Alternative Method for the Identification of Critical Nodes Leading to Voltage Instability in a Power System

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Abstract— Introduction of new operation enhancement technologies plus increasing application of power electronics coupled with the continuous increase in load demand has increased the risk of power networks to voltage instability and susceptibility to voltage collapse. This frequent occurrence of voltage collapse in modern power system has been a growing concern to power system utilities. This paper proposes alternative techniques for the identification of critical nodes that are liable to voltage instability in a power system. The first method is based on the critical mode corresponding to the smallest eigenvalues, while the second technique is based on the centrality measure to identify the influential node of the networks. The eigenvector centrality measure is formulated from the response matrices of both the load and generator nodes of the networks. The effectiveness of the suggested approaches is tested using the IEEE 30 bus and the Southern Indian 10 bus power networks. The results are compared to the techniques based on the traditional power flow. The whole procedure of the results involved in the identification of critical nodes through the proposed methods is totally non-iterative and thereby save time and require less computational burden.

Keywords: **Graph theory, Network topology, Power flow, Power Network, Voltage Collapse Weak nodes,**

1. Introduction

The increasing rate of voltage collapse in a power system due to voltage instability has attracted the attention of numerous power system researchers in the open literatures [1] [2]. Factors such as loss of a heavily loaded transmission line, inadequate reactive power supply, among others could result in a voltage collapse or in more serious instances, may lead to cascading outages and blackouts [3]. Voltage collapse is described by a gradual decrease in the voltage magnitude of the system buses, and continues until these system voltages decline rapidly [4]. To prevent the incessant occurrence of this catastrophe, identification of critical nodes that are liable to voltage collapse is of importance in power system. A substantial research was carried out in the open literatures with the view to identifying weak nodes that have highest proximity to voltage collapse in a power system [5]-[7]. In the past, analysis of voltage collapse has only been considered as the problem of operating point of a power system. Based on this, a considerable number of algorithmic techniques have been suggested in the literature [8]-[16]. The use of optimization based techniques for the assessment of voltage stability has also been reported by a considerable number of authors [17]. However, some of these power flow based techniques are time consuming and laborious, especially for large and complex power networks. Besides, in reality, the use of conventional power flow techniques does not tell the whole story surrounding voltage instability assessment in a power system. Therefore, there is a need for a more useful technique that could be employed for voltage stability analysis in a power system.

The inherent structural network theory proposed by [18] has been applied to solve some power system problems [19]-[21]. The idea proposed by [19], however, did not take into consideration, the effect of the special distinctions that exists between load and generator nodes in an interconnected power system network. In order to solve this problem, the method proposed by [19]-[21] was further explored by [22] to evaluate suitable locations for generation expansion in restructured power systems. Although, the approach adopted by [22] seems promising and gives insight as per the determination of a new location for generator, however, it is still time consuming. This is because each node must first be changed to a load node before an optimal location of a new generator could be found.

A considerable effort was made by [23] to further investigate and apply the idea based on the use of basic circuit theory law. The idea developed by [23] was used to solve some power system problems such as classification of power system networks, optimal location of network devices and so on [24]. Although, quite appreciable work was done by the authors of [24], notwithstanding, information contained in the work is not sufficient to proffer a lasting solution to the prevalent occurrence of voltage instability in a power system. For instance, although, authors of [24] carried out a study to identify an optimal location for reactive power compensator using inherent network structure based approach; however, critical node was identified based on the node with smallest singular value (eigenvalue), whereas the contributions of the minimum singular value (eigenvalue) to the entire power system network were not considered. This is a serious drawback.

Another area of research has recently showed that incident of voltage collapse in a modern power system may be analyzed through the use of complex network theory [25]-[27]. Central nodes will have more influence than non-central nodes as they can reach the whole network more quickly. The relative importance of a vertex within a graph have been demonstrated through the use of centrality measures [28]-[30]. These measures include betweenness centrality, closeness centrality, eigenvector centrality and degree centrality.

Thus, this paper contributes significantly to two dynamic streams of research. Firstly, alternative method to the existing eigenvector centrality measure which is often used in complex networks to identify most central nodes is suggested. The proposed measure is established from the network topological structural point of view, by considering the electrical interconnection between various elements of the network. These nodes are so named and are important in power system because if suddenly disconnected (in case of contingency) from the power network, may weaken the robustness of the power system considerably and this may cause major possible damage to network performance or in most times result in voltage collapse [31]. Thus, the need for prior identification of these critical nodes in a power network.

Secondly, to identify a structurally weak node in power networks that can lead to voltage instability, a non-iterative technique called Network Response Structural Characteristic Participation Factor Index (NRSPF) based on the topological nature of the network is further investigated. This approach is captured by Kirchhoff Matrix (admittance matrix) of a power system [32]. This method is also independent of the network loading conditions and thus can aid the system operator in proper planning and operation of the power network in case of occurrence of any contingencies. Worth-noting, is the fact that the critical nodes in the context of this paper are classified as the weak (low voltage profile) and influential (most central) nodes.

Comparison of all the approaches presented is done with the existing power flow based technique of L-index, and voltage collapse proximity index (VCPI) techniques as proposed by [33] and [16], respectively.

The effectiveness of all the methodology presented is tested using the IEEE 30 bus and the Southern Indian 10-bus power networks, whose schematic diagrams are shown in Figures 1a and b, respectively.

