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Supplementary Material

Francisco J. Rubio[1], Laurent Remontet[2], Nicholas P. Jewell[3], and Aurélien Belot[1]

1 Comparison of the different hazard structures

We aim to compare in this section the different hazard shapes according to the assumed hazard structure. We focus on the Accelerated hazard model (AH) and the General Hazard (GH) model, and compare it with the well-know Proportional Hazard (PH) model. For these comparisons, we assume one binary covariate x. As a reminder (see main document), the GH model is defined as

$$h^{\text{GH}}(t; x) = h_0 (t \exp(\beta_1 x)) \exp(\beta_2 x). \tag{S1}$$

Both the AH and PH model are particular cases of the GH model, *i.e.* the GH model coincides with the AH model when $\beta_2 = 0$, and the GH model coincides with the PH model when $\beta_1 = 0$.

[1] Cancer Survival Group, Faculty of Epidemiology and Population Health,
Department of Non-Communicable Disease Epidemiology,
London School of Hygiene & Tropical Medicine,
London, WC1E 7HT, United Kingdom.
[2]Hospices Civils de Lyon,
Service de Biostatistique-Bioinformatique,
Lyon, 69003, France.
[3] University of California, Berkeley,
School of Public Health,
Berkeley, CA 94720-7360.

Corresponding author:
Francisco J. Rubio, Cancer Survival Group, Faculty of Epidemiology and Population Health,
Department of Non-Communicable Disease Epidemiology,
London School of Hygiene & Tropical Medicine,
London, WC1E 7HT, United Kingdom.
Email: Francisco.Rubio@lshtm.ac.uk

We defined 4 different scenarios, depending on the values assumed for β_1 and β_2, and in each scenario (Figures S1-S4), we distinguish 4 different baseline hazard shapes (monotonic increasing -panels (a)-, monotonic decreasing -panels (b)-, bathtub -panels (c)-, and unimodal -panels (d)).

In each panel of each figure (Figures S1-S4), we plotted the baseline hazard (*i.e.* for group with $x = 0$) -grey solid lines-, and the hazard for group with $x = 1$ assuming the (i) PH model (thus β_1 set to 0) -dotted black lines-, (ii) AH model (thus β_2 set to 0) -dashed black lines-, (iii) GH model -dotdashed black lines-.

For full details on the interpretation of these figures, we refer the reader to the Section 2.3 of the main document.

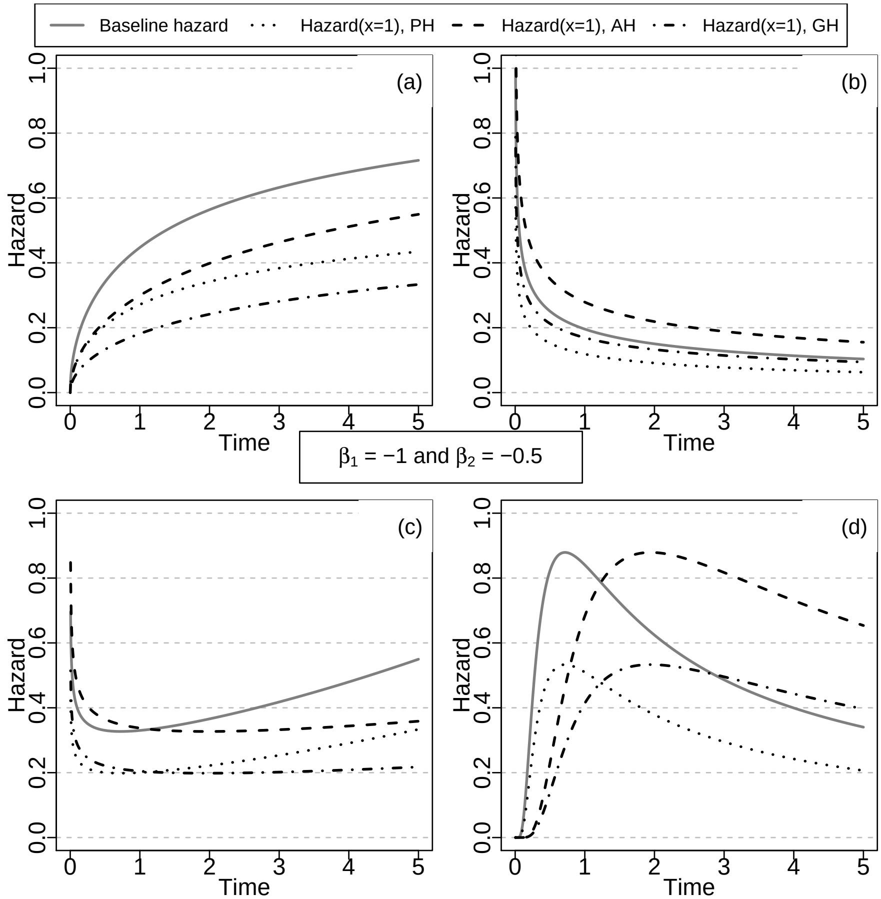

Figure S1. Baseline hazard ($X = 0$) and hazards for group with covariate $x = 1$ according to the hazard structure assumed (PH, AH, GH) - scenario with β_1 and β_2 are both negatives.

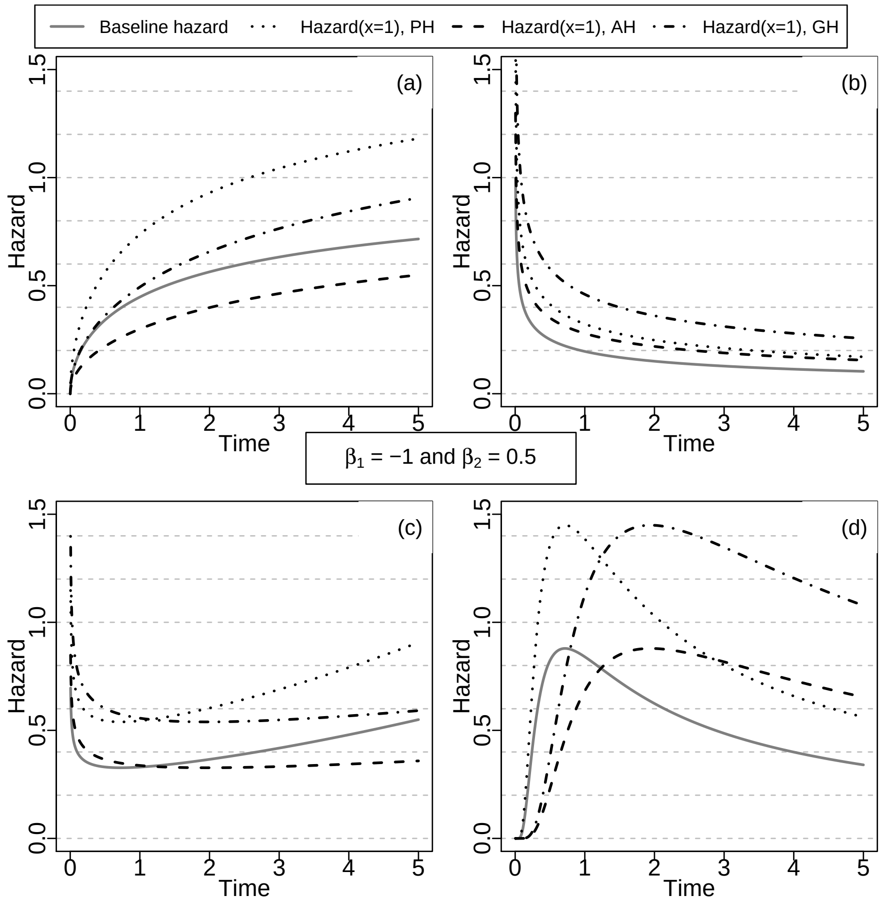

Figure S2. Baseline hazard ($X = 0$) and hazards for group with covariate $x = 1$ according to the hazard structure assumed (PH, AH, GH) - scenario with β_1 negative and β_2 positive.

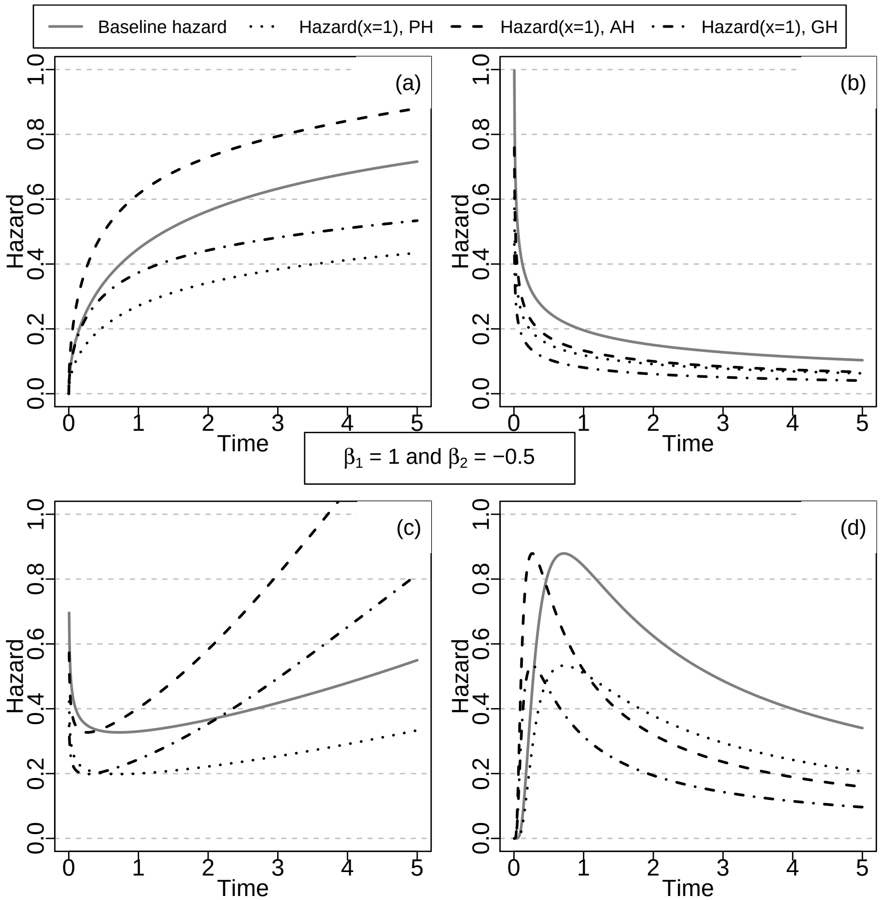

Figure S3. Baseline hazard ($X = 0$) and hazards for group with covariate $x = 1$ according to the hazard structure assumed (PH, AH, GH) - scenario with β_1 positive and β_2 negative.

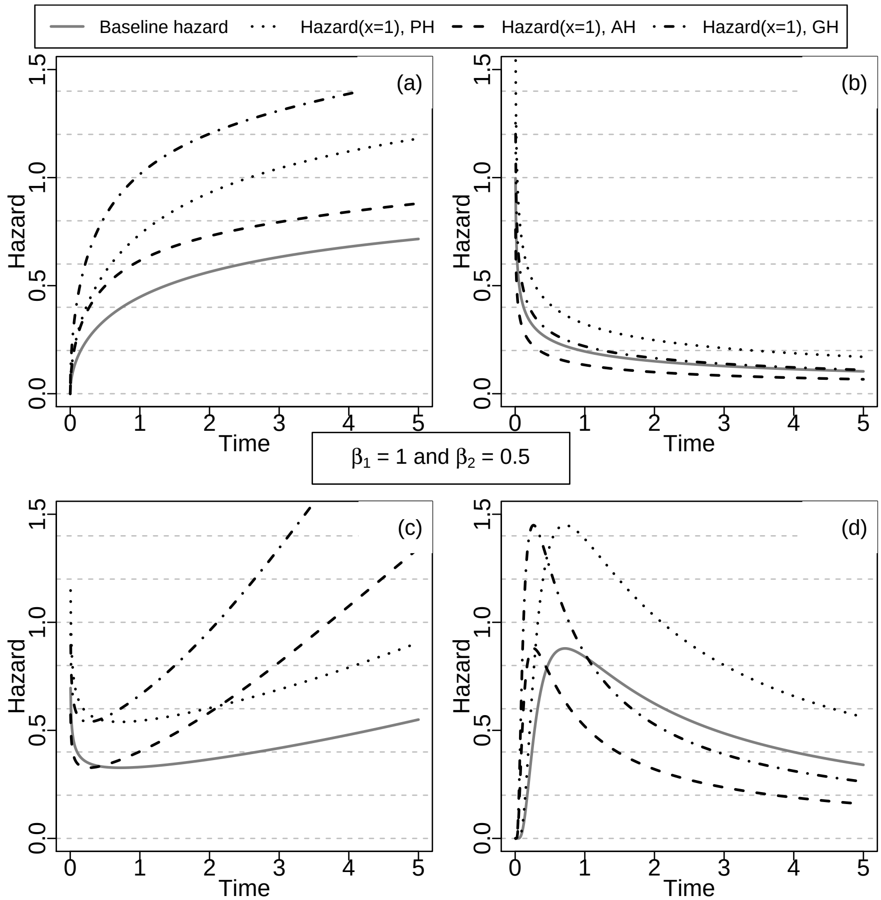

Figure S4. Baseline hazard ($X = 0$) and hazards for group with covariate $x = 1$ according to the hazard structure assumed (PH, AH, GH) - scenario with β_1 and β_2 are both positives.

2 Theoretical results

Proposition 1. *The Exponentiated Weibull distribution is identifiable.*

Proof. The strategy of the proof is by contradiction. Suppose that there are two sets of parameters $(\sigma, \kappa, \alpha) \neq (\sigma', \kappa', \alpha')$ such that

$$F_{\text{EW}}(t; \sigma, \kappa, \alpha) = F_{\text{EW}}(t; \sigma', \kappa', \alpha'), \quad \text{for all } t > 0. \tag{S2}$$

Consider the cases: (i) $\alpha = \alpha'$, (ii) $\alpha > \alpha'$, and (iii) $\alpha < \alpha'$.

(i) Suppose that $\alpha = \alpha'$. Given that $F_{\text{EW}}(t; \sigma, \kappa, \alpha) = F_{\text{W}}(t; \sigma, \kappa)^{\alpha}$, where $F_{\text{W}}(t; \sigma, \kappa)$ is the Weibull cumulative distribution function, it follows that (S2) implies that $F_{\text{W}}(t; \sigma, \kappa) = F_{\text{W}}(t; \sigma', \kappa')$. Since the Weibull distribution is identifiable, this implies that $\sigma = \sigma'$ and $\kappa = \kappa'$, which contradicts the assumption $(\sigma, \kappa, \alpha) \neq (\sigma', \kappa', \alpha')$.

