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Large amplitude solitary waves in four component dusty plasma

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Abstract : Nonlinear dust acoustic waves are studied in a four component dusty plasma The existence of soliton solution is determined by pseudo-potential approach It is shown that in small amplitude approximation our result reproduces the result obtained by Sayed and Mamun [*Phys Plas* **14** 014501 (2007)]

Keywords : Solitary waves, dusty plasma, pseudopotential, four components

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1. Introduction

Dust and plasmas are omnipresent in the universe. It plays significant role in space plasma, astrophysical plasma, laboratory plasma and environment. The presence of dusty plasmas in cometary tails, asteroid zones, planetary ring, intersteller medium, lower part of earth's ionosphere and magnetosphere [1–8] makes this subject increasingly important. Dusty plasmas also play a vital role in low temperature physics, radio frequency plasma discharge [9], coating and etching of thin films [10], plasma crystal [11] *etc.*

Nonlinear wave phenomena like soliton, shocks and vortices in dusty plasmas have also been studied by several investigators for the last two decades or so [12–22]. Bliokh and Yarashenko [14] first theoretically observed such waves while dealing with waves in Saturn's ring. Later the discovery of dust-acoustic wave (DAW) [15,16], dust ion-acoustic wave (DIAW) [17,18] and dust lattice (DL) waves [19,20], gave a new impetus to the study of waves in dusty plasmas. Due to the dust grain dynamics few new eigen modes like Dust-Berstain-Greene-Kruskal (DBGK) mode, Shukla-Verma

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mode [21], Dust-drift mode [22] *etc.* are also introduced.

Dust-acoustic solitary waves in the one dimensional and unmagnetized plasma have been investigated by several authors. However, most of them considered the three-component dusty plasma system consisting of ions and electrons and negatively charge dust particles [23–25]. But both negative as well as positive dust particles also present in different areas of space [26–28]. Fortov *et al* [29] explained the mechanism by which a dust grain can be positively charged. Chow *et al* [28] also explained the situations under which smaller dust particles become positively charged and larger particles become negatively charged. It was also investigated that both positively and negatively charged dust present simultaneously in different space plasmas [28–30]. Recently Sayed and Mamun [31] investigated solitary waves in four component plasmas where they considered both positively and negatively charged dust particles. To obtain the solitary wave solution they used Reductive Perturbative Technique (RPT). But few years ago, Malfliet and Wieers [32] reviewed the studies of solitary waves in plasma and found that the RPT is based on the assumption of smallness of amplitude and so this technique can explain only small amplitude solitary waves. But there are situations where the excitation mechanism gives rise to large amplitude waves, and to study such situation one should employ a non-perturbative technique. Sagdeev's [33] pseudo-potential method is one such method to obtain solitary wave solution. This method has been successfully applied in various cases [34,35].

In this paper, we consider a four component unmagnetized dusty plasma system consisting of Boltzmann distributed electrons and ions and also positively (smaller size) and negatively (larger size) charged dust grains. The existence of solitary waves is studied by Sagdeev's pseudo-potential technique. It is shown that in small amplitude approximation our result reproduces that of Sayed and Mamun [31].

The organization of this paper is as follows. In Section 2 basic equations are written for four component dusty plasma and Sagdeev's pseudo-potential is derived. Conditions for the existence of soliton solution are also discussed in Section 3. Small amplitude approximation solutions are given in Section 4. Section 5 is kept for result and discussions and Section 6 is kept for conclusion.

2. Basic equations

We consider a four-component dusty plasma consisting of Boltzmann distributed ions and electrons and also negatively and positively charged dust grains. The basic equations are (see Ref. [31])

$$\frac{\partial n_1}{\partial t} + \frac{\partial}{\partial x}\left(n_1 u_1\right) = 0\,, \tag{1}$$

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}=\frac{\partial \psi}{\partial x}, \tag{2}$$

$$\frac{\partial n_2}{\partial t}+\frac{\partial}{\partial x}(n_2u_2)=0, \tag{3}$$

$$\frac{\partial u_2}{\partial t}+u_1\frac{\partial u_2}{\partial x}=-\alpha\beta\frac{\partial \psi}{\partial x}, \tag{4}$$

$$\frac{\partial^2 \psi}{\partial x^2}=n_1-(1-\mu_i+\mu_e)\,n_2+\mu_e e^{\sigma\psi}-\mu_i e^{-\psi}, \tag{5}$$