The rest of the paper is organized as follows: Section 2 gives the formulation of the NRSPF being investigated while section 3 presents the mathematical illustration for the proposed eigenvector centrality measure. The mathematical formulations for the existing eigenvector centrality measure are also presented in this section. The results obtained during simulation and discussions are shown in sections 4 and 5. Section 6 gives a concise conclusion of the work.

2. Mathematical Formulations of NRSPF

The concept behind the technique of NRSPF is expressed in accordance with the inverse problem of electrical network suggested in [34]. Given an electrical network, $\Pi = (G, W)$, where $G = (V, E)$ is a graph, V is the set of vertices depicting nodes and E is the set of edges formed by pairs of vertices. W is the complex value function defined on all edges for each $e \in E$. If U is the voltage magnitude of each vertex, K the network admittance or a Kirchhoff's matrix and φ is the current injected into the networks, then, the relationship that exist between them can be determined by the Kirchhoff's and Ohm's law equation as

$$[\varphi] = [K_{network}][U] \tag{1}$$

Suppose Π is a connected electrical network with boundary, the response matrix Λ_K can be determined as the Schur complement in K of the square matrix corresponding to the interior nodes of Π. In this paper, the network elements, that is, generator and load nodes are arranged sequentially in the form of boundary and interior nodes, respectively as shown in Figures 1a and b. The main aim of this paper is to explore the structural topological characteristics that are inherent in the electrical network Π. These properties, as earlier stated are captured by the Kirchhoff matrix of a power network. Thus, eq.(1) may be further expressed in terms of the network elements as

$$\begin{bmatrix} \varphi^G \\ \varphi^L \end{bmatrix} = \begin{bmatrix} K^{GG} & K^{GL} \\ K^{LG} & K^{LL} \end{bmatrix} \begin{bmatrix} U^G \\ U^L \end{bmatrix} \tag{2}$$

where

The subscripts G and L represent symbols for generator and load nodes, respectively. $[K^{GG}]$ is the square admittance matrix that shows the connectivity between generator buses (boundary nodes), $[K^{GL}]$ is a $G \times L$ admittance matrix that represents electrical interconnection between generator and load buses, $[K^{LG}]$ is a $L \times G$ admittance matrix that represents the electrical interconnection between load and generator buses and $[K^{LL}]$ is the square admittance matrix that shows electrical interconnectivity that exist between the load buses. $[\varphi^{G}]$, $[\varphi^{L}]$ are the complex bus current injection vectors, $[U^{G}]$, $[U^{L}]$ are the complex bus voltage vectors.

Algebraic manipulations of eq. (2) give eq. (3)

$$\begin{bmatrix} U^{G} \\ \varphi^{L} \end{bmatrix} = \begin{bmatrix} [K^{GG}]^{-1} & M^{GL} \\ M^{LG} & D^{LL} \end{bmatrix} \begin{bmatrix} \varphi^{G} \\ U^{L} \end{bmatrix} \tag{3}$$

where $[M^{GL}] = -[K^{GG}]^{-1}[K^{GL}]$ is a matrix that gives the relation between the load bus and the generator bus voltages. It represents the impact of generators over the load buses. $[M^{LG}] = [K^{LG}][K^{GG}]^{-1}$ is the negative transpose of the matrix $[M^{GL}]$.

$[D^{LL}] = [K^{LL}] - [K^{LG}][K^{GG}]^{-1}[K^{GL}]$ is the electrical interconnection between load – load buses in a power network. $[K^{GG}]^{-1} = [Z^{GG}]$ and it shows the impedances between generator buses.

2.1 Network Response Matrix

The network response matrix Λ^{LL} can be obtained as the Schur Complement of the sub matrix K^{GG} in the Kirchhoff matrix of eq. (2). The Kirchhoff matrix K can be expressed as

$$K = \begin{bmatrix} K^{GG} & K^{GL} \\ K^{LG} & K^{LL} \end{bmatrix} \tag{4}$$

From eq. (4), the Schur complement of K^{GG} in K can be written as:

$$K/K^{GG} = [K^{LL}] - [K^{LG}][K^{GG}]^{-1}[K^{GL}] \tag{5}$$

where

$$\Lambda^{LL} = D^{LL} = K/K^{GG} = \left[K^{LL}\right] - \left[K^{LG}\right]\left[K^{GG}\right]^{-1}\left[K^{GL}\right] \tag{6}$$

The response matrix of the network is shown in eq.(6) and we termed it in this paper as the Inter-Load Buses Interconnection Response Matrix (ILBIRM). It gives information on how the loads at the interior nodes of the electrical networks $\Pi = (G, W)$ are interconnected. In this work, the absolute value of matrix of eq. (6) is determined to ensure a real and symmetrical matrix.

The eigenvalue decomposition of Λ^{LL} gives,

$$\Lambda^{LL} = D^{LL} = \mathrm{X}\,\xi\,\mathrm{X} = \sum_{i=1}^{\Phi} x_i\,\eta_i\,x_i^{*} \tag{7}$$

$$\Lambda^{LL} = \begin{bmatrix} x_1 & \cdots & x_\Phi \end{bmatrix} \begin{bmatrix} \eta_1 & & & \\ & . & & \\ & & . & \\ & & & . \\ & & & & \eta_\Phi \end{bmatrix} \begin{bmatrix} x_1^{*} \\ . \\ . \\ . \\ x_\Phi^{*} \end{bmatrix} \tag{8}$$

The columns of x being the eigenvectors of the response matrix defined in eq. (6), which is associated with the eigenvalues $\eta_1 \cdots \eta_\Phi$. X is the orthonormal matrix with the associated eigenvectors x_i. The diagonal matrix ξ is associated with a diagonal eigenvalues η_i. Since x is non-singular, $\eta = \eta^{*}$ and η is therefore a diagonal matrix.

