# Block sourcing

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# Block sourcing*

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**Abstract**

We study how a buyer should structure his demand in the presence of diseconomies of scale in production. Compared to an efficient market with $n$ (identical) suppliers, he benefits from auctioning large blocks of contracts and sourcing only the remainder via the market. Optimally, he sets $n-2$ or $n-1$ lots, depending on his bargaining power vs. a single supplier. The distortion leads to overproduction and to the misallocation of production. When he has commitment power and can strategically set the quantity, block sourcing is still beneficial, but – unless his bargaining power is very high – it leads to underproduction.

**JEL Classification:** C72, D44, L14

**Key words:** Procurement; Price competition; Split awards; Strategic sourcing.

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# 1 Introduction

We consider a buyer of a divisible homogeneous input and – upstream – a finite number of suppliers who have increasing marginal costs of production. The buyer's default option is to purchase his requirements from the suppliers via a "competitive" (reverse) auction, where in equilibrium price equals marginal cost and production is efficient. We ask the question: Can he increase his surplus by block sourcing? We define block sourcing as grouping together part of the requirements into sole-sourced lots (blocks) and sourcing the residual demand competitively. We find that effective lots must be larger than the efficient quantity of a single supplier, thus they can only benefit the buyer if he manages to squeeze the suppliers' profit margin[1] sufficiently to compensate for the deadweight loss created. We show that when lots – of the appropriate number and size – are introduced, suppliers indeed end up bidding "average" prices for these that are sufficiently below their marginal cost to benefit the buyer.

Our model captures a host of situations, but our focal application is that of procurement of a large company consisting of many sub-units, where block sourcing means the centralization of some of its supply management. A typical example would be a Health Board running a number of hospitals.[2] We analyze the decision of the Board on how to group the requirements of its hospitals into sole-sourced lots. That is, we consider centralization[3] as an enabler of partial coordination of procurement demands rather than as a blunt instrument that agglomerates all requirements into a single contract – what

[1] Indeed, since the competitive price is determined by marginal cost, which is increasing, the suppliers do make positive profits in the efficient Bertrand equilibrium.

[2] Alternatively, a tour operator looking for hotel rooms for its clients, or a produce wholesaler sourcing from many growers, etc.

[3] Actual procurement policies are highly heterogeneous regarding their degree of centralization and the structure of the supply contracts. As reported by Dimitri et al. (2006), while public sector tendency is to use centralized systems, the private sector is more heterogenous in this respect: centralization and decentralization coexist. Baldi and Vannoni (2014) provide some examples of such heterogeneity. "During the 1990s, many big companies went through important reorganization of their activities, including purchasing, and adopted different combinations of centralized and decentralized procurement. Some of them, as Motorola, General Electric, United Technologies and Fiat, decentralized this function, whereas some others, such as Honda and General Motors, centralized it."

would (especially) make sense if there were economies of scale in production.[4] We show, in this paper, that centralizing purchasing decisions continues to be an optimal policy when there are diseconomies of scale.

Our insight is that the competitive pressure facing the suppliers is in part determined by the lot policy chosen. How keen a supplier is to obtain a block contract depends on her outside option. This outside option is actually an inside option in our model: it is determined by what she can expect to supply in the residual market. The size and competitiveness of this "aftermarket"[5] are endogenous: the more demand is satisfied through lots, the less residual demand there is; in parallel, the more suppliers price themselves out of the aftermarket by signing lot contracts (thereby increasing their marginal costs), the less competition there is in the aftermarket. By setting many large lots, the buyer can squeeze the suppliers' profit margins in the aftermarket. As this makes their inside option worse, they also bid more aggressively for the block contracts, in the aggregate more than compensating for the inefficiency.[6]

Under the simplifying assumption of $n$ identical suppliers, we show that the optimal lot policy is always one of two options: either $n-2$ or $n-1$ lots of equal size. Which of the two is better is a function of the buyer's bargaining power when he faces a single remaining supplier to satisfy his residual demand. If it is high, he can "afford" to set the less inefficient $n-1$ lots, otherwise not. This means that – with the unique exception of the case of two suppliers and low buyer bargaining power – it is always optimal to bundle some (most) of the buyer's requirements.

The optimal lot policy leads to a suboptimal distribution of production across suppliers as the optimal lot size exceeds the quantity sold by a supplier in the aftermarket. To exacerbate this inefficiency, the aggregate volume of trade is excessive as well: there is overproduction. This raises the question of what happens when the buyer can credibly commit to limit his demand in the aftermarket. To keep a fair comparison, we simultaneously replace the efficient market with classical monopsony as the default option.

[4] Indeed, the ability to receive quantity discounts is one of the most cited reasons in favor of centralization (see Munson, 2007).

[5] It is not necessary that the residual market clear after the lot auction, but it sounds intuitive.

[6] To highlight the forces at play, following this Introduction we will go through a simple example.

Remarkably, setting lots (optimally) is still superior when the buyer can strategically distort his demand (and therefore also compared to the competitive benchmark). At the same time – while the optimal policy is still setting either $n-2$ or $n-1$ lots above the competitive production size – there is nearly always[7] underproduction.

We complement our general results with more detailed comparative static exercises for specific families of cost and utility functions. We display how the marginal bargaining power (that determines the optimal number of lots), prices, and buyer profits vary with changes in the parameters, providing further intuition for our results.

In previous studies on optimal multi-sourcing, the procurement policy of the strategic buyer is restricted to either single sourcing or multi-sourcing for the entire demand. Few of these studies consider diseconomies of scale. The two main exceptions are Anton and Yao (1989) and Inderst (2008). In their model, the way the award is split is endogenous: sellers submit bids for different possible splits of the whole requirement of the buyer. This set-up, together with diseconomies of scale, leads to the "annoying" result that, when there are two suppliers, any split can be supported as an equilibrium.[8] However, there is one, involving the efficient split, where suppliers maximize their individual profits. This is a clear candidate for collusive bidding. Compared to this focal equilibrium, the buyer will prefer to auction off a sole-sourcing contract. Inderst (2008) amends this result – reinforcing the equilibrium selection by applying a refinement first proposed by Bernheim and Whinston (1986) – by pointing out that, when there is more than one buyer, single sourcing results if and only if the buyer controls a sufficiently large share of the market. The intuition is that single sourcing would increase competition among suppliers only when the alternative to winning is bad enough (that is, the residual demand is low). Otherwise, because of convex costs, the losing supplier will be able to obtain a large share

[7] The exception is when the buyer's bargaining power is sufficiently high so that he captures enough of the residual surplus to compensate for the higher lot prices resulting form the higher supplier profit in the aftermarket. In this case, the residual quantity sold is efficient, but only conditional on the quantity bought via lots, and therefore there is still overproduction (in fact, the optimal lot policy is the same as without commitment).

[8] We avoid the multiplicity encountered when bidding is in offer curves, by sticking with the – commonly observed – procedure of setting the lot sizes before the auction takes place.

of the residual demand and therefore she will bid less aggressively for the sole-sourcing contract. Note that the presence of competition makes single sourcing worse rather than making split awards better for our buyer.

In the next section we work out an example that conveys the main intuition. In Section 3 we present and justify our model. Section 4 contains some preliminary results. In Section 5 we analyze the optimal lot policy of the buyer without commitment power. In Section 6 we discuss the case where the buyer can commit to a demand function of his choice. Both Sections 5 and 6 contain comparative statics exercises for a restricted family of cost and utility functions. Finally, we present some concluding remarks in Section 7. We collect the proofs not presented in the main text in Appendix A, while Appendix B contains justifications for our equilibrium selection.

## 2 Setting the stage

Let us go through an example to display the intuition for why block sourcing may lead to gains for the buyer. Consider a buyer with a concave utility function, $U(.)$, wishing to buy a commodity supplied by three identical sellers with convex cost functions, $c(.)$. We compare two cases here: the default option is turning to the competitive market; the block sourcing option is auctioning off a lot, $z$, before going to the competitive market with the two remaining[9] suppliers. In the first case, he will buy quantity $Q^c$ at (unit) price $p^c$, where, by definition, demand – given by marginal utility – equals marginal cost: $U'(Q^c) = c'(Q^c/3) = p^c$. In the latter case, let us first determine the outcome of the second stage. Here, the remaining suppliers produce $q$ and expect a profit of $p_2 q - c(q)$ each, where *residual* demand equals marginal cost: $U'(z + 2q) = c'(q) = p_2$. Note two things: first, that the second period price and quantity depend (negatively) on $z$; second, that, when $z = q(z)$, the outcome will be exactly the same as in the default case: block sourcing makes no difference at all.

A key observation is that, in equilibrium, the winner of the lot cannot make strictly higher profit than the losers (otherwise a loser could underbid her by $\varepsilon$ and obtain almost

[9] We only exclude the winner of the lot for simplicity. In our model this restriction is not imposed.

the same profit).[10] Crucially, this means that squeezing the losers' profit also squeezes the winner's. Thus, reducing the demand available to the losers – by increasing the lot's size – reduces the profits of all suppliers. Of course, this effect is countervailed by the inefficiencies introduced, but as we will see, a well-chosen lot size always benefits the buyer. By the above observation, the (worst case) consumer surplus under block sourcing can be written as the total surplus minus 3 supplier profits:

$$\Pi = U(z+2q) - c(z) - 2c(q) - 3\left[c'(q)q - c(q)\right].$$

Differentiating with respect to $z$, we have

$$U'(z+2q) - c'(z) + \frac{dq}{dz}\left[2U'(z+2q) - 2c'(q) - 3c''(q)q\right].$$

Evaluating the above derivative at $z = q(z) = Q^c/3$, which is equivalent to directly going to the market, and rearranging, we obtain

$$\frac{d\Pi}{dz} = \left[U'(Q^c) - c'(Q^c/3)\right]\left(1 + 2\frac{dq}{dz}\right) - \frac{dq}{dz}c''(Q^c/3)Q^c > 0.$$

As the first term is zero, by definition, and the quantity sold in the second stage is clearly decreasing in the lot set aside – totally differentiating the market clearing condition $U'(z+2q) = c'(q)$ and substituting in we have $\frac{dq}{dz} = \frac{U''(Q^c)}{c''(Q^c/3) - 2U''(Q^c)} < 0$ –, this derivative is positive, indicating that it is profitable to separate out a block contract that is (somewhat) larger than the competitive quantity.

