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Vector model of quantum interference laser

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Abstract The present work reports a vector model of quantum interference laser in terms of phasor diagrams which involve the motion of organized vectors in terms of detuning The time evolutions of these vectors have been worked out using different decay parameters in the usual expression for transition probabilities versus time in arbitrary units The vector model of density matrix is used to describe inversion with reference to Poincare sphere

Keywords Vector model, Quantum Interference Laser

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1. Introduction

Quantum interference is a challenging principle of quantum theory and the essentials of quantum mechanics could be grasped from an exploration of the double slit experiment According to Feynman[1], each photon not only goes through both slits simultaneously but traverses every possible trajectory on the way to the target, not just in theory, but in fact. In order to see how this might possibly occur, experiments have focused on tracking the paths of individual photons. What happens in this case is that the measurement in some way disrupts the photon trajectories (in accordance with uncertainty principle), and somehow, the results of the experiment become what would be predicted by classical physics: two bright lines on photographic plate, aligned with the slits in the barrier. If we cease the attempt to measure, however, the pattern will become multiple lines in varying degrees of lightness and darkness.

From what has been described above it is apparent that uncertainty principle is involved in explaining the phenomenon of interference and therefore the phenomenon is more appropriately termed as Quantum interference. Quantum interference research is being

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applied to a growing number of fields such as superconductivity quantum interference device (SQUID), quantum cryptography, quantum computing, quantum beat, the Hanle effect, Self induced transparency and Lasing without inversion or Quantum interference laser[2-9].

Under special conditions coherent atomic transitions can cancel absorption. The main idea of LWI is that absorption cancellation provides the possibilities to obtain light amplification even if the population of the upper level is less than the population of the lower level. Such a situation can be realized, for instance, in a three level system, when two coherent atomic transitions destructively interfere and, hence, cancel absorption.

To present the basic physics of LWI it is best to consider the theory of this effect in a three level Λ-configuration and then to demonstrate how the concept of lasing without inversion can be realized experimentally. The configuration of Λ-three levels atomic system is presented in Figure 1. It is formed by upper levels $|a>$ and $|c>$ through interaction with electromagnetic fields E_1 and E_2, respectively, in such way that only atomic transitions $|a> - |c>$ and $|a> - |b>$ are allowed. The physical reason for canceling absorption in this system is the uncertainty in atomic transitions $|c> - |a>$ and $|b> - |a>$ which results in destructive interference between them. This situation is similar to Young's double slit problem, where interference is a consequence of uncertainty in determining through which of two slits the photon passed.

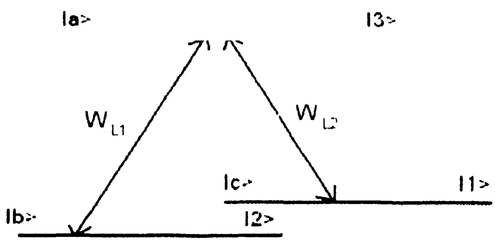

Figure 1. Three level atom in the Λ-configuration interacting with two fields of frequencies W_{L1} and W_{L2}

In the present work we report a vector model to represent quantum interference laser in terms of phasor diagrams which have been worked out from the concept of transition probabilities and their time evolutions with suitable decay parameters being introduced. We have also the occasion to use the formalism of vector model of density matrix and Poincare sphere to indicate their possible correlation with quantum interference lasers.

2. Atom field interaction

The usual representation of the energy level diagram for two level atom indicating decay rates γ_a and γ_b is shown in Figure 2.

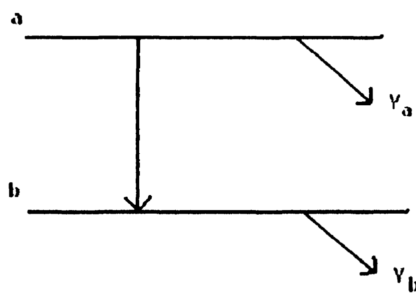

Figure 2. Energy level diagram for two level atom indicating decay rates γ_a and γ_b

It may be noted that the atom decays to the lower level as a result of its interaction with vacuum. The theory of spontaneous emission of Weisskopf – Wigner justified the inclusion of phenomenological decay rates γ_a and γ_b in the Schrodinger equation for the atomic probability amplitudes [10] The probability of stimulated absorption including damping is given by

$$|C_a(t)|^2 = \frac{1}{4}\left(\frac{\wp E_0}{\hbar}\right)^2 \exp(-\gamma_i t)\left[\frac{\sin(w-\nu)t/2}{(w-\nu)/2}\right]^2 \tag{1}$$