where n_1 and n_2 are the negative and positive number density normalized by the equilibrium values n_{10} and n_{20} respectively. u_1 and u_2 are negative and positive dust fluid speed normalized by $\frac{Z_1k_BT_i}{m_1}$. ψ, the electric potential is normalized by $\frac{k_BT_i}{e}$. x and t are normalized by $\lambda_D=\left(Z_1k_BT_i/4\pi Z_1^2e^2n_{10}\right)^{1/2}$ and $\omega_{p1}^{-1}=\left(m_1/4\pi Z_1^2e^2n_{10}\right)^{1/2}$. $\alpha=\frac{Z_2}{Z_1}$, $\beta=\frac{m_1}{m_2}$, $\mu_e=\frac{n_{e0}}{Z_1n_{10}}$, $\mu_i=\frac{n_{i0}}{Z_1n_{10}}$, $\sigma=\frac{T_i}{T_e}$, Z_1 and Z_2 are the number of electrons or protons residing on a negative and positive dust particle respectively. m_1 and m_2 are masses of the negative and positive dust particle respectively. T_i and T_e are ion and electron temperatures respectively, k_B is the Boltzmann constant and e is the charge of the electrons. In order to search for solitary waves which solves eqs. (1) to (5), we introduce a linear substitution $\xi = x - Mt$ admitting only solution which depend in space and time in the form of the wavy variable $x - Mt$. By substitution $\frac{\partial}{\partial x}=\frac{d}{d\xi}$ and $\frac{\partial}{\partial t}=-M\frac{d}{d\xi}$ eqs. (1)–(5) reduce to

$$-M\frac{dn_1}{d\xi}+\frac{d(n_1u_1)}{d\xi}=0, \tag{6}$$

$$-M\frac{du_1}{d\xi}+u_1\frac{du_1}{d\xi}=\frac{d\psi}{d\xi}, \tag{7}$$

$$-M\frac{dn_2}{d\xi}+\frac{d(n_2u_2)}{d\xi}=0, \tag{8}$$

$$-M\frac{du_2}{d\xi}+u_2\frac{du_2}{d\xi}=-\alpha\beta\frac{d\psi}{d\xi}, \tag{9}$$

$$\frac{\partial^2\psi}{\partial\xi} = n_1 - (1-\mu_i+\mu_e)\, n_2 + \mu_e e^{\sigma\psi} - \mu_i e^{-\psi}, \tag{10}$$

The boundary conditions are : ψ, u_1, $u_2 \to 0$, n_1, $n_2 \to 1$, $n_i \to \mu_i$ and $n_e \to \mu_e$ as $|\xi| \to \infty$.

From (6) we get,

$$n_1 = \frac{M}{M-u_1}. \tag{11}$$

Similarly from (8) we get,

$$n_2 = \frac{M}{M-u_2}. \tag{12}$$

From (7) we get,

$$\psi = -Mu_1 + \frac{u_1^2}{2}, \tag{13}$$

and from (9)

$$\alpha\beta\psi = Mu_2 - \frac{u_2^2}{2}, \tag{14}$$

Now using (11)–(14) in (10) we get

$$\frac{d^2\psi}{d\xi^2} = -\frac{\partial V(\psi)}{\partial\psi}, \tag{15}$$

where

$$V(\psi) = M^2\left[1-\left(1+\frac{2\psi}{M^2}\right)^{1/2}\right] + \frac{M^2}{\alpha\beta}(1-\mu_i+\mu_e)\left[1-\left(1-\frac{2\alpha\beta\psi}{M^2}\right)^{1/2}\right]$$

$$+\frac{\mu_e}{\sigma}\left(1-e^{\sigma\psi}\right)+\mu_i\left(1-e^{-\psi}\right).$$

Multiplying both side of eq. (15) by $2\frac{d\psi}{d\xi}$ and integrating w.r.t. ξ with the boundary conditions $|\xi| \to \infty$, $V \to 0$ and $\frac{d\psi}{d\xi} \to 0$ we get,

$$V(\psi)+\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2=0. \tag{17}$$

3. Pseudo-potential and soliton solution

Eq. (15) can be considered as a motion of a particle (whose pseudo-position is ψ at pseudo-time ξ) with pseudo-velocity $(d\psi/d\xi)$ in a pseudo-potential well $V(\psi)$. That is why Sagdeev's potential is called pseudo-potential. Here the pseudo-particle starts at a position $\psi = 0$ with a small velocity $\frac{d\psi}{d\xi}$ and it will be reflected back at some $\psi = \psi_m$ and then come back to $\psi = 0$.

