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Quark contribution in the evolution equation of the scaling violation of F_2

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Abstract : In a number of analyses in the literature, relations connecting the gluon density with the scaling violation of the structure function are reported In all these approaches, the DGLAP equation for the gluon is considered after neglecting the quark term In this work, we calculate the contribution coming from the quark term and estimate the error that enters when the quark contribution is neglected

Keywords DGLAP evolution equations, low x, Taylor expansion

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1. Introduction

In the literature, there are a number of approximate relations connecting the gluon density with the scaling violation of the structure function F_2 [1-3] These relations are obtained by evaluating the convolution integral involving the gluon density in the DGLAP equation [4] after neglecting the convolution integral involving the singlet structure function *i.e.* the quarks. In the above methods the gluon convolution integral is evaluated by Taylor expanding the gluon distribution about a chosen point.

In this work, we estimate the quark contribution in the DGLAP evolution equation for the singlet structure function using the framework of [5]. We then perform a numerical analysis in which we compare the analytically obtained value of the quark contribution using a test input distribution with the numerically obtained ones. We then determine the amount of error involved in neglecting the quark contribution for a number of standard non-perturbative parametrisations available in the literature [7-12].

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In Section 2 we describe the formalism and obtain the analytical expression for the quark contribution. In Section 3 we perform the numerical analysis to compare the analytically and numerically obtained values of the quark contribution using a test distribution. In Section 4 we evaluate the amount of error involved in neglecting the quark contribution for various input distributions and in Section 5 we conclude with some discussions.

2. Formalism

The DGLAP integrodifferential equation for the singlet structure function is given by

$$\frac{\partial F_2^s(x,Q^2)}{\partial t}=\frac{\alpha_s(t)}{3\pi}\Bigg[\{3+4\ln(1-x)\}F_2^s\left(x,Q^2\right)$$

$$+2\int_x^1\frac{dz}{1-z}\left\{\left(1+z^2\right)F_2^s\left(\frac{x}{z},Q^2\right)-2F_2^s\left(x,Q^2\right)\right\}+\frac{3}{2}N_f\int_x^1 dz\left\{z^2+(1-z)^2\right\}G\left(\frac{x}{z},Q^2\right)\Bigg]. \quad (1)$$

We consider the following term in the RHS of eq. (1) as the quark contribution

$$qc=\left[\{3+4\ln(1-x)\}F_2^s\left(x,Q^2\right)+2\int_x^1\frac{dz}{1-z}\left\{\left(1+z^2\right)F_2^s\left(\frac{x}{z},Q^2\right)-2F_2^s\left(x,Q^2\right)\right\}\right. \quad (2)$$

Writing $z = 1 - u$, the integral in (2) becomes

$$I=\int_0^{1-x}\frac{du}{u}\left\{1+(1-u)^2\right\}F_2^s\left(\frac{x}{1-u},Q^2\right)-2\int_0^{1-x}\frac{du}{u}F_2^s\left(x,Q^2\right)$$

$$=I_1-I_2. \quad (3)$$

We evaluate the I_1 in (3) as follows. The function $F_2^s\left(x/1-u,Q^2\right)$ is expanded about a point $z=\alpha$ where $0<\alpha<1-x$ such that $|u-\alpha|<R$, R being the radius of convergence. The n$^{\text{th}}$ term in the expanded integrand will be

$$T_n=(u-\alpha)^{n-1}\frac{F^{n-1}}{(n-1)!}\frac{1+(1-u)^2}{u} \quad (4)$$

where $F^{n-1}=\left.\frac{\partial^{n-1}F_2{}^s\left(\frac{x}{u}\right)}{\partial u^{n-1}}\right|_{u=\alpha}$. We suppress the Q^2 dependence of $F_2{}^s$.

The series can be integrated term by term and the resulting series converge. Integrating T_n we get at low x

$$J_n = \int_0^{1-x} T_n du = \sum_{r=0}^{n-1} a_r \left[\frac{(1-x)^{n-r+1}}{n-r+1} - \frac{2(1-x)^{n-r}}{n-r} + \frac{2(1-x)^{n-r-1}}{n-r-1} \right]$$

$$\approx \sum_{r=0}^{n-1} a_r \left[\frac{1}{n-r+1} - \frac{2}{n-r} + \frac{2}{n-r-1} \right] \tag{5}$$

where $a_r = \frac{(-\alpha)^r}{r!(n-r-1)!} F^{n-1}$.

