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Estimation of a Stochastic Spatio-temporal Model of the Flow-front Dynamics with Varying Parameters

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Abstract. For control and monitoring purposes, knowledge of the current state of the flow-front in a vacuum assisted resin transfer moulding (VARTM) process is essential. The permeability of the medium and viscosity of the epoxy can change during the infusion process. Especially for online monitoring of the infusion process there is a need for a fast and fairly accurate, possibly virtual sensor system which can handle such parameter variations. Stochastic-differential equations (SDEs) based estimation of the flow-front dynamics can offer a good trade-off between physics and data-driven estimators. In this paper, we analyze the effect of parameter variations on an SDE based spatio-temporal estimator of the flow-front dynamics in a VARTM process.

Introduction

Large scale composite shell structures like wind turbine blades are manufactured using a VARTM process. Some blades manufactured at the *Siemens Gamesa Renewable Energy* factories are casted as one piece using the patented *IntegralBlades*® technology [1]. The main disadvantage of this technology is that, there is no possibility to visually inspect the infusion process. Furthermore, variations in the material layup can cause an inhomogeneous flow of the epoxy resin inside a blade mould. Inhomogeneities in the flow-front increases the risk of casting defects such as voids or dry spots which may deteriorate the structural properties of the blade [2]. Hence, estimation of the flow-front during the infusion process is essential for online monitoring of the process.

In addition, micro-structural properties, operating conditions such as temperature, change in pressure can affect the permeability and viscosity of the epoxy during the infusion process. Estimating a good model for the change in viscosity of the epoxy during the infusion process is not trivial. Similarly estimation of the permeability model through numerical simulation or experimental characterization of the permeability is a cumbersome task [3, 4]. Previously, we used the partial differential equations (PDE) (derived using the Darcy's law) based flow-front model for our simulation study [5] and proposed a parsimonious grey-box model of the flow-front dynamics using coupled SDEs [6]. However, in our previous work we did not address the problem of parameter variation such as e.g. continuously increasing viscosity and random perturbations of the permeability. In this paper, we analyze the effect of such parameters variation on the performance of coupled SDEs based estimator of the flow-front.

Flow in a Porous Medium

To generate the flow patterns of the flow-front, the dynamics of the flow-front progression in a VARTM process is simulated by considering the epoxy as a Newtonian fluid and applying the Darcy's law in two spatial dimensions with relevant boundary conditions. This flow-front pattern data is a good representative of the data which can be measured experimentally using commercially available sensors and instrumentation [5]. By applying the Darcy's law in two spatial dimensions the pressure field as a function of space and time can be described as:

$$\dot{h} = \frac{\mathrm{d}h}{\mathrm{d}p}\dot{p} = \nabla \cdot \left(\frac{\kappa H}{\mu}\nabla p\right) \tag{1}$$

where $h = h(x, y, t) \leq H$ is the thickness of the fluid layer and $\dot{h}$ indicates time derivative of h, $\kappa = \kappa(x, y, t)$ is the permeability of the porous medium, H is a combined porosity and cross-sectional thickness measure, μ is the dynamic viscosity of the fluid, $p = p(x, y, t)$ is the pressure, and $\nabla = (\partial_x, \partial_y)$ is the spatial gradient. Using the relationship $h(p) = \min(H, p/(\rho g))$, the PDE (1) governs the pressure p. The boundary conditions are a pressure of zero at the outlet, a pressure of $p_0 = 1$ bar at the inlet, and no flux along the sides. The PDE model simplifies the physics in the flow-front itself; this is justified as the material transport is mainly governed by the flow behind the front, where the PDE reduces to the Laplace equation in p. The PDE is solved by using the *FEniCS* software [7] for a 80 cm × 90 cm rectangular casting. The numerical solution is obtained by time marching, using a semi-implicit Euler step where the derivative $\frac{dh}{dp}$ is evaluated at the previous time step, and the right hand side is evaluated at the next time step. Note that $\frac{dh}{dp}$ vanishes in those parts of the domain that have already been filled with the fluid, so the system can be viewed as differential-algebraic. The numerically simulated data obtained is considered as the true data for this case study.