We can better appreciate the effects at play graphically.

Figure 1a depicts the lot auction, while Figure 1b the second stage. The second stage determines $p_2$ and also the second stage consumer surplus (CS), the red triangle in Figure 1b. The profits of each supplier are then the blue triangle in Figure 1a (which is half the black triangle in Figure 1b). The CS from the lot auction – total surplus minus supplier profit – is the black pentagon in Figure 1a.

Now, observe that the red triangle in Figure 1a is the same as the red triangle in Figure 1b, translated to the right by the distance $z$. Consequently, total CS is given by the sum

[10] In fact, the profits of the winner and the losers must be equal, but that is a bit harder to prove, and not necessary for our argument here.

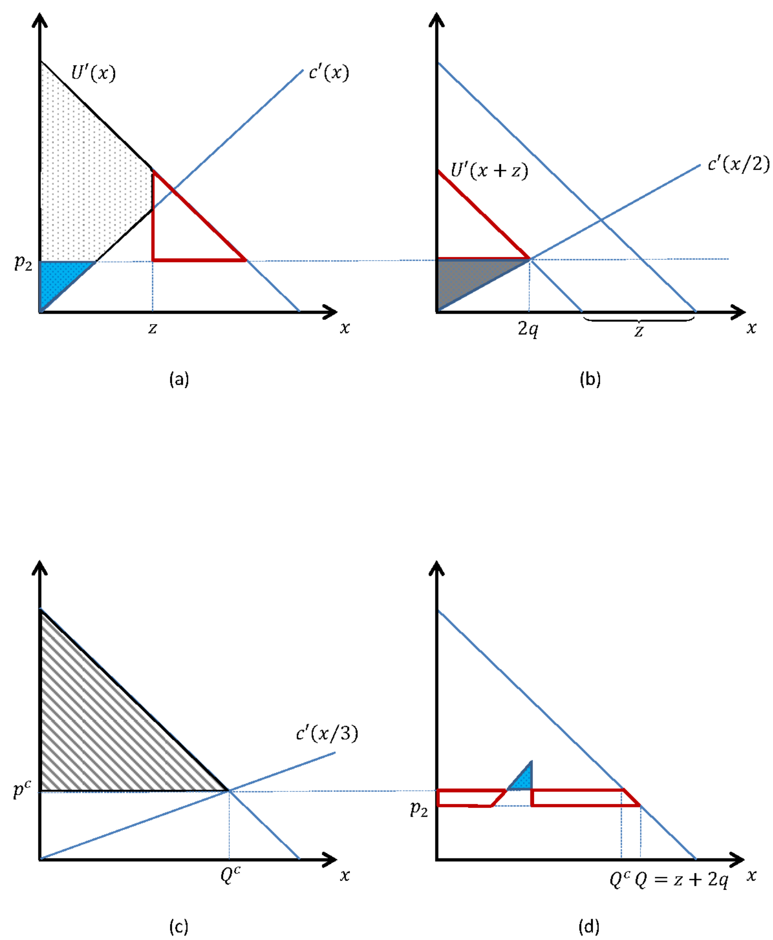


Figure 1: Effects of the lot on the market price and on consumer surplus.

of the black pentagon and the red triangle in Figure 1a. Figure 1c depicts the outcome of buying the requirements from a one-shot competitive market, and the black triangle is the resulting CS. Finally, Figure 1d displays the difference between the two CS's: the buyer has to pay for the additional cost due to the inefficient organization – and quantity – of production (the blue triangle) but he managed to lower the average price (the two red trapezoids). As the red area is larger than the black one, setting the lot increases the buyer's profit.

In the remainder of the paper we establish the generality of this result, and also characterize the optimal lot policy.

# 3 The base model

Consider $n > 1$ identical suppliers producing an infinitely divisible homogeneous good/service with a strictly increasing, strictly convex and thrice differentiable cost function $c(x)$.[11] There is a single buyer, $B$, with a twice continuously differentiable, quasi-linear vNM utility function, $V(x, \$) = U(x) + \$$, with $U'(x) > 0$, $U''(x) < 0$ for $x \in [0, 1]$, with the normalization $U'(1) = 0$. The cost and utility functions are common knowledge. We study the following two-stage procurement game: First, $B$ announces $m \in \{0, 1, ..., n-1\}$ contracts for (indivisible) lot sizes $z_1 \geq ... \geq z_m$, where $\sum_{i=1}^{m} z_i = Z \leq 1$. Next, these contracts are sequentially auctioned, in decreasing order of size.[12] As standard in multi-sourcing arrangements, each supplier can win at most one block contract. Following the lot auctions, each supplier decides whether to compete for the residual demand in the aftermarket.[13] We distinguish two cases of the aftermarket. If there is a single remaining supplier, then we have a bilateral monopoly. In this case, the parties engage in Coasian bargaining and share the residual surplus in proportion $(\alpha, 1-\alpha)$ between the supplier

[11] To ensure that second-order conditions for optimality are globally satisfied, we assume that $c'''(x)x + c''(x) > 0$ for $x \in [0, 1]$.

[12] Given complete information, the exact format of the auctions does not matter, even sequentiality we only assume for simplicity.

[13] To start with, we assume that the buyer cannot commit not to satisfy his residual demand. We analyze the consequences of strategic demand distortion in Section 6.

and the buyer, respectively. When there are multiple remaining suppliers, they compete in (unit) prices.[14] We assume that they play the equilibrium that leads to the efficient outcome at the competitive (unit) price.

In constructing our model, we have striven for a balance between realism and parsimony that best puts into relief our contribution. Still, some of our modelling assumptions may benefit from justification.

First, we carry out our analysis under complete information. On the one hand, this obviously simplifies and focuses the analysis. On the other hand, we can afford to forgo the presence of informational rents, unnecessary in our model to give profits to the suppliers. Moreover, we are not conducting a mechanism design exercise here (where the exact nature of asymmetric information is crucial, see, for example, Manelli and Vincent, 2006), rather, we analyze the usefulness of some standard practices.

Second, we assume that suppliers are identical. This is just for simplicity. Our qualitative results, especially that setting lots larger than the sellers' competitive supplies leads to increased buyer profit, do not depend on the symmetry assumption.

Third, we assume that – conditional on the number of suppliers left – the residual market is efficient: we either have Coasian bargaining, or a competitive market. This allows for a fair comparison. There is a host of reasons why – faced with the multiplicity of equilibria in the Bertrand game with convex costs – the competitive market outcome is a sensible assumption, and we discuss these in detail in Appendix B. In any case, at the cost of complicating the algebra, our model could easily be adapted to (additional) inefficiencies arising in the residual market.

Fourth, we posit that the auction for the lots precedes the market/negotiation for the residual demand. Yet again, our pursuit of simplicity is complemented by another argument: in this case, realism. It is likely that at the time of auctioning the lots the final demand is not yet certain, so the residual demand is satisfied later. Explicitly modelling the uncertainty and its resolution would unnecessarily complicate the analysis, taking us away from the focus of this paper.

[14]Note that if the buyer decides not to set a lot – his default option – then he directly goes to the competitive market.

Finally, we do not model explicitly the outside options of the suppliers. A realistic way of modelling them would be to assume that each supplier has the option to sell some amount of its production at a high price outside the market (think of guests showing up at a hotel without reservation). This would simply shift the supply curves to the left by the quantity sold off-market. As a result, our qualitative results would not be affected.

# 4 Preliminaries

In a competitive equilibrium with $g$ suppliers and $Z$ units already committed to (by other suppliers), where both the buyer and the suppliers behave as price takers, the total production would be $Q^c(g;Z) = gq^c(g;Z)$ that satisfies $U'(Z + Q^c(g;Z)) = c'(q^c(g;Z))$. When $g = n$ and $Z = 0$, we drop the arguments: $Q^c = nq^c$ with $U'(Q^c) = c'(q^c)$ is the efficient benchmark that we will keep referring back to.

We start by an important observation that simplifies our search for the optimal block sourcing policy. Though at first it may sound counter-intuitive, the buyer's optimal policy does not have all suppliers participating in the aftermarket. He sets his lots in a way that the lot winners are effectively priced out of the aftermarket – due to their high marginal costs.

**Lemma 1** *The buyer must exclude lot winners from the aftermarket, in order to (potentially) increase his profit. He can only achieve this by setting lots satisfying* $z_i \geq q^c(n+1-i; \sum_{j=1}^{i-1} z_j)$, $i = 1, ..., m$.