Therefore,

$$\Lambda^{LL} x_i = \eta\, x_i \;,\; i = 1 \cdots \Phi \tag{9}$$

We may resolve the response matrix defined in eq. (6) as a sum of orthogonal matrix and also determine the NRSPF. To do this, eq. (8) is further expanded to obtain eq. (10)

$$\Lambda^{LL} = D^{LL} = \eta_1\, x_1\, x_1^{*} + \cdots + \eta_\Phi\, x_\Phi\, x_\Phi^{*} \tag{10}$$

$$\Lambda^{LL} = D^{LL} = \eta_1\, PF_1 + \cdots + \eta_\Phi\, PF_\Phi \tag{11}$$

where

$$PF_1 = x_1 x_1^* \quad \text{and} \quad PF_\Phi = x_\Phi x_\Phi^*$$

$$PF_i = x_i x_i^* \; , i = 1 \ldots \Phi \tag{12}$$

Thus,

$$NRSPF_{ij} = PF_{ij} = x_{ij} x_{ij}^* \tag{13}$$

From eq. (13), it could be seen that, the network node participation, measuring the contributions of the i^{th} node to the j^{th} node is found by multiplying both the right and left eigenvectors together. It follows that; the node that has the highest value of $NRSPF$ corresponds to the critical mode identified using eq. (7). This node is considered as the structurally weak node of the network and has the maximum proximity to voltage instability. This method is similar to but different from the modal analysis technique proposed by [15]. Both techniques involve the determination of critical mode of the system by the application of eigenvalue decomposition technique. The difference, however, is in the fact that, the technique of NRSPF depends mainly on the structural properties that exist between various buses in power network, which is captured by the admittance matrix. Whereas, modal analysis depends on the Jacobian matrix of the system real and reactive power which is also based on power flow solutions.

To find the voltage magnitude of each interior node, eq. (3) may be expanded to give

$$[U^L] = [D^{LL}]^{-1} \{[\varphi^L] - [M^{LG}][\varphi^G]\} \tag{14}$$

Upon substitution, we have

$$[U^L] = \sum_{i=1}^{\Phi} \frac{x_i \; x_i^*}{\eta_i} [\, [\varphi^L] - [M^{LG}][\varphi^G]\,] \tag{15}$$

Large values of η_i as can be seen from the eq. (15) indicate small changes in the interior node voltage. The eigenvalues however, may become smaller and the interior node voltage becomes weaker if the network is stressed. If the magnitude of the eigenvalues decreases continuously until it reaches zero, the corresponding interior voltage will collapse. We can as well find the response matrix Λ^{GG} termed Inter-Generator Buses Interconnection Matrix Index (IGBIMI), by following similar procedures as in the above equations.

Algebraic manipulation of eq. (2) also gives:

$$\begin{bmatrix} U^{L} \\ \varphi^{G} \end{bmatrix} = \begin{bmatrix} \left[K^{LL}\right]^{-1} & H^{GL} \\ H^{LG} & D^{GG} \end{bmatrix} \begin{bmatrix} \varphi^{L} \\ U^{G} \end{bmatrix} \tag{16}$$

From which we can determine the current φ^{G} injected into the boundary nodes of the network as:

$$\varphi^{G} = H^{LG}\varphi^{L} + D^{GG}U^{G} \tag{17}$$

D^{GG} being the response matrix represented as Λ^{GG} can be determined by finding the Schur complement of K^{LL} in the Kirchhoff matrix. Electrical interconnection that exists between the boundary nodes of the electrical network $\Pi = (G, W)$ is captured in Λ^{GG}. In this case, the influence of the interior nodes is eliminated. Thus,

$$\left[D^{GG}\right] = \left[\Lambda^{GG}\right] = \left[K^{GG}\right] - \left[K^{GL}\right]\left[K^{LL}\right]^{-1}\left[K^{LG}\right] \tag{18}$$

We may further apply eigenvalue decomposition techniques to the matrix of eq. (18) in a similar way to to eq (7) as follows:

$$\left[\Lambda^{GG}\right] = \left[D^{GG}\right] = Z\lambda Z^{*} = \sum_{i=1}^{r} z_i\, \lambda_i\, z_i^{*} \tag{19}$$

where Z represents an orthogonal matrix corresponding to z_i eigenvectors. λ_i is the diagonal matrix which represents the eigenvalues of the elements present in the networks.

3. Conventional Centrality Measure

Centrality measures are mostly used to rank the relative importance of edges and vertices in a graph. The most common centrality measures are the degree centrality, closeness centrality, eigenvector centrality and betweenness centrality. Detailed description of each of these measures is presented in [28]. The main focus of this paper is, however, on the alternative algorithm to finding influential nodes using eigenvector centrality measure.

