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Memory effects in language acquisition and attrition processes

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Abstract. We recall some models and computer simulations that employed statistical physics and complex systems approach for language competitions, such as the model of Abrams and Strogatz, its natural extension by Minett and Wang, which added bilinguals into the dynamics. We also take a look at a game theoretical approach to language competition made by Iriberri and Uriarte. We introduce a new model that also brings a notion of memory into the dynamics, mimicking how agents acquire a language and potentially go through a language attrition processes, while being able to continually refresh their memory, if they are given the opportunity. We discuss the algorithm and implementation of the model.

The goal of this work is to explore what effects memory may have on language acquisition and attrition processes. In particular, memory effects may help to explain why many bilingual communities survive despite odds. Variants where the language used between bilingual agents is chosen by random decision making process and prior memory-based decision making processes are considered. Based on the results obtained it can be concluded, that in this model, competing languages can survive within a bilingual community, when agents possess a memory and are on a fully connected graph. Similar results were observed for asymmetrical languages and for populations that were initially split into unequal number of representatives of competing languages. Additionally, given the simple structure of the model proposed, some proposals for a more advanced model are given for further exploration.

Keywords: complex systems, language dynamics, bilingualism

CERCS: P190 - Mathematical and general theoretical physics, classical mechanics, quantum mechanics, relativity, gravitation, statistical physics, thermodynamics

Mälu mõjud keele omandamise ja unustamise protsessides

Lühikokkuvõte. Me vaatleme mõningaid mudeleid, mis rakendasid statistilise füüsika ja kompleksussüsteemide vahendeid keelte dünaamika modeleerimiseks, nagu näiteks Abrams ja Strogatz'i mudel ning ka Minett ja Wang'i mudel, mis hõlmas dünaamikasse lisaks kakskeelsed. Samuti vaatleme ka mänguteoreetilist lähenemist Iriberri ja Uriarte mudeli näol keelte dünaamikale. Esitleme oma välja arendatud uut mudelit, mis lisab agentidele ka mälu, jäljendades sellega keele omandamise ja unustamise protsesse. Mudelis on lubatud agentidel ka oma mälu värskendada, kui selle jaoks peaks olema tekkinud interaktsioonide käigus võimalus.

Antud töö eesmärgiks on uurida mälu mõjusid keele omandamise ja unustamise protsessides. Mälu mõju võib aidata ka selgitada, miks paljud keeled saavad kakskeelsetes ühiskondades vastupidiselt ootustele jääda ellu. Vaatleme olukordi, kus kakskeelsed valivad omavahel kasutatava keele juhuslikult ning ka olukordi, kus vastav otsus tehtakse eelnevaid eelistusi arvestades. Saame järeldada, et mudelis, kus kõik agendid on üksteise naabrid ning nende mälu saab pidevalt värskendada, siis on võimalik, et omavahel võistlevad keeled jäävad kakskeelses ühiskonnas ellu. Analoogilised tulemused olid ka olukordades, kus arvestati keelte omavahelise asümmeetriaga. Samuti vaadeldi olukordi, kus populatsioonide suurused kahe keele esindajate vahel olid algtingimustel erinevalt jaotatud ning leiti, et ka sellistes olukordades võivad mälu mõjude arvestamisel omvahel võistlevad keeled kakskeelsetes ühiskondades jääda ellu. Arvestades esitletud mudeli lihtsat olemust, pakume ka mõningaid võimalusi mudeli täiustamiseks edasise uurimustöö jaoks.

Märksõnad: komplekssüsteemid, keelte dünaamika, kakskeelsus

CERCS: P190 - Matemaatiline ja üldine teoreetiline füüsika, klassikaline mehaanika, kvantmehaanika, relatiivsus, gravitatsioon, statistiline füüsika, termodünaamika

Contents

1. Introduction

Introduction

Tools and ideas from statistical physics and complex systems theory have been applied successfully in various scientific disciplines outside traditional physics, such as biology, medicine, computer science and social sciences [1, 2]. They have also been used to model language dynamics and competition [3, 4, 5, 6].

The modelling of languages began already with the works of Baggs and Freedman in 1990's [7, 8]. However, the work that started a rapid development of multiple models which used computer simulations to gain insight into the competition between various languages was done by Abrams and Strogatz in 2003 [3]. Aside from the models done by Baggs and Freedman, those models only considered monolingual groups N_X and N_Y and the existence of bilingual group N_{XY} was not considered within the dynamics. Therefore, as a natural extension of the Abrams-Strogatz model, Minett and Wang introduced the concept of bilingual agents into the language dynamics [4, 9]. The role of bilinguals in language competition have been modelled from multiple approaches, including from a game theoretical approach and detailed overviews of some of them are given in Refs. [5, 6].

According to UNESCO [10], out of the approximately 6000 languages currently used, about 43% of them are endangered and linguists are predicting that by as early as the year 2050, almost all of them would be extinct [11]. These predictions are further reinforced by the conclusions made by both the Abrams and Strogatz model and the Minett and Wang models, unless special tactics are applied [3, 4]. Considering the complexity of interactions between competing languages, it is natural that questions about the dynamics about language acquisition and attrition arise. Is there a way that languages can survive or are they invariably doomed to die until only one remains? Under which criteria can an equilibria be reached between competing systems, for example, can stable bilingual communities be the answer? In the hope to provide answers to these questions, computer simulations are extremely important because they allow us to take a look at languages and their general trends on a microscopic, agent-based level and by fitting the simulations to real life data, the simulations can even provide hints at how to properly protect an endangered language from going extinct or how viable different strategies can be.

Within this work, we first give a short overview into some language dynamics models introduced to model language competition and additionally we also give a short introduction into general ideas about language acquisition and attrition processes from a linguistical perspective.

In Chapter 3 we provide a model of language acquisition and attrition processes. The goal of the present work is to determine what effects memory may have in language competition. Memory effects could be important in explaining the existence of many stable bilingual communities and are known to have an important role in opinion dynamics [12]. We implement the model in C++. In Chapter 4 we present and discuss the results, to gain insight into the aforementioned processes in language competition dynamics. We study memory effects in two different respects: in the first case, memory affects the learning and forgetting of a language. Furthermore, we study what happens when an additional memory effect is present, which determines the choice of the language used by a bilingual individual, as the language which has been used more. Additionally, we study both symmetrical models where languages X and Y follow the same dynamics, and asymmetrical models, where the acquisition of one of the competing languages is easier.

We discuss possibilities for further research and improvements in complex systems models of language competition in Chapter 5.

2. Theoretical background

In this section we present some prototypical examples of language dynamics models and some notions of linguistics about language acquisition and attrition processes.

2.1 The Abrams and Strogatz model

The seminal work that started an effort in a statistical physics and complex systems approach to langugage dynamics was one by Abrams and Strogatz [3]. In order to model language competition they constructed the first order differential equation

$$\frac{dx}{dt} = yP_{yx}(x, s) - xP_{xy}(x, s), \tag{2.1}$$

where x is the fraction of the population speaking language X, $y = 1 - x$ is the complementary fraction of Y at a time and $P_{yx}(x, s)$ is the probability with which an individual from Y converts to X. The parameter s takes values from $0 \leqslant s \leqslant 1$ and represents the prestige of a corresponding language. Assuming the transition functions to have forms $P_{yx}(x, s) = cx^a s$, $P_{xy}(x, s) = c(1 - x)^a(1 - s)$ and substituting $y = 1 - x$, the equation (2.1) takes the following form

$$\frac{dx}{dt} = c(1 - x)x^a s - cx(1 - x)^a(1 - s). \tag{2.2}$$

The general scheme for the Adams and Strogatz model is illustrated in Figure 2.1.

For the exponent a, which represents the volatility parameter, Abrams and Strogatz tested their model against collected data for language competitions between Welsh-English, Gaelic-English and Quechua-Spanish and found the value for it to be $a = 1.31$ [3]. The volatility parameter can produce for three different regimes. For $a = 1$ (neutral situation), the transition probabilities depend linearly on local population density. For $a < 1$ (high volatility), the probability of an agent changing their language is higher than in the neutral case and for $a > 1$ (low volatility), the probability of changing a language is lower than in the neutral case. It could be shown that in general, low volatility leads to the domination of a single language and no coexistence between two languages can take place [13] while for $a < 1$ a fragmentation process can happen.

In general, the Abrams and Strogatz model is in a good agreement with the general notion that endangered languages will go extinct, unless measures are taken to improve the prestige s of an language through means such as education and advertisement or increase in the number of speakers through other policies is achieved for example [3].

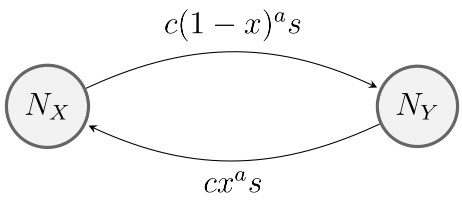

Figure 2.1: General scheme for the Abrams and Strogatz.