The logic of this result is simple: if a lot winner participated (selling an additional $q'$ units) in the aftermarket – which indeed must be a market as the lot winner is joining at least one loser (selling $q$ units) – then, in equilibrium, marginal costs would equalize, $c'(z_i + q') = c'(q) \Rightarrow z_i + q' = q$, leading to the same outcome as if the lot won had not been offered. In order for a lot winner not to want to participate, her marginal cost given the commitment to produce the lot must exceed the price that would arise in the aftermarket (if she participated), which equals the marginal cost of the competitive quantity. Given this result, we can safely assume that the buyer indeed sets $m$ lots so that lot winners do not participate in the aftermarket:

$$z_i \geq q^c(n+1-i; \sum_{j=1}^{i-1} z_j), \; i = 1, ..., m. \quad (1)$$

**Remark 1** *The larger the inside option is, the larger the minimum size of a lot that can make a difference. This puts into perspective the result of Inderst (2008), that buyer competition makes single-sourcing less likely: if there were additional buyers, then the competitive quantity would be higher, so our buyer would have to increase his lot size. As that would increase the inefficiency, he would be pushed to set fewer lots than when he is in a monopsony position. Basically, when the buyer has competition he has less control over the supplier profits, so while setting lots has the same inefficiency cost, its effect on decreasing supplier rents is lower.*

Our next observation is that – since all suppliers are identical – in equilibrium it must be the case that a seller that does not win any lot earns the same profit as any of the winners (and the other losers). Let $\mathbf{z} = (z_1, ..., z_m)$. Denote the profits of a supplier that does not get any of the $m$ lots – and therefore, by Lemma 1, competes with $n - m - 1$ suppliers for the residual demand – by $\pi(\mathbf{z})$.

**Lemma 2** *Given any feasible* $\mathbf{z}$*, the equilibrium profit of each supplier is equal to* $\pi(\mathbf{z})$*.*

Basically, the auction and the aftermarket serve as inside options for the suppliers, enforcing indifference in equilibrium. This result points to the important externalities between the lot auction(s) and the residual market: the more competitive is one the more competitive becomes the other. This interconnection is what the buyer exploits when setting his lot policy. The lemma also implies that – conditional on the lot sizes – the market/negotiation for the residual demand determines all supplier profits.

Next, note that to determine a loser's profit, we only need to know the number and aggregate size of the lots, individual lot sizes do not matter. As Lemma 1 implies that we can ignore cases when some lot winner participates in the aftermarket, the latter has only the losers participating. Therefore, the residual demand will only depend on the aggregate lot size, $Z$, and the aggregate (residual) supply will only depend on the number of losing suppliers, $n-m$. Consequently, we can write $\pi(n-m;Z)$ for $\pi(\mathbf{z})$.

From Lemma 2, the price, $b_i$, of lot $z_i$ leaves the winner of this lot with the same profits as suppliers that serve the aftermarket, $b_i - c(z_i) = \pi(n-m;Z)$ so the equilibrium cost to the buyer of procuring lot $z_i$ is

$$b_i = c(z_i) + \pi(n-m;Z). \tag{2}$$

When there are $m \in \{0,...,n-2\}$ lots, by definition, we have a competitive aftermarket with a unit price of $p^c(n-m;Z) = c'(q^c(n-m;Z))$.[15] Each loser has competitive profits $\pi(n-m;Z) = qc'(q) - c(q)$ and the buyer pays $(n-m)qp = (n-m)qc'(q)$. Consumer surplus is

$$\begin{aligned} & U(Z+(n-m)q) - \sum_{i=1}^{m} b_i - (n-m)qc'(q) \\ &= U(Z+(n-m)q) - \left[\sum_{i=1}^{m} c(z_i) + (n-m)c(q) + n\pi(n-m;Z)\right], \end{aligned} \tag{3}$$

where we have written $qc'(q)$ as $c(q)+\pi(n-m;Z)$. Note that the total procurement cost can be written as the sum of production costs $\sum c(z_i) + (n-m)c(q)$ and the rents for sellers in equilibrium $n\pi(n-m;Z)$.

When there are $n-1$ lots, the single loser negotiates with the buyer and has profits $\pi(1;Z) = \alpha[U(Z+q) - U(Z) - c(q)]$ and the buyer pays a total amount $T = \pi(1;Z) + c(q)$ for quantity $q$ in the negotiation. Consumer surplus is

$$U(Z+q) - \sum_{i=1}^{n-1} b_i - T = U(Z+q) - \left[\sum_{i=1}^{n-1} c(z_i) + c(q) + n\pi(1;Z)\right]. \tag{4}$$

Total procurement costs can be written again as the sum of production cost $\sum c(z_i) + c(q)$ and the rents for sellers in equilibrium $n\pi(1;Z)$. In both cases (3) or (4), losers' profits $\pi(n-m;Z)$ depend only on the number of lots and on the total lot procurement

[15] To simplify notation, from here on we drop the arguments and the $c$ superindex from the competitive quantities and prices.

quantity $Z$, not on the distribution of lot's sizes. Therefore, by appealing to efficiency, it is immediate to obtain the following result:

**Lemma 3** *Optimally, all lots are of equal size,* $z_i \equiv z = Z/m$.

With this observation (1) can be simplified: it is without loss of generality to facilitate the exposition by only considering block sourcing policies such that no winner of a lot will want to participate in the aftermarket: $z \geq q^c$.

There are three variables that can affect consumer surplus: $z$, $m$ and $q$. The last of these can be – indirectly – chosen via the announcement of a demand function at the beginning of the game. As whether or not the buyer has the ability to commit to a demand that is not his true demand is not a choice – rather, it depends on institutional constraints – we consider both cases separately. We start with the situation where the buyer cannot pretend that he has a lower demand than he actually has. Say, a Health Board cannot credibly threaten not to hire nurses for one of its hospitals.

# 5 Results when the buyer cannot hide his demand

In the absence of commitment power, the buyer must report his true preferences and thus the quantity produced by a loser, $q$, satisfies

$$U'(mz + (n-m)q) = c'(q), \tag{5}$$

whether it is a negotiation ($m = n-1$) or a market ($m < n-1$) as both lead to the efficient outcome (conditional on the residual demand), by assumption.

**Remark 2** *It is useful to observe that setting* $m \leq n-2$ *lots of* $z = q^c$ *is equivalent to setting no lots at all:* $U'(mq^c + (n-m)q) = c'(q)$ *is solved by* $q = q^c$.

Using Lemma 3, we can write (3) and (4) as

$$CS = U(mz + (n-m)q) - [mc(z) + (n-m)c(q) + n\pi(n-m; mz)], \tag{6}$$

with

$$\pi\left(n-m;mz\right)=c'(q)q-c(q)\quad\text{when } m<n-1\text{, and}$$
$$\pi\left(1;(n-1)z\right)=\alpha\left[U((n-1)z+q)-U((n-1)z)-c(q)\right].$$

Before turning to the selection of the optimal block sourcing policy and to the question of when it is beneficial for the buyer to bundle some of his requirements, let us characterize what the implications of imposing lots larger than the competitive quantity ($z>q^c$) are.

**Proposition 1** *When the buyer cannot misrepresent his demand, setting $m\in\{1,...,n-1\}$ lots of $z>q^c$ implies that losers produce less than the competitive quantity, $q<q^c$, but overall there is overproduction, $Q=mz+(n-m)q>Q^c$.*

Overproduction is possible despite the efficient aftermarket as what *ex post* is efficient *ex ante* need not be. By the fact that the lots are larger than the competitive quantities, the division of requirements between the lots and the aftermarket is inefficient. The decrease in the second-stage quantities relative to the competitive ones does not fully compensate for the inflated lot size (c.f. Figure 1d, noting that $U'(Q)=p_2$, even though the unit price paid for the lot is higher than $p_2$. This can be seen on Figure 1b: the total quantity is $2q+z$, where $U'(2q+z)=c'(q)$).

The key insight is that efficiency implies that the marginal utility of the buyer determines the aftermarket quantity sold by each loser via $U'(Q)=c'(q)$. As a result, $Q$ and $q$ are negatively related. Thus, since, $q$ has gone down relative to the competitive benchmark, $Q$ must have gone up.

Marginal cost pricing in the aftermarket implies that the lot winners, who produce more, must be paid a higher (unit) price to compensate for their higher costs (otherwise, every extra unit they would sell at a loss, contradicting that they make the same profit as the losers in the lot auction).

**Corollary 1** *When the buyer leaves at least two suppliers for the aftermarket, the unit price of a lot is higher than the unit price in the aftermarket.*

The optimal number of lots, in principle, could be any between 0 and $n-1$. However, it is straightforward to show that the buyer's profit is increasing in the number of lots, at least up to $n-2$ of them. He will never want to leave more than two suppliers without a lot (and thus to compete for the residual demand).

**Lemma 4** *Without commitment power, the buyer will never set fewer than $n-2$ lots.*

The key argument behind this result is that the way the buyer can squeeze the suppliers' profit margin is by restricting the quantity available in the aftermarket. The inefficiency cost of raising the lot sizes to achieve the same increase in the aggregate lot size is clearly decreasing in the number of lots. This is complemented by the fact that the buyer makes higher profits on the lots as the lot suppliers make the same profit as the losers but they produce more. The following example illustrates, in a crisp manner, that reducing the number of suppliers in the aftermarket is always profitable, while there remain at least two of them.

