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# QUANTUM CORRECTIONS TO THE GRAVITATIONAL POTENTIAL AND ORBITAL MOTION

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*Abstract*. GRT predicts the existence of relativistic corrections to the static Newtonian potential, which can be calculated and verified experimentally. The idea leading to quantum corrections at large distances consists of the interactions of massless particles, which only involve their coupling energies at low energies. Using the quantum correction term of the potential we obtain the perturbing quantum acceleration function. Next, with the help of the Newton-Euler planetary equations, we calculate the time rates of changes of the orbital elements per revolution for three different orbits around the primary. For one solar mass primary and an orbit with semimajor axis and eccentricity equal to that of Mercury we obtain that $\Delta\omega_{\mathrm{qu}} = 1.517\times10^{-81}$ °/cy, while $\Delta M_{\mathrm{qu}} = -1.840\times10^{-46}$ rev/cy.

*Key words:* celestial mechanics – perturbed two-body problem – quantum effects.

## 1. INTRODUCTION

The Newtonian potential that rules the motion of two point masses $M_{\mathrm{p}}$ (primary) and $m$ (secondary) separated by a distance $r$ is

$$V(r) = -\frac{GM_{\mathrm{p}}m}{r}, \tag{1}$$

where $G$ is the Newtonian constant of gravitation. This potential is of course only approximately valid (e.g., Donoghue 1994). For large masses and/or large velocities, GRT predicts that there exist relativistic corrections, which can be calculated and also verified experimentally (e.g., Bjorken and Drell 1964). In the microscopic distance domain, we could expect that quantum mechanics would predict a modification in the gravitational potential in the same way that the radiative corrections of quantum electrodynamics leads to a similar modification of the Coulomb interaction (t'Hooft and Veltman 1974).

Even though GRT constitutes a very well defined classical theory, it is not possible yet to combine it with quantum mechanics in order to create a satisfactory theory of quantum gravity. One of the basic obstacles that prevent this from happening is that general relativity does not actually fit the present paradigm for a fundamental theory, that of a renormalizable quantum field theory. Gravitational fields can be successfully quantized on smooth-enough spacetimes (Capper et al. 1973), but the form of gravitational interactions is such that they induce unwanted divergences which cannot be absorbed by the renormalization of the parameters of the minimal general relativity (Goroff and Sagnotti 1984). One can introduce new coupling constants and absorb the divergences then, but this unfortunately leads to an infinite number of free parameters. Despite the difficulty above, quantum gravity calculations can predict long distance quantum corrections.

The main idea leading to quantum corrections at large distances is due to the interactions of massless particles which only involve their coupling energies at low energies, something that it is known from the GRT, even though at short distances the theory of quantum gravity differs, resulting to finite correction of order $O\left(\frac{G\hbar}{c^3 r^3}\right)$, where $\hbar$ is Planck's constant, and $c$ is the speed of light. The existence of a universal long-distance quantum correction to the Newtonian potential should be relevant for a wide class of gravity theories. It is a well-known fact that the ultraviolet behaviour of Einstein's pure gravity can be improved, if higher derivative contributions to the action are added; in four dimensions they take the form (in usual notation):

$$\alpha R^{\kappa\lambda} R_{\kappa\lambda} + \beta R^2 , \tag{2}$$

where $\alpha$ and $\beta$ are dimensionless coupling constants. What makes the difference is that the resulting classical and quantum corrections to gravity are expected to significantly alter the gravitational potential at short distances comparable to that of Planck length $\ell_{\mathrm{P}} = \sqrt{\frac{G\hbar}{c^3}} = 1.616 \times 10^{-35}$ m, but it should not really affect its behaviour at long distances. At long distances it is the structure of the Einstein-Hilbert action that actually determines that. At this point we should mention that some of the calculations of the corrections to the Newtonian gravitational potential result in the absence of a cosmological constant $\Lambda$, which usually complicates the perturbative treatment to a significant degree because of the need to expand about a nonflat background.

In one-loop amplitude computation, one needs to calculate all first order corrections in $G$, which will include both the relativistic $O\left(\frac{G^2 m^2}{c^2}\right)$ and the quantum mechanical $O\left(\frac{G\hbar}{c^3}\right)$ corrections to the classical Newtonian potential (Hamber and Liu 1995).

As a short digression on this theme, we note that Gutzwiller (1971, 1973, 1977) defined and studied a type of anisotropic Kepler problem with an essential goal: to identify links between classical and quantum mechanics (see also Gutzwiller 1990). The same model was resumed by Devaney (1978) and Casasayas and Llibre (1984), who went deeper into the problem.

The anisotropic Manev problem, tackled by Craig et al. (1999), provided results that seem to build a bridge between classical mechanics, relativity, and quantum mechanics (as regards behavior in the neighbourhood of collision). Analogous results were obtained by Mioc et al. (2003) for the anisotropic Schwarzschild problem.