2.2 The Minett and Wang model

Minett and Wang proposed an improvement upon the Abrams and Strogatz model in Ref. [9] and published a new model, incorporating bilinguals into language dynamics in Ref. [4]. The original Minett-Wang model is given with the equations:

$$\begin{aligned}\frac{dN_X}{dt} &= k_{ZX} N_X^a N_Z - k_{XZ} N_Y^a N_X, \\ \frac{dN_Y}{dt} &= k_{ZY} N_Y^a N_Z - k_{YZ} N_X^a N_Y.\end{aligned} \tag{2.3}$$

These equations describe how the monolingual populations of N_X, N_Y and bilingual population N_Z change in time. The transition rates to change from one population to another are fixed with the rate constants $k_{ZX}, k_{ZX}, k_{ZY}, k_{YZ}$, the volatility parameter a and the size of the populations N_X, N_Y, N_Z. The rate constants $k_{ZX}, k_{ZX}, k_{ZY}, k_{YZ}$ contain information about the mortality rate, language status or other various sociolinguistic factors [4]. The Minett and Wang model allows for both vertical and horizontal transmissions within the population. Within the vertical transmissions, children inherit from their parents the original language that they possess at a rate corresponding to their attractiveness or both languages. In horizontal transmissions, it is assumed that all adults, after becoming bilingual, will stay bilingual and that adults of either language X or Y will acquire the opposing language according to its attractiveness. All other monolinguals stay monolinguals of their respective language. The general scheme for the Minett and Wang model is illustrated in Figure 2.2.

Overall, the Minett and Wang model reaches the same conclusion as the Abrams and Strogatz model, that eventually one language will dominate over the other and forces the minority into extinction unless specific strategies of intervention are adopted when the number of speakers of the endangered language decreases below a threshold where the language can naturally survive on its own. It should be noted that in the original Minett and Wang model [4] only the case where $a = 1$ was studied (neutral volatility). Regimes where $a \neq 1$ have also been studied for the Minett and Wang model in Ref. [13] with the conclusion, that different volatility parameters can lead to either language coexistence or dominance, with the prestige of a language also playing an important role.

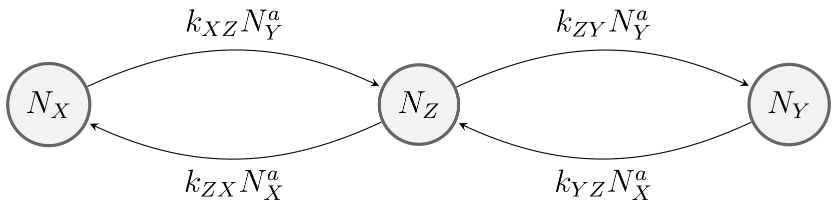

Figure 2.2: General scheme for the Minett and Wang model incorporating bilinguals.

2.3 The Iriberri-Uriarte model

Because the language competition model implemented within this work also incorporates game-theoretical elements, we would like to review also the model developed by Iriberri and Uriarte [14]. The language conversation game (LCG) is a simple game played by at least two individuals, out of whom one should be bilingual as a minimum initial condition. The model itself and the decisions that the players make are based on the following five assumptions [6, 14]:

- **Assumption I: Imperfect information.** The players know if they are a monolingual of either language X or language Y or a bilingual, however, do not possess any information about the type of the other player. The type of an agent is determined by randomly, but it is assumed that the probability distributions α for bilinguals and $1-\alpha$ for the monolinguals hold. Additionally $\alpha < 1-\alpha$, where α is considerably smaller than $1 - \alpha$.

- **Assumption II: Language preference.** Bilinguals prefer to use language Y.

- **Assumption III: Linguistic distance.** The languages X and Y are sufficiently different from one another, so that successful interaction between the two agents can only be possible in one language. In other words, the representative of language X is not a passive speaker of language Y and *vice versa*.

- **Assumption IV: Payoff.** For α, so that $\alpha < 1-\alpha$, the payoff ordering is defined as $m > n > c > 0$. Maximum payoff m is achieved when a bilingual is able to use their preferred language as defined in Assumption II. Equal payoff n is achieved when the monolingual or the bilingual uses the majority language X. This can happen when a bilingual is matched with a monolingual who uses X. However, there is an associated frustration cost c for the bilingual, due to having been 'forced' to use the majority language and not their own.

- **Assumption V: Frustration.** Frustration cost is smaller than the weighted average:
$$c < \frac{(m-n)\alpha}{1-\alpha},$$
because if the frustration cost is too high, a bilingual would refrain from using another language.

From these assumptions one can derive the following pure strategies:

S1: *Always use Y, whether you know for certain you are speaking to a bilingual individual or not.*

S2: *Use Y only when you know for certain that you are speaking to a bilingual individual; use X, otherwise.*

These assumptions and strategies lead to the following result:

Proposition: *There exists a mixed strategy Nash equilibrium in which the bilingual population plays S1 with probability* $x^* = \frac{1-c(1-\alpha)}{\alpha(m-n)}$. *This equilibrium is evolutionary stable - that is* x^* *is a language convention built by the bilingual population - and asymptotically stable in the associated one-population Replicator Dynamics.*

Proofs of the above proposition and a more extensive overviews of the Iriberri-Uriarte model are given in articles [6, 14].

2.4 The Naming Games

We also mention that in semiotic dynamics, the Naming Game [15] is a simple example of how complex processes can lead to the formation of a consensus, as in human-like language, without any central control. The Naming Game is characterized by a population of agents taking part in a pairwise game, where they try to negotiate words, meanings and how they are globally perceived. Within the Naming Game, the initial vocabularies are set up by individual agents themselves but as they are forced to interact with other agents in the population, globally shared vocabularies should emerge. It can be mentioned that The Naming Game has multiple variants such as the Naming Game restricted to two conventions (2c-Naming Game) [1], where the competition between only two conventions is looked at, which is similar to the language competition model considered here; or The Minimal Naming Game [16, 2], where the game is made up of a population on a fully connected graph trying to give a common name to an object, while choosing possible names from their individual repository of knowledge until a consensus is met.

2.5 Language acquisition and attrition

In language acquisition and attrition processes, a human being acquires or loses the ability to comprehend and utilize languages [17]. As these processes are generally regarded to belong to the scientific discipline of linguistics, it is also important to give a short overview of some of the aspects involved in those processes.

Language acquisition usually refers to first-language acquisition and how children acquire their native language. This is separate from the study of second-language acquisition, by which everyone acquires any number of additional languages and are thereby referred to as multilingual speakers. While this distinction is made by linguists, then usually in the study of language competition it is not considered.

Multilingualism

A multilingual person is an individual, who can communicate, either actively or passively, in more than one language. Multilingualism is very common and considered important, especially in European countries, where the domestic market is rather constricted and international trade is a necessity [18, 19]. Additionally, in order to master a language, one also has to obtain an extensive understanding of the culture associated with it as well. Therefore, bilinguals have a tangibly broader view of the world itself. Globalization and the advent of the Internet has also lead to an ever increasing interconnectedness and established English as the current *lingua franca*, the bridge or trade language. It should be mentioned however, that while currently English is perhaps the most common *lingua franca*, it is not the only one and can vary by region.

However, in order to become a multilingual person, one first has to acquire a new language besides their first language, which can bring with it multiple attrition effects, among which, at least theoretically, can be the loss of the entire first language. Language acquisition and attrition processes are described in the following sections.

Acquisition

How exactly individuals, especially children, acquire a language is a subject of great debate. For example, it is argued on one hand, that childrens ability to acquire a language must be guided biologically by the human brain, while others argue, that it is more akin to a social phenomena [20]. It is, however, noted with the use of modern technologies such as functional magnetic resonance imaging (fMRI) and positron-emission tomography (PET), that the acquisition and maintenance of a first and subsequent languages are processed differently within the cortex and seem to concentrate into areas known as Broca's area, which is in the left frontal lobe and Wernicke's area, which is in the temporal lobe. The areas are associated with the roles of syntactic and lexical processing, respectively. The importance of Broca's and Wernicke's area can be illustrated by mentioning, that any damage (such as from a stroke or a head trauma) to those brain regions can lead to signifcant language disorders, such as aphasia [20, 21].

There are tangible differences between learning a language in a controlled class environment and learning through complete immersion, which is the situation considered in this model.

Attrition

Language attrition, as the process of losing a language or aspects of it, is usually caused by an individual becoming isolated from the language in question, as a side effect of acquiring a second language. While there are many disputed aspects that may affect the process of attrition, such as frequency of use and overall attractiveness of language communities, it has been shown that the probability of attrition is linked to age [22, 23]. It is currently a held belief, that a language attrition process begins notably with the decline of a persons lexical ability while phonological and grammatical aspects of language seem to be more stable [24, 25].

There are various ways how language attrition happens. Lexical attrition is a standard deterioration of an agents ability to properly use a language. The cause for this is believed to be the side effect of constant use of a competing language, since words from the first language are in direct competition with their translated counterparts [26]. Another type of attrition is phonological and it consists of the decreased ability to reproduce the correct pronunciation. Additionally, grammatical attrition is characterized as the disintegration of the syntax structure, while in contact situations with a second language.

As was mentioned before, language attrition also depends on various factors such as age, frequency of use and motivation.

Among all the factors mentioned in both language acquisition and attrition, which determines transitions between the state of being a bilingual and a monolingual, in the following we study a simple model of language dynamics, focusing on the role of usage frequency and attractiveness of language.

3. Model

3.1 Description of the model

We consider a model in which the population of size N is composed of two monolingual communities of size N_X and N_Y and a bilingual community N_{XY} that is initially zero. During the interactions, we give the monolingual agents an ability to acquire the other language, therefore becoming bilingual. Our model is different from the model of Minett and Wang due to an additional memory for each individual agent, allowing them to refresh their knowledge about the language that they are using or trying to learn. In order to learn a new language, a monolingual agent has to use it K times within a time interval Δt_b, without refreshing the memory. Analogously, an agent has to use a certain language L times within a time interval Δt_m without refreshing the memory, otherwise forgetting the language. Bilinguals can choose their language for interacting either randomly or by taking into account their previous interactions and preferences regarding the use of language X or Y.