**Example 1** *Suppose that – in contradiction to Lemma 4 – the buyer sets a lot policy of $(z, z; q, q, q)$: two lots of $z$ and three suppliers in the aftermarket. Now consider changing the policy by moving a supplier from the aftermarket to the lot auction maintaining her quantity at $q$: $(z, z, q; q', q')$. Observe that $q' = q$, since $U'(2z + 3q) = c'(q)$ and $U'(2z + q + 2q') = c'(q')$. In fact, by the same token, this alternative policy leads to the same consumer surplus. But now the buyer can trivially improve by equalizing the size of her lots, $(z', z', z'; q, q)$. As the aftermarket outcome only depends on the aggregate lot size and number, when $3z' = 2z + q$, the second period profits and quantities are unaffected, so the buyer can pocket all of the efficiency gain.*

We are now ready to provide a characterization of the optimal buyer strategy that applies to any number of suppliers and for all cost and utility functions satisfying our assumptions. By Lemma 4, it is evidently a two-horse race: the buyer either sets $n-2$ or $n-1$ lots. That is, the buyer chooses between negotiating with a single supplier or leaving two of them to compete in the aftermarket. Since both options are (constrained) efficient, the decisive factor must be the distribution of bargaining power. When the supplier is powerless ($\alpha = 0$) and $m = n-1$ the buyer takes away all the surplus from the aftermarket, while (all) the suppliers have zero profit for any aftermarket quantity of trade. As a result, the buyer has no incentive to distort from efficiency, so there is efficient trade – all lots, as well as the aftermarket quantity traded are equal to $q^c$ – and the buyer only pays for the actual production cost.[16] This is his first best, which dominates setting $n-2$ lots (what always leads to profits for the suppliers). As $\alpha$ increases, the consumer surplus with $m = n-1$ monotonically decreases and we have the following simple characterization:

**Proposition 2** *For any $n$, strictly convex cost function $c(.)$ and decreasing demand function $U'(.)$, there exists $\widehat{\alpha} \in (0,1]$, such that if $\alpha < \widehat{\alpha}$, then the optimal number of lots is $n-1$ and if $\alpha > \widehat{\alpha}$, then the optimal number of lots is $n-2$.*

Note that the proposition implies that, except for the case of two suppliers with "large"[17] bargaining power, the buyer will always be able to increase his utility by block sourcing. By Proposition 1 the buyer's increased profit is accompanied by a double inefficiency. Too much is produced and the production is distributed inefficiently across suppliers. As a result, the suppliers are worse off.

## 5.1 Illustration

In this subsection we explicitly calculate the optimal policy for a family of utility and cost functions, what enables us to carry out insightful comparative statics exercises. Assume that the buyer's utility function is $U(Q) = (1 - Q/2)\,Q$, while the cost function is $c(q;r,n) = \frac{nr}{2}q^2$. Note that with this cost function aggregate capacity is independent of $n$: the competitive equilibrium features total production $Q^c(r) = \frac{1}{1+r}$, consumer surplus $CS^c(r) = \frac{1}{2(1+r)^2}$ and total welfare $W^c(r) = \frac{1}{2(1+r)}$.

With $n-2$ lots of size $z$, there are 2 firms in the aftermarket. Each one produces $q = \frac{1}{2+nr}(1-(n-2)z)$. The optimal size of the lots is the one that satisfies (17), which

[16] Note that while the quantities produced by each supplier are the competitive ones, the prices are not, the buyer retains all the surplus.

[17] In the following subsection, we provide examples for the value of $\widehat{\alpha}$ as a function of the number of suppliers and the characteristics of the demand and cost functions.

in this quadratic example is

$$z(n,r) = \frac{1}{n} \cdot \frac{n(1+r)+2}{n(1+r^2)+(n+2)r}. \tag{7}$$

Firms in the aftermarket produce $q(n,r) = \frac{1}{n} \cdot \frac{nr+2}{n(1+r^2)+(n+2)r}$, while total production is

$$Q(n,r) = \frac{n(1+r)}{n(1+r^2)+(n+2)r}. \tag{8}$$

We see that firms in the aftermarket produce less than firms that win a lot, $q(n,r) < Q(n,r)/n < z(n,r)$, and total production is above the efficient one, $Q(n,r) > Q^c(n,r)$, if and only if $n > 2$ ($n = 2$ means no lots and therefore $Q(2,r) = Q^c(r)$). In equilibrium, consumer surplus is

$$CS(n,r) = \frac{1}{2} \cdot \frac{n}{n(1+r^2)+(n+2)r}. \tag{9}$$

Consumer surplus is larger than in a competitive market, $CS^c(r)$, if and only if $n > 2$. The relative increase in consumer surplus, $\frac{CS(n,r)}{CS^c(r)} = \frac{n(1+r)^2}{n(1+r^2)+(n+2)r}$, is increasing in $n$ and single-peaked in $r$, attaining its maximum at $r = 1$. At this value of $r$ it is $\frac{4n}{3n+2}$; therefore the gain in consumer surplus by setting $n-2$ lots is bounded by 1/3 of the original amount.

With $n-1$ lots of size $z'$, there is one firm left to negotiate a price for producing the efficient residual quantity $q' = \frac{1}{1+nr}(1-(n-1)z')$. The optimal size of lots is the one that satisfies (17), which in this quadratic example is

$$z(n,r;\alpha) = \frac{r+\alpha}{nr(1+r)+(n-1)\alpha}, \tag{10}$$

and the firm left to negotiate produces $q(n,r;\alpha) = \frac{r}{nr(1+r)+(n-1)\alpha}$, while total production is

$$Q(n,r;\alpha) = \frac{nr+(n-1)\alpha}{nr(1+r)+(n-1)\alpha}. \tag{11}$$

We see that the firm that negotiates produces less than the firms that win a lot, $q(n,r;\alpha) < Q(n,r;\alpha)/n < z(n,r,\alpha)$ and total production is again above the efficient one, $Q(n,r;\alpha) > Q^c(r)$ if and only if $\alpha > 0$ ($\alpha = 0$ means that the buyer receives all the surplus in the negotiation and it makes sense to set efficient lots, $z' = q' = Q^c(r)/n$) and is increasing in $\alpha$. This last observation has a straightforward explanation: as the buyer loses bargaining power, he wishes to decrease the last (and therefore all) supplier's profit. As he cannot reduce the quantity negotiated over *ex post*, he uses the lot policy to achieve the same goal. In equilibrium, consumer surplus is

$$CS(n, r; \alpha) = \frac{1}{2} \cdot \frac{nr(1-\alpha) + (n-1)\alpha}{nr(1+r) + (n-1)\alpha} \tag{12}$$

and it is decreasing in $\alpha$, as expected. At $\alpha = 0$, the buyer reaps all the surplus created in his relationship with the suppliers and also structures production efficiently, leading to his first best: $CS(n, r; 0) = \frac{1}{2(1+r)} = W^c(r)$. The relative increase in consumer surplus for a buyer with high bargaining power can be very large in an industry with large diseconomies of scale (high values of $r$) as $\frac{CS(n,r;0)}{CS^c(r)} = 1 + r$.

Comparing (12) with (9), we obtain that $CS(n, r) > CS(n, r, \alpha)$ – that is, the buyer prefers to leave two suppliers without a lot – if and only if $\alpha > \widehat{\alpha}(n, r) \equiv \frac{nr(nr+2)}{(nr+1)(nr+2)-n}$. Note that $\widehat{\alpha}(n, r) \in (0, 1)$ if and only if $r > (n-2)/n$. In other words, for a given number of suppliers, a negotiation is the more likely the lower $r$ is: a less elastic supply favors negotiation. Only for very low values of $r$ – that is, mild diseconomies of scale – will the buyer always prefer to leave a single supplier without a lot. Finally, in the particular case of $n = 2$, $\widehat{\alpha}(2, r) < 1$ is always satisfied. That implies that there exist some values of $\alpha$ (specifically, $\alpha > \frac{2+2r}{3+2r}$) for which the default option of the market (that is, $n - 2 = 0$ lots) dominates setting a lot.

Figure 2 below displays $\widehat{\alpha}(n, r)$ for different values of the parameter[18] $h = \frac{1}{1+r}$ in the interval $(0, 1)$ and for the number of sellers $n$ between 2 and 5.

Figure 3 shows the relative increase in consumer surplus when the buyer follows the optimal lot policy.

# 6 Results when the buyer has commitment power

In the preceding analysis, we have assumed that the buyer cannot strategically manipulate his (residual) demand. While in many public procurement situations this is the realistic

[18] We transform $r$, that can take any positive value, into $h$ that is always between 0 and 1.

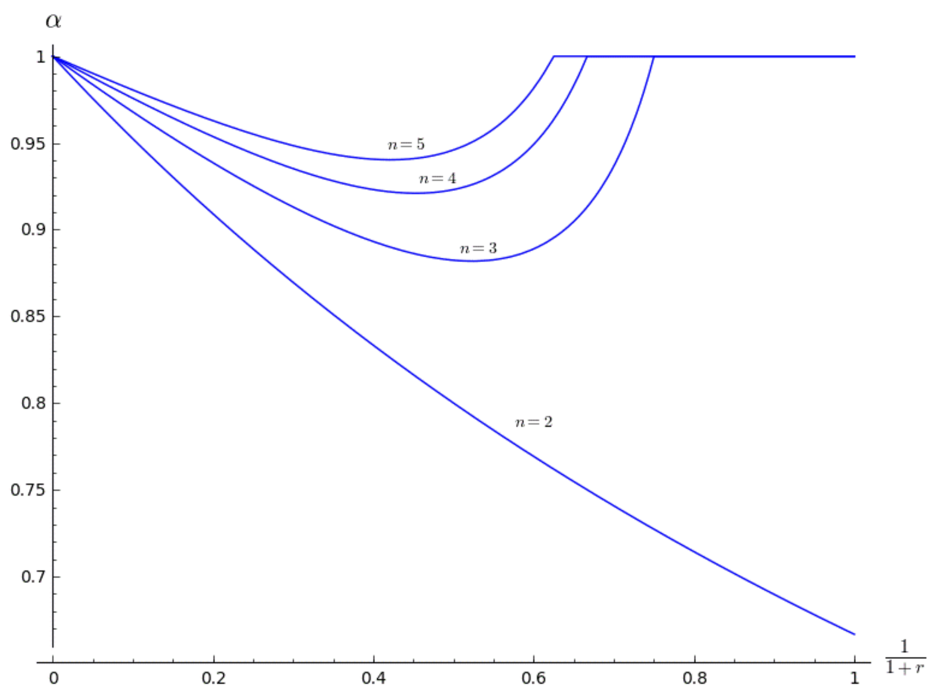


Figure 2: Without commitment, regions $\alpha > \widehat{\alpha}(n, r)$ for which an aftermarket ($n-2$ lots) dominates negotiation ($n-1$ lots)

assumption, a buyer often has some margin to distort his demand, similarly to a standard monopsony. In this section, we analyze the consequences of the ability to commit to a (lower) requirement. Note that, as only its intersection with the aggregate supply function matters, given complete information, committing to a particular (residual) demand function is equivalent to committing to a fixed amount bought in the second stage. We assume that the buyer commits to a total demand before the lot auction takes place.[19]

Commitment power in itself increases consumer surplus, so we need to set up a different default option before considering the effects of block sourcing. Now, the buyer can decrease the price – which is still given by the marginal cost of suppliers – by buying less. He

[19]This is the strongest form of commitment. An alternative would be that the buyer only commits to a (residual) demand after the lot auction.