(ii) Suppose that $\alpha > \alpha'$. Given that $F_{\text{EW}}(t; \sigma, \kappa, \alpha) = F_{\text{W}}(t; \sigma, \kappa)^{\alpha}$, it follows that $F_{\text{W}}(t; \sigma, \kappa)^{\alpha} \leq F_{\text{W}}(t; \sigma, \kappa)^{\alpha'}$. Consequently, $F_{\text{W}}(t; \sigma, \kappa)^{\alpha} = F_{\text{W}}(t; \sigma', \kappa')^{\alpha'} \leq F_{\text{W}}(t; \sigma, \kappa)^{\alpha'}$, and $F_{\text{W}}(t; \sigma', \kappa') \leq F_{\text{W}}(t; \sigma, \kappa)$. By using the expression of the Weibull distribution function, we obtain the condition $\dfrac{t^{\kappa'}}{\sigma'^{\kappa'}} \leq \dfrac{t^{\kappa}}{\sigma^{\kappa}}$, for all $t > 0$. Equivalently, we have $t^{\kappa'} \leq \dfrac{\sigma'^{\kappa'}}{\sigma^{\kappa}} t^{\kappa}$, for all $t > 0$. Given that these are polynomials of different power, this inequality cannot hold on both regions $t \in (0, A)$ and $t \in [A, \infty)$, for some $A = A(\sigma, \sigma', \kappa, \kappa') > 0$, simultaneously for either $\kappa > \kappa'$ or $\kappa < \kappa'$. Thus, we need $\kappa = \kappa'$, which in turns implies the need for $\sigma' > \sigma$.

Now, using (S2) and that $\alpha > \alpha'$, $\kappa = \kappa'$ and $\sigma' > \sigma$, it follows that:

$$F_{\text{EW}}(t; \sigma, \kappa, \alpha) = F_{\text{EW}}(t; \sigma', \kappa, \alpha'), \quad \text{for all } t > 0.$$

Consequently,

$$\alpha \log \left[1 - \exp\left\{-\left(\frac{t}{\sigma}\right)^{\kappa}\right\}\right] = \alpha' \log \left[1 - \exp\left\{-\left(\frac{t}{\sigma'}\right)^{\kappa}\right\}\right],$$

and

$$\frac{\alpha}{\alpha'} = \frac{\log \left[1 - \exp\left\{-\left(\frac{t}{\sigma'}\right)^{\kappa}\right\}\right]}{\log \left[1 - \exp\left\{-\left(\frac{t}{\sigma}\right)^{\kappa}\right\}\right]} > 1.$$

Now, taking the limit as $t \to 0$, we get (after sequentially applying L'Höpital's rule)

$$\lim_{t \to 0} \frac{\log \left[1 - \exp\left\{-\left(\frac{t}{\sigma'}\right)^{\kappa}\right\}\right]}{\log \left[1 - \exp\left\{-\left(\frac{t}{\sigma}\right)^{\kappa}\right\}\right]} = \frac{\sigma^{\kappa}}{\sigma'^{\kappa}} \lim_{t \to 0} \frac{\exp\left\{\left(\frac{t}{\sigma}\right)^{\kappa}\right\} - 1}{\exp\left\{\left(\frac{t}{\sigma'}\right)^{\kappa}\right\} - 1} = \lim_{t \to 0} \frac{\exp\left\{\left(\frac{t}{\sigma}\right)^{\kappa}\right\}}{\exp\left\{\left(\frac{t}{\sigma'}\right)^{\kappa}\right\}} = 1,$$

which contradicts the assumption that $\alpha > \alpha'$.

(iii) The case $\alpha < \alpha'$ follows analogously as (ii).

3 True values of the parameters and corresponding net survival simulated in the different scenarios

	PH	AFT	HH	AH	GH	CH
σ	1.75	1.4	1.8	1.75	1.75	0.114
κ	0.5	0.5	0.5	0.5	0.6	0.366
α	2.5	2.5	2.5	1.75	2.5	18.306
β_{11}	–	0.05	0.1	-0.1	0.1	–
β_{12}	–	0.2	–	-0.3	0.1	–
β_{13}	–	0.3	–	-0.5	0.1	0.5
β_{21}	0.035	0.05	0.05	–	0.05	0.05
β_{22}	0.2	0.2	0.2	–	0.2	0.3
β_{23}	0.3	0.3	0.5	–	0.25	-0.6
r	0.230	0.275	0.230	0.325	0.200	0.150

Table S1. True values of the parameters used in each scenario, where $\beta_{\cdot 1}$ corresponds to variable age, $\beta_{\cdot 2}$ corresponds to sex, and $\beta_{\cdot 3}$ corresponds to W.

sex	comorb.	age	NS(1)	NS(5)
0	0	60	0.85	0.52
0	1	60	0.80	0.42
1	0	60	0.82	0.45
1	1	60	0.77	0.34
0	0	70	0.80	0.40
0	1	70	0.73	0.29
1	0	70	0.76	0.33
1	1	70	0.69	0.22
0	0	80	0.72	0.27
0	1	80	0.64	0.17
1	0	80	0.67	0.20
1	1	80	0.58	0.11

Table S2. Simulation design: PH with $(\sigma, \kappa, \alpha) = (1.75, 0.5, 2.5)$ and $\beta = (0.035, 0.2, 0.3)$

sex	comorb.	age	NS(1)	NS(5)
0	0	60	0.84	0.48
0	1	60	0.79	0.39
1	0	60	0.81	0.42
1	1	60	0.75	0.34
0	0	70	0.75	0.34
0	1	70	0.69	0.26
1	0	70	0.71	0.28
1	1	70	0.64	0.20
0	0	80	0.64	0.21
0	1	80	0.57	0.14
1	0	80	0.59	0.16
1	1	80	0.51	0.11

Table S3. Simulation design: AFT with $(\sigma, \kappa, \alpha) = (1.4, 0.5, 2.5)$ and $\beta = (0.05, 0.2, 0.3)$.

sex	comorb.	age	NS(1)	NS(5)
0	0	60	0.87	0.53
0	1	60	0.80	0.35
1	0	60	0.85	0.46
1	1	60	0.76	0.27
0	0	70	0.80	0.41
0	1	70	0.69	0.22
1	0	70	0.76	0.33
1	1	70	0.64	0.16
0	0	80	0.72	0.32
0	1	80	0.58	0.15
1	0	80	0.67	0.25
1	1	80	0.51	0.10

Table S4. Simulation design: HH with $(\sigma, \kappa, \alpha) = (1.8, 0.5, 2.5)$, $\beta_1 = 0.1$, and $\beta_2 = (0.05, 0.2, 0.5)$.

sex	comorb.	age	NS(1)	NS(5)
0	0	60	0.74	0.44
0	1	60	0.71	0.37
1	0	60	0.72	0.39
1	1	60	0.68	0.33
0	0	70	0.67	0.30
0	1	70	0.63	0.24
1	0	70	0.65	0.26
1	1	70	0.61	0.21
0	0	80	0.60	0.19
0	1	80	0.56	0.14
1	0	80	0.57	0.16
1	1	80	0.53	0.12

Table S5. Simulation design: AH with $(\sigma, \kappa, \alpha) = (1.75, 0.5, 1.75)$ and $\beta = (-0.1, -0.3, -0.5)$.

sex	comorb.	age	NS(1)	NS(5)
0	0	60	0.90	0.51
0	1	60	0.87	0.42
1	0	60	0.88	0.44
1	1	60	0.85	0.35
0	0	70	0.81	0.34
0	1	70	0.76	0.25
1	0	70	0.77	0.27
1	1	70	0.72	0.19
0	0	80	0.69	0.22
0	1	80	0.63	0.15
1	0	80	0.64	0.16
1	1	80	0.56	0.10

Table S6. Simulation design: GH with $(\sigma, \kappa, \alpha) = (1.75, 0.6, 2.5)$, $\beta_1 = (0.1, 0.1, 0.1)$, and $\beta_2 = (0.05, 0.2, 0.25)$.

sex	treatment	age	NS(1)	NS(5)
0	0	60	0.93	0.47
0	1	60	0.94	0.67
1	0	60	0.90	0.36
1	1	60	0.92	0.59
0	0	70	0.88	0.29
0	1	70	0.90	0.52
1	0	70	0.84	0.19
1	1	70	0.87	0.41
0	0	80	0.81	0.13
0	1	80	0.84	0.34
1	0	80	0.75	0.06
1	1	80	0.80	0.23

Table S7. Simulation design: CH with $(\sigma, \kappa, \alpha) = (0.114, 0.366, 18.306)$, $\beta_1 = (0, 0, 0.5)$, and $\beta_2 = (0.05, 0.3, -0.6)$.

4 Performance of the MLEs

4.1 PH structure

$$h_{\mathrm{E}}^{\mathrm{PH}}(t;\mathbf{x}_j) = h_{EW}(t;1.75,0.5,2.5) \times \exp\left(0.035 \times \text{age} + 0.2 \times \text{sex} + 0.3 \times \text{W}\right).$$

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
GHEW	σ (1.75)	1.652	1.508	1.055	1.078	1.059	0.947
	κ (0.5)	0.489	0.477	0.129	0.138	0.129	0.947
	α (2.5)	3.221	2.705	1.818	1.675	1.954	0.958
	β_{11} (0)	0.004	0.000	0.042	0.037	0.042	0.932
	β_{12} (0)	-0.285	-0.006	2.117	4.756	2.136	0.972
	β_{13} (0)	-0.422	-0.110	2.413	6.452	2.449	0.967
	β_{21} (0.035)	0.035	0.035	0.007	0.007	0.007	0.931
	β_{22} (0.2)	0.187	0.203	0.188	0.314	0.189	0.957
	β_{23} (0.3)	0.271	0.284	0.195	0.283	0.197	0.952
PHEW	σ (1.75)	1.717	1.571	1.085	1.078	1.085	0.954
	κ (0.5)	0.495	0.483	0.126	0.126	0.126	0.969
	α (2.5)	3.082	2.682	1.565	1.487	1.669	0.977
	β_1 (0.035)	0.035	0.035	0.004	0.004	0.004	0.954
	β_2 (0.2)	0.198	0.198	0.088	0.090	0.088	0.954
	β_3 (0.3)	0.300	0.302	0.092	0.091	0.092	0.949
			50% censoring				
GHEW	σ (1.75)	1.486	1.321	1.065	1.130	1.097	0.957
	κ (0.5)	0.465	0.459	0.138	0.157	0.142	0.948
	α (2.5)	3.835	2.901	3.131	2.653	3.403	0.969
	β_{11} (0)	0.009	0.006	0.048	0.040	0.049	0.927
	β_{12} (0)	-0.519	-0.103	2.732	9.247	2.780	0.984
	β_{13} (0)	-0.419	-0.104	2.272	3.757	2.309	0.979
	β_{21} (0.035)	0.035	0.035	0.007	0.007	0.007	0.935
	β_{22} (0.2)	0.180	0.186	0.213	0.498	0.214	0.960
	β_{23} (0.3)	0.279	0.285	0.193	0.257	0.194	0.954
PHEW	σ (1.75)	1.583	1.406	1.118	1.163	1.130	0.970
	κ (0.5)	0.476	0.464	0.137	0.145	0.139	0.959
	α (2.5)	3.529	2.856	2.594	2.210	2.790	0.987
	β_1 (0.035)	0.035	0.035	0.005	0.005	0.005	0.950
	β_2 (0.2)	0.199	0.195	0.108	0.106	0.108	0.939
	β_3 (0.3)	0.300	0.299	0.107	0.106	0.107	0.947

Table S8. Simulation from PH structure with $(\sigma,\kappa,\alpha) = (1.75, 0.5, 2.5)$, $\beta = (0.035, 0.2, 0.3)$, and $n = 1000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
GHEW	σ (1.75)	1.784	1.749	0.597	0.561	0.597	0.944
	κ (0.5)	0.505	0.504	0.068	0.064	0.068	0.943
	α (2.5)	2.580	2.489	0.544	0.523	0.550	0.946
	β_{11} (0)	0.000	-0.000	0.017	0.015	0.017	0.949
	β_{12} (0)	0.017	0.029	0.370	0.362	0.370	0.962
	β_{13} (0)	0.007	0.017	0.388	0.367	0.387	0.953
	β_{21} (0.035)	0.035	0.035	0.003	0.003	0.003	0.937
	β_{22} (0.2)	0.205	0.208	0.070	0.069	0.070	0.951
	β_{23} (0.3)	0.302	0.306	0.073	0.070	0.073	0.932
PHEW	σ (1.75)	1.788	1.756	0.553	0.531	0.554	0.943
	κ (0.5)	0.503	0.503	0.058	0.056	0.059	0.950
	α (2.5)	2.568	2.492	0.513	0.492	0.517	0.954
	β_1 (0.035)	0.035	0.035	0.002	0.002	0.002	0.936
	β_2 (0.2)	0.202	0.201	0.041	0.040	0.041	0.952
	β_3 (0.3)	0.301	0.302	0.041	0.041	0.041	0.955
			50% censoring				
GHEW	σ (1.75)	1.750	1.696	0.632	0.629	0.631	0.956
	κ (0.5)	0.501	0.495	0.078	0.076	0.077	0.949
	α (2.5)	2.634	2.537	0.655	0.623	0.668	0.962
	β_{11} (0)	0.000	-0.000	0.019	0.018	0.019	0.939
	β_{12} (0)	-0.001	-0.004	0.457	0.435	0.457	0.960
	β_{13} (0)	-0.038	-0.033	0.452	0.433	0.453	0.957
	β_{21} (0.035)	0.035	0.035	0.003	0.003	0.003	0.929
	β_{22} (0.2)	0.201	0.205	0.073	0.072	0.073	0.953
	β_{23} (0.3)	0.296	0.296	0.075	0.073	0.076	0.941
PHEW	σ (1.75)	1.774	1.751	0.580	0.597	0.580	0.958
	κ (0.5)	0.502	0.501	0.065	0.066	0.065	0.961
	α (2.5)	2.595	2.512	0.577	0.573	0.585	0.959
	β_1 (0.035)	0.035	0.035	0.002	0.002	0.002	0.938
	β_2 (0.2)	0.201	0.201	0.046	0.047	0.046	0.957
	β_3 (0.3)	0.302	0.300	0.050	0.047	0.050	0.946

Table S9. Simulation from PH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.5, 2.5)$, $\beta = (0.035, 0.2, 0.3)$, and $n = 5000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

4.2 AFT structure

$$\begin{aligned} h_{\text{E}}^{\text{AFT}}(t; \mathbf{x}_j) &= h_{EW}\left(t \times \exp\left(0.05 \times \text{age} + 0.2 \times \text{sex} + 0.3 \times \text{W}\right); 1.4, 0.5, 2.5\right) \\ &\times \exp\left(0.05 \times \text{age} + 0.2 \times \text{sex} + 0.3 \times \text{W}\right). \end{aligned}$$