Whether the solitary wave solution of eq. (15) exists or not, can be determined from the nature of the pseudo-potential $V(\psi)$. To discuss the possibility of solution we may consider the cases in classical mechanics when a conservative force field is given. It is known that between to single roots (0 and ψ_m here) of $V(\psi)$ if $V(\psi) < 0$ then ψ is periodic and that leads to a periodic nonlinear waves. But when a single root on the one side of the interval is considered, and a double root on the other end, a solitary wave is generated (see Ref. [6]). Due to initial conditions incorporated in $V(\psi)$, the double root is in $\psi = 0$. It takes an infinite large time to get away from it and the ψ reaches a maximum or minimum in ψ_m and then again taking infinite long time to returned to 0. Hence, conditions for the existence of soliton solution are

(i) $V(\psi) = 0$ at $\psi = 0$ and $\psi = \psi_m$

(ii) $\left.\frac{dV}{d\psi}\right|_{\psi=0} = 0$ but $\left.\frac{dV}{d\psi}\right|_{\psi=\psi_m} \neq 0.$

There is also one requirement that the double root at $\psi = 0$ corresponds to a local maximum at $\psi = 0$.

Hence another condition is

(iii) $\left.\frac{d^2V}{d\psi^2}\right|_{\psi=0} < 0.$

Also, it can be noted that physically complex V will not be allowed because this implies complex dust density which is not allowed. Hence there exists a nonzero ψ_m, the maximum (or minimum) value of ψ where $V(\psi_m) = 0$ (*i.e.* $V(\psi_m)$ crosses the ψ axis from below). Then ψ_m is the amplitude of the solitary wave. If ψ_m is positive then the solitary wave is called compressive solitary wave and if ψ_m is negative then the solitary wave is called refractive solitary wave. Obviously, $V(\psi)$ is negative in the interval $(0, \psi_m)$.

4. Small amplitude approximation

To obtain KdV type solution we obtain the small amplitude approximation of $V(\psi)$ we expand $V(\psi)$ about $\psi = 0$. Using the boundary condition $V \to 0$ and $\frac{dV}{d\psi} \to 0$ as $\psi \to 0$, we get

$$V(\psi) = A_1 \frac{\psi^2}{2} + A_2 \frac{\psi^3}{6}, \tag{18}$$

when

$$A_1 = \frac{1}{M^2} + (1 - \mu_i + \mu_e) \frac{\alpha\beta}{M^2} - \sigma\mu_e + \mu_i, \tag{19}$$

$$A_2 = \frac{3}{M^4} + (1 - \mu_i + \mu_e) \frac{3\alpha^2\beta^2}{M^4} - \sigma^2\mu_e/2 - \mu_i/2. \tag{20}$$

Hence the KdV type soliton solution is given by

$$\psi = \psi_0 \sec h^2 \frac{\xi}{\delta} \tag{21}$$

when

$$\psi_0 = -\frac{3A_1}{A_2} \tag{22}$$

is the amplitude of the solitary wave and

$$\delta = \frac{2}{\sqrt{-A_1}}. \tag{23}$$

is the width of the solitary wave. Sayed and Mamun [31] studied the same model for small amplitude solitary wave using RPT and they obtained the KdV equation as

$$\frac{\partial\psi_1}{\partial\tau} + A\psi_1 \frac{\partial\psi_1}{\partial\xi} + B \frac{\partial^3\psi_1}{\partial\xi^3} = 0 \tag{24}$$

when

$$A = \frac{1}{2V_0\left[(1 - \mu_i + \mu_e)\,\alpha\beta\right]}\left[(1 - \mu_i + \mu_e)\,\alpha^2\beta^2 - 3 - V_0^4(\mu_e\sigma^2 - \mu_i)\right] \tag{25}$$

$$B = \frac{V_0^3}{2\left[1 + (1 + \mu_e - \mu_i)\,\alpha\beta\right]}, \tag{26}$$

where V_0, the phase speed of the DA wave is given by

$$V_0^2 = \frac{1+(1+\mu_e - \mu_i)\alpha\beta}{\sigma\mu_e + \mu_i} \tag{27}$$

To get the steady state solution they use a transformation $\xi = \eta - U_0\tau$ and the usual boundary conditions, where U_0 is the velocity of the frame of the transformed coordinate. By usual technique one can obtain

$$\frac{d^2\psi_1}{d\xi^2} = \frac{2}{B}\left(U_0\psi_1 - \frac{\psi_1^2}{2}\right) = \frac{\partial V_1}{\partial \psi_1} \tag{28}$$

whence

$$V_1(\psi_1) = -\frac{U_0}{B}\psi_1^2 + \frac{1}{3B}\psi_1^3 \tag{29}$$

Now to compare the small amplitude approximation of $V(\psi)$ of eq. (18) with the values of $V_1(\psi)$ of eq. (29) obtained by RPT (Ref. [31]), we first replace M by $V_0 + U_0$ when U_0 is small. Then Keeping only first-order terms (in U_0) it can easily be verified that $V(\psi)$ in eq. (18) reduces to the $V_1(\psi_1)$ given in eq. (29). Hence the $V_1(\psi_1)$ obtined by RPT in Ref. ([31]) is nothing but a small amplitude approximation of $V(\psi)$ of eq. (18).