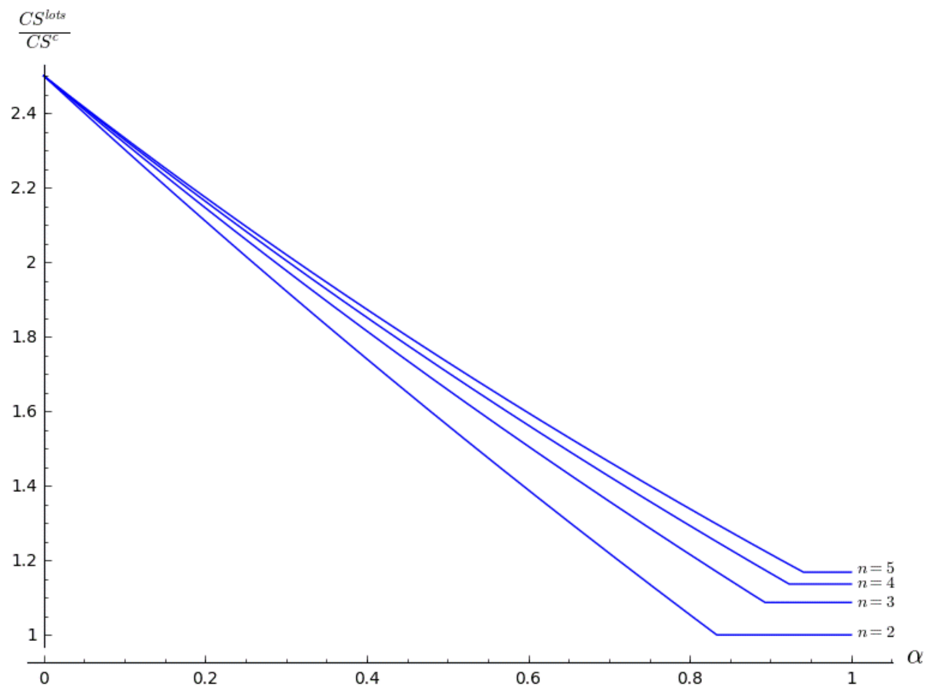


Figure 3: Relative increase in consumer surplus when $r = 3/2$.

maximizes $U(Q) - Qc'(Q/n)$ in $Q$, resulting in the first-order condition[20]

$$U'(Q^m) = c'(Q^m/n) + \frac{Q^m}{n}c''(Q^m/n). \tag{13}$$

It is the second term on the right-hand side that is new, relative to the case without commitment. As, by the convexity of the cost function, it is positive it is straightforward that: $Q^m < Q^c$. Just standard monopsony.

The result that the buyer will always want to set at least $n - 2$ lots continues to hold with commitment. However, this is not a direct corollary of the result without commitment as now the buyer can distort his demand.

**Lemma 5** *With commitment power the buyer will never set fewer than $n - 2$ lots either.*

We can now give a characterization of the equilibria for each of the two remaining cases.

[20]The second-order condition is satisfied by our assumption that $c'''(x)x + c''(x) > 0$.

**Proposition 3** *Suppose the buyer has commitment power. Then,*

**(i)** *With $n-1$ lots, if $\alpha < 1/n$ then the buyer leaves the same quantity for negotiation – and sets the same lots – as without commitment (leading to overproduction); otherwise, he chooses not to have an aftermarket, $q = 0$,[21] and there is underproduction, $Q < Q^c$.*

**(ii)** *With $n-2$ lots, we always have underproduction.*

When the buyer is strong in the negotiation, he does not wish to exercise his commitment power, as the loss of efficiency would mostly come out of his share of the surplus, even taking into account the effect on the lot winners' profits. Consequently, the optimal lot sizes are the same as if he had no commitment power and, by Proposition 1, we have overproduction. Otherwise, at any level of the bargaining surplus he has the incentive to reduce it, and consequently, he commits not to negotiate at all. As he has reduced the number of producers (from $n$ to $n-1$) this necessarily leads to underproduction: when there is no aftermarket, the lot winners make zero profit and thus the buyer buys the lots at cost. This means that he internalizes the cost fully and chooses the efficient quantity (with $n-1$ suppliers), what – due to the increasing marginal cost – is clearly less than the efficient quantity with $n$ suppliers.

When he leaves two suppliers for the aftermarket, in this market he has an incentive, like a standard monopsonist, to decrease demand. However, in our case he is more interested in decreasing the sellers' surplus than in increasing his own. As this demand reduction is a strategic substitute for setting large lots, he ends up consuming less than in the competitive equilibrium.

By Proposition 3, Proposition 2 carries over to the commitment case (with a different threshold, of course): consumer surplus with $n-1$ lots is the same as before for $\alpha \in [0, 1/n]$ – and, therefore it is strictly decreasing in $\alpha$ – and it is constant thereafter, while with $n-2$

[21] Actually, there might exist an equivalent solution with a very high negotiated quantity that also yields zero surplus. We consider this completely unrealistic and discard it. It would also disappear with any transaction costs attached to bargaining.

lots it is obviously constant in $\alpha$. The fact that, when the buyer has all the bargaining power ($\alpha = 0$), $n-1$ lots are clearly superior, completes the argument.

**Proposition 4** *There exists $\widetilde{\alpha} \in (0, 1/n] \cup \{1\}$, such that if $\alpha \leq \widetilde{\alpha}$ then the optimal procurement strategy is to set $n-1$ lots. If $\alpha > \widetilde{\alpha}$, then the optimal strategy is to set $n-2$ lots. When $\widetilde{\alpha} = 1/n$, then for all $\alpha \in [1/n, 1]$ both solutions are equivalent.*

Just as in the case without commitment, the threshold value $\widetilde{\alpha}$ cannot be explicitly calculated in general. In the next section we investigate its comparative statics for a family of cost functions.

### 6.1 Illustration

Assume that the buyer's utility function is $U(Q) = (1 - Q/2)\, Q$, while the suppliers' cost function is $c(q; n, k) = \frac{n^k}{1+k} q^{1+k}$ with $k > 0$. We have $c' = n^k q^k > 0$, $c'' = k n^k q^{k-1} > 0$. This cost function implies that – for any quantity symmetrically distributed among the suppliers – total production costs do not vary with the number of suppliers: $nc(Q/n; n, k) \equiv \frac{r}{1+k} Q^{1+k}$. As a consequence, we can investigate the effects of changing industry structure (the number of suppliers) holding industry "capacity" constant.

Form Proposition 4, we know that for sufficiently low value of $\alpha$, $n-1$ lots are preferred. We also know – by Proposition 3 – that when $\alpha > 1/n$, the buyer commits to $q = 0$ after auctioning $n-1$ lots, leaving the suppliers with no surplus. Therefore,[22] when $\alpha > 1/n$ and $n-1$ lots are auctioned, total cost is equal to $n-1$ production costs and consumer surplus is

$$U(Q) - (n-1)c\left(\frac{Q}{n-1}\right) = \left(1 - \frac{Q}{2}\right) Q - \left(\frac{n}{n-1}\right)^k \frac{Q^{1+k}}{1+k}. \tag{14}$$

The optimal quantity procured, $Q_1$, satisfies the first-order condition $1 - Q_1 - \left(\frac{n}{n-1}\right)^k Q_1^k = 0$.

[22] By Proposition 3 and Lemma 6 when $\alpha < 1/n$ the buyer has an additional bias in favor of $n-1$ lots.

On the other hand, consumer surplus when quantity $Q$ is procured through $n-2$ equal sized lots is

$$U(Q)-(n-2)c\left(\frac{Q-2q}{n-2}\right)-2c\left(q\right)-n\left[c'\left(q\right)q-c\left(q\right)\right]$$
$$=U(Q)-\frac{n^k}{1+k}\left[(n-2)\left(\frac{Q-2q}{n-2}\right)^{1+k}+(2+nk)q^{1+k}\right],$$

where we write the quantity $z$ procured through a lot as $z=\frac{Q-2q}{n-2}$. Given $Q$, the quantity $q$ sold by sellers in the market is $q^*=\frac{Q}{2+(n-2)\left(\frac{2+nk}{2}\right)^{1/k}}$; the optimal size of a lot is $z^*=\frac{\left(\frac{2+nk}{2}\right)^{1/k}}{2+(n-2)\left(\frac{2+nk}{2}\right)^{1/k}}Q$. Therefore, when $n-2$ lots are auctioned, consumer surplus is

$$\left(1-\frac{Q}{2}\right)Q-\frac{2+nk}{2(1+k)}\left(\frac{n}{2+(n-2)\left(\frac{2+nk}{2}\right)^{1/k}}\right)^kQ^{1+k} \tag{15}$$

and the optimal quantity procured, $Q_2$, satisfies $1-Q_2-\frac{2+nk}{2}\left(\frac{n}{2+(n-2)\left(\frac{2+nk}{2}\right)^{1/k}}\right)^kQ_2^k=0$.