3.1 Eigenvector Centrality Measure

The eigenvector centrality is of great importance in the determination of the relative influences of a node within a graph. Given a graph G: = (V, E), one eigenvalue, μ , with its adjacency matrix A , the corresponding eigenvector m satisfy

$$\mu m = Am \tag{20}$$

The adjacency matrix A may be extracted from the Laplacian as:

$$Am = -K + D(K) \tag{21}$$

where K is the admittance of the network and $D(K)$ represents the diagonal of matrix K .

Thus, the eigenvector centrality of the node υ as the υth entry of the eigenvector m which corresponds to the maximum eigenvalue as:

$$C_E{}^K(\upsilon) = \|m_\upsilon\| = \left\| \frac{1}{\mu_{\max}} \sum_{j=1}^{N} A_k(\upsilon, j) m_j \right\| \tag{22}$$

3.2 Proposed Eigenvector Centrality

The standard way of measuring network centrality has been through the use of eigenvector. The information contained in the eigenvalue decomposition of the response matrices Λ^{LL} and Λ^{GG} expressed in eqs. (7) and (19), respectively, is of great importance in the determination of the most influential node of the power network. These equations can be re-written as:

$$\Lambda^{LL} x = \eta x \tag{23}$$

$$\Lambda^{GG} z = \lambda z \tag{24}$$

In this case, Λ^{LL} and Λ^{GG} are obtained from eqs. (7) and (19), respectively.

The conventional eigenvector centrality depends on the adjacency matrix of the network as expressed in eq. (22). This implies that, Laplacian matrix has to be determined first, before the adjacency matrix of the network is found. The proposed eigenvector centrality however, is a function of the electrical interconnection that exists between the various elements of the network.

Suppose m_i is the score (eigenvector) of the i^{th} node of the connected and weighted network $G=(V,E)$ corresponding to the maximum eigenvalue $\eta_{\max}$, Λ^{LL} and Λ^{GG} are the network structural indices captured by the admittance of the edges of the network. These indices also show the electrical interconnection that exists between the interior and boundary nodes of the network, respectively. Thus, the proposed eigenvector centrality of a node i taking into consideration the interior and boundary nodes is given as:

$$C_{mE}^{\Lambda^{LL}}(i) = \|m_i\| = \left\| \frac{1}{\eta_{\max}} \sum_{j=1}^{N} \Lambda^{LL}(i,j) m_j \right\| \tag{25}$$

$$C_{mE}^{\Lambda^{GG}}(i) = \|m_i\| = \left\| \frac{1}{\eta_{\max}} \sum_{j=1}^{N} \Lambda^{GG}(i,j) m_j \right\| \tag{26}$$

This requires application of eigen – analysis on both Λ^{LL} and Λ^{GG}. The magnitude of the entries of the eigenvector as the centrality measures need also to be determined. The definition of the proposed eigenvector centrality also selects the eigenvector corresponding to the maximum eigenvalue to maintain all the centrality scores to be positive. The interior and boundary node i that gives maximum value of $C_{mE}^{\Lambda^{LL}}$ and $C_{mE}^{\Lambda^{GG}}$, respectively is considered as the critical node (influential node) of the system.

4. Simulation results and discussion

The effectiveness of the performance of all the approaches presented is tested using the IEEE 30 bus and the Southern Indian 10-bus power networks as shown in Figure 1 (a) and (b), respectively. In the course of simulations, the network nodes are arranged sequentially in the form of interior (load) and boundary (generator) nodes. The IEEE 30 bus is made up of six generators, forty- one transmission lines with four (4) tap ratios and eighteen load nodes. Of all the interior nodes, nodes 24, 26, 29 and 30 have been reported in the literature to be most critical and vulnerable to voltage collapse [35] [36]. These nodes also have minimum allowable loads compared with other interior nodes of the IEEE 30 bus network. Thus, these nodes are randomly selected for further investigation in this paper. Similarly, for the three generators and seven load nodes Southern Indian 10-bus test system, nodes 4,6,7,8 and 10 of the seven interior nodes are randomly chosen. All simulations were performed using MATLAB R2014a software. Windows 7, HP, 64 bit operating system, with 500GB hard disc and 4GB random access memory laptop are used.

4.1 Identification of Weak Node through the use of NRSPF, L-Index and VCPI

Identification of weak nodes of the IEEE 30 bus and 10-bus test systems using the suggested approach of NRSPF begins by first finding the critical modes (node with the smallest eigenvalue) of the power networks. This is done by applying eigenvalue decomposition techniques on the Inter-Load Buses Interaction Response Matrix (ILBIRM) of the networks as defined in eq. (6). Next, the right and left eigenvectors of the networks are computed. These eigenvectors are then used to compute the NRSPF for each of the nodes based on the critical mode identified. Results of the simulations obtained for both IEEE 30 bus and the Southern Indian 10 bus power networks are presented in the form of test cases I and II respectively. Results of identification of critical mode and the corresponding eigenvalues for both the IEEE 30 bus and 10 bus power networks are presented in Figures 2, Table 1 and Figure 3, Table 5, respectively. Presented in Tables 2, 3 and Tables 6, 7, respectively are the results of the traditional techniques of L-Index and VCPI for the IEEE 30 bus and 10 bus test systems. Results of simulation which show the comparison of the proposed approach of NRSPF with the traditional technique for the IEEE 30 bus and the 10-bus test systems are also presented in Tables 4 and 8, respectively.