For important results about the links between classical and quantum physics, we direct the reader to the paper of DeWitt-Morette (1979).

The main goal of this contribution is to use the acceleration resulting from the quantum correction to the potential into the Newton-Euler planetary equations, and calculate the changes in the orbital elements for various two-body scenarios, and, given their magnitude, to determine if such corrections are detectable with today's satellite technology.

## 2. CORRECTIONS TO GRAVITATIONAL POTENTIAL

Our goal is not to present the details of the one-loop treatment that leads to the corrections of the Newtonian gravitational potential, but rather state the result and then use it in our calculations. Valid to order $G^2$, we have that the corrected potential now becomes (Hamber and Liu 1995):

$$V(r) = -\frac{GM_{\mathrm{p}}m}{r}\left(1 - \frac{G(M_{\mathrm{p}}+m)}{2c^2 r} - \frac{122 G\hbar}{15\pi c^3 r^2}\right). \tag{3}$$

Perusing (3), we see that in the correction of the static Newtonian potential two different length scales are involved. First, the Planck length $\ell_{\mathrm{P}} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35}$ m, and second, the Schwarzschild radius of the massive source $r_{\mathrm{Sch}} = -\frac{2GM_{\mathrm{p}}}{c^2}$. Furthermore, there are two independent dimensionless parameters that appear in the correction term, and involve the ratio of these two scales with respect to the distance $r$ . Presumably for meaningful results the two length scales are much smaller than $r$ .

### 3. GENERAL PERTURBING FACTORS AND NEWTON–EULER EQUATIONS

In a two-body orbital motion the secondary body moves under the dominant force of the primary one. However, other bodies exert forces, which change with the relative positions of the objects and perturb the two-body motion. The resulting deviations from the actual orbit are usually very small, and given the well-known Keplerian orbital elements $\{i, \Omega, \omega, a, e, M\}$ at any instant, we can calculate the perturbations or changes of these elements as functions of time. Recall that these parameters are: inclination, longitude of the ascending node, argument of pericenter, semimajor axis, eccentricity and mean anomaly, respectively. They completely feature the relative orbit of the secondary body.

Some of the most important effects responsible for these perturbations are: (a) gravitational forces exerted by other celestial bodies; (b) gravitational forces resulting from the nonspherical character and nonuniform mass repartition of the central primary body; (c) surface forces resulting from radiation pressure; (d) surface forces resulting from atmospheric drag.

In classical celestial mechanics, the most general system of ODE describing the perturbed motion consists of Lagrange's planetary equations (see any classical textbook of celestial mechanics). These equations are valid no matter which the nature of the perturbing force is, or whether this force derives from a potential or not.

But, if the perturbing force derives from a potential (or a perturbing function), it is much more convenient to resort to the Newton-Euler equations (also called sometimes Gauss' equations). They use the well-known components of the perturbing acceleration (perturbing force per unit mass): $R$ (radial), $S$ (transverse), and $W$ (binormal). This is the way we will tackle our problem.

The Newton-Euler equations were largely used by one of the authors of the present paper, especially to study the artificial Earth satellite dynamics under the most various perturbations. We quote arbitrarily: Mioc (1980, 1991), Mioc and Radu (1977, 1979, 1982, 1991a, b),

To emphasize the usefulness of these equations, they were also used by the respective author to the study of other dynamical problems. We also quote arbitrarily: Blaga and Mioc (1992), Delgado et al. (1996), Diacu et al. (1995), Mioc (1994), Mioc and Radu (1991c), Mioc et al. (1991, 1992).

In general, when the components of the perturbing acceleration do not depend explicitly on time, it is more convenient to resort to other independent variables (angular), e.g.: the true anomaly $(f)$, the eccentric anomaly $(E)$, the mean anomaly $(M)$, or the argument of latitude $(u)$. The above quoted papers used such independent variables, especially the argument of latitude.

However, in our present approach, we shall use for the first step the time as independent variable. The general Newton-Euler equations in this case (no matter which the nature of the perturbation is) read (e.g., Blanco and McCuskey 1961):

$$\frac{\mathrm{d}a}{\mathrm{d}t}=\frac{2}{nr\sqrt{1-e^2}}[(er\sin f)R+a(1-e^2)S], \tag{4}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t}=\frac{\sqrt{1-e^2}}{nea}\left[(e\sin f)R+\left[\frac{a(1-e^2)}{r}-\frac{r}{a}\right]S\right], \tag{5}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t}=-\frac{\sqrt{1-e^2}}{nea}\left[(\cos f)R-\sin f\left[1+\frac{r}{a(1-e^2)}\right]S+\frac{er\sin u\cot i}{a(1-e^2)}W\right], \tag{6}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t}=\frac{r\cos u}{na^2\sqrt{1-e^2}}W, \tag{7}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t}=\frac{r\sin u}{na^2\sin i\sqrt{1-e^2}}W, \tag{8}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t}=n+\frac{1}{na}\left[\left[\frac{(1-e^2)\cos f}{e}-\frac{2r}{a}\right]R-\frac{(1-e^2)\sin f}{nea}\left[1+\frac{r}{a(1-e^2)}\right]S\right]. \tag{9}$$