3. Moving vectors

The transition probability for arbitrary value of time may be plotted for different values of detuning $(\omega - \nu)$ Using the eq (1) we have worked out the graph which represents the time evolution of population in the upper state for different values of detuning at 90 100 110, 120 and 130 MHz This is shown in Figure 3

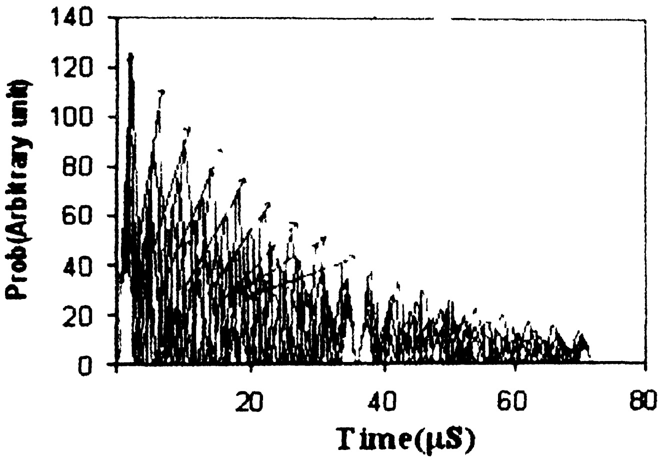

Figure 3 Transition probability *vs* time for different values of detuning and decay co... 05

As shown in Figure 3 we may join all the five maxima in a group of detunings as an arrow and a number of these arrows can be drawn in this way We observe that these vectors evolve in time. These vectors can be used to represent transition probabilities for the changes of population in the upper state The zero transition probabilities at arbitrary time of $6^2 n$ (n = 1, 2, 3 ..) indicate some type of saturation A phasor diagram has been constructed with the help of the moving vectors The graphs obviously represent some kind of resonance.

As may be seen in Figure 3, these vectors change directions as well as magnitudes, a situation which is analogous to the process of lasing without inversion or quantum interference laser.

Let us now follow a wave as it bounces back and forth between two mirrors inside a laser cavity as shown in Figure 4.

Consider the initial field at the plane just to the right of M_1, labeled by E_0. It propagates to M_2 and back to the starting plane and undergoes a change in amplitude $a_1 a_2$ and a phase factor $e^{[-ik2d]}$ as it travels that round trip and thus generates the field labeled E_1. This field experiences the same amplitude change and phase factor as E_0 and in turn generates E_2^+ and so on.

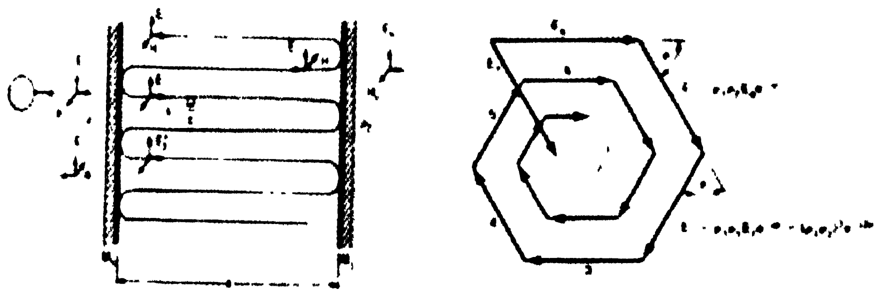

Figure 4. Resonating cavity and phasor diagram

The phasor diagram does not occur if $\Phi = 0$, but ϕ should be sufficiently small. The total field propagating to the right may be represented by

$$E_T^+ = \sum_0^{\infty} E_n^+ = E_0 = E_0 a_1 a_2 e^{-ik2d} + E_0 \left[a_1 a_2 E^{-ik2d} \right]^2 + \ldots\ldots\ldots$$

$$= \frac{E_0}{1 - a_1 a_2 e^{-i2\theta}} \qquad (2)$$

The field returning from M_2 is just a_2 times the round-trip phase factor $e^{[-ik2d]}$ multiplying the wave going to the right.

We now proceed to analyze the Phasor diagrams as shown in Figure 5 which are drawn with the help of the moving vectors.