5. Results and discussion

Figure 1 shows the plot of $V(\psi)$ *vs.* ψ for v = 1.57, 1.75 and 1.925. Other parameters are and α = .01, β = 50, μ_i = .5, μ_e = .2, σ = .5. It is seen that $V(\psi)$ crosses the ψ axis for negative values of ψ for $1.57 \leq v \leq 1.925$. Hence rarefractive solitary waves exists for $1.57 \leq v \leq 1.925$. For v = 1.75, $V(\psi)$ crosses the ψ axis at $\psi = -1.35$. Hence $|\psi_{min}| = \psi_0 = 1.35$ is the amplitude of the rarefractive solitary waves for the above set of parameters with v = 1.75. It is also seen from this figure that the amplitude of the solitary waves increases with the increase of velocity.

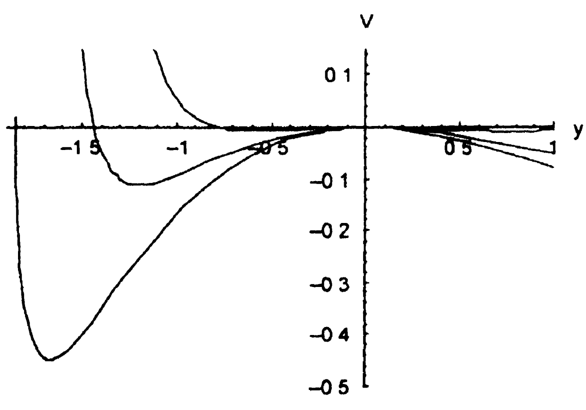

Figure 1. Plot of $V(\psi)$ *vs.* ψ for v = 1.57, 1.75 and 1.925 Other paremeters are and α = .01, β = 50, μ_i = .5, μ_e = .2, σ = .5. (read ψ in place of y, ξ in place of z and β in place of b in all Figures).

Figure 2 shows the plot of $V(\psi)$ *vs.* ψ for v = 1.85, 2.85 and 3.41. β = 150 Other parameters are the same as those in Figure 1. It is seen that $V(\psi)$ crosses the ψ axis for positive values of ψ for $1.85 \leq v \leq 3.41$. Hence compressive solitary waves exists for $1.85 \leq v \leq 3.41$. For $v = 2.85$, $V(\psi)$ crosses the ψ axis at $\psi = 2.45$. Hence $\psi_{max} = \psi_0 = 2.45$ is the amplitude of the compressive solitary waves. It is also seen from this figure that the amplitude of the solitary waves increases with the increase of velocity.

The shape of the solitary wave is obtained from the formula

$$\pm\xi = \int_{\psi_0}^{\psi} \frac{1}{\sqrt{-2V(\psi)}}, \tag{30}$$

and Figure 3 depits the soliton solution $\psi(\xi)$ plotted against ξ for v = 2.85.

The other parameters are same as those in Figure 2.

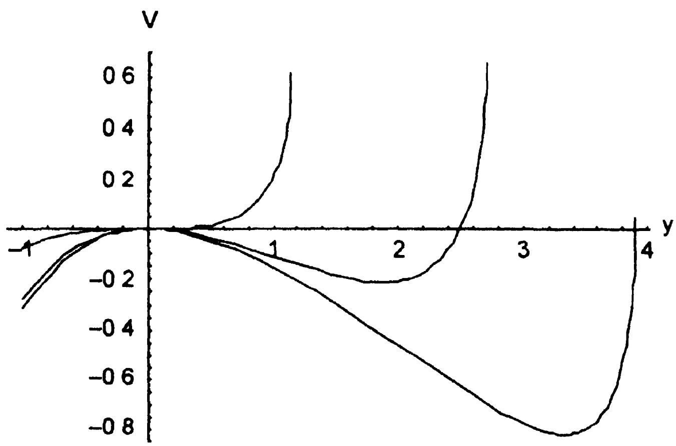

Figure 2. Plot of $V(\psi)$ *vs* ψ for v =1 85, 2 85 and 3 41 β = 150 Other parameters are same as those in Figure 1