Here $n$ is the daily mean motion ($n=2\pi/P=\sqrt{GM_{\mathrm{p}}}/a^{3/2}$), $P$ is the orbital period of the secondary, all other notations being already specified. Of course, in order to use the equations (4)–(9), the components of the perturbing acceleration must be expressed in terms of the osculating orbital elements at some particular epoch.

## 4. NEWTON–EULER EQUATIONS FOR QUANTUM EFFECTS

Given the corrections to the Newtonian potential in (3), we have that the corresponding force acting in the radial direction is

$$F(r) = -\frac{\partial V}{\partial r} = -\frac{GM_{\mathrm{p}}m}{r^2} + \frac{G^2M_{\mathrm{p}}^2m}{c^2r^3} + \frac{G^2M_{\mathrm{p}}m^2}{c^2r^3} + \frac{366G^2M_{\mathrm{p}}m\hbar}{15\pi c^3r^2}. \tag{10}$$

Considering only the radial component of the perturbing acceleration due to the quantum effects needed in the Newton-Euler equations, we obtain that:

$$X(r) := R_{\mathrm{qu}} = \frac{366G^2M_{\mathrm{p}}\,\hbar}{15\pi c^3r^4}. \tag{11}$$

The two perturbing terms in (3) are radial terms. Hence we can use the equations (4)–(9) with $S = 0 = W$. In this case the equations corresponding to the quantum effect become

$$\frac{\mathrm{d}a_{\mathrm{qu}}}{\mathrm{d}t} = \frac{2e}{n\sqrt{1-e^2}}\sin\ f\ X(r), \tag{12}$$

$$\frac{\mathrm{d}e_{\mathrm{qu}}}{\mathrm{d}t} = -\frac{\sqrt{1-e^2}}{na}\sin f\ X(r), \tag{13}$$

$$\frac{\mathrm{d}\omega_{\mathrm{qu}}}{\mathrm{d}t} = -\frac{\sqrt{1-e^2}}{nae}\cos f\ X(r), \tag{14}$$

$$\frac{\mathrm{d}i_{\mathrm{qu}}}{\mathrm{d}t} = 0, \tag{15}$$

$$\frac{\mathrm{d}\Omega_{\mathrm{qu}}}{\mathrm{d}t} = 0, \tag{16}$$

$$\frac{\mathrm{d}M_{\mathrm{qu}}}{\mathrm{d}t} = n + \frac{2}{na}\left[\frac{(1-e^2)\cos f}{e} - \frac{2r}{a}\right] X(r). \tag{17}$$

One can easily see that the quantum effect does not influence the position of the orbital plane ($i$ and $\Omega$).

In order to simplify and explicit the remaining equations (12), (13), (14), and (17), we consider that the perturbations due to the quantum corrections (and relativistic, as well) are very small. So we may safely affirm that, to a certain extent, the orbit will be more or less Keplerian, but within a good approximation.

## 5. QUANTUM EFFECTS OVER ONE ANOMALISTIC PERIOD

For our purposes, we shall consider that the orbit is of elliptic type. In order to estimate the quantum effects in the motion of the secondary over one anomalistic period of this one, we shall use the orbit equation in polar coordinates

$$r(f) = \frac{a(1-e^2)}{1+e\cos f}, \tag{18}$$

and the fact that $GM_\mathrm{p} = n^2 a^3$.

Also, we choose the true anomaly as independent variable via the change

$$\mathrm{d}t = \frac{1}{n}\left(\frac{r}{a}\right)^2 \frac{\mathrm{d}f}{\sqrt{1-e^2}}. \tag{19}$$

Now, introducing (18) and (19) into equations (12)–(13), then integrating the resulting equations between the limits 0 and $2\pi$, one easily obtains

$$\Delta a_\mathrm{qu} = 0, \tag{20}$$

$$\Delta e_\mathrm{qu} = 0, \tag{21}$$

This means that, after one anomalistic period (from $f = 0$ to $f = 2\pi$), the semimajor axis and the eccentricity come back to their initial values. In other words, the shape and the dimensions of the orbit do not experience secular changes.