3.2 Algorithm of the model

The general algorithm and description for the simulation is as follows:

1. Initial conditions: A population is generated, containing fractions of agents representing languages X and Y such that $N = N_X + N_Y + N_{XY} = 1$ is normalized to 1. The fraction N_{XY} represents the bilingual agents within the population and is initially set to zero, $N_{XY}(0) = 0$. For the purpose of the simulation, all data and time evolution about the population will be held in an interaction matrix, which is explained in further detail in the following Section 3.3.

2. Parameters:

    - The maximum number of iterations the simulation would ideally run Δt_{MAX}.
    - The upper value for the time allowed for learning a new language Δt_b without refreshing the memory.

- The number of interactions K needed for becoming a bilingual or the values for K_X and K_Y if an asymmetry between the languages is also being considered.
- The number of interactions L needed for retaining a language before reverting back to a monolingual without memory refreshing.
- The upper time limit Δt_m without memory refreshing, after which the number of acquired interactions will be checked against the previously given value for L.

3. Interactions: At each time step t two agents are randomly selected:
   - If they are both from the same X or Y subset of the population, then they are monolinguals of the same language and nothing happens.
   - If one is from group X and the other is from Y, then they are representatives of different language groups and through their interaction, they will learn something about the other agents language and that interaction will be recorded for both and their memory refreshed. If K interactions within a time interval Δt_b are met by an agent from either group, that member of the group becomes a bilingual in the group N_{XY}. The same holds if an asymmetry is considered between the languages, by substituting K with K_X and K_Y.
   - When a bilingual N_{XY} and a monolingual agent (either N_X or N_Y) interact, they select the language that the monolingual uses. The interaction is recorded for the bilingual and their memory refreshed for that language but the monolingual learns nothing new.
   - When two bilinguals of the population group N_{XY} interact they can either choose the language for their communication completely randomly or make their decision based on memory about which language they prefer to use. The memory-based decision making is elaborated upon in Section 3.4. If, however, the two agents had the same original language, they will prefer to use it. The interaction will be recorded between the agents and their corresponding memories refreshed.
   - After each interaction between two agents, time is updated. And two new agents will be selected for the next interaction until the simulation ends.

4. Learning and attrition:
   - During the simulation bilinguals need to have at least L interactions within Δt_m for both their original language and the learned language. If this condition is not met, then their state of being a bilingual will be erased and based on the language they have used more while being a bilingual, they become a monolingual.

- During the learning process, every time an agent spends time learning a new language, the time counter for learning a new language is updated. If the agent has K interactions within Δt_b, then they become bilingual.

5. All data: time t, number of agents for language X, N_X, number of agents for language Y, N_Y and the number of bilinguals N_{XY} will be gathered and outputted into a file for later statistical analysis. The data will be appended into a file of the following form:

t	N_X	N_Y	N_{XY}
0	$N_X(0)$	$N_Y(0)$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t_i	$N_X(t_i)$	$N_Y(t_i)$	$N_{XY}(t_i)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$N_X(t)$	$N_Y(t)$	$N_{XY}(t)$

6. During the simulations runtime, the following criterion is being constantly checked:

$$\sum_{i=1}^{N} \frac{N_i}{N} = \begin{cases} 1, & \text{if everyone is a monolingual of X} \\ 0, & \text{if everyone is a monolingual of Y} \\ 0 < x < 1, & \text{otherwise,} \quad x \in \mathbb{R} \end{cases}$$

If either of the first two conditions is met, the program will terminate because one of the languages has completely dominated the other and the total population N has become a homogeneous representation of either N_X or N_Y where no learning of another language can occur. Otherwise the program will run until the maximum number of iterations have passed.

3.3 The interaction matrix

For a given population of size N, all the data and interactions can be summarized with the following interaction matrix of size $(N, 8)$ that includes submatrices of sizes $(N_X, 8)$ and $(N_Y, 8)$ so that $N = N_X + N_Y$ for the simulation is given as:

$$\left.\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}1\\ \vdots\\ 1\end{matrix}}\right\} N_X \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} N_Y \end{matrix} \right\} N \tag{3.1}$$

where the columns $1, 2, \ldots, 8$ hold the following meanings:

1. The first column holds the data about the agents original language in the form of $(\mathrm{X}, \mathrm{Y}) \mapsto (1, 0)$ where X and Y are the original languages for either the agent N_X or N_Y respectively.

2. The second column holds the data about the state of whether the corresponding agent is a bilingual of languages X and Y or not. The state of being a bilingual is marked with the following relation $(\text{true}, \text{false}) \mapsto (1, 0)$.

3. If the state of bilingualism is true (set to 1), then the third column will hold the data about how many times the agent has used their own original language while being a bilingual of language X and Y.

4. If the state of bilingualism is true (set to 1), then the fourth column will hold the data about how many times the agent has used the learned language of either X or Y while being a bilingual.

5. If the state of bilingualism is true (set to 1), then the fifth column counts the time the agent has been a bilingual with respect to their original language.

6. If the state of bilingualism is true (set to 1), then the sixth column counts the time the agent has been a bilingual with respect to the language that they learned.

7. The seventh column counts the time one has spent learning a language before becoming a bilingual.

8. The eight column counts the number of interactions an agent has spent learning a new language during the learning period.

For a simulation that also takes into account the possible asymmetry between two language groups the eight column that stands for K learning interactions must be split into two columns that will hold the data for K_X and K_Y or the 8th and 9th column respectively.

3.4 Memory-based decision making for bilinguals

Within our model there are two possible approaches to the language selection between interacting bilinguals. Either they select the language completely by random or the language is selected by using data from their previous interactions and taking into account their personal preferences. For the later purpose a memory must be built. From the 3rd and 4th column of the interaction matrix (3.1) we can build a 2×2 submatrix, where the first row is for a representative of group N_X and the second row is for the representative from group N_Y.

Generation of the decision matrices

In order to account for all the possible variants that two interacting agents may have and what language they choose to use next, then by having the elements $a, b, c, d \in \mathbb{N}_0$ we can write in total 47 different matrices for decision making and the algorithm for generating them as the following sets:
The first 24 matrices, where all elements are different ($a \neq b \neq c \neq d$):

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & a \\ d & b \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & c \\ b & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} b & d \\ a & c \end{bmatrix}$$

$$\begin{bmatrix} b & a \\ c & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & b \\ d & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & c \\ a & b \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & d \\ b & c \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ d & c \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & a \\ c & b \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & d \\ b & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} b & c \\ a & d \end{bmatrix}$$

$$\begin{bmatrix} c & b \\ a & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & c \\ d & b \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & a \\ b & c \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} b & d \\ c & a \end{bmatrix}$$

$$\begin{bmatrix} a & d \\ c & b \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & a \\ b & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} b & c \\ d & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & b \\ a & c \end{bmatrix}$$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} b & a \\ d & c \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & b \\ c & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

The next 12 matrices with three differing elements ($a = b \neq c \neq d$):

$$\begin{bmatrix} a & a \\ c & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & a \\ d & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & c \\ a & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & d \\ a & c \end{bmatrix}$$

$$\begin{bmatrix} a & a \\ d & c \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & a \\ c & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & d \\ a & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & c \\ a & d \end{bmatrix}$$

$$\begin{bmatrix} a & d \\ c & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} c & a \\ a & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & c \\ d & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & a \\ a & c \end{bmatrix}$$

The set of 6 matrices with two differing elements ($a = b \neq c = d$):

$$\begin{bmatrix} a & a \\ d & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & a \\ d & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & d \\ a & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & d \\ a & d \end{bmatrix}$$

$$\begin{bmatrix} a & d \\ d & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & a \\ a & d \end{bmatrix}$$

Set of 4 matrices with only a single differing element ($a = b = c \neq d$):

$$\begin{bmatrix} a & a \\ a & d \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & a \\ d & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} d & a \\ a & a \end{bmatrix} \xrightarrow{\text{Rotate by } \frac{\pi}{2}} \begin{bmatrix} a & d \\ a & a \end{bmatrix}$$

Matrix with no elements differing ($a = b = c = d$) from one another has only a single possible variant:

$$\begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

3.4.1 Examples of decision making

For a clearer picture, we can choose the elements a, b, c, and d to be $1, 2, 3$ and 4 respectively and then view some of the following configurations for the matrices:

1. For the decision matrix, where all the elements are the same:

$$\begin{bmatrix} a & a \\ a & a \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

   we'll have no preference between the agents toward either language and thus the language will be chosen randomly so that the submatrix within the interaction matrix would change as follows:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} & \text{if chose X} \\ \\ \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} & \text{if chose Y} \end{cases}$$

2. For the decision matrix, where all but one of the elements is the same we'll have:

$$\begin{bmatrix} a & a \\ a & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

   In this case, the first agent has no preference but the second agent does have one for the language that they have obtained through the learning process. Thus the first agent will use their original language and the second agent shall use the language that they prefer:

$$\begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} \xrightarrow{\text{Select language X}} \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$$

   It is easy to see that depending on the specific configuration of the matrix, that this type of decision making matrix will always select the language that either the first or the second agent prefers to use.

3. For the decision matrix that contains two different elements in the following way:

$$\begin{bmatrix} a & a \\ d & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix}$$

there will be two possible ways of selecting the language based on the matrix configuration. For the matrix type that is already shown, the language will be chosen randomly, since neither of the agents has a preference.
The second way of selecting a language arises when a and d in one of the columns are interchanged and thus there will be a clear preference toward either language X or Y. Example when language X would be chosen:

$$\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \xrightarrow{\text{Select langauge X}} \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$$

4. For the decision matrix that has three different elements we can write:

$$\begin{bmatrix} a & a \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$$

there are again both scenarios possible, where either the language selection will be straightforward or will have to be decided upon randomly. For the already given example, language X will be decided upon, because while the first agent doesn't have a preferred language, the second agent does. It is also possible, that both the agents that happen to interact would like to use their own original language or the one that they have learned as illustrated with the following examples:

$$\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 4 & 1 \\ 4 & 2 \end{bmatrix} & \text{if chose X} \\ \\ \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} & \text{if chose Y} \end{cases}, \quad \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix} & \text{if chose X} \\ \\ \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} & \text{if chose Y} \end{cases}$$

The first case is where both agents prefer to use their own original language and the second scenario is when the opposite is true.