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (1.75)	1.756	1.730	0.409	0.399	0.409	0.943
	κ (0.5)	0.501	0.498	0.046	0.045	0.046	0.947
	α (2.5)	2.547	2.515	0.371	0.360	0.374	0.945
	β_{11} (0)	0.000	0.000	0.010	0.011	0.010	0.960
	β_{12} (0)	-0.001	0.005	0.246	0.248	0.246	0.962
	β_{13} (0)	0.005	0.002	0.257	0.249	0.257	0.938
	β_{21} (0.035)	0.035	0.035	0.002	0.002	0.002	0.942
	β_{22} (0.2)	0.201	0.200	0.048	0.048	0.047	0.951
	β_{23} (0.3)	0.301	0.301	0.049	0.049	0.049	0.948
PHEW	σ (1.75)	1.765	1.738	0.391	0.376	0.391	0.937
	κ (0.5)	0.501	0.498	0.041	0.039	0.041	0.944
	α (2.5)	2.537	2.503	0.349	0.339	0.351	0.938
	β_1 (0.035)	0.035	0.035	0.001	0.001	0.001	0.938
	β_2 (0.2)	0.201	0.202	0.028	0.028	0.028	0.952
	β_3 (0.3)	0.300	0.300	0.029	0.029	0.029	0.942
50% censoring							
GHEW	σ (1.75)	1.775	1.763	0.454	0.453	0.455	0.949
	κ (0.5)	0.504	0.503	0.054	0.054	0.054	0.957
	α (2.5)	2.540	2.479	0.420	0.415	0.421	0.958
	β_{11} (0)	-0.000	0.000	0.014	0.013	0.014	0.941
	β_{12} (0)	-0.006	-0.004	0.287	0.292	0.287	0.959
	β_{13} (0)	0.004	0.012	0.292	0.294	0.292	0.969
	β_{21} (0.035)	0.035	0.035	0.002	0.002	0.002	0.940
	β_{22} (0.2)	0.201	0.202	0.050	0.050	0.050	0.950
	β_{23} (0.3)	0.300	0.301	0.051	0.051	0.051	0.956
PHEW	σ (1.75)	1.785	1.776	0.420	0.428	0.421	0.956
	κ (0.5)	0.504	0.504	0.046	0.047	0.046	0.962
	α (2.5)	2.527	2.472	0.387	0.387	0.388	0.961
	β_1 (0.035)	0.035	0.035	0.001	0.001	0.001	0.954
	β_2 (0.2)	0.201	0.202	0.033	0.033	0.033	0.949
	β_3 (0.3)	0.300	0.301	0.034	0.033	0.034	0.948

Table S10. Simulation from PH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.5, 2.5)$, $\beta = (0.035, 0.2, 0.3)$, and $n = 10000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

4.3 HH structure

$$h_{\text{E}}^{\text{HH}}(t; \mathbf{x}_j) = h_{EW}\left(t \times \exp\left(0.1 \times \text{age}\right); 1.8, 0.5, 2.5\right) \times \exp\left(0.05 \times \text{age} + 0.2 \times \text{sex} + 0.5 \times \text{W}\right).$$

Model	30% censoring	50% censoring
	$n = 1000$	
PHEW	65.4	59.8
AHEW	0.8	1.3
AFTEW	22.3	28.7
GHEW	11.5	10.2
	$n = 5000$	
PHEW	83.3	78.4
AHEW	0	0
AFTEW	6.8	10.5
GHEW	9.9	11.4
	$n = 10000$	
PHEW	88.2	85.2
AHEW	0	0
AFTEW	1.5	5.7
GHEW	10.3	9.1

Table S11. Simulation from PH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.5, 2.5)$, and $\beta = (0.035, 0.2, 0.3)$. Percentage of models selected with AIC.

4.4 AH structure

$$h_{\text{E}}^{\text{AH}}(t; \mathbf{x}_j) = h_{EW}\left(t \times \exp\left(-0.1 \times \text{age} - 0.3 \times \text{sex} - 0.5 \times \text{W}\right); 1.75, 0.5, 1.75\right).$$

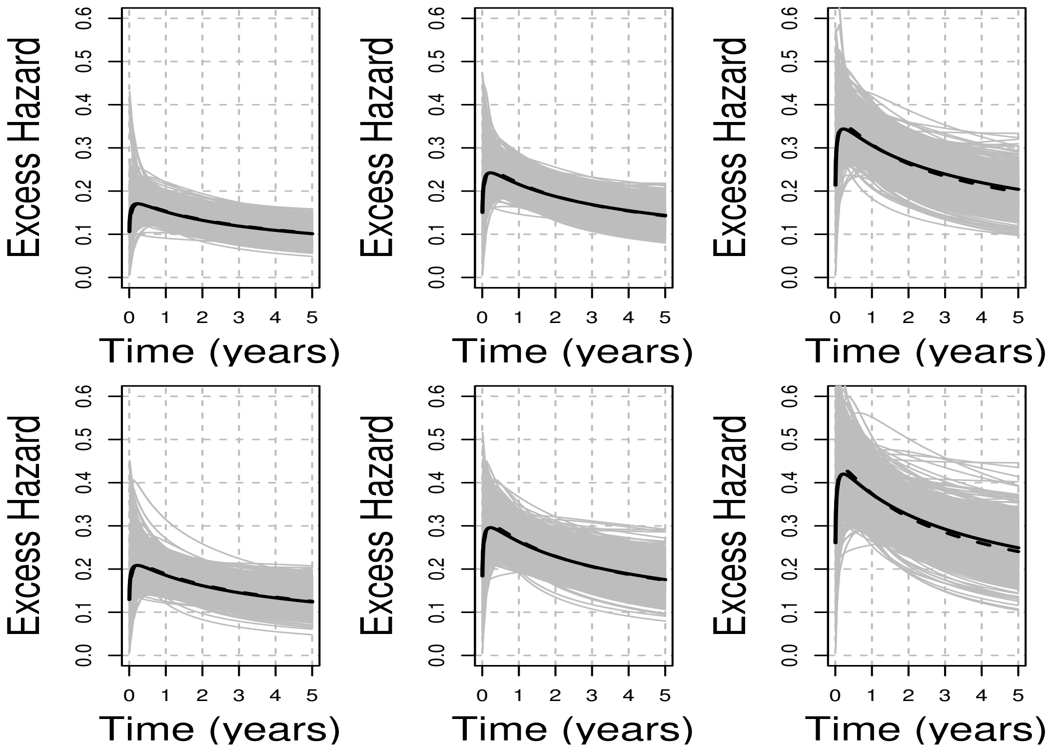

Figure S5. Best hazards in terms of AIC for $n = 1000$ and 50% censoring PH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

4.5 GH structure

$$
\begin{aligned}
h_{\text{E}}^{\text{GH}}(t; \mathbf{x}_j) &= h_{EW}\left(t \times \exp\left(0.1 \times \text{age} + 0.1 \times \text{sex} + 0.1 \times \text{W}\right); 1.75, 0.6, 2.5\right) \\
&\times \exp\left(0.05 \times \text{age} + 0.2 \times \text{sex} + 0.25 \times \text{W}\right).
\end{aligned}
$$

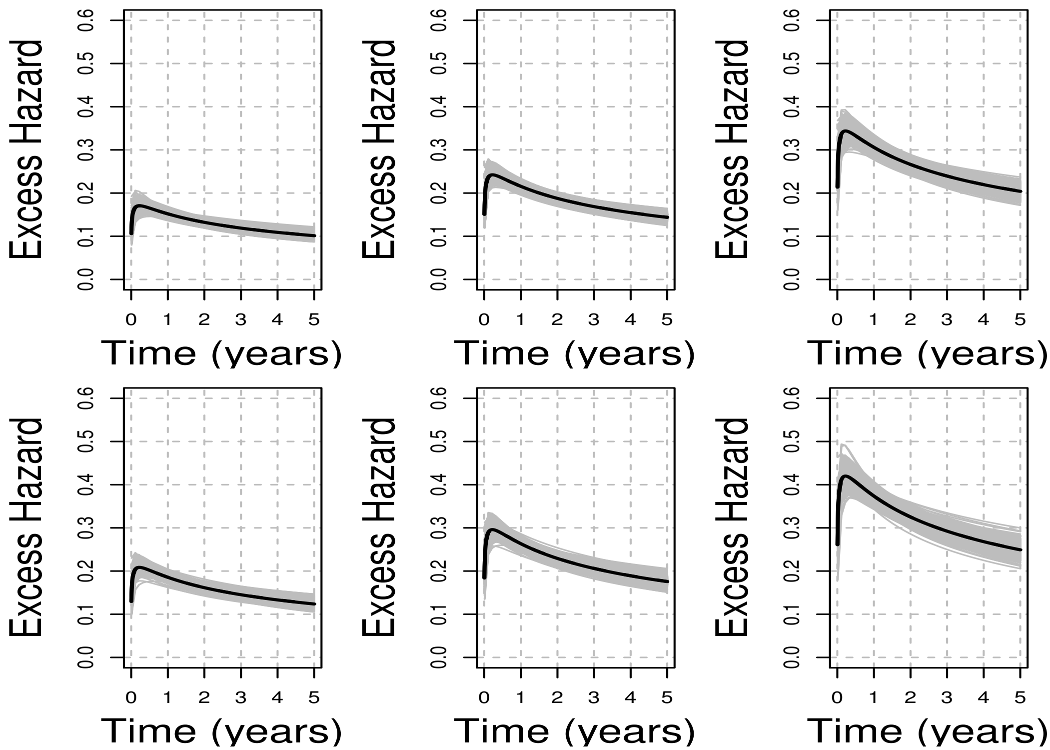

Figure S6. Best hazards in terms of AIC for $n = 10000$ and 30% censoring PH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

4.6 CH structure

$$h_{\mathrm{E}}^{\mathrm{CH}}\left(t; \mathbf{x}_j\right) = h_{EW}\left(t \times \exp\left(0.5 \times \mathrm{W}\right); 0.114, 0.366, 18.306\right) \times \exp\left(0.05 \times \mathrm{age} + 0.3 \times \mathrm{sex} - 0.6 \times \mathrm{W}\right)$$

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
GHEW	σ (1.4)	1.306	1.206	0.758	0.797	0.763	0.969
	κ (0.5)	0.486	0.484	0.111	0.118	0.112	0.940
	α (2.5)	3.127	2.715	1.623	1.426	1.739	0.965
	β_{11} (0.05)	0.049	0.049	0.039	0.034	0.039	0.926
	β_{12} (0.2)	0.063	0.212	1.498	2.529	1.503	0.978
	β_{13} (0.3)	0.161	0.205	1.279	1.398	1.286	0.973
	β_{21} (0.05)	0.049	0.050	0.007	0.006	0.007	0.953
	β_{22} (0.2)	0.186	0.198	0.206	0.278	0.206	0.953
	β_{23} (0.3)	0.285	0.289	0.172	0.182	0.172	0.958
AFTEW	σ (1.4)	1.395	1.324	0.746	0.741	0.745	0.963
	κ (0.5)	0.496	0.489	0.095	0.095	0.095	0.956
	α (2.5)	2.901	2.598	1.326	1.101	1.384	0.964
	β_1 (0.05)	0.050	0.050	0.004	0.004	0.004	0.957
	β_2 (0.2)	0.197	0.199	0.108	0.107	0.108	0.943
	β_3 (0.3)	0.303	0.304	0.105	0.107	0.105	0.954
			50% censoring				
GHEW	σ (1.4)	1.232	1.147	0.791	0.868	0.808	0.970
	κ (0.5)	0.475	0.473	0.126	0.140	0.128	0.948
	α (2.5)	3.461	2.796	2.322	2.014	2.512	0.975
	β_{11} (0.05)	0.050	0.049	0.047	0.040	0.047	0.924
	β_{12} (0.2)	-0.009	0.106	1.699	2.281	1.711	0.973
	β_{13} (0.3)	0.076	0.227	1.559	2.005	1.575	0.964
	β_{21} (0.05)	0.049	0.049	0.008	0.007	0.008	0.953
	β_{22} (0.2)	0.180	0.186	0.207	0.241	0.208	0.952
	β_{23} (0.3)	0.276	0.284	0.195	0.204	0.197	0.950
AFTEW	(1.4)	1.350	1.241	0.816	0.833	0.817	0.967
	κ (0.5)	0.489	0.479	0.110	0.114	0.111	0.963
	α (2.5)	3.098	2.704	1.617	1.437	1.723	0.973
	β_1 (0.05)	0.050	0.050	0.005	0.005	0.005	0.948
	β_2 (0.2)	0.197	0.196	0.124	0.123	0.124	0.942
	β_3 (0.3)	0.299	0.297	0.125	0.123	0.125	0.947

Table S12. Simulation from AFT structure with $(\sigma, \kappa, \alpha) = (1.4, 0.5, 2.5)$, $\beta = (0.05, 0.2, 0.3)$, and $n = 1000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (1.4)	1.419	1.386	0.409	0.405	0.410	0.943
	κ (0.5)	0.503	0.500	0.056	0.055	0.056	0.940
	α (2.5)	2.563	2.519	0.472	0.461	0.476	0.950
	β_{11} (0.05)	0.051	0.050	0.015	0.015	0.015	0.936
	β_{12} (0.2)	0.212	0.227	0.324	0.311	0.324	0.949
	β_{13} (0.3)	0.304	0.297	0.327	0.315	0.327	0.957
	β_{21} (0.05)	0.050	0.050	0.003	0.003	0.003	0.955
	β_{22} (0.2)	0.203	0.202	0.075	0.072	0.075	0.943
	β_{23} (0.3)	0.301	0.301	0.074	0.073	0.074	0.943
AFTEW	σ (1.4)	1.428	1.403	0.348	0.351	0.349	0.951
	κ (0.5)	0.503	0.500	0.042	0.042	0.042	0.955
	α (2.5)	2.534	2.506	0.390	0.390	0.392	0.956
	β_1 (0.05)	0.050	0.050	0.002	0.002	0.002	0.952
	β_2 (0.2)	0.201	0.199	0.049	0.048	0.049	0.944
	β_3 (0.3)	0.302	0.303	0.048	0.048	0.048	0.952
50% censoring							
GHEW	σ (1.4)	1.407	1.375	0.472	0.464	0.472	0.946
	κ (0.5)	0.502	0.500	0.069	0.067	0.069	0.937
	α (2.5)	2.604	2.504	0.583	0.559	0.592	0.946
	β_{11} (0.05)	0.051	0.051	0.019	0.018	0.019	0.945
	β_{12} (0.2)	0.206	0.203	0.378	0.368	0.378	0.946
	β_{13} (0.3)	0.298	0.276	0.382	0.373	0.382	0.958
	β_{21} (0.05)	0.050	0.050	0.003	0.003	0.003	0.944
	β_{22} (0.2)	0.200	0.200	0.076	0.077	0.076	0.950
	β_{23} (0.3)	0.300	0.300	0.079	0.077	0.079	0.950
AFTEW	σ (1.4)	1.423	1.411	0.400	0.404	0.401	0.953
	κ (0.5)	0.503	0.502	0.051	0.051	0.051	0.951
	α (2.5)	2.558	2.497	0.471	0.462	0.474	0.951
	β_1 (0.05)	0.050	0.050	0.002	0.002	0.002	0.939
	β_2 (0.2)	0.199	0.202	0.053	0.054	0.053	0.957
	β_3 (0.3)	0.301	0.301	0.057	0.055	0.057	0.939