4.2 Identification of Influential Nodes through the proposed Degree and Eigenvector Centrality measures

Identification of influential node in a power system is so much important. This is because sudden disconnection (in case of contingency) of this node may lead to voltage collapse, system damages, cascading failures to mention a few. The simulation results of the traditional eigenvector centrality measure and the corresponding proposed centrality measure for the IEEE 30 bus and Southern Indian 10-bus power networks are as presented in Tables 9 and 10, respectively.

5. Discussion

Discussion of results obtained for all the techniques are presented in this section.

5.1 The NRSPF approach

It can be seen from the results presented in Figure 2 of Test case I, that mode 19 of the IEEE 30 bus test system has the smallest NRSPF eigenvalues and thus, it is identified as the critical mode of the system. To ensure a real and symmetrical matrix, absolute value of eqn. (6) are used. Both the right and left eigenvectors are then computed to determine the values of NRSPF for each interior node (load bus) of the IEEE 30 bus system. The node with the highest value of NRSPF is identified as node 26. This node is thus considered as a structurally weak node of the IEEE 30 bus system that can be liable to voltage instability. It takes the total computational time of **0.748765** second to identify this weak node. In the same vein, as shown in Figure 3, mode 3 of the Southern 10 bus India network has the least eigenvalues and thus, it is considered as the critical mode of the network. Table 5 indicates node 10 as the weakest of all the nodes due to its maximum value of NRSPF. It takes the total computational time of 0.409865 seconds to identify this node.

Noteworthy is the fact that, the proposed approach of NRSPF is non-iterative and therefore does not depend on specifying particular loading conditions before the weak bus liable to voltage collapse is known. However, to demonstrate the effectiveness of the suggested approach, traditional power flow based techniques of voltage stability index (L-index) and the voltage collapse proximity index (VCPI) are used.

5.2 The traditional power flow based L-Index and VCPI techniques

The results of the values of L-index for each load node are computed for in a single iteration to determine the weak node of the systems. For the L- Index approach, load node 26 of the IEEE 30 bus test system has the lowest voltage magnitude and also, it is the bus that has maximum value of L-index as shown in Table 2. With the L-index technique, it takes the total computational time of 0.984587 second to identify the node susceptible to voltage instability. For the VCPI method, effect of load variations on each load node of the system has to be considered to identify the critical node of the system. This has to be done repeatedly taking one load node at a time until the power flow solution fails to converge. The simulation result of the VCPI values for each load node of the IEEE 30 bus system is presented in Table 3. The bus with the least allowable load and maximum value of VCPI is identified as bus 26. This bus is indeed the weakest of the IEEE 30 bus test system as it is identified by all the approaches considered. It takes the total computational time of 51.486806 seconds to identify node 26 as most critical using the VCPI technique.

For the 10 bus system, whose bus data and line data are shown in Tables 11 and 12 respectively, with L-index technique, bus 10 has the highest value of L-index and thus considered as the weak node of the system as shown in Table 6. This bus was identified in just 0.523409 second. Similarly, for the VCPI approach, load bus 10 was also identified as the weak node of the system as shown in Table 7. To identify this, it takes the total computational time of 98.509321 seconds.

5.3 A brief Comparison of all the approaches

The results of comparison of all the approaches presented in Tables 4 and 8, respectively, are in agreement, as nodes 26 and 10 of the IEEE 30 bus and 10 bus test systems respectively are identified as weak nodes. For the structurally based approach, it takes just 0.748765 second and 0.409865 second to identify nodes 26 and 10 as the weak nodes of the IEEE 30 bus and 10 bus test systems, respectively. Whereas, for the traditional approach of L-Index, it takes the total computational time of 0.984587 second and 0.523409 second to identify the weak node of the IEEE 30 bus and 10 bus test systems, respectively. Longer period of time was taken to identify weak nodes with VCPI method.

5.4 Proposed eigenvector centrality measure

Results of the influential node obtained using the traditional eigenvector centrality and the proposed eigenvector centrality for the IEEE 30 bus and the 10 bus test systems are presented in Tables 9 and 10. In this case, for the existing technique, adjacency matrix is first formed using eq. (21). We then applied eigenvalue decomposition technique on the adjacency matrix. The eigenvector of each node corresponding to the largest eigenvalue is found using eq. (22). It must be noted that, in this section of the work, only the PQ buses are considered for voltage stability analysis. This is because, there is no voltage-regulated bus in the system due to the fact that, when getting close to voltage collapse a PV bus is injecting its maximum reactive power into the system to maintain the voltage magnitude, so its voltage is no longer under control. For the traditional approach, node 13 of the IEEE 30 bus system has the highest values of the eigenvector centrality as can be seen in column 2 of Table 9.

In the case of the proposed eigenvector centrality, we do not need to find the Laplacian first. Rather, we applied eigenvalue decomposition technique to the response matrices formulated in eqs. (7) and (19). We then found the eigenvector of each node that correspond to the maximum eigenvalue using equations eqns. (25) and (26), respectively. The result of the suggested eigenvector centrality obtained from the IEEE 30 bus system, considering only PQ nodes is in agreement with that obtained with the traditional method as node 13 was identified as the most central PQ node of the IEEE 30 bus system. This is shown in column 5 of Table 9. The interpretation of this is that, if for instance the PQ node 13 is suddenly disconnected from the power network, this may result in system voltage instability due to high relative influence of this node compared with other nodes. As such, in case of contingency whereby node 13 is suddenly disconnected from the network, the coupling strength of the network will adversely be affected and thus, affect the network topology. Results obtained from the practical Southern Indian 10-bus power network shown in Table 10 is also in conformity with that which was obtained using the conventional method. For instance. PQ bus 7 is found to have the highest value of both the conventional eigenvector centrality and the proposed eigenvector centrality measures. Thus, confirming the significance of the proposed approach.