For the argument of pericenter, (14), (18) and (19) lead to

$$\frac{\mathrm{d}\omega_\mathrm{qu}}{\mathrm{d}f} = -\frac{366 G^2 M_\mathrm{p}\hbar}{15\pi c^3 n^2 a^5 e(1-e^2)^2}\left[\cos f(1+e\cos f)^2\right], \tag{22}$$

where we took into account (11). Integrating (22) between 0 and $2\pi$, we get

$$\Delta\omega_\mathrm{qu} = \frac{244\, G^2 M_\mathrm{p}\,\hbar}{5\, c^3 n^2 a^5 (1-e^2)^2} = \frac{244\, G\hbar}{5\, c^3 a^2 (1-e^2)^2}. \tag{23}$$

For the mean anomaly we proceed exactly in the same way. The relations (11), (17), (18) and (19) provide an ODE, whose integration from $f = 0$ to $f = 2\pi$, gives

$$\Delta M_{\mathrm{qu}} = -\frac{488\, G^2 M_{\mathrm{p}}\, \hbar}{5\, c^3 n^2 a^5 (1-e^2)^{3/2}} = -\frac{488\, G\hbar}{5\, c^3 a^2 (1-e^2)^{3/2}} . \tag{24}$$

For a last step before numerical estimations, we recall that the Planck length is given by $\ell_{\mathrm{P}} = \sqrt{\dfrac{G\hbar}{c^3}}$ . Using this, (23) and (24) respectively turn to

$$\Delta\omega_{\mathrm{qu}} = \frac{244}{5\,(1-e^2)^2} \left[\frac{\ell_{\mathrm{P}}}{a}\right]^2 . \tag{25}$$

$$\Delta M_{\mathrm{qu}} = \frac{488}{5\,(1-e^2)^{3/2}} \left[\frac{\ell_{\mathrm{P}}}{a}\right]^2 . \tag{26}$$

We see that both changes due to the quantum correction of the Newtonian potential in the argument of the perigee and mean anomaly over one revolution are independent of the mass of the primary body and scales as the square of the ratio of the Planck length over the semimajor axis of the orbiting body

To estimate numerically the magnitude of such changes, we have chosen some concrete cases belonging to our solar system. Using appropriate values for semimajor axes and eccentricities of the orbiting bodies, we found:

– Moon/lunar orbiter: $\Delta\omega_{\mathrm{qu}} = 1.800 \times 10^{-74}$; $\Delta M_{\mathrm{qu}} = -2.180 \times 10^{-39}$ ;

– Jupiter/Europa: $\Delta\omega_{\mathrm{qu}} = 6.873 \times 10^{-81}$; $\Delta M_{\mathrm{qu}} = -8.506 \times 10^{-46}$ ;

– Sun/Mercury: $\Delta\omega_{\mathrm{qu}} = 1.517 \times 10^{-81}$; $\Delta M_{\mathrm{qu}} = -1.840 \times 10^{-46}$ ,

where the changes of the argument of periastron $\omega$ are measured in °/cy, while those of the mean anomaly $M$ in rev/cy.

## 6. COMMENTS AND SUMMARY

The presence of the Planck length indicates that quantum effects will be extremely small but not identically zero. From formulae (25) and (26) one observes that quantum effects should "relatively increase" when the semimajor axis $a$ becomes smaller, but, given the size of real orbits, they still remain extremely small, and impossible to measure with today's technology. Mathematically speaking, the expressions (25) and (26) maximize when $\ell_{\mathrm{P}} = a$ , but this by no means constitutes a valid orbit in celestial mechanics.

In the case where the idea of quantized redshifts proves to be valid, somebody might have to introduce a new cosmic quantum of action $\hbar_{\rm g} = 6.322\times10^{67}$ J s (see Haranas and Harney 2009), and therefore a new cosmic Planck length $\ell^2_{\rm P,\,cos} = G\hbar_{\rm g}/c^3 = 1.315\times10^{16}$ m. If this new quantum of action operates in the large-scale universe, it might affect distant orbital phenomena. Lastly, verifying the quantum corrections to the potential resulting from today's quantum gravity theories associated with solar-system orbital phenomena, we say that satellite orbits are definitely not a "viable tool" since they are limited by today's technology.

We considered the idea of a possible correction to the Newtonian gravitational potential predicted by the theory of general relativity along with the idea that leads to quantum corrections at large distances. Using the radial perturbing acceleration that corresponds to quantum correction of the Newtonian gravitational potential over large distances, we derived and solved the Newton-Euler planetary equations for the time rate of change of the orbital elements. From the six orbital elements that define the orbit, quantum effects only affect the argument of the periastron and the mean anomaly. Both these changes per anomalistic period do not depend on the mass of the primary body, and scale as the Planck length over the semimajor axis of the orbit square. Quantum correction effects are extremely small and cannot be detected using satellites in orbit and today's technology.

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