Table S13. Simulation from AFT structure with $(\sigma, \kappa, \alpha) = (1.4, 0.5, 2.5)$, $\beta = (0.05, 0.2, 0.3)$, and $n = 5000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (1.4)	1.407	1.401	0.295	0.289	0.295	0.940
	κ (0.5)	0.501	0.500	0.040	0.039	0.040	0.945
	α (2.5)	2.536	2.499	0.327	0.320	0.329	0.939
	β_{11} (0.05)	0.050	0.050	0.010	0.010	0.010	0.954
	β_{12} (0.2)	0.201	0.203	0.215	0.216	0.215	0.959
	β_{13} (0.3)	0.305	0.297	0.222	0.218	0.222	0.948
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.941
	β_{22} (0.2)	0.200	0.201	0.051	0.051	0.051	0.958
	β_{23} (0.3)	0.302	0.300	0.051	0.051	0.051	0.942
AFTEW	σ (1.4)	1.414	1.399	0.255	0.248	0.256	0.944
	κ (0.5)	0.501	0.500	0.031	0.030	0.031	0.946
	α (2.5)	2.520	2.504	0.278	0.272	0.279	0.940
	β_1 (0.05)	0.050	0.050	0.001	0.001	0.001	0.936
	β_2 (0.2)	0.200	0.201	0.034	0.034	0.034	0.950
	β_3 (0.3)	0.302	0.304	0.034	0.034	0.034	0.949
50% censoring							
GHEW	σ (1.4)	1.419	1.405	0.337	0.335	0.337	0.940
	κ (0.5)	0.504	0.501	0.048	0.048	0.049	0.941
	α (2.5)	2.532	2.494	0.381	0.378	0.382	0.939
	β_{11} (0.05)	0.051	0.050	0.013	0.013	0.014	0.936
	β_{12} (0.2)	0.204	0.203	0.258	0.255	0.258	0.944
	β_{13} (0.3)	0.311	0.309	0.265	0.259	0.265	0.957
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.948
	β_{22} (0.2)	0.200	0.204	0.056	0.054	0.056	0.944
	β_{23} (0.3)	0.301	0.302	0.055	0.054	0.055	0.948
AFTEW	σ (1.4)	1.420	1.400	0.286	0.288	0.286	0.951
	κ (0.5)	0.502	0.501	0.036	0.036	0.036	0.960
	α (2.5)	2.520	2.503	0.320	0.318	0.320	0.951
	β_1 (0.05)	0.050	0.050	0.002	0.002	0.002	0.948
	β_2 (0.2)	0.200	0.200	0.040	0.039	0.040	0.938
	β_3 (0.3)	0.300	0.300	0.038	0.039	0.038	0.950

Table S14. Simulation from AFT structure with $(\sigma, \kappa, \alpha) = (1.4, 0.5, 2.5)$, $\beta = (0.05, 0.2, 0.3)$, and $n = 10000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	30% censoring	50% censoring
	$n = 1000$	
PHEW	19.8	22.3
AHEW	0.1	0
AFTEW	71.8	65.9
GHEW	8.3	11.8
	$n = 5000$	
PHEW	1.9	5.1
AHEW	0	0
AFTEW	86.9	84.7
GHEW	11.2	10.2
	$n = 10000$	
PHEW	0.2	0.9
AHEW	0	0
AFTEW	89.2	87.9
GHEW	10.6	11.2

Table S15. Simulation from AFT structure with $(\sigma, \kappa, \alpha) = (1.4, 0.5, 2.5)$, and $\beta = (0.05, 0.2, 0.3)$. Percentage of models selected with AIC.

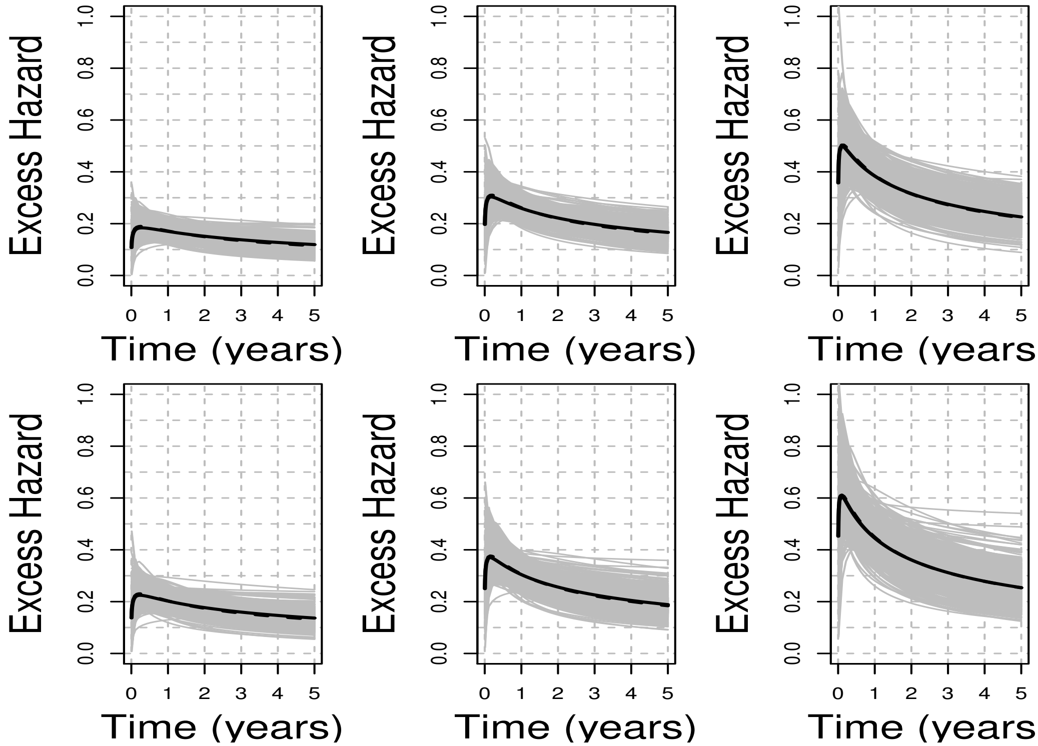

Figure S7. Best hazards in terms of AIC for $n = 1000$ and 50% censoring AFT: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

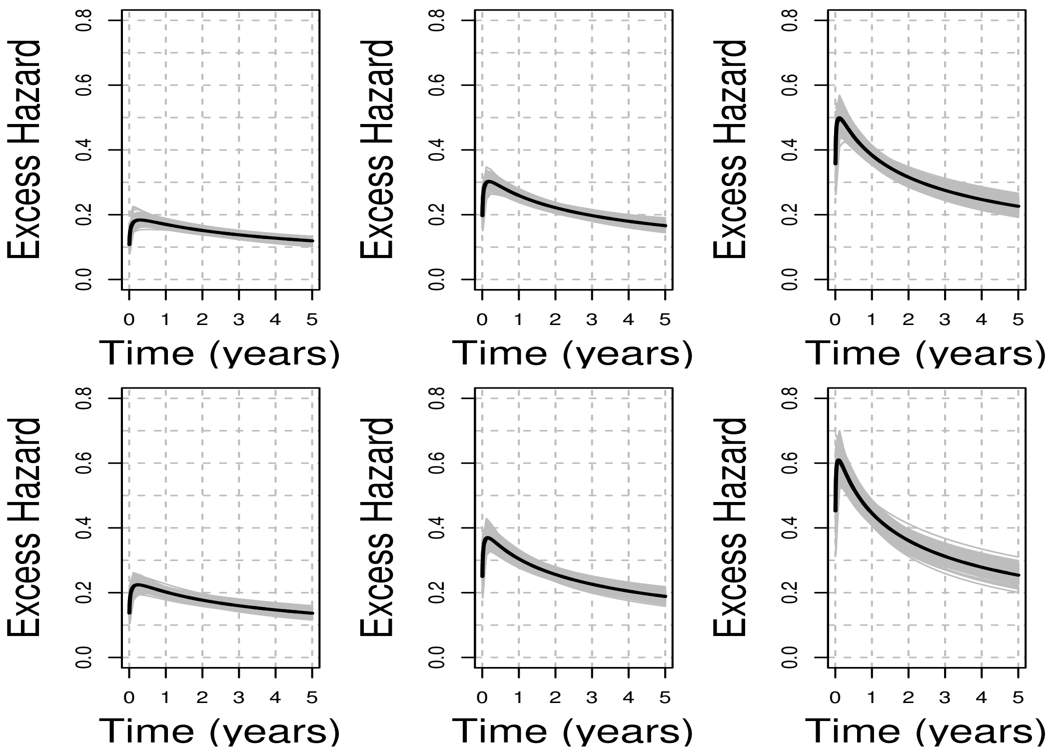

Figure S8. Best hazards in terms of AIC for $n = 10000$ and 30% censoring AFT: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (1.8)	1.647	1.577	0.940	1.026	0.952	0.976
	κ (0.5)	0.483	0.477	0.114	0.125	0.115	0.960
	α (2.5)	3.184	2.719	1.779	1.562	1.905	0.982
	β_{11} (0.1)	0.099	0.097	0.042	0.038	0.042	0.926
	β_{12} (0)	-0.066	-0.038	0.859	0.911	0.861	0.973
	β_{13} (0)	-0.081	-0.050	1.000	0.843	1.003	0.966
	β_{21} (0.05)	0.049	0.049	0.006	0.006	0.006	0.965
	β_{22} (0.2)	0.195	0.199	0.144	0.154	0.144	0.958
	β_{23} (0.5)	0.495	0.500	0.149	0.149	0.149	0.962
HHEW	σ (1.8)	1.688	1.611	0.931	1.004	0.937	0.972
	κ (0.5)	0.487	0.481	0.109	0.116	0.109	0.954
	α (2.5)	3.082	2.680	1.587	1.419	1.689	0.976
	β_{11} (0.1)	0.101	0.096	0.039	0.038	0.039	0.937
	β_{21} (0.05)	0.050	0.049	0.005	0.006	0.005	0.966
	β_{22} (0.2)	0.201	0.200	0.091	0.090	0.091	0.946
	β_{23} (0.5)	0.506	0.510	0.088	0.091	0.088	0.956
50% censoring							
GHEW	σ (1.8)	1.575	1.467	1.011	1.118	1.035	0.963
	κ (0.5)	0.474	0.466	0.129	0.147	0.131	0.967
	α (2.5)	3.464	2.805	2.346	2.131	2.535	0.980
	β_{11} (0.1)	0.098	0.096	0.049	0.046	0.049	0.927
	β_{12} (0)	-0.144	-0.039	1.425	1.419	1.432	0.965
	β_{13} (0)	-0.102	-0.037	1.049	1.134	1.053	0.962
	β_{21} (0.05)	0.049	0.049	0.007	0.007	0.007	0.953
	β_{22} (0.2)	0.194	0.196	0.168	0.173	0.168	0.958
	β_{23} (0.5)	0.488	0.494	0.167	0.161	0.167	0.951
HHEW	σ (1.8)	1.630	1.517	1.002	1.109	1.016	0.969
	κ (0.5)	0.479	0.471	0.121	0.135	0.123	0.967
	α (2.5)	3.336	2.778	2.235	1.931	2.385	0.982
	β_{11} (0.1)	0.098	0.097	0.044	0.044	0.044	0.939
	β_{21} (0.05)	0.049	0.049	0.007	0.007	0.007	0.958
	β_{22} (0.2)	0.200	0.201	0.107	0.105	0.107	0.943
	β_{23} (0.5)	0.499	0.497	0.107	0.106	0.107	0.945

Table S16. Simulation from HH structure with $(\sigma, \kappa, \alpha) = (1.8, 0.5, 2.5)$, $\beta_{11} = 0.1$, $\beta_2 = (0.05, 0.2, 0.5)$, and $n = 1000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
GHEW	σ (1.8)	1.812	1.810	0.510	0.519	0.510	0.959
	κ (0.5)	0.502	0.501	0.057	0.058	0.057	0.953
	α (2.5)	2.571	2.506	0.472	0.476	0.477	0.957
	β_{11} (0.1)	0.101	0.100	0.017	0.016	0.017	0.952
	β_{12} (0)	0.001	0.004	0.290	0.284	0.290	0.957
	β_{13} (0)	-0.003	-0.009	0.299	0.289	0.298	0.950
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.945
	β_{22} (0.2)	0.202	0.204	0.060	0.060	0.060	0.959
	β_{23} (0.5)	0.500	0.501	0.063	0.062	0.063	0.949
HHEW	σ (1.8)	1.822	1.831	0.491	0.500	0.491	0.957
	κ (0.5)	0.503	0.502	0.052	0.053	0.053	0.960
	α (2.5)	2.558	2.486	0.455	0.457	0.458	0.960
	β_{11} (0.1)	0.101	0.100	0.016	0.016	0.016	0.951
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.945
	β_{22} (0.2)	0.202	0.202	0.039	0.040	0.039	0.959
	β_{23} (0.5)	0.502	0.500	0.041	0.041	0.041	0.952
			50% censoring				
GHEW	σ (1.8)	1.813	1.800	0.584	0.589	0.584	0.949
	κ (0.5)	0.503	0.500	0.070	0.070	0.070	0.947
	α (2.5)	2.599	2.489	0.601	0.566	0.609	0.955
	β_{11} (0.1)	0.102	0.101	0.020	0.020	0.020	0.950
	β_{12} (0)	-0.018	-0.001	0.342	0.338	0.342	0.964
	β_{13} (0)	-0.000	0.002	0.348	0.345	0.348	0.960
	β_{21} (0.05)	0.050	0.050	0.003	0.003	0.003	0.953
	β_{22} (0.2)	0.198	0.200	0.064	0.066	0.064	0.969
	β_{23} (0.5)	0.501	0.502	0.070	0.068	0.070	0.946
HHEW	σ (1.8)	1.824	1.817	0.553	0.568	0.553	0.953
	κ (0.5)	0.504	0.502	0.063	0.064	0.063	0.951
	α (2.5)	2.579	2.479	0.557	0.538	0.562	0.958
	β_{11} (0.1)	0.102	0.101	0.020	0.020	0.020	0.956
	β_{21} (0.05)	0.050	0.050	0.003	0.003	0.003	0.953
	β_{22} (0.2)	0.200	0.200	0.045	0.046	0.045	0.956
	β_{23} (0.5)	0.502	0.500	0.048	0.047	0.048	0.949