5. For the decision matrix where all the elements are different we can write:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

which again is a scenario, where both agents prefer to use the language that they have acquired and thus the tie will be solved by randomly choosing between X or Y. For decision matrices of this type all possible variations must be considered and through different configurations we can easily have interactions where the language X is preferred, where the language Y comes out as a winner and also scenarios where the agents would rather use their own original or learned language and a tie will be solved by randomly selecting the language for a given interaction between two agents.

All possible 47 variants are explicitly shown in Appendix A and the more general rules for deciding between languages are given in the Table A.1.

4. Results

Selection of the study parameters

The main goal of this exploratory work is to study the effects of memory on the language acquisition ability of an agent. For this reason, we selected the following parameters:

- We assume a concentrated time unit $\Delta t = 1$ and we define a single day within our model as $\Delta t_{\mathrm{DAY}} = 4 \cdot \Delta t$ so that a year would be equal to $\Delta t_{\mathrm{YEAR}} = 4 \cdot \Delta t \cdot 365 = 1460$. This means, that on average, an individual agent can effectively have a minimum of 4 language contacts during the day.

- We take Δt_{YEAR} as a reference value for Δt_b, within which an agent can become bilingual in principle, depending additionally on the selection of the parameter K, the number of interactions required to learn a new language. It should be noted, that within our model Δt_b is actually a dynamic parameter, as it is continuously refreshed as the agent is learning

- We take the maximum number of iterations for the simulation as $\Delta t_{\mathrm{MAX}} = 100 \cdot \Delta t_{\mathrm{YEAR}}$.

- For the parameter Δt_m, the maximum time allowed without continuous use of a language before forgetting it, we assume $\Delta t_m = 7300$ or 5 years within the context of this simulation, since it can be argued that after becoming a bilingual, a bilingual should be able to retain their status without using the learned language again for at least some years. Δt_m works similarly to Δt_b because as people interact and reinforce their knowledge of the obtained language their memory is also refreshed.

- For K, the number of contacts needed to learn a language, and L, the number of contacts needed to retain a language without using it during time interval Δt_m, we also use the reference value Δt_b, since selecting a value for K that is too close to Δt_b, we can assume there will little or no chance for someone to realistically become a bilingual. Thus, we set $K < \Delta t_b$. As a natural assumption, an agent uses a language at least once a day in order to learn a new one over the course of a year and we also naturally assume, that $L < K$, where L should at least have a minimum value $L \approx \frac{K}{4} \approx 92$. Therefore, we can summarize it in the following fashion for all values that were checked:

$$365 \leqslant K < 1460$$
$$92 \leqslant L < K$$

All of the above described parameters and the values that were used in the symmetrical model within Section 4.1 and in asymmetrical model within Section 4.2 are summarized in Table 4.1.

Short summary of study parameters

Description	Parameter	Initial values
Concentrated unit of time	Δt	1
Assumed unit for a day	Δt_{DAY}	$4 \cdot \Delta t$
Assumed unit for a year	Δt_{YEAR}	$365 \cdot \Delta t_{\mathrm{DAY}}$
Maximum number of iterations simulation would ideally run	Δt_{MAX}	$100 \cdot \Delta t_{\mathrm{YEAR}}$
Number of interactions needed to learn a new language	K	$365 \leqslant K < \Delta t_{\mathrm{YEAR}}$
Time allowed for learning a new language without refreshing memory	Δt_b	Δt_{YEAR}
Number of interactions needed after becoming bilingual to remember a language	L	$92 \leqslant L < K$
Maximum time allowed after becoming a bilingual without memory refreshing, after which L will be checked, if a language is forgotten or not	Δt_m	$5 \cdot \Delta t_{\mathrm{YEAR}}$

Table 4.1: Short summary of parameters used in the study.

4.1 Symmetrical model

In a symmetrical model both competing languages are considered to be equally difficult to acquire and can therefore be characterized with a single parameter K.

4.1.1 Random and memory-based decision making for bilinguals

We observe that for a population distribution, $N_X + N_Y + N_{XY} = 1$ where $N_X(0) = N_Y(0)$ both languages over the course of a generation will survive indefinitely within a completely bilingual population $N_{XY} = 1$. The time evolution of such a population is shown on Figure 4.1a.

For population where either $N_X(0) > N_Y(0)$ or $N_X(0) < N_Y(0)$, we can observe that populations can also survive arbitrarily long with the caveat, that the minority language will do so within the population of N_{XY} and the majority language will exist on its own, as it can be seen on Figure 4.1b. Within these types of symmetrical interactions it can be shown that one of the languages can eventually take over if the population of minority language representatives is below a critical threshold N^*. For $N = 100$ it was found to be about $N^* = 0.02$. These results hold both for a system, where all decisions about language selection are made randomly, and where memory-based decisions are made by taking into account the previous interactions.

We also checked the model in the limit of no memory within the dynamics, in which we recovered similar results to the Minett and Wang model.

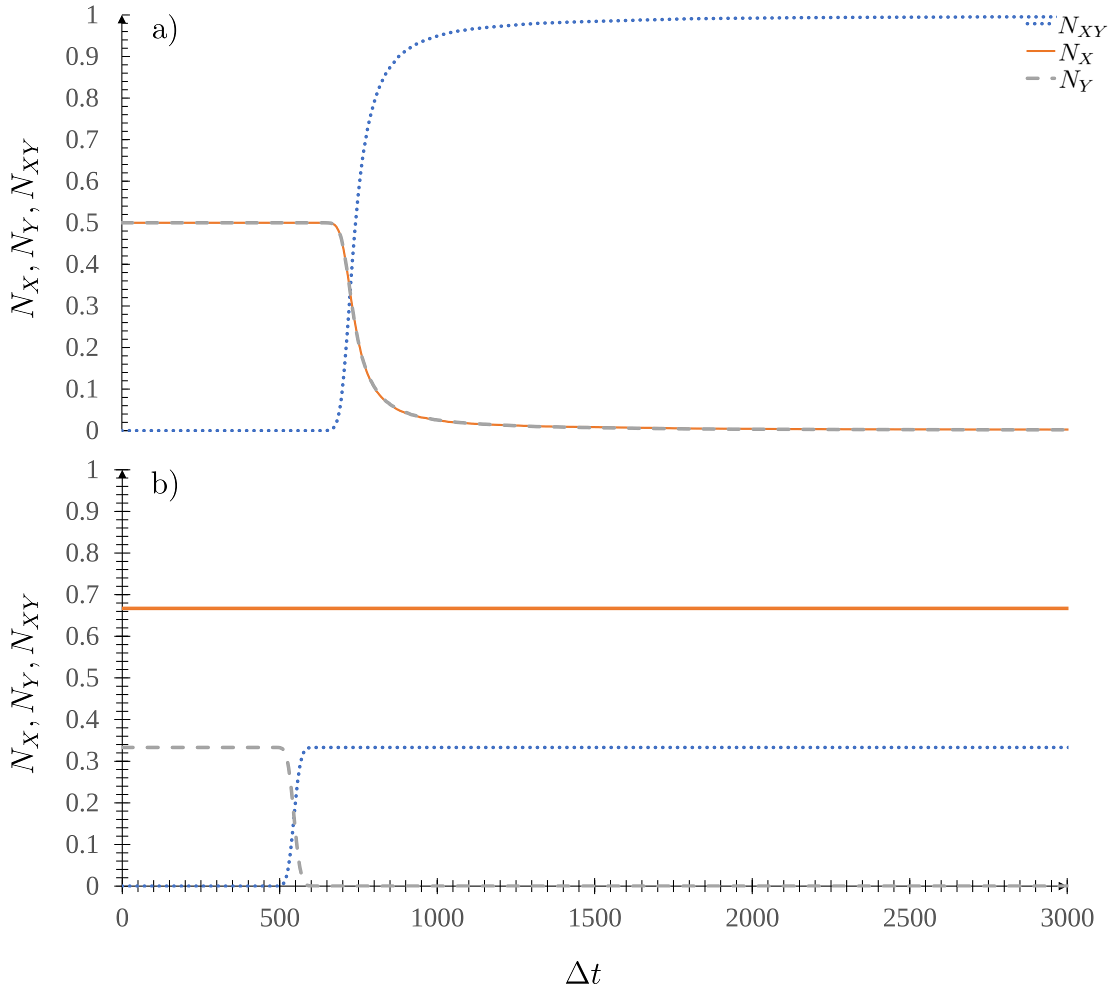

Figure 4.1: Time evolution of population fractions above a) for initial population ratio between X and Y, $N_X/N_Y = 1/2$ and b) $N_X/N_Y = 1/3$. Only the first 3000 iterations have been shown.

4.2 Asymmetrical model

Results were different when an additional layer of asymmetry was added into the dynamics by splitting the number of interactions needed to learn a new language, K, into K_X and K_Y within the interaction matrix (3.1), for the respective language. In other words, when different levels of difficulty in acquiring a language is given to language X and Y. This produces interesting non-monotonous dependence of the final state on the difference between K_X and K_Y. In all the examples considered, language X is easier one with $K_Y > K_X$.

