon. With a typical Raman gain $G^2 = 10$, to make the signal-to-noise ratio $R = 4G^2\kappa^2 I_{ps} \sim 10$ for better single-photon resolution, we obtain the phase sensing spin wave number (I_{ps}) is 8.91×10^5, which is much smaller than the number of atoms on the $|g\rangle$ level $(N) \sim 10^9 - 10^{10}$ in interaction region [28, 29], and corresponding power of $100ns$ optical sgnal pulse (P_{sig}) is $2.27uW$, which is also much smaller than the pump power (P_{pump}) of several to tens of milliwatts [28, 29]. This power is easy to realize in experiment. And in future experiments, the suggested ratios of I_{ps}/N and P_{sig}/P_{pump} should be both smaller than 0.1%, which could guarantee that the pump power and the number of atoms on the $|g\rangle$ level can be treated as constants in wave splitting and recombination processes in theory.

When such interferometer is used to detect the photon number in probe pulse, we can assume the probe pulse is at Fock state to calculate the expectation and fluctuation of quadrature amplitude of the output meter signals of the interferometer. The output quadrature amplitude expectation and fluctuation are $\langle \hat{X}_S^{out} \rangle = 2G\kappa n_p \sqrt{I_{ps}}$ and $\sqrt{\langle \Delta^2 \hat{X}_S^{out} \rangle} = \sqrt{1 + G^2(G^2 + g^2)\kappa^2 n_p^2}$, in which $\langle \hat{X}_S^{out} \rangle \propto n_p$ (the photon number contained in the probe pulse) but $\sqrt{\langle \Delta^2 \hat{X}_S^{out} \rangle} \approx 1$ due to small κ value. The calculation result is given in Fig. 4. This figure shows the photon-number-resolving ability of the atom-light hybrid SU(1,1) interferometer.

Funding

National Key Research and Development Program of China under Grant number 2016YFA0302001; the National Natural Science Foundation of China (grant numbers 91536114, 11654005, and 11234003); and Natural Science Foundation of Shanghai (No. 17ZR1442800).