Comparing (14) with (15), it is easy to check that for any total quantity, $Q$, purchased, the total cost to the buyer is higher with $n-1$ lots if and only if $n<f(k)\equiv 2\frac{2^k-1}{k}$. Therefore, when $\alpha>1/n$, consumer surplus with $n-2$ lots is higher than with $n-1$ lots if and only if $n<f(k)$.[23] We see in particular that, when $n=2$ and production costs are quadratic, $k=1$, the market $(n-2=0)$ performs equal to the procurement of one lot; similarly, setting one lot leads to the same procurement costs as two lots when $n=3$ and production costs are cubic, $k=2$. The following corollary to Proposition 4 is now immediate.

**Corollary 2** *If $n<f(k)$ then $\widetilde{\alpha}\in(0,1/n)$, if $n=f(k)$ then $\widetilde{\alpha}=1/n$ and for any $\alpha\geq\widetilde{\alpha}$ the buyer is indifferent between setting $n-1$ or $n-2$ lots. Finally, if $n>f(k)$ then $\widetilde{\alpha}=1$.*

Figure 4 below displays $f(k)$. When $\alpha>1/n$, to reduce procurement cost it is advisable to set $n-1$ $(n-2)$ lots when $n>(<)f(k)$.

[23] To see this, pick the optimal quantity for the higher cost option. The lower cost one will provide a higher surplus at that quantity, which is not even the best option.

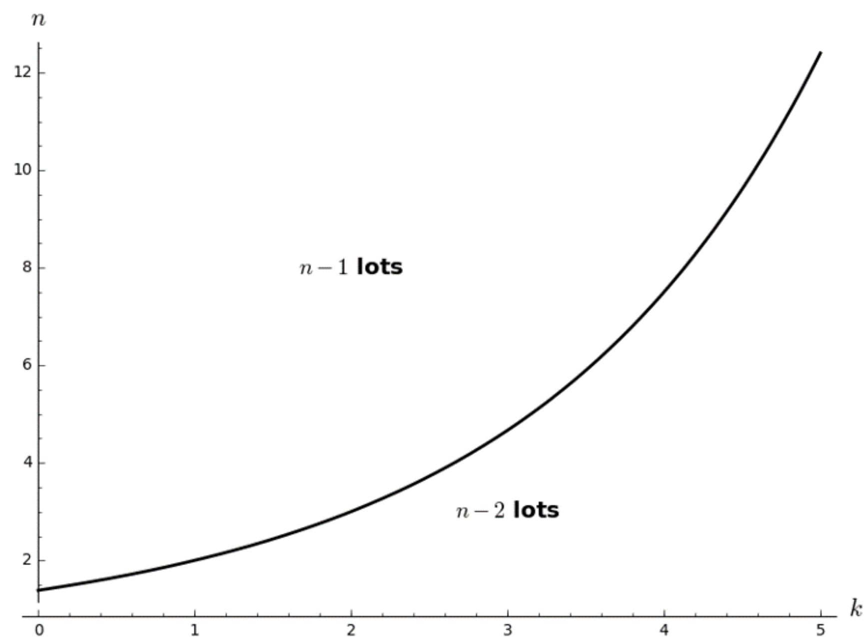


Figure 4: Regions for which $n-1$ or $n-2$ lots minimize total procurement costs when $\alpha > 1/n$.

To get a better idea about the gains from the optimal policy as well as about the effects of a change in market structure, Figures 5 and 6 show the consumer surplus achieved for the cost function $c(x,n) = \frac{n^3}{4}q^4$ when the number of suppliers moves from $n = 3$ to $n = 5$, and compares them to the one achieved in the textbook monopsony – which is the same as our competitive market with commitment. In Figure 5 we see the level of consumer surplus that can be achieved in the presence of $n = 3$ suppliers. Total procurement costs with 1 lot are lower than with 2 lots when $\alpha > 1/3$, since we have $n < f(k) = 14/3$. For $\alpha < 1/3$, 1 lot is still preferred if $\alpha > \widetilde{\alpha} \approx 0.273$.

In Figure 6 we see the level of consumer surplus that can be achieved in the presence of $n = 5$ suppliers. Total procurement costs with 3 lots are higher than with 4 lots when $\alpha > 1/5$, since we have $n > f(k) = 14/3$. Therefore, when $n = 5$ we have $\widetilde{\alpha} = 1$ and the optimal procurement policy is always to set 4 lots (and additionally negotiate with the remaining supplier if $\alpha < 1/5$).

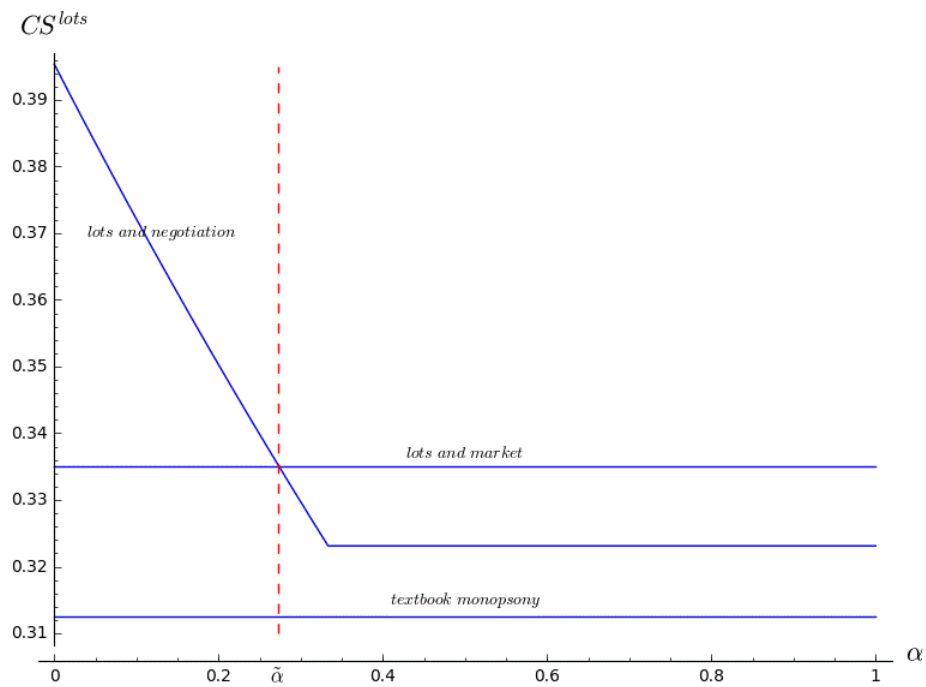


Figure 5: Consumer surplus in the presence of $n = 3$ suppliers with cost functions $c(q) = \frac{27}{4}q^4$.

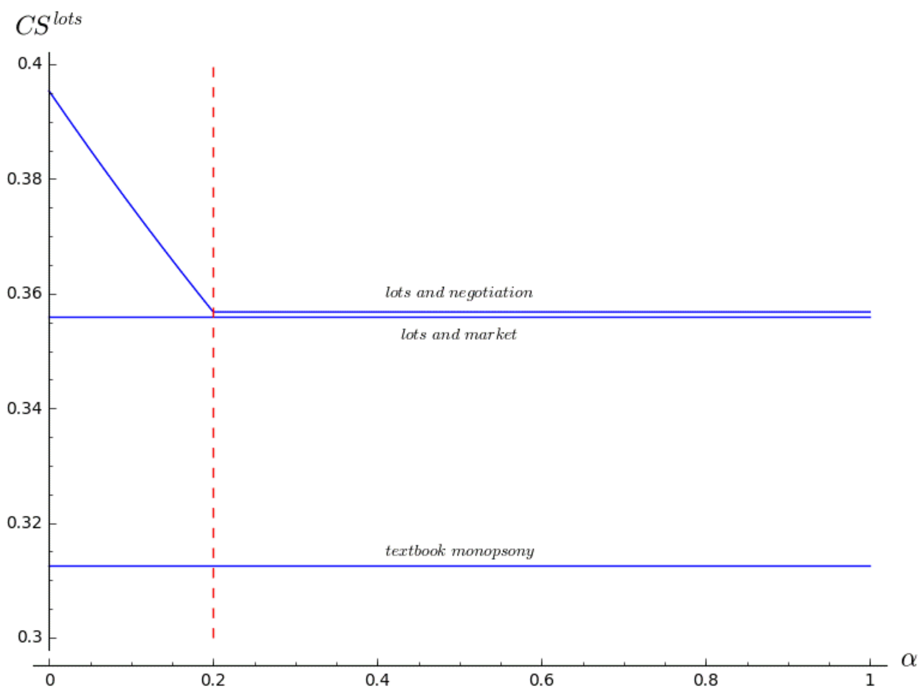


Figure 6: Consumer surplus in the presence of $n = 5$ suppliers with cost functions $c(q) = \frac{125}{4}q^4$.

With $0 \leq m \leq n-2$ lots of optimal size, consumer surplus is

$$U(Q) - TPC(Q;m) = \left(1 - \frac{Q}{2}\right) Q - \frac{A(m)}{1+k} Q^{1+k}, \tag{16}$$

where we have defined $A(m) \equiv \frac{n-m+nk}{n-m} \left( \frac{n}{n-m+m\left(\frac{n-m+nk}{n-m}\right)^{1/k}} \right)^k$ and the optimal quantity procured, $Q_{n-m}$, satisfies $1 - Q_{n-m} - \frac{n-m+nk}{n-m} \left( \frac{n}{n-m+m\left(\frac{n-m+nk}{n-m}\right)^{1/k}} \right)^k Q_{n-m}^k = 0$.