4.2.1 Random-based decision making for bilinguals

Initial populations of $\boldsymbol{N_X(0) = N_Y(0) = 0.5}$. When $N_X(0) = N_Y(0)$ the simulations converged to a point where the language that was easier to acquire became the only monolingual population and the other survived as part of N_{XY}. As expected, it can be noted that when $K_X = K_Y$, as was the case in a symmetrical simulation, the simulation ended with everyone becoming bilingual. However, defining a difference between K_X and K_Y as

$$\Delta K_{XY} = |K_X - K_Y|, \tag{4.1}$$

for values where

$$1 \leqslant \Delta K_{XY} \leqslant 53$$

one could see a steady decline in the number of bilinguals N_{XY} and the slow increase in the population whose language was easier to acquire, until an dynamical equilibrium is reached, where bilinguals $N_{XY} = 50\%$, the more complicated language had disappeared in terms of monolingual representatives and the easier language exists as the remaining 50%. This is illustrated on Figure 4.2a.

Initial population of $\boldsymbol{N_X(0) = 0.4}$ and $\boldsymbol{N_Y(0) = 0.6}$. When the simulation was also taking into account the difference between the number of original speakers of X or Y, in this case they were distributed so that N_X made up 40% of the total population and N_Y made up 60% of the total population N, a change in ΔK_{XY} was observed expectedly, due to one of the population having a clear advantage over the other in terms of the number of speakers alone. As was the case in the symmetrical simulations, one of the populations in its pure form quickly disappeared and survived within the population of N_{XY} proportionally. However, once ΔK_{XY} reached the following range

$$107 \leqslant \Delta K_{XY} \leqslant 243$$

a slow decline within the monolingual representatives of the majority population was observed, eventually reaching a point of inversion, where the minority representatives survived as monolinguals and all others had to become part of bilinguals N_{XY}. Within the inversion process a peak value for bilinguals N_{XY} was observed at $\Delta K_{XY} = 183$ when the minimum K_X value for language X was taken to be 365, where approximately all members of N were bilingual. It was further observed, that for all initial lower K values a ratio of about $\frac{3}{2}$ can be detected when looking for a value of ΔK_{XY}, where the number of bilinguals would reach a maximum. This is a rather natural result when one thinks of the ratio between number of speakers in both initial monolingual population (for example for $N_X = 40$ and $N_Y = 60$, then the ratio is $\Delta K_{XY}^{max} = \frac{60}{40} = \frac{3}{2}$). This is illustrated on Figure 4.3a.

Initial population of $N_X(0) = 1/3$ and $N_Y(0) = 2/3$. The general dynamics were similar to the case where one of the initial populations formed 40% of the total population N. Therefore we only summarize the different values of ΔK_{XY}, within which the inversion process took place and give the peak ratio at,

$$255 \leqslant \Delta K_{XY} \leqslant 437, \quad \Delta K_{XY}^{max} = \frac{67}{33}.$$

This is illustrated on Figure 4.4a.

Initial population of $N_X(0) = 1/4$ and $N_Y(0) = 3/4$. The dynamics observed for a population ratio where $N_X = 25\%$ (see Figure 4.5a), gave the results following,

$$545 \leqslant \Delta K_{XY} \leqslant 799, \quad \Delta K_{XY}^{max} = \frac{75}{25}.$$

Furthermore, it can be shown that these changes in both ΔK_{XY} and the ratio ΔK_{XY}^{max} hold for other fractions of the population as well such as $\frac{1}{5}, \frac{1}{6}$ and etc. In general, ΔK_{XY}^{max} turns out be as the ratio between the two initial populations, resulting in a multiplier that one can use on the K value for the minority population to find ΔK_{XY}, where the maximum number of bilinguals will emerge in an asymmetrical model. Considering this and using $N = N_X + N_Y$ with equation (4.1) we can write:

$$\begin{aligned}\Delta K_{XY}^{max} = \frac{N_{\text{majority}}}{N_{\text{minority}}} \Rightarrow \Delta K_{XY}|_{N_{XY}=max} &= K_{\text{minority}}(\Delta K_{XY}^{max} - 1) \\ &= K_{\text{minority}}\left(\frac{N}{N_{\text{minority}}} - 2\right).\end{aligned}$$

4.2.2 Memory-based decision making for bilinguals

Because the overall tendency of the dynamics was very similar to simulations, where the language chosen between bilinguals was decided by chance, then we can summarize the results as follows

Population ratio	Inversion range	Peak ratio ΔK_{XY}^{max} within the inversion range
50%	$1 \leqslant \Delta K_{XY} \leqslant 66$	1
40%	$163 \leqslant \Delta K_{XY} \leqslant 246$	3/2
33.33%	$360 \leqslant \Delta K_{XY} \leqslant 451$	67/33
25%	$720 \leqslant \Delta K_{XY} \leqslant 807$	3

When comparing the values where inversion took place, one can notice that the values for ΔK_{XY} are quite different than before, resulting in a much smaller area where such transition took place. Also, after entering the inversion range, the maximum number of bilinguals is reached rather quickly followed by a slow convergence toward a situation, where all members of N_{XY} are the initial members of the majority population. This is contrary to the case that could be observed in a purely random system without a memory-based decision making process for the bilinguals, where a gradual increase in the number of bilinguals could be noted after entering the inversion range and then after going past the maximum value for bilinguals, a quicker conclusion was reached, where the majority population made up the entire N_{XY} and the initial minority continued to exist as their own monolingual group. It can also be noted from the results, that while the random asymmetrical model started to gradually shift toward a society with maximum number of bilinguals and then towards the reversal of initial groups, the dynamics in memory asymmetrical were completely different. For the memory asymmetrical model, ΔK_{XY} values seem to produce a much quicker and a more violent inversion in the group roles. Additionally, the range of inversion is typically also smaller than it is for the completely random asymmetrical model. All graphs illustrating the inversion ranges can be found on Figures 4.2b, 4.3b, 4.4b and 4.5b for population ratios of 50%, 40%, 33.33% and 25%, respectively.

Varying the parameter L

Due to the fact that in our model both Δt_b and Δt_m are dynamic parameters in the sense that they are being continually refreshed during interactions as explained before and the considerably large value used for Δt_m, then within the parameter values allowed for L in the context of this simulation L played little to no role. Therefore, once a person was able to become a bilingual, then due to continued use and constant refreshing of their memory, they never went back to being a monolingual of either language unless extreme conditions were met. For example, when an agent doesn't have enough contact with representatives of their own language as would be the case where N_{majority} is sufficiently larger than N_{minority}.

Random- versus memory-based decisions for bilinguals

From these results we could see that despite the vast difference between making a decision by pure random chance and via complicated decision matrix, no real difference can be noted in the final results. When taking a detailed look at individual time intervals and the state of the interaction matrix (Section 3.3), we could see that while initially as populations started becoming bilingual, all variants of the memory-based decision matrices were used to reach a consensus between two agents about which language to use. However, as time advanced, almost all agents on average still used their own original language more than the learned one and thus their following decisions were again made by a random chance (Decision matrix in the second row, third column in Table A.1). This result can most likely be explained by viewing how agents are located within the simulation as they form two large groups and in an interaction between two bilinguals who originate from the same population, their original language is always preferred. Since this interaction has a high probability of happening, then it is natural that eventually a completely random regime must be achieved. It can be hypothesized, that if the agents were either moving around or made to interact on a 2D lattice (only with their immediate neighbours), then the results could be different.

For a model with asymmetrical learning, in general, similar tendency could be observed for ΔK_{XY} values not within the range of inversion. For values within the inversion range the transition from one dominant language group to the other seems to be a lot smoother for the model, where decisions are made by random chance. For models where previous memory was taken into account for decision making, more noise could be seen prior to reaching maximum value of bilingual population within the transition and then a slow convergence to the reversal of initial proportions.

In general, it could be concluded that memory-based decision making adds a layer of inertia into the dynamics as one can seen from the nearly twice smaller inversion ranges, when compared to a model where all decisions were made by chance. However, when a critical value for ΔK_{XY} was reached, which was larger than in the completely random decision making model, the transition appeared to be quite rapid followed by a slow stabilization.