Table S17. Simulation from HH structure with $(\sigma, \kappa, \alpha) = (1.8, 0.5, 2.5)$, $\beta_{11} = 0.1$, $\beta_2 = (0.05, 0.2, 0.5)$, and $n = 5000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (1.8)	1.804	1.793	0.375	0.371	0.375	0.953
	κ (0.5)	0.501	0.498	0.041	0.041	0.041	0.945
	α (2.5)	2.541	2.510	0.336	0.329	0.338	0.948
	β_{11} (0.1)	0.100	0.100	0.011	0.011	0.011	0.950
	β_{12} (0)	0.005	0.007	0.201	0.199	0.201	0.946
	β_{13} (0)	0.001	0.003	0.211	0.202	0.211	0.935
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.945
	β_{22} (0.2)	0.201	0.200	0.043	0.042	0.043	0.943
	β_{23} (0.5)	0.502	0.502	0.045	0.044	0.045	0.935
HHEW	σ (1.8)	1.804	1.788	0.361	0.356	0.361	0.949
	κ (0.5)	0.500	0.500	0.038	0.038	0.038	0.947
	α (2.5)	2.539	2.504	0.326	0.318	0.328	0.947
	β_{11} (0.1)	0.100	0.100	0.011	0.011	0.011	0.953
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.949
	β_{22} (0.2)	0.200	0.200	0.029	0.028	0.029	0.950
	β_{23} (0.5)	0.502	0.501	0.029	0.029	0.029	0.953
50% censoring							
GHEW	σ (1.8)	1.819	1.803	0.429	0.424	0.429	0.940
	κ (0.5)	0.503	0.500	0.050	0.049	0.050	0.944
	α (2.5)	2.537	2.500	0.390	0.382	0.392	0.955
	β_{11} (0.1)	0.102	0.101	0.014	0.014	0.014	0.955
	β_{12} (0)	0.000	0.002	0.242	0.235	0.242	0.938
	β_{13} (0)	-0.000	0.003	0.243	0.239	0.243	0.952
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.955
	β_{22} (0.2)	0.202	0.202	0.049	0.046	0.049	0.941
	β_{23} (0.5)	0.500	0.500	0.049	0.048	0.049	0.946
HHEW	σ (1.8)	1.822	1.809	0.405	0.406	0.405	0.955
	κ (0.5)	0.503	0.501	0.045	0.045	0.045	0.957
	α (2.5)	2.530	2.488	0.368	0.366	0.369	0.962
	β_{11} (0.1)	0.102	0.101	0.014	0.014	0.014	0.957
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.957
	β_{22} (0.2)	0.202	0.201	0.034	0.033	0.034	0.941
	β_{23} (0.5)	0.500	0.501	0.034	0.033	0.034	0.946

Table S18. Simulation from HH structure with $(\sigma, \kappa, \alpha) = (1.8, 0.5, 2.5)$, $\beta_{11} = 0.1$, $\beta_2 = (0.05, 0.2, 0.5)$, and $n = 10000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	30% censoring	50% censoring
$n = 1000$		
PHEW	3.1	5.5
AHEW	0	0.1
AFTEW	23.5	30.0
GHEW	11.7	11.5
HHEW	61.7	52.9
$n = 5000$		
PHEW	0	0
AHEW	0	0
AFTEW	0.4	2.4
GHEW	14.4	13.2
HHEW	85.2	84.4
$n = 10000$		
PHEW	0	0
AHEW	0	0
AFTEW	0	0.1
GHEW	15.5	14.4
HHEW	84.5	85.5

Table S19. Simulation from HH structure with $(\sigma, \kappa, \alpha) = (1.8, 0.5, 2.5)$, $\beta_{11} = 0.1$, $\beta_2 = (0.05, 0.2, 0.5)$. Percentage of models selected with AIC.

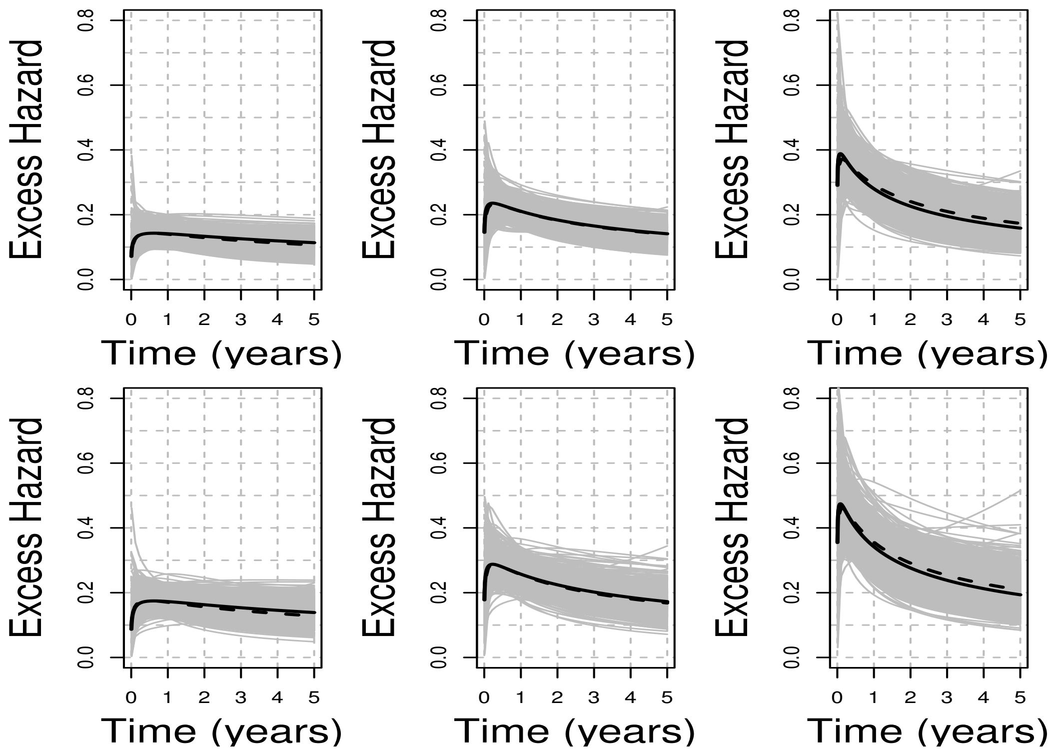

Figure S9. Best hazards in terms of AIC for $n = 1000$ and 50% censoring HH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

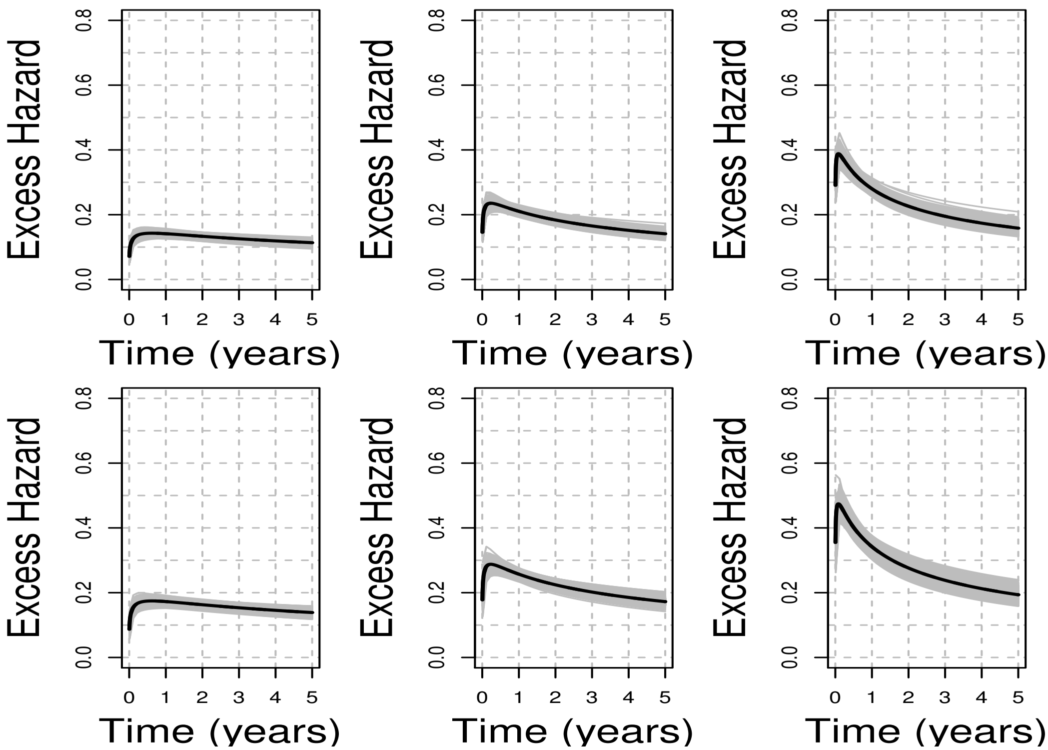

Figure S10. Best hazards in terms of AIC for $n = 10000$ and 30% censoring HH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (1.75)	1.206	1.154	0.573	0.718	0.790	0.994
	κ (0.5)	0.429	0.431	0.070	0.227	0.100	0.942
	α (1.75)	2.391	2.182	0.805	0.356	1.029	0.960
	β_{11} (-0.1)	-0.075	-0.079	0.042	0.044	0.049	0.886
	β_{12} (-0.3)	-0.613	-0.371	1.908	2.052	1.933	0.973
	β_{13} (-0.5)	-0.863	-0.554	2.050	2.642	2.080	0.988
	β_{21} (0)	0.007	0.006	0.015	0.016	0.017	0.911
	β_{22} (0)	-0.082	-0.039	0.498	0.532	0.505	0.967
	β_{23} (0)	-0.097	-0.025	0.529	0.677	0.538	0.982
AHEW	σ (1.75)	1.420	1.404	0.657	0.749	0.735	0.989
	κ (0.5)	0.457	0.457	0.077	0.091	0.088	0.951
	α (1.75)	2.165	2.002	0.712	0.710	0.824	0.975
	β_1 (-0.1)	-0.095	-0.095	0.013	0.016	0.014	0.914
	β_2 (-0.3)	-0.301	-0.288	0.281	0.271	0.281	0.951
	β_3 (-0.5)	-0.508	-0.508	0.282	0.274	0.282	0.958
50% censoring							
GHEW	σ (1.75)	1.100	1.065	0.612	0.952	0.893	0.999
	κ (0.5)	0.407	0.412	0.082	0.291	0.124	0.926
	α (1.75)	2.670	2.323	1.207	0.446	1.517	0.974
	β_{11} (-0.1)	-0.065	-0.071	0.055	0.053	0.065	0.880
	β_{12} (-0.3)	-0.779	-0.476	2.429	3.987	2.475	0.983
	β_{13} (-0.5)	-1.126	-0.664	2.849	4.149	2.915	0.981
	β_{21} (0)	0.010	0.008	0.018	0.017	0.020	0.893
	β_{22} (0)	-0.116	-0.050	0.587	0.895	0.598	0.973
	β_{23} (0)	-0.147	-0.046	0.679	0.945	0.695	0.968
AHEW	σ (1.75)	1.372	1.320	0.745	0.870	0.836	0.996
	κ (0.5)	0.444	0.443	0.094	0.114	0.109	0.948
	α (1.75)	2.337	2.088	1.014	0.985	1.171	0.975
	β_1 (-0.1)	-0.094	-0.093	0.015	0.019	0.016	0.923
	β_2 (-0.3)	-0.305	-0.298	0.331	0.338	0.331	0.960
	β_3 (-0.5)	-0.524	-0.511	0.344	0.343	0.344	0.961

Table S20. Simulation from AH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.2, 1.75)$, $\beta = (-0.1, -0.3, -0.5)$, and $n = 1000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
GHEW	σ (1.75)	1.660	1.654	0.355	0.247	0.367	0.977
	κ (0.5)	0.489	0.489	0.042	0.099	0.043	0.974
	α (1.75)	1.842	1.807	0.242	0.146	0.259	0.977
	β_{11} (-0.1)	-0.096	-0.096	0.019	0.024	0.020	0.956
	β_{12} (-0.3)	-0.317	-0.299	0.454	0.473	0.454	0.956
	β_{13} (-0.5)	-0.525	-0.515	0.475	0.477	0.475	0.966
	β_{21} (0)	0.001	0.001	0.007	0.008	0.007	0.964
	β_{22} (0)	-0.004	-0.004	0.142	0.145	0.142	0.962
	β_{23} (0)	-0.006	-0.005	0.149	0.147	0.149	0.965
AHEW	σ (1.75)	1.719	1.704	0.352	0.363	0.353	0.959
	κ (0.5)	0.496	0.494	0.040	0.042	0.040	0.964
	α (1.75)	1.798	1.778	0.220	0.228	0.226	0.961
	β_1 (-0.1)	-0.100	-0.100	0.007	0.008	0.007	0.949
	β_2 (-0.3)	-0.303	-0.301	0.126	0.125	0.126	0.953
	β_3 (-0.5)	-0.505	-0.505	0.127	0.126	0.127	0.952
			50% censoring				
GHEW	σ (1.75)	1.594	1.600	0.390	0.299	0.420	0.983
	κ (0.5)	0.479	0.483	0.049	0.125	0.053	0.956
	α (1.75)	1.901	1.839	0.308	0.177	0.343	0.972
	β_{11} (-0.1)	-0.094	-0.095	0.022	0.027	0.023	0.945
	β_{12} (-0.3)	-0.393	-0.366	0.563	0.586	0.570	0.971
	β_{13} (-0.5)	-0.571	-0.546	0.612	0.590	0.616	0.955
	β_{21} (0)	0.002	0.001	0.007	0.009	0.008	0.951
	β_{22} (0)	-0.027	-0.027	0.161	0.169	0.164	0.970
	β_{23} (0)	-0.019	-0.015	0.178	0.171	0.179	0.943
AHEW	σ (1.75)	1.678	1.651	0.394	0.417	0.400	0.971
	κ (0.5)	0.491	0.490	0.049	0.052	0.050	0.958
	α (1.75)	1.835	1.810	0.277	0.283	0.290	0.964
	β_1 (-0.1)	-0.099	-0.099	0.009	0.009	0.009	0.951
	β_2 (-0.3)	-0.299	-0.307	0.155	0.156	0.154	0.955
	β_3 (-0.5)	-0.505	-0.501	0.164	0.157	0.164	0.951