The completed integrated series can be obtained by evaluating the terms J_1, J_2, J_3 . The second integral I_2 in (3) can be trivially evaluated. The integral I thus being evaluated, the quark contribution becomes,

$$qc = \frac{8}{3} F_2^{s(1)}\left(\frac{x}{1-\alpha} - \alpha \right) + \frac{7}{12} F_2^{s(2)}\left(\frac{x}{1-\alpha} - \alpha \right) + \frac{11}{90} F_2^{s(3)}\left(\frac{x}{1-\alpha} - \alpha \right)$$

$$+ \frac{1}{45} F_2^{s(4)}\left(\frac{x}{1-\alpha} - \alpha \right) + \frac{11}{150} F_2^{s(5)}\left(\frac{x}{1-\alpha} - \alpha \right) + \frac{29}{60480} F_2^{s(6)}\left(\frac{x}{1-\alpha} - \alpha \right) +$$

$$= \frac{8}{3}\left[F_2^{s(1)}\left(\frac{x}{1-\alpha} - \alpha + a \right) + R(a) \right] \tag{6}$$

where a is a parameter and $R(a)$ is given by

$$R(a) = \left(\frac{7}{32} - \alpha \right) F_2^{s(2)}\left(\frac{x}{1-\alpha} - \alpha \right) + \left(\frac{11}{240} - \frac{\alpha^2}{2!} \right) F_2^{s(3)}\left(\frac{x}{1-\alpha} - \alpha \right)$$

$$+ \left(\frac{1}{120} - \frac{\alpha^3}{3!} \right) F_2^{s(4)}\left(\frac{x}{1-\alpha} - \alpha \right) + \left(\frac{11}{8400} - \frac{\alpha^4}{4!} \right) F_2^{s(5)}\left(\frac{x}{1-\alpha} - \alpha \right)$$

$$+ \left(\frac{29}{161280} - \frac{\alpha^5}{5!} \right) F_2^{s(6)}\left(\frac{x}{1-\alpha} - \alpha \right) + \cdots \tag{7}$$

Let a_0 be the value of a for which $R(a) \approx 0$. This gives

$$qc \approx \frac{8}{3} F_2^{s(1)}\left(\frac{x}{1-\alpha} - \alpha + a_0 \right).$$

The function $F_2^{s(1)}$ will be sensitive to the variation of x only when $x/1-\alpha \approx \alpha - a_0$. Hence for $x \to 0, \alpha \to a_0$ and we get

$$qc \approx \frac{8}{3} F_2^{s(1)}\left(\frac{x}{1-\alpha} - \alpha\right) \tag{8}$$

where α satisfies the condition (from (7)),

$$\left(\frac{7}{32} - \alpha\right) F_2^{s(2)}\left(\frac{x}{1-\alpha}\right) + \left(\frac{11}{240} - \frac{\alpha^2}{2!}\right) F_2^{s(3)}\left(\frac{x}{1-\alpha}\right) + \left(\frac{1}{120} - \frac{\alpha^3}{3!}\right) F_2^{s(4)}\left(\frac{x}{1-\alpha}\right)$$

$$+\left(\frac{11}{8400} - \frac{\alpha^4}{4!}\right) F_2^{s(5)}\left(\frac{x}{1-\alpha}\right) + \left(\frac{29}{161280} - \frac{\alpha^5}{5!}\right) F_2^{s(6)}\left(\frac{x}{1-\alpha}\right) + \cdots = 0 \tag{9}$$

The value of α is determined by solving (9) with a parametrisation for F_2^s.