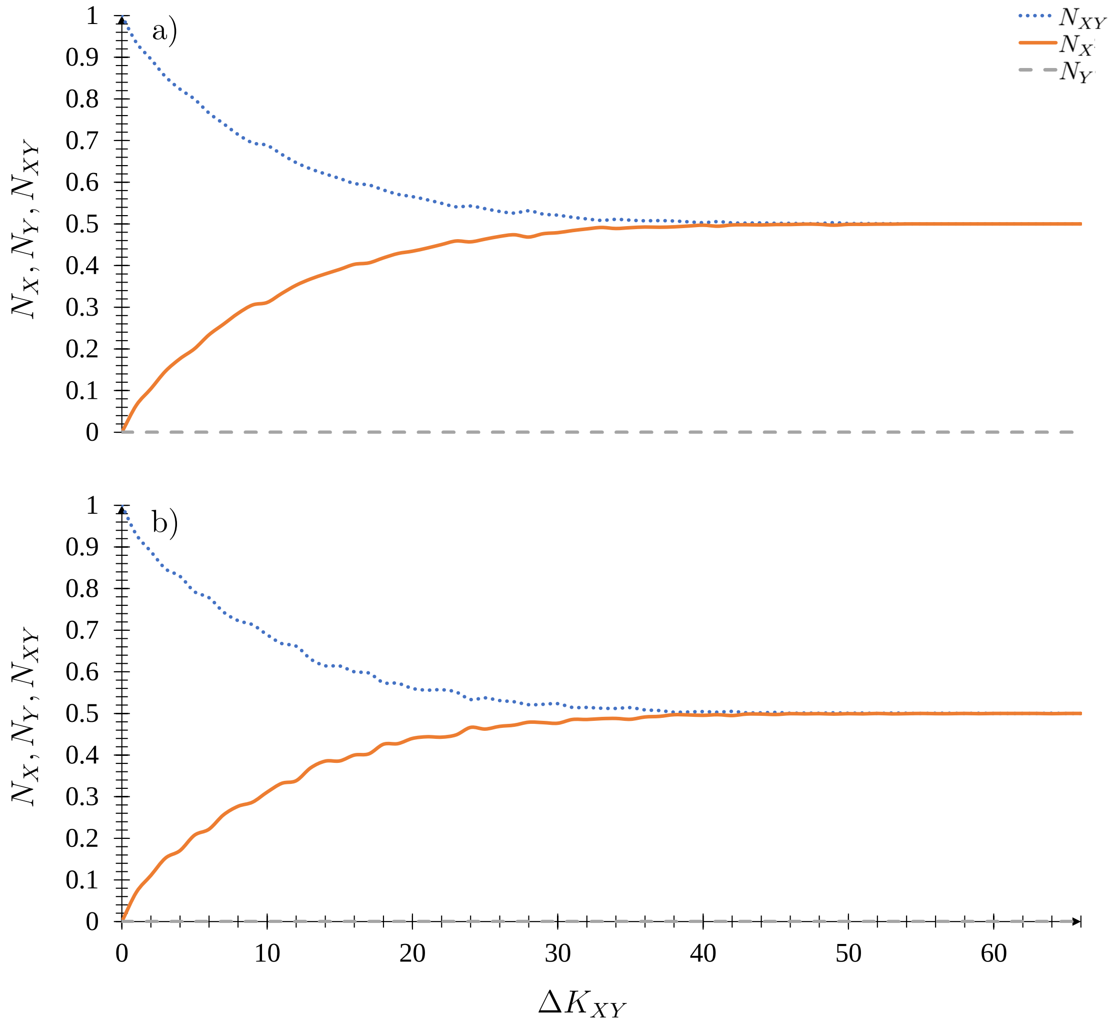

Figure 4.2: Population fractions versus ΔK_{XY} with initial conditions $N_X/N_Y = 1/2$ for a) random-based decision making (above) and b) memory-based decision making (below) model. For a) interesting values for ΔK_{XY} were found in the range $1 \leqslant \Delta K_{XY} \leqslant 53$ and for b) the corresponding values were $1 \leqslant \Delta K_{XY} \leqslant 66$. It can be seen that within these values the majority language starts to lose its dominance as the monolingual population and is eventually forced to become the sole bilingual community, while the initial minority language can survive on as a monolingual group.

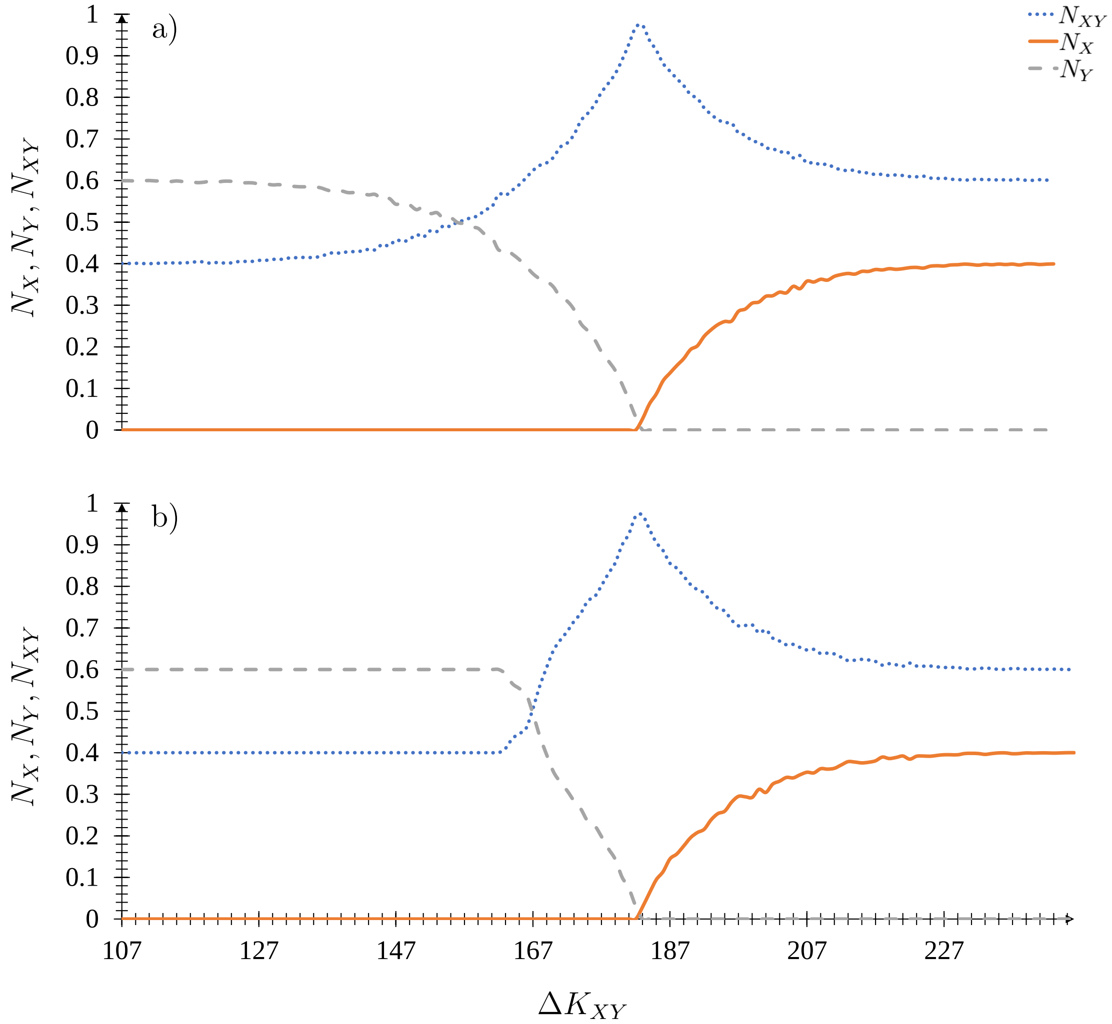

Figure 4.3: Population fractions versus ΔK_{XY} with initial conditions $N_X/N_Y = 2/5$ for a) random-based decision making (above) and b) memory-based decision making (below) model. For a) interesting values for ΔK_{XY} were found in the range $107 \leqslant \Delta K_{XY} \leqslant 243$ and for b) the corresponding values were $163 \leqslant \Delta K_{XY} \leqslant 246$. It can be seen that within these values the majority language starts to lose its dominance as the monolingual population and is eventually forced to become the sole bilingual community, while the initial minority language can survive on as a monolingual group.