We know from Lemma 5 that for any strictly convex cost function $TPC(Q;m)$ is decreasing in $m$. Thus, for the particular cost funtions we consider here, $A(m)$ is decreasing in $m$. We thus have marginal procurement costs $TPC_Q(Q;m) = A(m)Q^k$ that satisfy $TPC_Q(Q; n-2) < TPC_Q(Q;0)$. So it is immediate that, if the buyer sets $n-2$ lots, then consumption is larger than the one chosen by a monopsonist that does not set lots, $Q^m$.

On the other hand, when $\alpha > 1/n$, if the buyer prefers $n-1$ to $n-2$ lots it is because the marginal procurement cost, $\left(\frac{n}{n-1}\right)^k Q^k$, are below $TPC_Q(Q; n-2)$; therefore we obtain again that consumption is larger than $Q^m$.

# 7 Conclusions

We have studied the optimal way for a buyer to group together part of his requirements into block contracts. We have identified a critical block size, equalling the efficient quantity, that determines whether the block contract affects the market outcome. With more than two suppliers, block sourcing is always profitable and the only question is whether the buyer leaves one or two suppliers without a lot. At the same time – except in the case where he has commitment power but not too high bargaining power and there are many suppliers – the buyer tends not to divide all his requirements into lots and leaves some for the aftermarket.

While we have framed our discussion around the decisions of a single buyer, our model can also be interpreted as one where many independent buyers consider grouping together to improve their bargaining position versus the suppliers. Given the absence of quantity discounts, as we have increasing marginal costs, this does not sound an obvious strategy.[24]

[24] Group purchasing organizations (GPOs) have recently received some special attention, specially in

However, if we interpret setting a block as the establishment of a purchasing group, our results imply that the prices obtained for the block contracts are indeed lower than the competitive price. Interestingly, the buyers not part of the block improve their prices even more,[25] so the formation of the group may be a complicated process. We leave the analysis of that for further research.

what refers to the health care reform debate in the US. In particular, the analysis is concerned about their effects on the total health care costs and the competitive effects of these groups (see, e.g. Blair and Durrance, 2014 and Rooney, 2011). Gong et al. (2012) cite examples in Canada and Australia that see significantly lower prices vis-à-vis the U.S. for the same drugs.

[25] Recall Corollary 1.

# Appendix A

**Proof of Lemma 1.** We start by showing that if, in equilibrium, the winner of the smallest lot is interested in participating in the aftermarket then the buyer's profits would be the same as if the smallest lot was not offered. The quantity traded in the second-stage market by a loser is $q'$, where $U'\left(Z+(n-m)q'+\sum_{i=1}^{m}[q'-z_i]^+\right)=c'(q')$, as, of course, $q'$ must be larger than $z_i$, for the winner of lot $i$ to participate in the aftermarket. Without lot $m$, the quantity traded in the second-stage market by a loser would be $q$, where $U'\left(Z-z_m+(n-m+1)q+\sum_{i=1}^{m-1}[q-z_i]^+\right)=c'(q)$. It is immediate that $q=q'$ and therefore $c'(q)=c'(q')$ and, consequently, the two outcomes are the same. Therefore, the only way the buyer can hope for a better outcome than the competitive one is by setting the smallest lot larger than the competitive quantity for $n-m+1$ suppliers: $z_m \geq q^c\left(n+1-m;\sum_{j=1}^{m-1}z_j\right)$. By induction, he needs lot $i$ larger than $q^c\left(n+1-i;\sum_{j=1}^{i-1}z_j\right)$. As this increase decreases the residual demand, the resulting aftermarket unit price will be lower than the competitive one, $c'\left(q^c(n+1-i;\sum_{j=1}^{i-1}z_j)\right)$, so the lot winners – who produce more than the competitive amount and therefore have a marginal cost above $c'\left(q^c(n+1-i;\sum_{j=1}^{i-1}z_j)\right)$ are effectively priced out of the aftermarket. ■

**Proof of Lemma 2.** Let the equilibrium profit of the winner of lot $m-k+1$ be denoted by $\pi_w^{(k)}$.

Induction hypothesis (IH): If there are $k$ lots left then $\pi_w^{(k)}=\pi(\mathbf{z})$.

Step 1: The IH holds when $k=1$. Since $m<n$ there are $n-m+1\geq 2$ remaining suppliers. It is immediate that in equilibrium neither $\pi_w^{(1)}>\pi(\mathbf{z})$ nor $\pi_w^{(1)}<\pi(\mathbf{z})$. In the first case any losing bidder could do better by bidding slightly below the winner's bid (which must have been the (weakly) lowest), whereas in the second case the winner could increase her profits by increasing her offer in order to lose. The latter argument, presupposes that there is another valid bid for the lot in equilibrium. This must be the case as otherwise the winner could increase her offer and still win.

Step 2: If the IH holds for $k$ then it is also true for $k+1$. By the IH, all the suppliers who do not win lot $m-k$ will earn $\pi(\mathbf{z})$. Thus, the argument used in Step 1 can be directly applied to show that $\pi_w^{(k+1)} = \pi(\mathbf{z})$. ■

**Proof of Lemma 3.** For any demand, given a total quantity $Z$ obtained through $m$ lots, to maximize (3) or (4) individual lot sizes must be set to minimize total (lot) production costs $\sum c(z_i)$. Given convexity and symmetry of the cost functions, it is immediate that identical lots minimize them. ■

**Proof of Proposition 1.** By (5) $z > q^c$ directly implies $q < q^c$. For the overproduction result, note that we cannot have $Q < Q^c$: If we write (5) as $U'(Q) = c'(q)$ we have $\frac{dq}{dQ} = \frac{U''(Q)}{c''(q)} < 0$ (less production in the aftermarket if there is more total production since this is related with lower market prices), then $Q < Q^c$ would imply $q > q^c$, which is a contradiction. ■

**Proof of Corrollary 1.** By Proposition 1, $z > q$. As both winning and losing suppliers make the same profit of $\pi = c'(q)q - c(q)$, we have that the difference in unit prices is

$$\frac{c(z)+\pi}{z} - c'(q) = \frac{c(z) - c(q) + c'(q)(q-z)}{z} = \int_q^z \frac{c'(s) - c'(q)}{z} ds > 0.$$

■

**Proof of Lemma 4.** Let $n > 2$, as otherwise the Lemma holds by default. By (6) the optimal size of a lot, $z^*$, satisfies (for a given number of lots, $m \geq 1$)

$$\frac{dCS(q,z,m)}{dz} = m\left[U'((n-m)q + mz) - c'(z)\right] - n\pi'(q)\frac{\partial q}{\partial z} = 0, \tag{17}$$

where $q$ is given by (5), satisfying $\frac{\partial q}{\partial z} = -\frac{mU"}{(n-m)U" - c"(q)} < 0$. Note that $z^* > q^c$ for any $m \geq 1$, since by Remark 2, $\frac{dCS(q,q^c,m)}{dz} = -n\pi'(q)\frac{\partial q}{\partial z} > 0$. By Remark 2, (for any $m > 0$) setting lots at $q^c$ is equivalent to setting none. Thus, since we have shown that optimally the lot size is strictly larger than $q^c$, the consumer surplus is also strictly larger than in the competitive market (without lots). As a result, it is always strictly optimal to set at least one lot.

To look for the optimal number of lots that leave at least two sellers in the market, $1 \leq m \leq n-2$, we differentiate the consumer surplus with respect to $m$, momentarily disregarding that it is an integer and taking into account that $\frac{dCS(q,z^*,m)}{dz} = 0$:

$$\frac{dCS(q, z^*, m)}{dm} = (z^* - q)U'((n-m)q + mz^*) - [c(z^*) - c(q)] - n\pi'(q)\frac{\partial q}{\partial m};$$

we then use (17) to write

$$\frac{dCS(q, z^*, m)}{dm} = (z^* - q)U'((n-m)q + mz^*) - [c(z^*) - c(q)] - \\ - m\left[U'((n-m)q + mz^*) - c'(z^*)\right]\frac{\partial q/\partial m}{\partial q/\partial z}.$$

Note that, by (5), we can write $\frac{\partial q/\partial m}{\partial q/\partial z} = \frac{z^*-q}{m}$. Therefore,

$$\begin{aligned}\frac{dCS(q, z^*, m)}{dm} &= (z^* - q)U'((n-m)q + mz^*) - \\ &- [c(z^*) - c(q)] - m\left[U'((n-m)q + mz^*) - c'(z^*)\right]\frac{z^* - q}{m} \\ &= -[c(z^*) - c(q)] + c'(z^*)(z^* - q) = \int_q^{z^*}\{c'(z^*) - c'(s)\}ds > 0.\end{aligned}$$

That is – since $z^* > q$, by Proposition 1 – buyer profits are strictly increasing in the number of (optimally sized) lots, as long as the aftermarket is competitive. ■

**Proof of Proposition 2.**

Let us start with a lemma.

**Lemma 6** *The consumer surplus at the optimal lot size with* $n-1$ *lots is strictly decreasing in the suppliers' bargaining power:* $\frac{dCS^*}{d\alpha} < 0$.