Table S21. Simulation from AH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.2, 1.75)$, $\beta = (-0.1, -0.3, -0.5)$, and $n = 5000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
	σ (1.75)	1.743	1.737	0.211	0.167	0.211	0.984
	κ (0.5)	0.499	0.499	0.025	0.070	0.025	0.984
	α (1.75)	1.767	1.763	0.130	0.101	0.131	0.985
	β_{11} (-0.1)	-0.099	-0.099	0.013	0.017	0.013	0.978
GHEW	β_{12} (-0.3)	-0.283	-0.276	0.288	0.335	0.288	0.981
	β_{13} (-0.5)	-0.496	-0.487	0.310	0.339	0.310	0.963
	β_{21} (0)	0.000	0.000	0.005	0.006	0.005	0.969
	β_{22} (0)	0.006	0.008	0.088	0.103	0.089	0.974
	β_{23} (0)	0.003	0.006	0.094	0.104	0.094	0.962
	σ (1.75)	1.750	1.755	0.256	0.259	0.256	0.949
	κ (0.5)	0.500	0.500	0.030	0.030	0.030	0.944
	α (1.75)	1.766	1.747	0.157	0.156	0.158	0.946
AHEW	β_1 (-0.1)	-0.100	-0.100	0.005	0.005	0.005	0.946
	β_2 (-0.3)	-0.302	-0.302	0.092	0.088	0.092	0.941
	β_3 (-0.5)	-0.505	-0.502	0.093	0.089	0.093	0.947
			50% censoring				
	σ (1.75)	1.728	1.734	0.226	0.196	0.227	0.987
	κ (0.5)	0.498	0.499	0.028	0.088	0.028	0.988
	α (1.75)	1.775	1.761	0.149	0.123	0.151	0.990
	β_{11} (-0.1)	-0.100	-0.101	0.016	0.020	0.015	0.981
GHEW	β_{12} (-0.3)	-0.297	-0.291	0.361	0.413	0.361	0.979
	β_{13} (-0.5)	-0.506	-0.496	0.355	0.417	0.355	0.982
	β_{21} (0)	0.000	-0.000	0.005	0.006	0.005	0.976
	β_{22} (0)	0.003	0.005	0.105	0.119	0.105	0.976
	β_{23} (0)	-0.001	0.002	0.102	0.120	0.102	0.984
	σ (1.75)	1.739	1.728	0.289	0.297	0.289	0.953
	κ (0.5)	0.499	0.499	0.036	0.037	0.036	0.955
	α (1.75)	1.775	1.759	0.182	0.188	0.183	0.951
AHEW	β_1 (-0.1)	-0.100	-0.100	0.006	0.006	0.006	0.965
	β_2 (-0.3)	-0.305	-0.307	0.113	0.111	0.114	0.943
	β_3 (-0.5)	-0.500	-0.503	0.112	0.112	0.112	0.943

Table S22. Simulation from AH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.2, 1.75)$, $\beta = (-0.1, -0.3, -0.5)$, and $n = 10000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	30% censoring	50% censoring
	$n = 1000$	
PHEW	13.1	16.6
AHEW	78.6	73.0
AFTEW	0.4	2.8
GHEW	7.9	7.6
	$n = 5000$	
PHEW	0.1	1.3
AHEW	91.7	87.5
AFTEW	0	0
GHEW	8.2	11.2
	$n = 10000$	
PHEW	0	0
AHEW	94.1	94.8
AFTEW	0	0
GHEW	5.9	5.2

Table S23. Simulation from AH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.2, 1.75)$, and $\beta = (-0.1, -0.3, -0.5)$. Percentage of models selected with AIC.

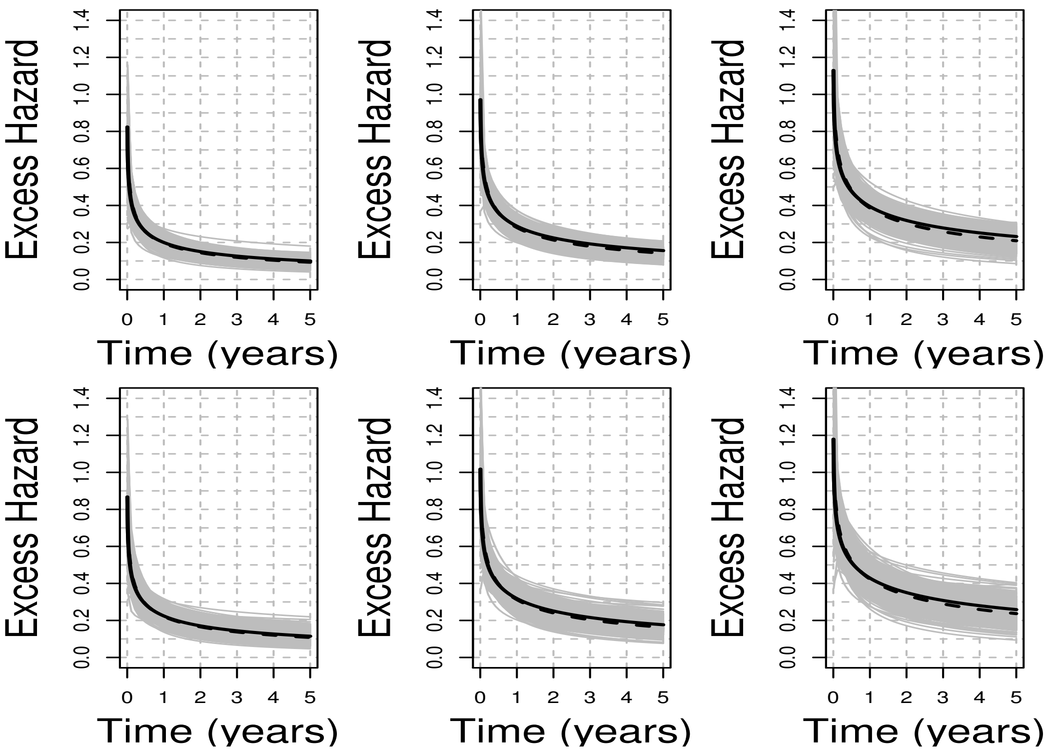

Figure S11. Best hazards in terms of AIC for $n = 1000$ and 50% censoring AH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

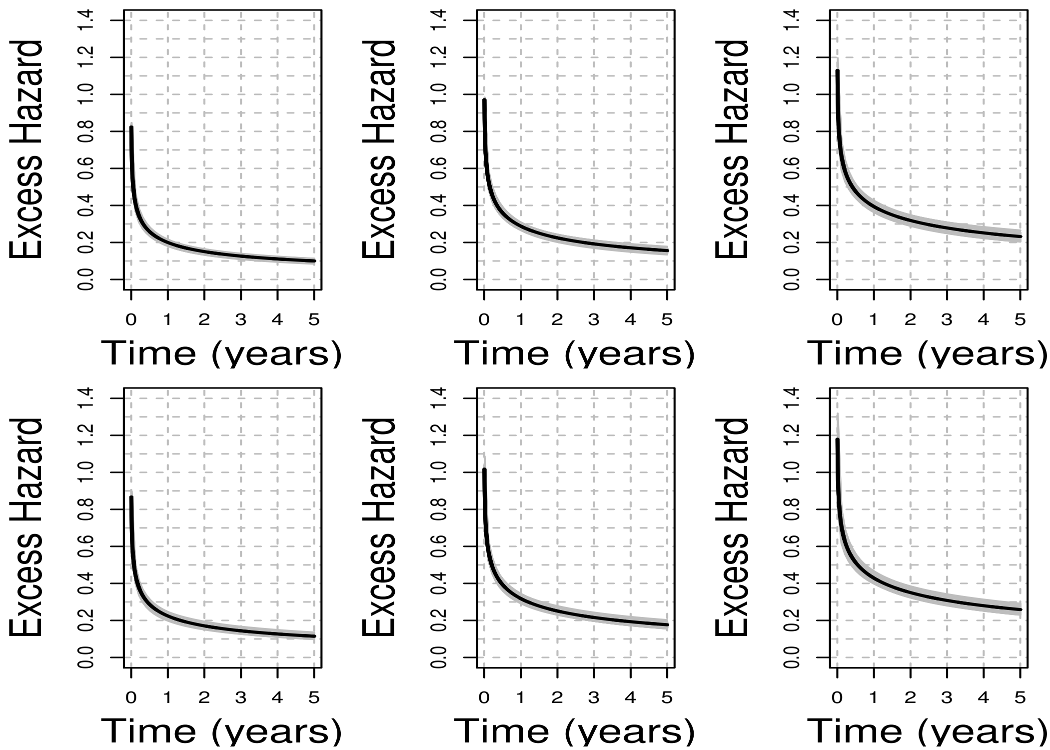

Figure S12. Best hazards in terms of AIC for $n = 10000$ and 30% censoring AH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
	σ (1.75)	1.580	1.550	0.787	0.834	0.805	0.981
	κ (0.6)	0.576	0.576	0.128	0.138	0.130	0.958
	α (2.5)	3.181	2.733	1.671	1.501	1.803	0.976
GHEW	β_{11} (0.1)	0.099	0.098	0.033	0.033	0.033	0.941
	β_{12} (0.1)	0.028	0.051	0.547	0.546	0.552	0.970
	β_{13} (0.1)	0.032	0.054	0.720	0.679	0.723	0.969
	β_{21} (0.05)	0.050	0.050	0.007	0.007	0.007	0.955
	β_{22} (0.2)	0.202	0.204	0.093	0.098	0.093	0.959
	β_{23} (0.25)	0.251	0.252	0.094	0.103	0.094	0.961
			50% censoring				
	σ (1.75)	1.560	1.527	0.841	0.928	0.862	0.975
	κ (0.6)	0.573	0.580	0.143	0.163	0.146	0.956
	α (2.5)	3.467	2.687	2.621	2.094	2.793	0.980
GHEW	β_{11} (0.1)	0.099	0.097	0.038	0.039	0.038	0.932
	β_{12} (0.1)	0.044	0.082	0.809	0.719	0.810	0.977
	β_{13} (0.1)	0.039	0.058	0.663	0.647	0.666	0.977
	β_{21} (0.05)	0.050	0.050	0.008	0.008	0.008	0.963
	β_{22} (0.2)	0.208	0.206	0.133	0.128	0.133	0.958
	β_{23} (0.25)	0.247	0.245	0.111	0.114	0.111	0.961

Table S24. Simulation from GH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.6, 2.5)$, $\beta_1 = (0.1, 0.1, 0.1)$, $\beta_2 = (0.05, 0.2, 0.25)$, and $n = 1000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (1.75)	1.747	1.745	0.293	0.292	0.293	0.948
	κ (0.6)	0.600	0.599	0.045	0.045	0.045	0.951
	α (2.5)	2.541	2.516	0.321	0.316	0.323	0.946
	β_{11} (0.1)	0.100	0.100	0.010	0.009	0.010	0.947
	β_{12} (0.1)	0.100	0.096	0.163	0.159	0.163	0.948
	β_{13} (0.1)	0.100	0.109	0.163	0.160	0.163	0.946
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.946
	β_{22} (0.2)	0.201	0.201	0.029	0.029	0.029	0.949
	β_{23} (0.25)	0.252	0.250	0.029	0.029	0.029	0.952
50% censoring							
GHEW	σ (1.75)	1.753	1.746	0.334	0.332	0.334	0.946
	κ (0.6)	0.602	0.600	0.054	0.054	0.054	0.951
	α (2.5)	2.540	2.509	0.366	0.364	0.368	0.953
	β_{11} (0.1)	0.101	0.100	0.011	0.011	0.011	0.954
	β_{12} (0.1)	0.101	0.107	0.185	0.186	0.185	0.950
	β_{13} (0.1)	0.106	0.096	0.190	0.187	0.190	0.953
	β_{21} (0.05)	0.050	0.050	0.003	0.003	0.003	0.954
	β_{22} (0.2)	0.200	0.200	0.034	0.033	0.034	0.940
	β_{23} (0.25)	0.250	0.250	0.034	0.033	0.033	0.948

Table S25. Simulation from GH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.6, 2.5)$, $\beta_1 = (0.1, 0.1, 0.1)$, $\beta_2 = (0.05, 0.2, 0.25)$, and $n = 10000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

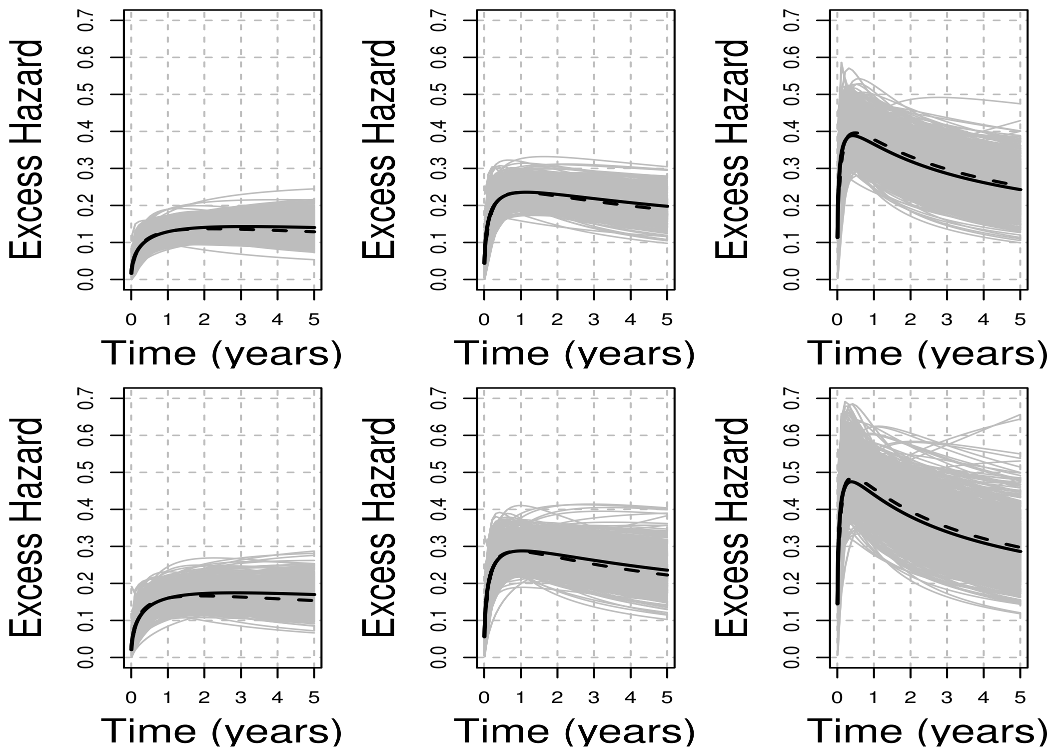

Figure S13. Best hazards in terms of AIC for $n = 1000$ and 50% censoring GH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