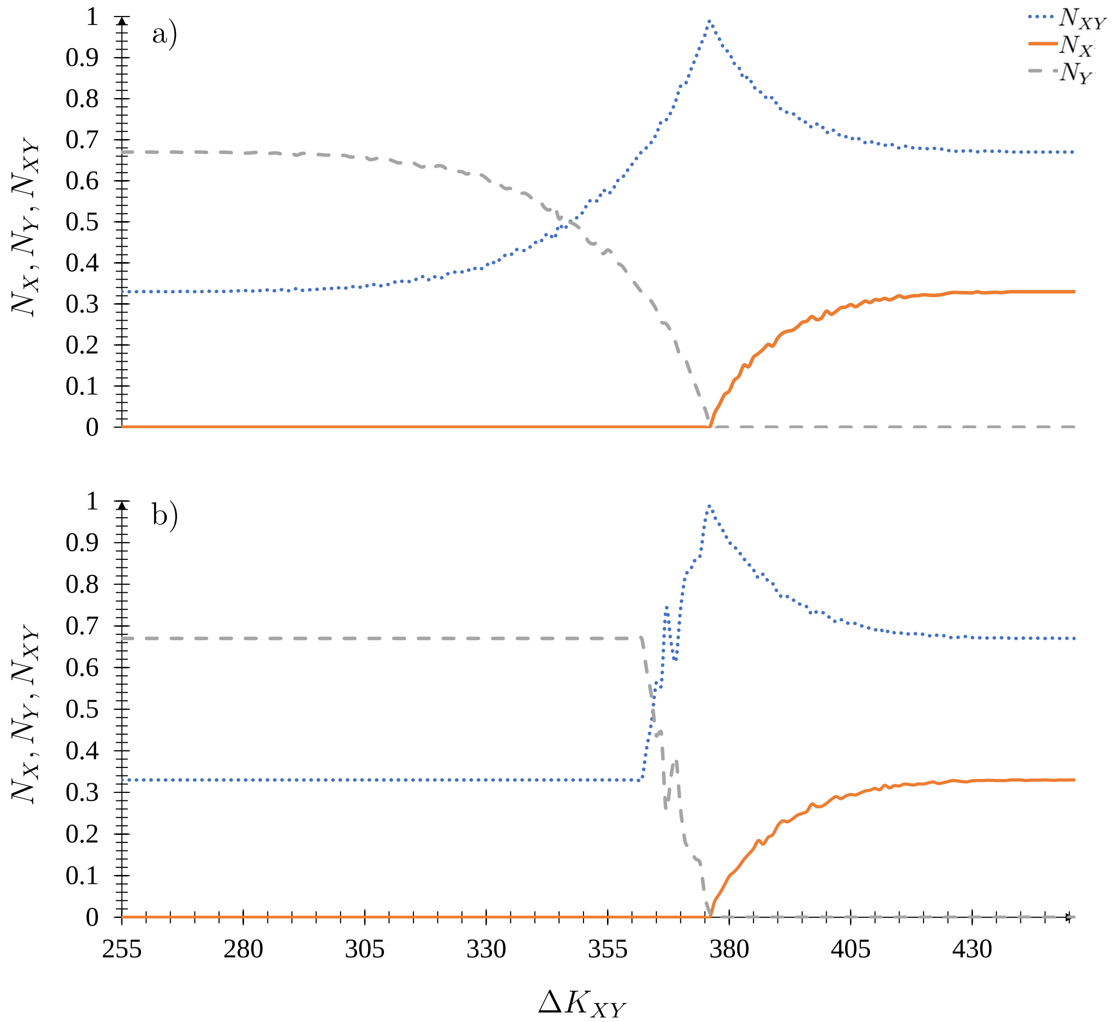

Figure 4.4: Population fractions versus ΔK_{XY} with initial conditions $N_X/N_Y = 1/3$ for a) random-based decision making (above) and b) memory-based decision making (below) model. For a) interesting values for ΔK_{XY} were found in the range $255 \leqslant \Delta K_{XY} \leqslant 437$ and for b) the corresponding values were $360 \leqslant \Delta K_{XY} \leqslant 451$. It can be seen that within these values the majority language starts to lose its dominance as the monolingual population and is eventually forced to become the sole bilingual community, while the initial minority language can survive on as a monolingual group.

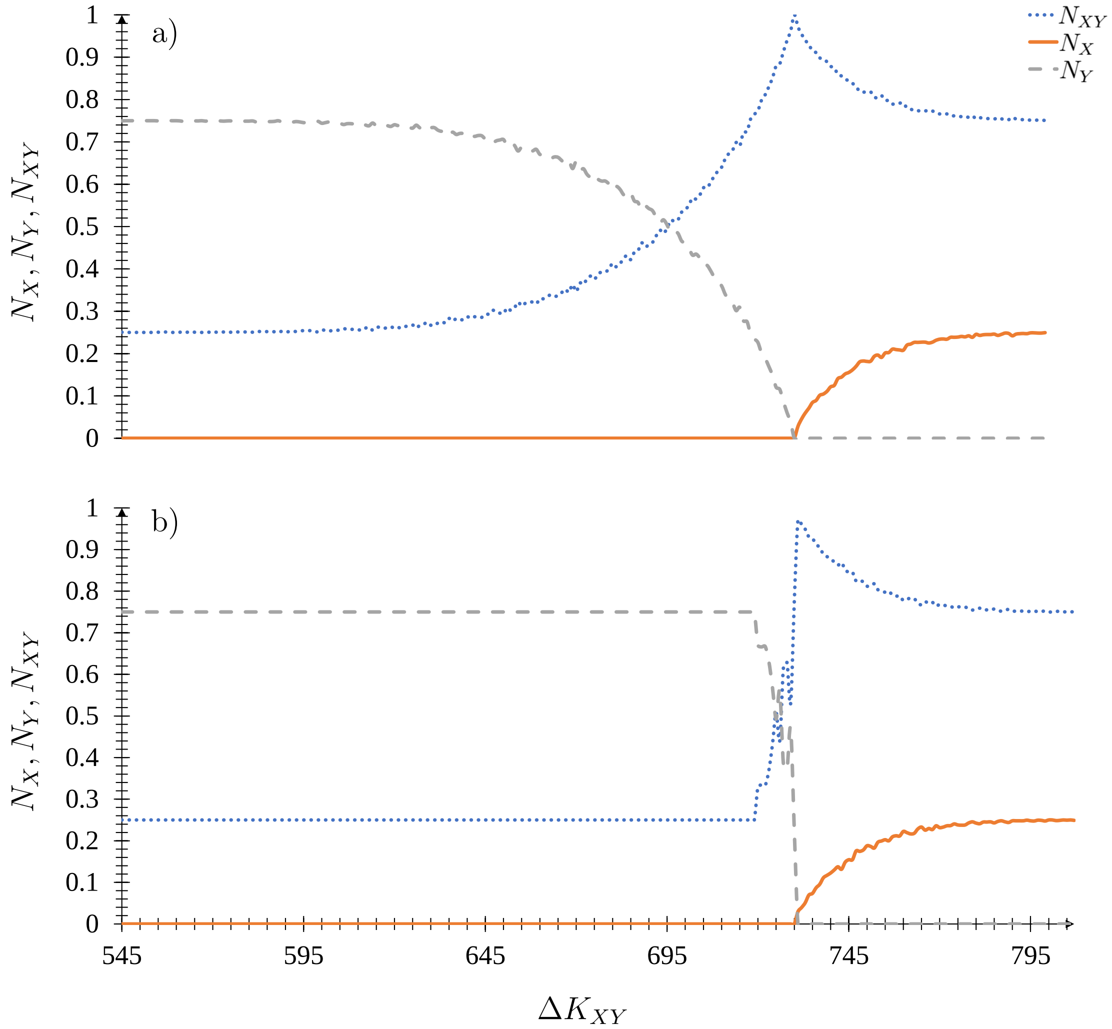

Figure 4.5: Population fractions versus ΔK_{XY} with initial conditions $N_X/N_Y = 1/4$ for a) random-based decision making (above) and b) memory-making decision making (below) model. For a) interesting values for ΔK_{XY} were found in the range $545 \leqslant \Delta K_{XY} \leqslant 799$ and for b) the corresponding values were $720 \leqslant \Delta K_{XY} \leqslant 807$. It can be seen that within these values the majority language starts to lose its dominance as the monolingual population and is eventually forced to become the sole bilingual community, while the initial minority language can survive on as a monolingual group.

5. Possible developments and summary

5.1 Proposals for model improvement

Due to the simple structure of the model studied, numerous natural improvements can be added for further investigations.

1. *Connectedness.* In the current model all agents are fully connected or in other words, everyone is a neighbour of everyone else. Therefore no geographical or other spatial effects are considered. Further models can improve upon this matter by locating the agents on a 2D lattice or a complex network for example, where they can only interact with their first neighbours. It can be advanced even more by implementing agents that also travel around in small steps and are thus forced to change their adjacent partners.

2. *Population dynamics.* The current model only describes the first hundred years of a population that is essentially immortal within this time interval. Therefore no mortality of agents is accounted for. In a more advanced model, this can also be an additional factor in the dynamics and can essentially model how language is passed on from a parent to a child and what percentage of it survives. One can here think of an example where the first generation of immigrants to a country with another language may retain their own language and acquire a new one in order to survive and their offspring, who will likely understand their heritage language but usually will not use it in everyday interaction. Then it can happen, that by the third generation offsprings, the heritage language has probably disappeared within that community.

3. *Probability of learning.* Our model currently assumes that everyone in the population, when in contact with a representative of another language is willing and able to learn a fraction of a new language. However, in real life, this is highly improbable and therefore at least at first, only some members of the population should be open to learning and after a certain threshold is reached, this willingness to learn should spread to others. In other terms it can be viewed as the gradual increase of a certain languages status or prestige or even usefulness.

4. *Effectiveness of learning.* It is natural to assume, that in most cases, a single contact with another language does does not result in absolutely effective incrementation of K. For example, for some it can be difficult to grasp and remember new information. A clear example of it could be the word *supercalifragilisticexpialidocious*, which is another way of saying extraordinarily good or wonderful but could also be seen as a rather difficult word to learn by few simple tries for every agent. Thus a different probability of learning effectiveness p_l can essentially be assigned to every agent. Although this factor is already accounted for by the minimum values of K and how time is defined in our model, it can still be further improved upon.

5. *Effectiveness of retaining.* The same idea can be assigned to the parameter L in a way that there is a different probability p_r for each agent, by which L decreases in value 'spontaneously' and can account for the natural phenomena of how some information may be 'there', but is eerily difficult to access lexically or in terms of overall memory.

5.2 Summary

In this work we first recalled the prior investigations done by Abrams and Strogatz, we took a look at the Minett and Wang model and how game-theory has been used by Iriberri and Uriarte to model language competition. Additionally, a look at how linguists view the acquisition and attrition processes of languages was described.

In Chapter 3, we introduced a detailed description of a new model, with the goal of studying memory effects in language competition. We found, that in this model memory plays a critical role in the dynamics between two competing languages. We found that both languages can survive indefinitely, provided that all agents form a fully connected graph (everyone is connected to everyone), they are open to acquiring a language from their partnering agents and that as they do so, their memory is continually being refreshed.

For symmetrical learning models, where neither language was more difficult to acquire than the other, everyone became a bilingual if the initial population ratio was set at 50%. However, for other population ratios the language representatives who held the majority stayed monolingual, while the minority formed the bilingual group N_{XY}, but their language was not driven into extinction by the majority. Only when the majority population was overwhelmingly larger than the minority, then the minority language was driven to disappear. Interestingly these conclusions hold for both a purely random-decision making process and a memory-based decision making process for bilinguals.

For asymmetrical models, where K was split into K_X and K_Y for the two languages X and Y, we could observe similar results: continual memory refreshing and interaction produced either completely bilingual populations for a population ratio of 50% or other ratios resulted in the monolingual groups of the majority population staying monolingual and the minority population forming the entirety of bilingual population. In any case, neither language X or Y was observed to disappear. However, it was further observed, when languages hold varying level of difficulty in acquisition, $\Delta K_{XY} = |K_X - K_Y| > 0$, compared to each other, that a range of inversion was revealed, where the minority population began gaining numbers of speakers until eventually overtaking the majority population.