6. Conclusion

This paper demonstrates the effectiveness of using network structurally based approaches as alternative algorithms for the identification of critical nodes that may lead to voltage instability in a power system. A detailed mathematical derivation of the technique based on the structural interconnections of the network elements (NRSPF) for the identification of weak nodes in power networks is presented. The suggested method of NRSPF is compared with the existing power flow based voltage stability Index and the VCPI techniques. The proposed eigenvector centrality measure is also formulated based on the response matrices of both load and generator nodes of the power networks. Their usefulness in identifying most influential node of the network is also investigated and compared to the traditional approach. Results of simulations obtained show that weak node which is susceptible to voltage instability in a power system network is better identified through a non-iterative based technique of NRSPF, than going through the traditional approach of VCPI and L-Index. Also, the proposed eigenvector centrality measure can serve as alternative tool to identify the most influential node of the network and can be of tremendous help to power system community.

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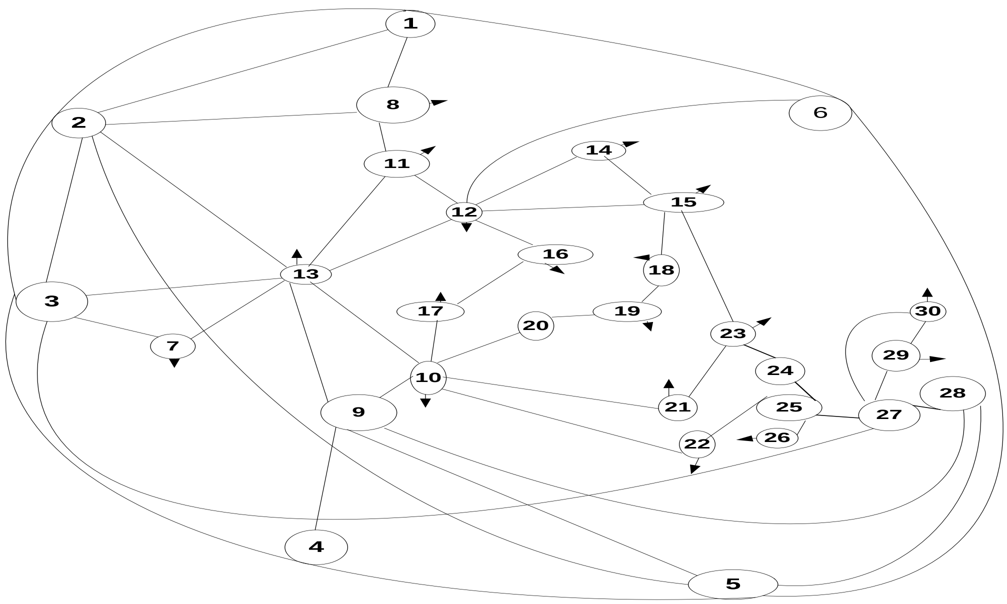