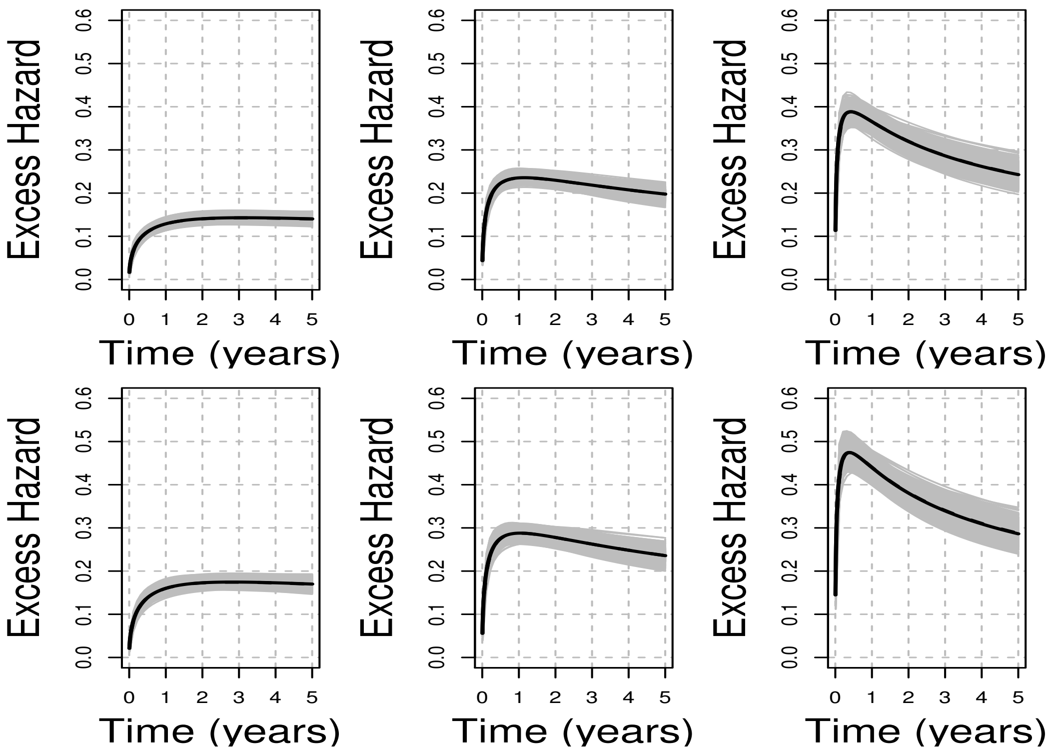

Figure S14. Best hazards in terms of AIC for $n = 10000$ and 30% censoring GH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
GHEW	σ (0.113)	0.239	0.121	0.303	0.278	0.328	0.914
	κ (0.366)	0.387	0.368	0.117	0.131	0.119	0.957
	α (18.305)	29.053	17.832	28.879	49.726	30.800	0.941
	β_{11} (0)	-0.000	0.000	0.012	0.011	0.012	0.945
	β_{12} (0)	-0.009	-0.008	0.230	0.223	0.230	0.946
	β_{13} (0.5)	0.518	0.495	0.255	0.260	0.256	0.959
	β_{21} (0.05)	0.050	0.050	0.005	0.005	0.005	0.949
	β_{22} (0.3)	0.302	0.302	0.099	0.104	0.099	0.958
	β_{23} (-0.6)	-0.610	-0.608	0.109	0.113	0.110	0.957
CHEW	σ (0.113)	0.249	0.134	0.304	0.281	0.333	0.905
	κ (0.366)	0.392	0.376	0.115	0.126	0.118	0.958
	α (18.305)	28.059	16.906	30.104	44.256	31.630	0.933
	β_{11} (0.5)	0.517	0.501	0.249	0.258	0.250	0.964
	β_{21} (0.05)	0.050	0.050	0.005	0.005	0.005	0.940
	β_{22} (0.3)	0.300	0.300	0.093	0.096	0.093	0.950
	β_{23} (-0.6)	-0.610	-0.608	0.109	0.112	0.110	0.958
			50% censoring				
GHEW	σ (0.113)	0.254	0.117	0.349	0.317	0.376	0.923
	κ (0.366)	0.391	0.368	0.130	0.155	0.132	0.967
	α (18.305)	30.104	18.644	29.471	65.195	31.732	0.947
	β_{11} (0)	0.000	0.001	0.013	0.012	0.013	0.940
	β_{12} (0)	0.001	0.011	0.250	0.249	0.250	0.957
	β_{13} (0.5)	0.525	0.512	0.287	0.291	0.288	0.965
	β_{21} (0.05)	0.050	0.050	0.006	0.006	0.006	0.951
	β_{22} (0.3)	0.298	0.295	0.125	0.126	0.125	0.955
	β_{23} (-0.6)	-0.612	-0.605	0.131	0.136	0.132	0.965
CHEW	σ (0.113)	0.261	0.134	0.335	0.323	0.366	0.922
	κ (0.366)	0.395	0.377	0.123	0.147	0.126	0.976
	α (18.305)	28.300	16.939	30.330	54.544	31.920	0.943
	β_{11} (0.5)	0.525	0.516	0.272	0.286	0.273	0.974
	β_{21} (0.05)	0.050	0.050	0.005	0.005	0.005	0.937
	β_{22} (0.3)	0.297	0.294	0.114	0.111	0.114	0.946
	β_{23} (-0.6)	-0.611	-0.606	0.129	0.134	0.130	0.966

Table S26. Simulation from CH structure with $(\sigma, \kappa, \alpha) = (0.113, 0.366, 18.305)$, $\beta_{13} = 0.5$, $\beta_2 = (0.05, 0.3, -0.6)$, and $n = 1000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
GHEW	σ (0.113)	0.138	0.114	0.111	0.101	0.114	0.948
	κ (0.366)	0.368	0.366	0.055	0.055	0.055	0.960
	α (18.305)	21.509	18.537	11.053	10.913	11.502	0.958
	β_{11} (0)	-0.000	-0.000	0.005	0.005	0.005	0.944
	β_{12} (0)	0.005	0.004	0.092	0.094	0.092	0.956
	β_{13} (0.5)	0.506	0.508	0.109	0.108	0.109	0.948
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.951
	β_{22} (0.3)	0.301	0.300	0.046	0.045	0.046	0.955
	β_{23} (-0.6)	-0.600	-0.600	0.049	0.048	0.049	0.952
CHEW	σ (0.113)	0.139	0.112	0.110	0.100	0.112	0.943
	κ (0.366)	0.368	0.366	0.054	0.054	0.054	0.952
	α (18.305)	21.353	18.546	11.091	10.612	11.497	0.958
	β_{11} (0.5)	0.506	0.506	0.109	0.108	0.109	0.952
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.947
	β_{22} (0.3)	0.301	0.300	0.044	0.043	0.044	0.956
	β_{23} (-0.6)	-0.600	-0.600	0.049	0.048	0.049	0.950
			50% censoring				
GHEW	σ (0.113)	0.143	0.105	0.125	0.116	0.128	0.936
	κ (0.366)	0.367	0.363	0.063	0.064	0.063	0.964
	α (18.305)	22.816	19.184	13.824	13.818	14.535	0.950
	β_{11} (0)	-0.000	-0.000	0.005	0.005	0.005	0.941
	β_{12} (0)	0.000	-0.000	0.103	0.105	0.103	0.955
	β_{13} (0.5)	0.497	0.488	0.117	0.119	0.117	0.961
	β_{21} (0.05)	0.050	0.050	0.003	0.002	0.003	0.945
	β_{22} (0.3)	0.299	0.300	0.052	0.054	0.052	0.960
	β_{23} (-0.6)	-0.600	-0.597	0.058	0.057	0.058	0.953
CHEW	σ (0.113)	0.145	0.111	0.124	0.115	0.127	0.934
	κ (0.366)	0.368	0.365	0.061	0.062	0.061	0.961
	α (18.305)	22.503	18.793	14.161	13.463	14.763	0.945
	β_{11} (0.5)	0.497	0.490	0.117	0.119	0.117	0.958
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.935
	β_{22} (0.3)	0.299	0.300	0.047	0.050	0.047	0.958
	β_{23} (-0.6)	-0.600	-0.598	0.058	0.057	0.058	0.952

Table S27. Simulation from CH structure with $(\sigma, \kappa, \alpha) = (0.113, 0.366, 18.305)$, $\beta_{13} = 0.5$, $\beta_2 = (0.05, 0.3, -0.6)$, and $n = 5000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
30% censoring							
GHEW	σ (0.113)	0.133	0.120	0.078	0.072	0.080	0.943
	κ (0.366)	0.370	0.370	0.040	0.039	0.040	0.949
	α (18.305)	19.414	17.848	6.931	6.484	7.016	0.945
	β_{11} (0)	-0.000	0.000	0.003	0.003	0.003	0.959
	β_{12} (0)	0.001	0.002	0.070	0.067	0.070	0.939
	β_{13} (0.5)	0.504	0.501	0.076	0.076	0.076	0.953
	β_{21} (0.05)	0.050	0.050	0.001	0.001	0.001	0.945
	β_{22} (0.3)	0.299	0.300	0.032	0.032	0.032	0.952
	β_{23} (-0.6)	-0.601	-0.602	0.036	0.034	0.036	0.944
CHEW	σ (0.113)	0.133	0.121	0.077	0.071	0.079	0.941
	κ (0.366)	0.370	0.370	0.039	0.038	0.039	0.949
	α (18.305)	19.255	17.706	6.902	6.311	6.963	0.947
	β_{11} (0.5)	0.505	0.503	0.076	0.076	0.076	0.957
	β_{21} (0.05)	0.050	0.050	0.001	0.001	0.001	0.949
	β_{22} (0.3)	0.300	0.301	0.031	0.030	0.031	0.949
	β_{23} (-0.6)	-0.601	-0.602	0.036	0.034	0.036	0.942
50% censoring							
GHEW	σ (0.113)	0.133	0.115	0.085	0.081	0.087	0.940
	κ (0.366)	0.369	0.367	0.045	0.045	0.045	0.952
	α (18.305)	19.961	18.307	7.911	7.618	8.078	0.953
	β_{11} (0)	0.000	0.000	0.004	0.004	0.004	0.942
	β_{12} (0)	-0.001	0.000	0.072	0.074	0.072	0.954
	β_{13} (0.5)	0.502	0.499	0.082	0.084	0.082	0.950
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.952
	β_{22} (0.3)	0.300	0.301	0.037	0.038	0.037	0.954
	β_{23} (-0.6)	-0.602	-0.601	0.040	0.040	0.040	0.951
CHEW	σ (0.113)	0.134	0.115	0.083	0.080	0.085	0.950
	κ (0.366)	0.370	0.367	0.043	0.044	0.043	0.957
	α (18.305)	19.714	18.163	7.620	7.401	7.745	0.956
	β_{11} (0.5)	0.502	0.499	0.082	0.084	0.082	0.951
	β_{21} (0.05)	0.050	0.050	0.002	0.002	0.002	0.950
	β_{22} (0.3)	0.300	0.301	0.036	0.035	0.035	0.947
	β_{23} (-0.6)	-0.602	-0.602	0.040	0.040	0.040	0.953

Table S28. Simulation from CH structure with $(\sigma, \kappa, \alpha) = (0.113, 0.366, 18.305)$, $\beta_{13} = 0.5$, $\beta_2 = (0.05, 0.3, -0.6)$, and $n = 10000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage).

Model	30% censoring	50% censoring
	$n = 1000$	
PHEW	19.8	25.4
AHEW	0	0
AFTEW	0.1	0.9
GHEW	13.4	11.9
CHEW	66.7	61.8
	$n = 5000$	
PHEW	0	0.1
AHEW	0	0
AFTEW	0	0
GHEW	13.4	14.4
CHEW	86.6	85.5
	$n = 10000$	
PHEW	0	0
AHEW	0	0
AFTEW	0	0
GHEW	14.3	13.1
CHEW	85.7	86.9

Table S29. Simulation from CH structure with $(\sigma, \kappa, \alpha) = (0.113, 0.366, 18.305)$, $\beta_{13} = 0.5$, $\beta_2 = (0.05, 0.3, -0.6)$. Percentage of models selected with AIC.

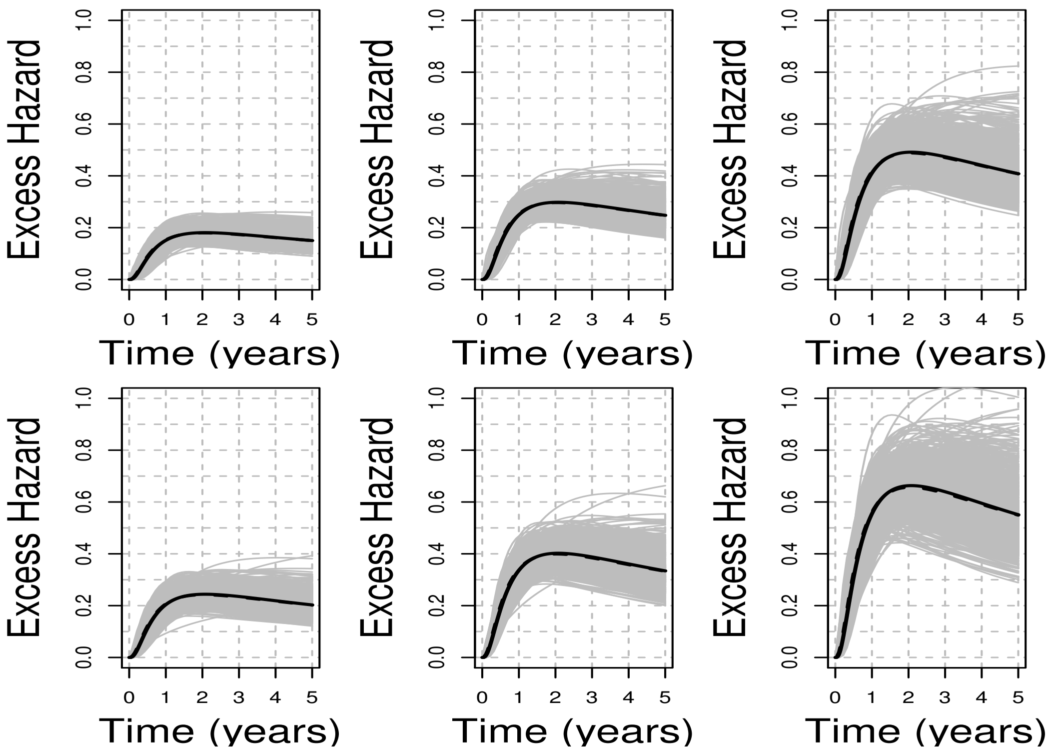

Figure S15. Best hazards in terms of AIC for $n = 1000$ and 50% censoring CH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

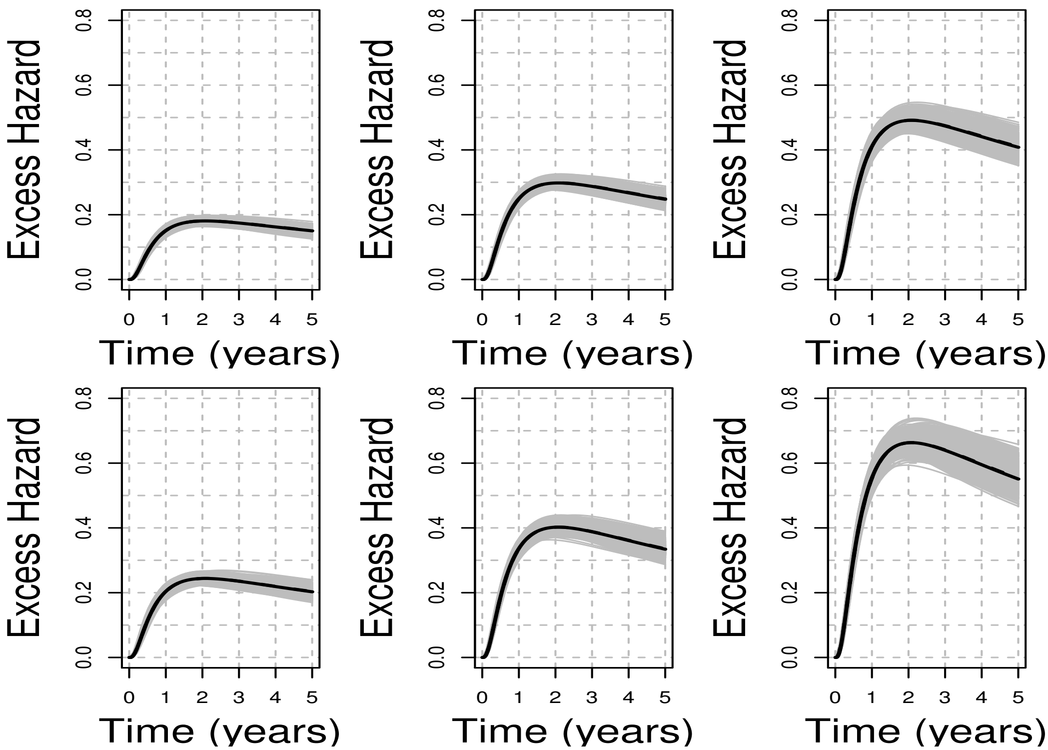

Figure S16. Best hazards in terms of AIC for $n = 10000$ and 30% censoring CH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively.