3. Numerical analysis

As a test distribution we take

$$F_2^s(x) \approx A_f x^{\lambda} (1-\lambda)^{\delta} (1+\gamma x)$$

which at low x

$$F_2^s(x) \approx A_f x^{\lambda} . \tag{10}$$

Inserting (10) in (9) and considering the first three terms of the series, we get the cubic

$$\left(1 + \frac{(\lambda+2)(\lambda+3)}{6} - \frac{\lambda+2}{2}\right)\alpha^3 - \left(\frac{71}{32} - \frac{\lambda+2}{2}\right)\alpha^2$$

$$\left(\frac{23}{16} + \frac{11(\lambda+2)}{240}\right)\alpha - \left(\frac{7}{32} + \frac{11(\lambda+2)}{240} + \frac{(\lambda+2)(\lambda+3)}{120}\right) = 0 . \tag{11}$$

For the representative values $\lambda = -0.5, -0.3$, and -0.08, this gives

$$\frac{7}{8}\alpha^3 - \frac{47}{32}\alpha^2 + \frac{241}{160}\alpha - \frac{51}{160} = 0 \tag{12}$$

$$\frac{180}{5}\alpha^3 - \frac{219}{4}\alpha^2 + \frac{3137}{60}\alpha - \frac{4019}{300} = 0 \tag{13}$$

$$\frac{1218}{125}\alpha^3-\frac{1007}{80}\alpha^2+\frac{3051}{200}\alpha-\frac{35347}{10000}=0 \tag{14}$$

respectively

The above cubics are solved numerically by Horner's method [6] to find the value of α Using the value of α the quark contribution from (8) is calculated The quark contribution is also calculated numerically using Simpson's rule from (2) The results are shown in Table 1

Table 1. Comparison of analytical and numerically evaluated values of qc

λ	α	Error in evaluating α	Value of qc (anal) x x^{λ}	Value of qc (num) x x^{λ}
–0 5	0 27211738583	–5 0 x 10^{10}	–1 562816773	–1 574607
–0 3	0 27774612055	6 0 x 10^{8}	–1 0046321418	- 1 026599
–0 08	0 28345901264	4 0 x 10^{7}	–0 2898924899	–0 2975729

The results clearly show that the analytically derived expression for the quark contribution is in accurate agreement to the numerically computed values

4. Results

The quark contribution (8) derived from the DGLAP evolution equation for singlet structure function (1) will be compared to the gluon contribution given by the following term in (1)

$$gc=\frac{3}{2}N_f\int^{1}dz\left\{z^2+(1-z)^2\right\}G\left(\frac{x}{z},Q^2\right) \tag{15}$$

which will allow us to determine the amount of error that enters when the quark contribution is neglected. We consider a number of standard non-perturbative parametrisations available in the literature [7 -12] and calculate the percentage error that enters when the quark contribution is neglected The results, for four quark flavours are shown in Table 2

Table 2. Estimation of error that enters when quark contribution is neglected for various inputs

Paramete-risations	λ	A_f	α	qc & gc	Actual value (x x^{λ})	Approx value (x x^{λ})	error (%)
MRS(A)	λq = - 0 3	Aq = 0 411	0 277746	qc = - 1 004632	1 96324619	2 37615	17 38
	λg = - 0 3	Ag = 0 775	0 58778	gc = 3 066			
MRS(R)	λq = - 0 04	Aq = 0 92	0 284445	qc = 0 147086	7 74825	7 88357	1 72
	λg = - 0 04	Ag = 2 07	0 706696	gc = 3 808488			
MRST(01)	λq = - 0 26	Aq = 0 222	0 29135	qc = - 1 018093	5 799305	6 0253218	3 75
	λg = - 0 33	Ag = 1 9	0 50518	gc = 3 171222			

In the above analysis we have observed that for inputs with positive values of λ [8, 10, 12], the values of *q.c.* are diminishingly small but for the inputs with negative values of λ [7, 9, 11], the values of *q.c.* are considerable. Neglecting the quark contribution thus brings in error particularly when the parton distributions have an x^{λ} dependence with negative values of λ.

5. Conclusions and discussions

We evaluated the convolution integral for the quark sector in the DGLAP equation (1) analytically using the framework of [5] and calculated the quark contribution *q c* defined in eq. (2). We found that the analytical result for *q c* is in excellent agreement with the numerically computed value of the same. The fact that the formalism allows us to obtain the point of expansion in terms of the parameter λ characterising various parton distributions available in the literature, was taken advantage of to calculate the amount of error entering in the relations like that in Ref. [1 – 3] where the quark contribution is neglected compared to the gluon contribution. We analysed various standard non-perturbative parameterizations in vogue indicating that the error for inputs having positive values of λ is diminishingly small while those having negative values of λ, the error is considerable.

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