Fig 1: (a) One Line diagram of the IEEE 30-bus power system

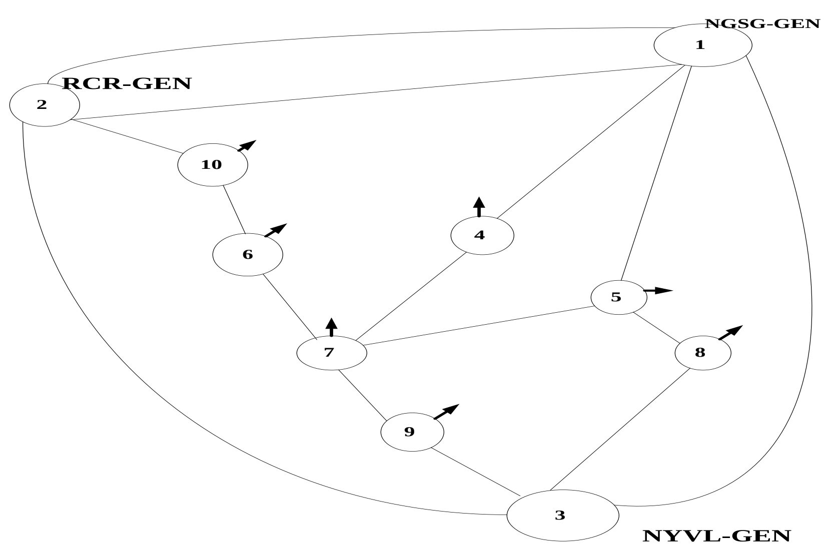

Fig. 1(b): Schematic diagram of the Southern Indian 10-bus power networks

Test Case I: Results of the IEEE 30 bus power Networks

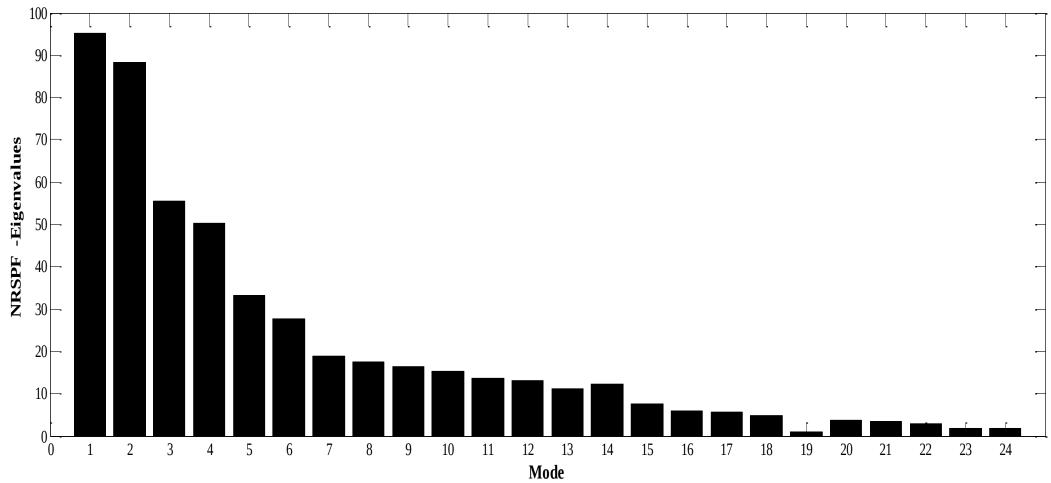

Figure 2: NRSPF eigenvalues for the IEEE 30 bus power system

Table 1: Results of NRSPF for the IEEE 30 bus test system

Rank	Load Bus	Voltage Mag.(p.u)	NRSPF	Computational Time (secs)
1st	26	1.0075	0.6588	
2nd	29	1.0189	0.0396	0.748765
3rd	24	1.0227	0.0301	
4th	30	1.0076	0.0105	

Table 2: Traditional power flow based L-Index approach for the IEEE 30 bus system

Ranking Order	Load Bus No	Traditional Approach L-Index	Voltage Mag.(p.u)	Computational Time (Secs)
1st	26	0.0794	1.0075	
2nd	30	0.0764	1.0076	0.984587
3rd	29	0.0675	1.0189	
4th	24	0.0622	1.0227	

Table 3: VCPI of the IEEE 30 bus system

Ranking Order	Load Bus No	Qmax (MVar)	Traditional Approach VCPI	Voltage Mag. (p.u)	Computational Time (Secs.)
1st	26	32	0.4441	0.5821	7.987209
2nd	30	34	0.3992	0.5945	8.709306
3rd	29	40	0.3213	0.6109	9.780415
4th	24	112	0.2389	0.6506	25.009876

Table 4: Results of the comparison between the approaches for the IEEE 30 bus test system

		Traditional Approach			Proposed Approach	
Rank	Bus No	L-Index	VCPI	Bus No	Rank	NRSPF
1st	**26**	**0.0794**	**0.4441**	**26**	**1st**	**0.6588**
2nd	30	0.0764	0.3992	29	2nd	0.0396
3rd	29	0.0675	0.3213	24	3rd	0.0301
4th	24	0.0622	0.2389	30	4th	0.0105

Test Case II: Results of the Southern Indian 10-bus Power networks

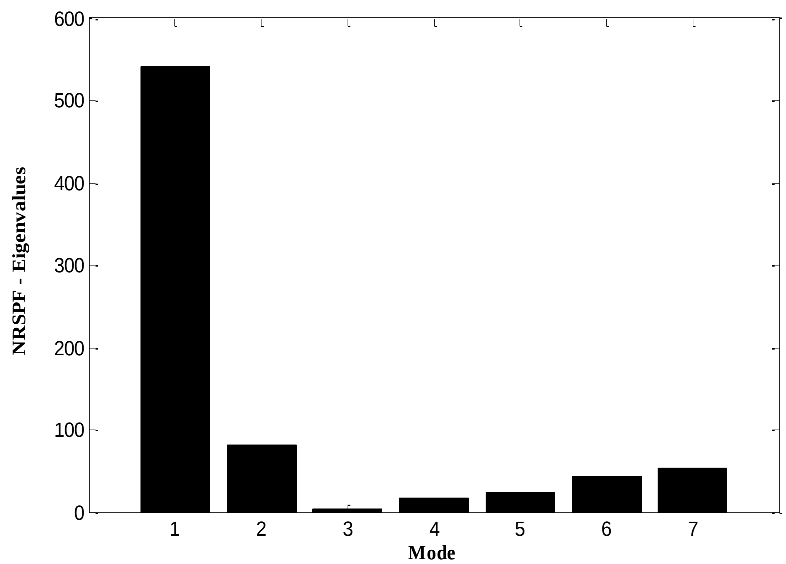

Figure 3: NRSPF eigenvalues for the 10 bus power system

Table 5: Results of the NRSPF for the 10 bus power system

Ranking Order	Load Bus No	NRSPF	Voltage Mag.(p.u)	Computational Time (Secs)
1st	10	0.1944	0.9039	
2nd	6	0.1668	0.9040	0.409865
3rd	8	0.1551	0.9642	
4th	7	0.1500	0.9076	
5th	5	0.1208	0.9436	
6th	4	0.0271	0.9066	
7th	9	0.0181	0.9859	