**Proof.** The efficient residual quantity, $q(Z)$, solves $U'(Z+q) = c'(q)$ where $Z = (n-1)z$.

$$CS = U(Z) - (n-1)\left\{\alpha\left[U(Z + q(Z)) - U(Z) - c(q(Z))\right] + c\left(\frac{Z}{n-1}\right)\right\} \\ + (1-\alpha)\left[U(Z + q(Z)) - U(Z) - c(q(Z))\right].$$

At the optimal size of the $n-1$ lots $CS_Z = 0$, which can be solved for $Z(\alpha; ...)$. Differentiating with respect to $\alpha$ at the optimum we obtain

$$CS_\alpha = CS_Z\frac{\partial Z}{\partial \alpha} - n\left[U(Z + q(Z)) - U(Z) - c(q(Z))\right].$$

As $U''(r) < 0$,

$$U(Z+q(Z)) - U(Z) - c(q(Z)) > U'(Z+q(Z)) \cdot q(Z) - c(q(Z)) = c'(q(Z)) \cdot q(Z) - c(q(Z)),$$

which is (strictly) positive by the (strict) convexity of costs. Thus, as $CS_Z = 0$, it follows that $CS_\alpha < 0$.

Putting Lemmas 4 and 6 together, and observing that when $\alpha = 0$, there is no opportunity cost to choosing negotiation and therefore setting $n-1$ lots is superior, the proposition is proven. ■

**Proof of Lemma 5.** Let $n > 2$, as otherwise the Lemma holds by default. Given any $Q$ he has chosen, the buyer will want to minimize costs. Note that if $z < Q/n$ all suppliers participate in the aftermarket and total procurement costs are the same as if no lot had been set. If the buyer sets $1 \leq m \leq n-2$ (equal) lots of $z \geq Q/n$, followed by buying $q = \frac{Q-mz}{n-m} \leq z$ from each loser in the aftermarket, total procurement costs are

$$TPC(Q,m,z) = mc(z) + (n-m)c(q) + n\left[c'(q)\,q - c(q)\right]. \tag{18}$$

First we show that, for any $m > 0$, the optimal lot size is interior – $z^* \in (Q/n, Q/m)$ – that directly implies that it is always optimal to set at least one lot. Note that the total procurement cost varies with the lot size according to

$$\frac{\partial TPC(Q,m,z)}{\partial z} = m\left[c'(z) - c'(q) - \frac{n}{n-m}c''(q)q\right]. \tag{19}$$

Substituting in $z = q = Q/n$, this is clearly negative. Similarly, at $z = Q/m$ (and $q = 0$) it is positive. Therefore, the optimal lot size satisfies $\frac{\partial TPC(Q,m,z)}{\partial z} = 0$ or

$$c'(q) + \frac{n}{n-m}c''(q)q = c'(z). \tag{20}$$

Treating the number of lots as a real number and differentiating with respect to it (evaluating at the optimal size, so that the terms multiplied by $dz/dm$ disappear) we have

$$\frac{dTPC(Q,m,z)}{dm} = c(z) - c(q) - \left[c'(q) + \frac{n}{n-m}c''(q)\,q\right][z-q]. \tag{21}$$

Substituting (20) in, we obtain

$$\frac{dTPC(Q,m,z)}{dm} = c(z) - c(q) - c'(z)[z-q] = -\int_q^z [c'(z) - c'(s)]\, ds < 0, \tag{22}$$

since $c'(z) - c'(s) > 0$ for $s < z$, by $c'' > 0$, and as $q < z$, the total procurement cost is decreasing in the number of lots, for all $0 < m \leq n-2$. ■

**Proof of Proposition 3.** (i) Consider that the buyer sets $n-1$ lots of size $z$, and commits to buy an extra amount $q$. The second stage surplus created is $\Delta(z,q) = U((n-1)z+q) - U((n-1)z) - c(q)$. The suppliers obtain a share $\alpha$ of this. Of course $\Delta(z,0) = 0$: no profits for the loser if $q = 0$, and as a consequence the lot price is $c(z)$. The buyer chooses $q$ (and $z$) to maximize

$$\begin{aligned} CS(z,q,\alpha) &= U((n-1)z+q) - [c(q) + (n-1)c(z) + n\alpha\Delta(z,q)] \\ &= (1-\alpha n)\Delta(z,q) + U((n-1)z) - (n-1)c(z). \end{aligned} \tag{23}$$

When $\alpha n > 1$, creating surplus in the bargaining stage hurts the buyer, so his optimal choice is to set $q = 0$. Also, there is underproduction: The buyer's optimal size of the lot $z = \frac{Q}{n-1}$ satisfies $U'(Q) = c'\left(\frac{Q}{n-1}\right)$, whereas the efficient level of production satisfies $U'(Q^c) = c'\left(\frac{Q^c}{n}\right)$, directly implying that $Q < Q^c$.

If $\alpha n < 1$, then the buyer wants to maximize second stage surplus, leading to the efficient second stage quantity, $q$. Therefore, the solution of the overall maximization problem is the same as without commitment, given by (5) and (17), leading to overproduction.

(ii) The buyer chooses consumption $Q$ to maximize

$$U(Q) - TPC(Q, n-2, z). \tag{24}$$

where, substituting $q$ for $\frac{Q-Z}{2}$ and $\frac{Q-2q}{n-2}$ for $z$

$$TPC(Q, n-2, z) = (n-2)c\left(\frac{Q-2q}{n-2}\right) + 2c(q) + n[c'(q)q - c(q)] \tag{25}$$

The optimal level of consumption satisfies the first-order condition $\frac{\partial[U(Q) - TPC(Q,n-2,z)]}{\partial Q} = 0$:

$$U'(Q) - \left[c'(q) + \frac{n}{2}c''(q)q\right] = 0. \tag{26}$$

The consumption that maximizes welfare (or the competitive consumption without lots) satisfies $U'(Q^c) = c'(q^c)$. On the other hand, since $\frac{\partial TPC(Q,m,z)}{\partial z} = 0$, we know that the optimal lot satisfies $c'(q) + \frac{n}{2}c''(q)q = c'(z)$, and therefore the buyer chooses a level of consumption $Q$ that satisfies $U'(Q) = c'(z) = c'(\frac{Q}{n} + s)$. Then $\frac{\partial Q}{\partial s} = \frac{c''(z)}{U''(Q)-\frac{1}{n}c''(z)} < 0$. If we had $z \leq Q/n$ it would mean $s \leq 0$, implying that $Q \geq Q^c$. However, we know that $z > q^c > q$, implying that $(n-2)z + 2q = Q < nz$, what is impossible. Therefore, $z > Q/n$ and so $s > 0$ and the buyer chooses $Q < Q^c$. Finally, $z > Q/n$ also implies that $q = \frac{Q-(n-2)z}{2} < \frac{Q-(n-2)\frac{Q}{n}}{2} = \frac{Q}{n}$. ■

# Appendix B

## Justification for the use of the competitive pricing equilibrium

There are two main classes of price competition models. We need not choose between them in order to justify the competitive outcome.

In the classical Bertrand approach, firms bid unit prices. The buyer then accepts the lowest price and satisfies his (residual) demand. If several suppliers make the same bid, then the buyer shares his demand equally among them. This game is known (c.f. Dastidar, 1995) to have multiple equilibria in the presence of increasing marginal cost. The highest equilibrium price is the one at which a supplier makes the same profit as she would by satisfying the entire demand on her own. Only for prices above this one does it pay to undercut a competitor. The lowest equilibrium price is the one at which all suppliers producing the same quantity just break even when they satisfy demand. For any lower price it is inefficient for trade to take place. It is easy to see that the competitive price is within the Dastidar interval: on the one hand, the positive profits imply that it is above the lower bound, on the other hand, at the competitive price the optimal quantity for a supplier is the competitive one (selling at marginal cost) therefore it is clearly better than satisfying the entire demand, so the competitive price must be below the upper bound. Consequently, our assumption could be simply thought of as a judicious selection – maintains efficiency, shares surplus[26] – from the set of equilibria. Nonetheless, we have stronger arguments as well. We could modify the extensive form, so that the competitive outcome is the unique equilibrium. Simply offering a price quantity pair does not do away with the multiplicity in a meaningful way: it leads to a mixed strategy equilibrium with a broad support of equilibrium prices. However, as Dixon (1992) showed, if firms commit not to a fixed quantity, but to a maximum quantity that they are willing to supply at the

[26] In principle, the buyer could manipulate the equilibrium selection by a strategically chosen demand sharing rule (for example, assigning all the production to a single, randomly chosen, supplier if their asking prices were the same, unless they asked for a given price, where the production would be equally shared). But this is neither realistic, nor does it take into account that the suppliers are likely to have outside options, which the buyer would have to match.

(unit) price they bid, then – setting rationing rules and demand sharing rules judiciously – the unique equilibrium is the competitive one.

The alternative approach is where firms bid supply schedules (or menus). The buyer aggregates these, and determines the outcome by crossing it with his demand. This game is also known to have multiple equilibria. Bernheim and Whinston (1986), develop an equilibrium refinement concept – Truthful Equilibrium – that basically requires that the relative differences between different points of the menu reflect the truth. Simply put, the bids are thought of as two-part tariffs and only the fixed part can be manipulated strategically. Again, only the efficient outcome can be supported in a Truthful Equilibrium.

In a model that is in between the above two classes, Burguet and Sákovics (2017) assume that firms are allowed to make separate offers for the supply of every infinitesimal unit. From the point of view of each such unit, they simply receive (unit) price offers, as in Bertrand, but from the point of view of the suppliers, they name supply schedules, not in terms of quantities but in terms of specific demand units. In the unique equilibrium, all suppliers offer the competitive price to supply all the units, with no need to exogenously specify rationing or demand sharing rules.

Finally, note that if in a specific institutional setting it were the case that the market outcome is different from the competitive one assumed here, there are two possibilities. If the equilibrium gave lower profits to the buyer, he would be even more in favor of setting lots. If it gave higher profits, this would change two trade-offs: it would bias the buyer towards $n-1$ lots as opposed to $n-2$, and it would make the overall likelihood of setting a lot decrease. However, the qualitative results of our analysis would remain unaltered.

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