5 Analysis of initial points

5.1 Performance of initial points

We now analyse the impact of using different initial points in the optimisation of the likelihood of the GH model, which corresponds to the most challenging structure. We compare the R commands 'nlminb' and 'optim' for the maximisation of the likelihood function, both with a maximum limit of $10,000$ iterations. The command 'optim' implements, by default, the Nelder-Mead algorithm, while 'nlminb' implements a quasi-Newton algorithm. For this purpose, we simulated $N = 1000$ data sets of size $n = 1000$ from the GH model with parameters $(\sigma, \kappa, \alpha) = (1.75, 0.6, 2.5)$, $\beta_1 = (0.1, 0.1, 0.1)$ and $\beta_2 = (0.05, 0.2, 0.25)$, with administrative censoring at $T_C = 5$ years, which induced approximately 30% censoring (simulation scenario (v)). We then checked the the performance of:

- Methods 1a and 1b. Initialising the optimisation algorithms ('nlminb' and 'optim') at $(\sigma, \kappa, \alpha) = (1, 1, 1)$, $\beta_1 = (0, \dots, 0)$, and $\beta_2 = (0, \dots, 0)$. Method 1a refers to the use of 'nlminb', while Method 1b refers to the use of 'optim'.
- Methods 2a and 2b. Initialising the optimisation algorithms at $(\sigma, \kappa, \alpha) = (\widehat{\sigma}, \widehat{\kappa}, 1)$, $\beta_1 = (0, \dots, 0)$, and $\beta_2 = \widehat{\beta}_2$, where the notation "$\widehat{\cdot}$" indicates the MLE obtained with the model with Weibull excess baseline hazard. That is, we are initialising at the estimated PH model with Weibull baseline hazard (an already known model). Method 2a refers to the use of 'nlminb', while Method 2b refers to the use of 'optim'. The MLE for the Weibull excess hazard model is, in turn, obtained using the initial points $(\sigma, \kappa) = (1, 1)$, $\beta_1 = (0, \dots, 0)$, and $\beta_2 = (0, \dots, 0)$, and the best outcome (highest maximum) from either 'optim' or 'nlminb'.

Table S30 shows the percentages when each of the methods is preferred. This study shows that initialising the optimisation algorithm as in Methods 1a and 1b tends to provide the best results. Although Methods 2a and 2b were not superior in general, in some cases they provided a better performance than Methods 1a and 1b. For the best methods, the convergence codes were 0 (successful convergence) in 89.5% of the cases, 1 (iteration limit *maxit* had been reached) in 8.4% of the cases, and 10 (degeneracy of the Nelder-Mead simplex) in 2.1% of the cases. Moreover, Table S31 shows that Methods 1a and 1b converged (Code 0) to a solution in 92.3% of the cases, 6.7% of the cases required more than $10,000$ iterations to converge (Code 1), and only 1% of the cases produced degeneracy of the Nelder-Mead simplex algorithm (Code 10), while Methods 2a and 2b tended to required more iterations to converge and produced Code 10 in a higher proportion. **In practice, we recommend checking different optimisation methods and different initial points, while allowing each method for a large enough number of iterations; and this study provides some guidelines to achieve an accurate optimisation**. In particular, one could also play with different initial values for α, say $\alpha = 0.5, 1, 2$, as we illustrate in the next section. Alternatively (to Methods 2a and 2b) or in addition, the GH model can be initialised at the AFT model with Weibull baseline hazard.

Method 1a	Method 1b	Method 2a	Method 2b
57.5	27.1	5.5	9.9

Table S30. Percentages of preferred methods.

Convergence Code	0	1	10
Methods 1a	89.7	10.3	0
Methods 1b	99.3	0	0.7
Methods 2a	54.5	45.6	0
Methods 2b	80.8	0	19.2

Table S31. Percentages of convergence codes for Methods 1a and 1b compared to Methods 2a and 2b.

5.2 Re-analysis of the GH model with different initial points

We now re-analyse the GH simulation scenario described in Table S1, with $n = 1000$ and censoring rates 30% and 50%, in order to assess the performance of using different "automatic" initial points instead of the true values of the parameters employed in the simulation study. We consider the following three initial points:

1. $\sigma = \kappa = \alpha = 1, \beta_1 = \beta_2 = 0.$
2. $\sigma = \kappa = 1, \alpha = 2, \beta_1 = \beta_2 = 0.$
3. $\sigma = \kappa = 1 \; \alpha = 0.5, \beta_1 = \beta_2 = 0.$

coupled with the R command 'nlminb()'. Thus, we initialise the optimiser at these three initial points, allowing for a maximum of $10,000$ iterations, and we select the estimates associated to the maximum value of the likelihood function. We emphasise that, in the simulation study, we have used the true values of the parameters as initial points as a way of making the estimation process more efficient, since we were only interested in the properties of the maximum likelihood estimators rather than those of the optimisation process. Table S32 shows the properties of the MLEs obtained using these three initial points. We can notice that the numbers in this table are virtually the same as those in Table S24, as expected. The model selection also has virtually the same performance using these three initial values compared to that obtained using the true values of the parameters as initial values. Figures S17 and S18 show the shapes of the best (in terms of AIC) fitted baseline excess hazards using the proposed initial points. These figures closely resemble the results obtained in Figure S13, which is reassuring. This study shows that the use of the three proposed "automatic" initial points has a very good performance in terms of producing accurate estimators, and therefore we recommend their use in practice as an initial guess.

References

Model	Parameter	MMLE	mMLE	ESD	Mean Std Error	RMSE	Coverage
			30% censoring				
	σ (1.75)	1.581	1.549	0.790	0.834	0.808	0.980
	κ (0.6)	0.576	0.575	0.128	0.138	0.131	0.957
	α (2.5)	3.181	2.733	1.672	1.500	1.804	0.975
	β_{11} (0.1)	0.099	0.098	0.033	0.033	0.033	0.941
GHEW	β_{12} (0.1)	0.010	0.048	0.730	0.775	0.735	0.970
	β_{13} (0.1)	0.028	0.054	0.738	0.682	0.741	0.970
	β_{21} (0.05)	0.050	0.050	0.007	0.007	0.007	0.957
	β_{22} (0.2)	0.204	0.205	0.106	0.129	0.106	0.959
	β_{23} (0.25)	0.251	0.251	0.095	0.105	0.095	0.961
			50% censoring				
	σ (1.75)	1.559	1.521	0.851	0.919	0.872	0.970
	κ (0.6)	0.572	0.578	0.145	0.162	0.147	0.954
	α (2.5)	3.503	2.712	2.743	2.202	2.919	0.975
	β_{11} (0.1)	0.098	0.096	0.039	0.039	0.039	0.927
GHEW	β_{12} (0.1)	0.027	0.078	0.898	0.835	0.901	0.978
	β_{13} (0.1)	0.025	0.060	0.847	1.388	0.850	0.976
	β_{21} (0.05)	0.050	0.050	0.008	0.008	0.008	0.963
	β_{22} (0.2)	0.210	0.206	0.147	0.149	0.147	0.959
	β_{23} (0.25)	0.247	0.244	0.117	0.176	0.117	0.962

Table S32. Simulation from GH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.6, 2.5)$, $\beta_1 = (0.1, 0.1, 0.1)$, $\beta_2 = (0.05, 0.2, 0.25)$, and $n = 1000$. Mean of the MLEs (MMLE), median of the MLEs (mMLE), empirical standard deviation (ESD), mean standard error, root-mean-square error (RMSE), and coverage proportions (Coverage). The estimates are obtained using 6 different automatic initial points.

Model	30% censoring	50% censoring
	$n = 1000$	
PHEW	1.6	2.4
AHEW	0	0
AFTEW	56.5	65.1
GHEW	41.9	32.5

Table S33. Simulation from GH structure with $(\sigma, \kappa, \alpha) = (1.75, 0.6, 2.5)$, $\beta_1 = (0.1, 0.1, 0.1)$, and $\beta_2 = (0.05, 0.2, 0.25)$. Percentage of models selected with AIC.

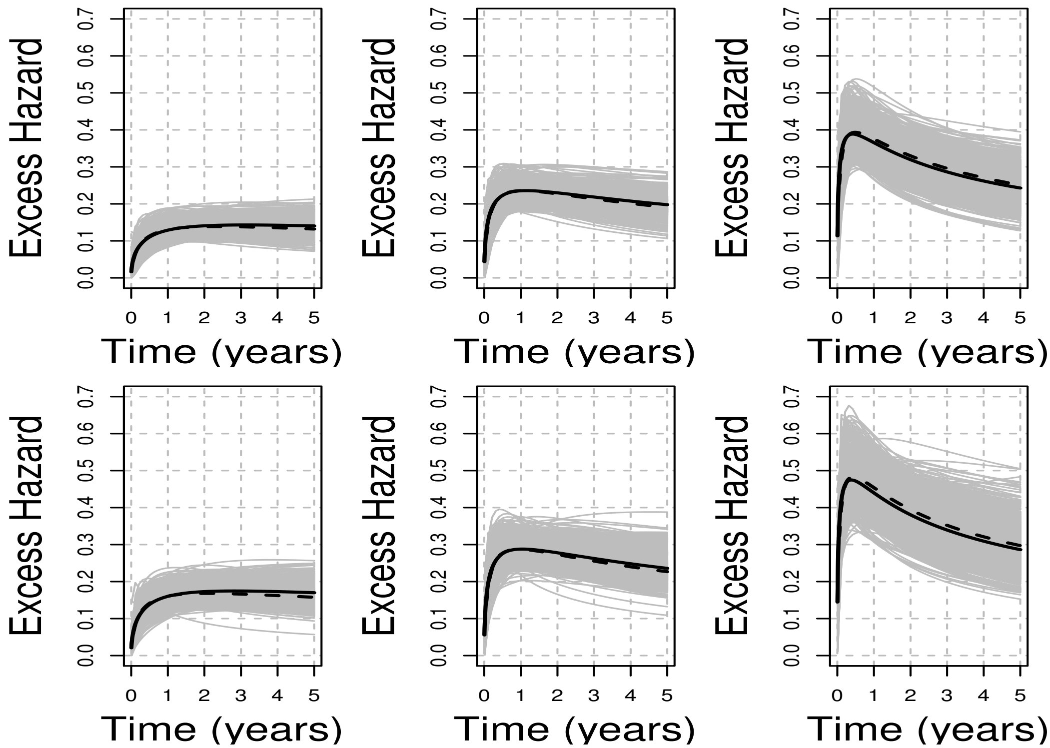

Figure S17. Best hazards in terms of AIC for $n = 1000$ and 30% censoring GH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0), (70, 0, 0), (80, 0, 0), (60, 1, 0), (70, 1, 0), (80, 1, 0)$, respectively. The estimate values are obtained using different automatic initial points.

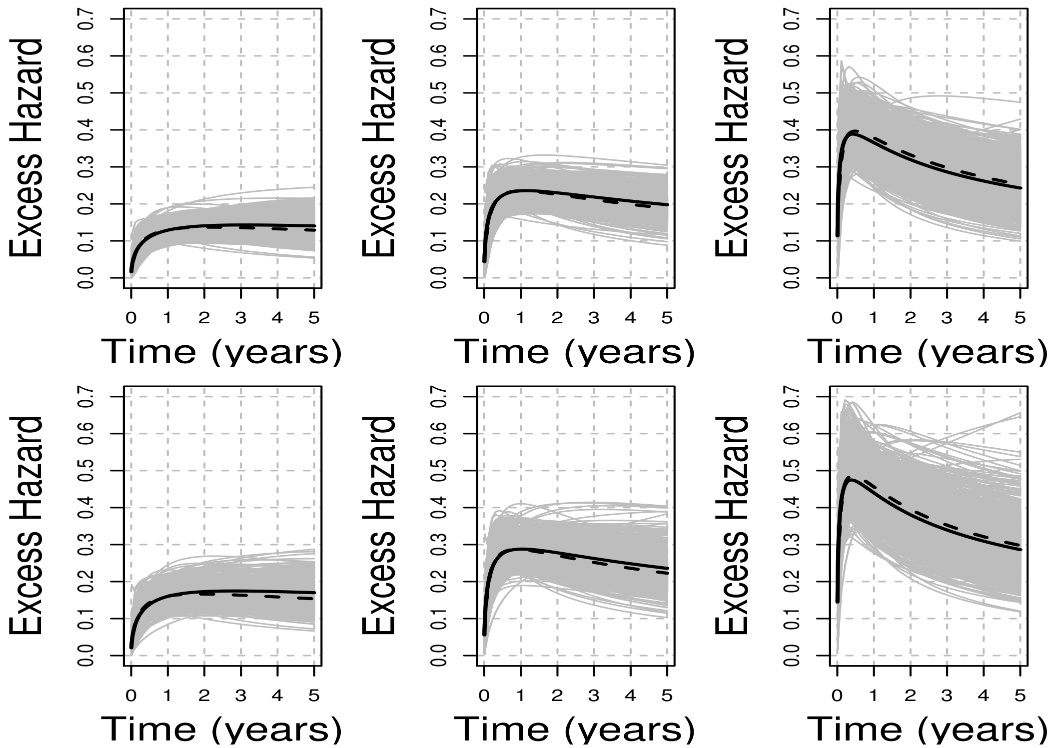

Figure S18. Best hazards in terms of AIC for $n = 1000$ and 50% censoring GH: true generating hazard (continuous line), mean of best fitted hazards (dashed lines), and individual fitted hazards (gray lines). Panels from left to right correspond to covariate values (age, sex, comorbidity)=$(60, 0, 0)$, $(70, 0, 0)$, $(80, 0, 0)$, $(60, 1, 0)$, $(70, 1, 0)$, $(80, 1, 0)$, respectively. The estimate values are obtained using different automatic initial points.