As may be inferred from Figure 5 the vectors in the phasor diagram first closes up as in the case of resonating cavity but uncoiled again and expands. It is also observed that when decay is small the expansion is slow but for bigger decay the expansion is more. This is a general nature of all the phasor diagrams we have constructed with different decay constants and with the same set of detuning values. Thus we find that there is a distinct difference between the phasor diagram for a resonating cavity and the phasor diagrams of moving vectors. The former is contracting while the later is initially contracting but expanding afterwards. The phasor diagrams also indicate that as the decay γ increases the area decreases exponentially which may be illustrated by plotting decay versus area. As regards its physical significance it may be noted that the decay parameter γ is related to the lifetime as $\tau = 1/\gamma$ This follows from the theory of spontaneous emission In the present case the area approximately behaves like lifetime

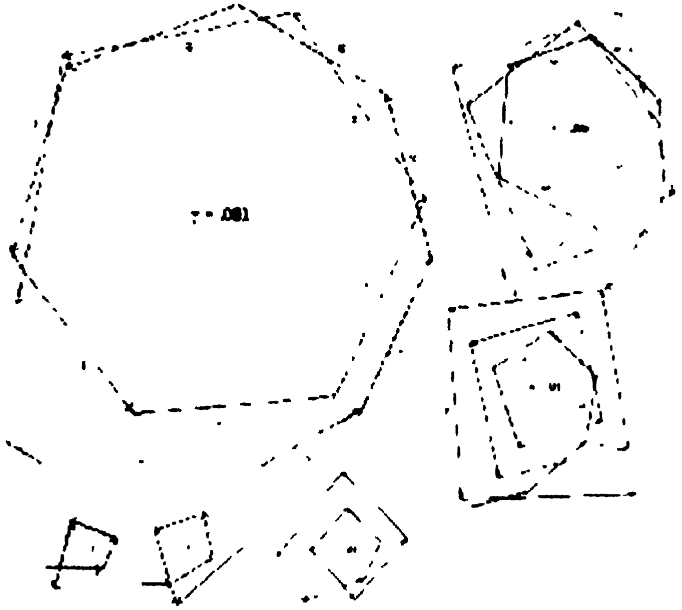

Figure 5 Phasor diagrams for moving vectors

4. Density matrix in motion

In this section we introduce the concept of the vector model of density matrix in motion [11]. This model has value not only in solving the equation of density matrix in motion but also in providing a physical picture of a particular system The equation are equivalent to the Bloch equation [12] appearing in nuclear magnetic resonance In this connection we may note that the phenomenon of the effect of an adiabatic rapid passage in inverting the population distribution of an assembly of nuclear spins has been recognized and predates the maser itself by approximately a decade. The technique is straightforward In a system in which a radiation field far from resonance with a transition frequency in a material sample interacts with a sample, the field frequency is swept through the material resonance until it is again far off resonance but with the frequency deviation reversed. If the sweep rate has been adjusted to lie within relatively easily attainable limits in certain materials, it is found that the population of the material sample after the passage is inverted relative to its value prior to the passage. For laser media when decays γ_a and γ_b are small compared to γ in the Bloch model which is accurate and may be easier to use. We have in the geometrical model of density matrix a moving vector defined as

$$R = -\gamma R + R \times B \tag{3}$$

The $\vec{R}$ vector precesses clockwise about the effective field $\vec{B}$ with diminishing magnitude. It is possible to follow the $\vec{R}$ vectors for some nonzero value of detuning as in the case of moving vectors and observe the condition of interference.

It is appropriate to conclude our discussion on moving vectors and phasor diagrams with reference to an elegant concept known as Poincare representation of describing the state of polarization of light. Conceptually Poincare representation is simple but it remained neglected for a long time until its utility was pointed out [13].In a so-called Poincare sphere there is one to one correspondence between all the points on the surface of a sphere (of unit radius called the Poincare sphere) and all possible forms of elliptic vibrations that can be conceived, may be represented by this sphere. One also notes that all surface points represent states of complete polarization. One could extend the concept and represent partially polarized states by points inside the sphere such that distance from the centre is a measure of the degree of polarization. Given the Poincare sphere, the change in the state of polarization of light as it passes through any medium can be visualized as a trajectory or moving vectors on the surface of the sphere. In fact, Pancharatnam [14] exploits this idea to actually design an achromatic quarter waveplate.

5. Conclusion

The present work is concerned with a vector model of transition probability which may be used to visualize quantum interference laser. The density matrix in motion and the concept of Poincare sphere has been introduced and their possible use in vector model is indicated.

Acknowledgment

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