Table 6: Traditional power flow based L-Index approach for the 10 bus system

Ranking Order	Load Bus No	Traditional Approach L-Index	Voltage Mag.(p.u)	Computational Time (Secs)
1st	10	0.1463	0.9039	
2nd	6	0.1373	0.9040	0.523409
3rd	7	0.1325	0.9076	
4th	4	0.1043	0.9066	
5th	5	0.1039	0.9436	
6th	8	0.0896	0.9642	
7th	9	0.0626	0.9859	

Table 7: Results of the VCPI for the 10 bus power system

Ranking Order	Load Bus No	Qmax (Mvar)	Traditional Approach VCPI	Voltage Mag.(p.u)	Computational Time (Secs)
1st	10	370	0.9728	0.5502	
2nd	4	450	0.9522	0.5667	98.509321
3rd	6	550	0.8981	0.5624	
4th	7	600	0.7762	0.5871	
5th	8	700	0.6075	0.5907	
6th	5	800	0.4596	0.5970	
7th	9	900	0.2106	0.7807	

Table 8: Results of the comparison between the approaches for the 10 bus test system

Rank	Bus No	L-Index	Bus No	VCPI	Bus No	NRSPF
1st	10	0.1463	10	0.9728	10	0.1944
2nd	6	0.1373	4	0.9522	6	0.1668
3rd	7	0.1325	6	0.8981	8	0.1551
4th	4	0.1043	7	0.7762	7	0.1500
5th	5	0.1039	8	0.6075	5	0.1208
6th	8	0.0896	5	0.4596	4	0.0271
7th	9	0.0626	9	0.2106	9	0.0181

Table 9: Results of the traditional eigenvector centrality and the proposed eigenvector centrality measures (IEEE 30-bus system)

Bus No	Bus Type	Basecase Voltage Mag.(p.u)	Traditional Eigenvector Centrality	Proposed Approach Eigenvector Centrality $C_{mE}^{\Lambda^{LL}}$
7	PQ	1.0141	0.1682	0.1177
8	PQ	1.0442	0.3011	0.2564
9	PQ	1.0615	0.0783	0.0474
10	PQ	1.0562	0.0498	0.0438
11	PQ	1.0407	0.5127	0.6178
12	PQ	1.0689	0.0516	0.0344
13	PQ	1.0298	0.6109	0.7028
14	PQ	1.0546	0.0046	0.0015
15	PQ	1.0504	0.0104	0.0047
16	PQ	1.0564	0.0067	0.0022
17	PQ	1.0510	0.0131	0.0062
18	PQ	1.0404	0.0012	0.0003
19	PQ	1.0376	0.0018	0.0005
20	PQ	1.0415	0.0054	0.0025
21	PQ	1.0420	0.0365	0.0348
22	PQ	1.0485	0.0069	0.0032
23	PQ	1.0419	0.0324	0.0273
24	PQ	1.0379	0.0031	0.0013
25	PQ	1.0377	0.0015	0.0003
26	PQ	1.0150	0.0001	0.0000
27	PQ	1.0459	0.0153	0.0060
28	PQ	1.0324	0.2579	0.1933
29	PQ	1.0266	0.0008	0.0001
30	PQ	1.0154	0.0005	0.0001

Table 10: Node type, Voltage Mag., and the proposed eigenvector centrality

Bus No	Bus Type	Basecase Voltage Mag.(p.u)	Traditional Eigenvector Centrality	Proposed Approach Eigenvector Centrality $C_{mE}^{\Lambda^{LL}}$
4	PQ	0.9066	0.0472	0.0248
5	PQ	0.9436	0.0606	0.0327
6	PQ	0.9040	0.6982	0.6673
7	PQ	0.9076	0.7056	0.7423
8	PQ	0.9642	0.0069	0.0028
9	PQ	0.9859	0.0815	0.0416

10	PQ	0.9387	0.0407	0.0192

Table 11: Bus data of the Southern 10 bus test system (Southern Indian 10 bus power system)

Bus No	Bus code	Voltage Mag.	Angle Degrees	Load		Generator			
				MW	MVAR	MW	MVAR	Qmin	Qmax
1	2	1.05	0	780	350	818	0	-50	400
2	2	1.05	0	0	0	132	0	-50	400
3	1	1.06	0	0	0	0	0	-90	400
4	3	1	0	100	150	0	0	0	0
5	3	1	0	250	125	0	0	0	0
6	3	1	0	180	100	0	0	0	0
7	3	1	0	250	100	0	0	0	0
8	3	1	0	320	150	0	0	0	0
9	3	1	0	340	120	0	0	0	0
10	3	1	0	50	25	0	0	0	0

Table 12: Line data of the Southern 10 bus test system

From bus	To bus	R (p.u)	X (p.u)	1/2B	Tap ratio
1	2	0.00477	0.05103	0.72673	1
3	8	0.00297	0.03706	0.47543	1
3	9	0.00145	0.01802	0.93968	1
7	5	0.0043	0.0477	0.637	1
2	10	0.00676	0.03029	0.75003	1
10	6	0.00546	0.02294	0.88836	1
6	7	0.004	0.044	0.15	1
1	4	0.00569	0.06008	0.79414	1
7	4	0.00589	0.05995	0.7841	1
7	9	0.00289	0.03603	0.46222	1
1	5	0.00272	0.02872	1.51829	1
5	8	0.00388	0.04834	0.6547	1