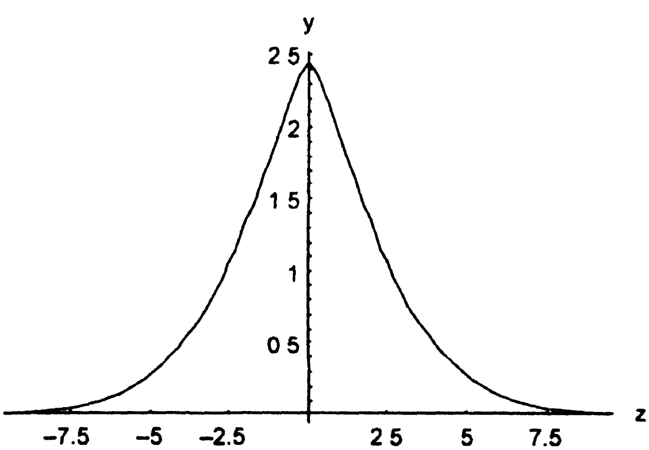

Figure 3. Plot of ψ *vs.* ξ for v = 2.85. Other parameters are same as those in Figure 2.

To see the effect of β on the amplitude of the solitary wave Figures 4(a) and 4(b) are drawn. For Figure 4(a) v is taken as 2.5. Other parameters are same as those in Figure 3. Here it is seen that for compressive solitary waves, the amplitude of the solitary waves increase with the increase of β.

For Figure 4(b) v = 1.75. Other parameters are same as those in Figure 1. Here it is seen that the amplitude of the rarefractive solitary waves decreases with the increase of β. Hence β has a significant effect on the shape of solitary waves. It can also be shown that the amplitude of the solitary waves also depends upon other parameters.

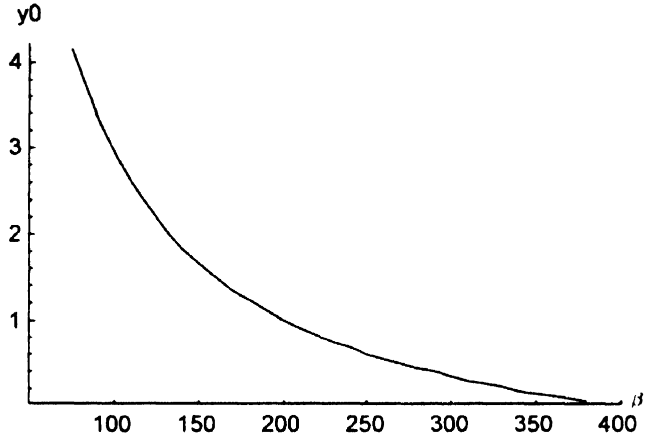

Figure 4a. ψ_0 is plotted *vs.* β for v = 2.5. Other parameters are same as those in Figure 3.

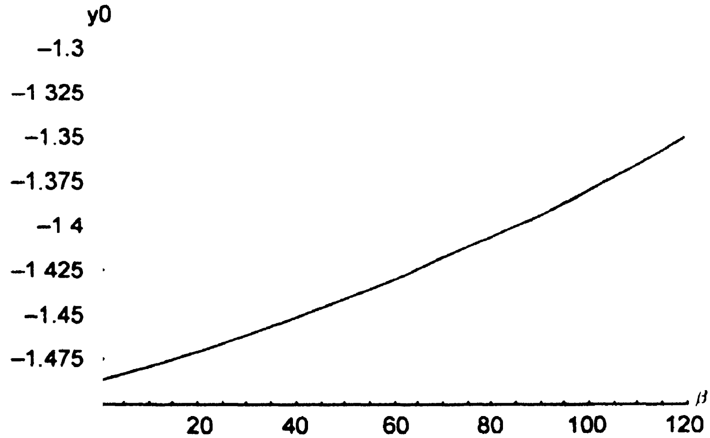

Figure 4b. ψ_0 is plotted vs. β for v = 1.75. Other parameters are same as those in Figure 1.

6. Conclusion

Existence of both the rarefractive and compressive solitary waves in four component dusty plasma are investigated using Sagdeev's pseudo-potential approach. It is also seen that in small amplitude approximation our result completely agrees with the RPT result obtained by Syaed and Mamun [31]. The shape of the solitary wave is done using the integration (30). It is shown that β has a significant effect on the amplitude of the solitary wave. This technique can be extended to the study of non-thermal distribution of electrons in four component plasmas. Work in this direction is in progress.

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