Additionally, within the range of inversion, a peak value for bilinguals among two competing asymmetrical language representatives with a difference between the number of speakers was observed to be associated with a coefficient $\Delta K_{XY}^{max} = N_{\text{majority}}/N_{\text{minority}}$.

While we can clearly see that two languages can indeed survive within bilingual communities as their memories are being refreshed, this is also a very simple model and the various mentioned proposals for further work can clarify and enlarge the results obtained.

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A. Decision matrices

This section contains all the possible variants for the decision matrices and the interactions that will be decided from them.
Letting a, b, c, and d to be $1, 2, 3$ and 4 respectively, the decisions will be made as follows for the first set of decision matrices where $a \neq b \neq c \neq d$:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 1 & 3 \\ 4 & 4 \end{bmatrix} & \text{if chose Y} \end{cases}, \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 4 & 1 \\ 4 & 3 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 3 & 2 \\ 5 & 2 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 4 & 4 \\ 3 & 1 \end{bmatrix} & \text{if chose Y} \end{cases}, \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 3 & 4 \\ 1 & 4 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 2 & 5 \\ 2 & 3 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix} 3 & 1 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 3 & 3 \\ 5 & 1 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 1 & 5 \\ 3 & 3 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix}1&2\\4&3\end{bmatrix}\xrightarrow{\text{Decides for Y}}\begin{bmatrix}1&3\\5&3\end{bmatrix},\begin{bmatrix}4&1\\3&2\end{bmatrix}\xrightarrow{\text{Select X or Y}}\begin{cases}\begin{bmatrix}5&1\\3&3\end{bmatrix}&\text{if chose X}\\[2ex]\begin{bmatrix}4&2\\4&2\end{bmatrix}&\text{if chose Y}\end{cases}$$

$$\begin{bmatrix}3&4\\2&1\end{bmatrix}\xrightarrow{\text{Decides for Y}}\begin{bmatrix}3&5\\3&1\end{bmatrix},\begin{bmatrix}2&3\\1&4\end{bmatrix}\xrightarrow{\text{Select X or Y}}\begin{cases}\begin{bmatrix}3&3\\1&5\end{bmatrix}&\text{if chose X}\\[2ex]\begin{bmatrix}2&4\\2&4\end{bmatrix}&\text{if chose Y}\end{cases}$$

$$\begin{bmatrix}3&2\\1&4\end{bmatrix}\xrightarrow{\text{Decides for X}}\begin{bmatrix}4&2\\1&5\end{bmatrix},\begin{bmatrix}1&3\\4&2\end{bmatrix}\xrightarrow{\text{Decides for Y}}\begin{bmatrix}1&4\\5&2\end{bmatrix}$$

$$\begin{bmatrix}4&1\\2&3\end{bmatrix}\xrightarrow{\text{Decides for X}}\begin{bmatrix}5&1\\2&4\end{bmatrix},\begin{bmatrix}2&4\\3&1\end{bmatrix}\xrightarrow{\text{Decides for Y}}\begin{bmatrix}2&5\\4&1\end{bmatrix}$$

$$\begin{bmatrix}1&4\\3&2\end{bmatrix}\xrightarrow{\text{Decides for Y}}\begin{bmatrix}1&5\\4&2\end{bmatrix},\begin{bmatrix}3&1\\2&4\end{bmatrix}\xrightarrow{\text{Decides for X}}\begin{bmatrix}4&1\\2&5\end{bmatrix}$$

$$\begin{bmatrix}2&3\\4&1\end{bmatrix}\xrightarrow{\text{Decides for Y}}\begin{bmatrix}2&4\\5&1\end{bmatrix},\begin{bmatrix}4&2\\1&3\end{bmatrix}\xrightarrow{\text{Decides for X}}\begin{bmatrix}5&2\\1&4\end{bmatrix}$$

$$\begin{bmatrix}1&3\\2&4\end{bmatrix}\xrightarrow{\text{Select X or Y}}\begin{cases}\begin{bmatrix}2&3\\2&5\end{bmatrix}&\text{if chose X}\\[2ex]\begin{bmatrix}1&4\\3&4\end{bmatrix}&\text{if chose Y}\end{cases},\begin{bmatrix}2&1\\4&3\end{bmatrix}\xrightarrow{\text{Select X or Y}}\begin{cases}\begin{bmatrix}3&1\\4&4\end{bmatrix}&\text{if chose X}\\[2ex]\begin{bmatrix}2&2\\5&3\end{bmatrix}&\text{if chose Y}\end{cases}$$

$$\begin{bmatrix}4&2\\3&1\end{bmatrix}\xrightarrow{\text{Select X or Y}}\begin{cases}\begin{bmatrix}5&2\\3&2\end{bmatrix}&\text{if chose X}\\[2ex]\begin{bmatrix}4&3\\4&1\end{bmatrix}&\text{if chose Y}\end{cases},\begin{bmatrix}3&4\\1&2\end{bmatrix}\xrightarrow{\text{Select X or Y}}\begin{cases}\begin{bmatrix}4&4\\1&3\end{bmatrix}&\text{if chose X}\\[2ex]\begin{bmatrix}3&5\\2&2\end{bmatrix}&\text{if chose Y}\end{cases}$$

For the set of matrices where $a = b \neq c \neq d$:

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 4 & 1 \\ 4 & 2 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 2 & 4 \\ 1 & 4 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix} \xrightarrow{\text{Decides for Y}} \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 4 & 2 \\ 4 & 1 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \xrightarrow{\text{Decides for Y}} \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases} \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} & \text{if chose X} \\ \begin{bmatrix} 1 & 4 \\ 2 & 4 \end{bmatrix} & \text{if chose Y} \end{cases}$$

$$\begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix} \xrightarrow{\text{Decides for Y}} \begin{bmatrix} 1 & 5 \\ 4 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} \xrightarrow{\text{Decides for Y}} \begin{bmatrix} 1 & 4 \\ 5 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}$$

For the case where $a = b \neq c = d$:

$$\begin{bmatrix}1 & 1\\4 & 4\end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases}\begin{bmatrix}2 & 1\\4 & 5\end{bmatrix} & \text{if chose X}\\ \\ \begin{bmatrix}1 & 2\\5 & 4\end{bmatrix} & \text{if chose Y}\end{cases}, \begin{bmatrix}4 & 1\\4 & 1\end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases}\begin{bmatrix}5 & 1\\4 & 2\end{bmatrix} & \text{if chose X}\\ \\ \begin{bmatrix}4 & 2\\5 & 1\end{bmatrix} & \text{if chose Y}\end{cases}$$

$$\begin{bmatrix}4 & 4\\1 & 1\end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases}\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix} & \text{if chose X}\\ \\ \begin{bmatrix}4 & 5\\2 & 1\end{bmatrix} & \text{if chose Y}\end{cases}, \begin{bmatrix}1 & 4\\1 & 4\end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases}\begin{bmatrix}2 & 4\\1 & 5\end{bmatrix} & \text{if chose X}\\ \\ \begin{bmatrix}1 & 5\\2 & 4\end{bmatrix} & \text{if chose Y}\end{cases}$$

$$\begin{bmatrix}4 & 1\\1 & 4\end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix}5 & 1\\1 & 5\end{bmatrix}, \begin{bmatrix}4 & 1\\1 & 4\end{bmatrix} \xrightarrow{\text{Decides for Y}} \begin{bmatrix}5 & 1\\1 & 5\end{bmatrix}$$

For the case where $a = b = c \neq d$:

$$\begin{bmatrix}1 & 1\\1 & 4\end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix}2 & 1\\1 & 5\end{bmatrix}, \begin{bmatrix}1 & 1\\4 & 1\end{bmatrix} \xrightarrow{\text{Decides for Y}} \begin{bmatrix}1 & 2\\5 & 1\end{bmatrix}$$

$$\begin{bmatrix}4 & 1\\1 & 1\end{bmatrix} \xrightarrow{\text{Decides for X}} \begin{bmatrix}5 & 1\\1 & 2\end{bmatrix}, \begin{bmatrix}1 & 4\\1 & 1\end{bmatrix} \xrightarrow{\text{Decides for Y}} \begin{bmatrix}1 & 5\\2 & 1\end{bmatrix}$$

And finally where $a = b = c = d$:

$$\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \xrightarrow{\text{Select X or Y}} \begin{cases}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix} & \text{if chose X}\\ \\ \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} & \text{if chose Y}\end{cases}$$

Table A.1: Summary of how decisions to use language X, language Y or for picking one randomly are made for bilinguals using logic operators. Variables $a, b, c, d \in \mathbb{N}_0$ represent the number of times the corresponding agent has used their native (a, c) or learned (b, d) language.

Use language X	**Use language Y**	**X or Y randomly**
$\begin{bmatrix} a > b \\ c = d \end{bmatrix}$	$\begin{bmatrix} a < b \\ c = d \end{bmatrix}$	$\begin{bmatrix} a = b \\ c = d \end{bmatrix}$
$\begin{bmatrix} a = b \\ c < d \end{bmatrix}$	$\begin{bmatrix} a = b \\ c > d \end{bmatrix}$	$\begin{bmatrix} a > b \\ c > d \end{bmatrix}$
$\begin{bmatrix} a > b \\ c < d \end{bmatrix}$	$\begin{bmatrix} a < b \\ c > d \end{bmatrix}$	$\begin{bmatrix} a < b \\ c < d \end{bmatrix}$

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