Simulating parameter variations

The manufacturing errors, less careful handling of the materials and the operating conditions may results in parameter variations (e.g. change in permeability and viscosity etc.) in the flow-front progression during the infusion process. To simulate the effect of these variations i.e. heterogeneity in the flow-front progression with respect to (w.r.t) time and displacement, κ is kept constant w.r.t. time and μ is kept constant w.r.t displacement such that

$$\kappa = \kappa(x_m, y_n) = \underbrace{\frac{c_0}{\left(1 - A \cdot \cos\left(\frac{2\pi x_m}{L_x}\right)\right)\left(1 - A \cdot \cos\left(\frac{2\pi y_n}{L_y}\right)\right)}}_{\kappa^*} + \mathfrak{w}(x_m, y_n) \tag{2}$$

where $\kappa(x_m, y_n) > 0$ is the permeability, $\mathfrak{w}(x_m, y_n)$ is a Gamma distributed perturbation i.e. $\mathfrak{w}(x_m, y_n) \sim \Gamma(\alpha, \beta)$ at spatial coordinate (x_m, y_n), α and β are shape and scale parameters of the distribution and $m = 1, \cdots M_x$; $n = 1, \cdots N_y$ are number of spatial discretisation points. Therefore the simulated permeability includes a spatial deterministic term κ^*, and a random perturbation $\mathfrak{w}(x_m, y_n)$. Furthermore, we introduce linearly increasing viscosity w.r.t. time i.e. $\mu(t) = \mu_0 + \mu_1 t$ here μ_0 is the initial viscosity and μ_1 is the viscosity change rate. It is clear that the formulation of the flow-front dynamics problem as a spatio-temporal modelling problem is very useful to simulate the propagation of the flow-front inside the mould for different scenarios and operating conditions. However, for control and monitoring purposes these high-dimensional PDEs based models are not well suited . Therefore in the section below, we propose to use a lumped model described as a set of coupled SDEs [6, 8].

Stochastic Spatio-Temporal Estimator

The SDEs are the preferred choice to model stochastic, complex, and nonlinear systems where only a partial information about the system dynamics is available. An SDE based state-space model can be written as [8, 9]:

$$dY_t = f(Y_t, U_t, t, \theta)dt + \sigma(U_t, t, \theta)dW_t \tag{3}$$

$$Z_k = h(Y_k, U_k, t_k, \theta) + e_k \tag{4}$$

The equations describing the dynamics of the states of the system, Y_t, are formulated in continuous-time and are separated in a drift term, $f(Y_t, U_t, t, \theta)$, and a diffusion term, $\sigma(U_t, t, \theta)$. Here W_t is a Wiener process of dimension d, with incremental covariance Q_t. The observations, Z_k, are linked to the states through the observation equation (4), which are typically formulated in discrete-time and include the measurement error e_k which is Gaussian white noise with covariance Σ_t. Here U_t represents the inputs and θ the parameters of the model. A clear separation of the residual error into diffusion and measurement noise results in a more correct description of the prediction error [8]. To obtain a parsimonious model coupled SDEs are used to model the flow-front dynamics for a limited amount of spatial discretisation points. The solution to the PDE in (1) in the homogeneous case, and for small H, can be written as:

$$p(x, y, t) = p_0 \cdot \max\left(0, 1 - \frac{y}{Y_t}\right), \tag{5}$$

where Y_t is the position of the front, given by:

$$\frac{dY_t}{dt} = \frac{\kappa p_0}{\mu} \frac{1}{Y_t}, \tag{6}$$

where the material is considered to be flowing in the direction of the y-axis throughout the domain. The spatial discretisation in the x direction is performed using M_x grid points. Let $Y_{n,t}$ denote the position of the front at the corresponding value of x_m co-ordinate. Considering no coupling (interaction) between neighbouring grid points, results in N ordinary differential equations. Furthermore, a constant diffusion term σ is introduced to capture the effect of the heterogeneity due to loose coupling between the lines and any difference between the model and the true system:

$$dY_{n,t} = \left(\frac{\kappa p_0}{\mu} \frac{1}{Y_{n,t}} + D \frac{Y_{n+1,t} - 2Y_{n,t} + Y_{n-1,t}}{(\Delta x_m)^2} \right) dt + \sigma_{n,t} dw \tag{7}$$

By defining $c_{0,n} = \frac{\kappa p_0}{\mu}$, and treating $\frac{D}{(\Delta x_m)^2}$ as a single global constant, D_0, for all state equations, the resulting equation is

$$dY_{n,t} = \left(\frac{c_{0,n}}{Y_{n,t}} + D_0 \cdot (Y_{n+1,t} - 2Y_{n,t} + Y_{n-1,t}) \right) dt + \sigma_{n,t} dw \tag{8}$$

Except for the boundary cases where it is modelled as

$$dY_{1,t} = \left(\frac{c_{0,1}}{Y_{1,t}} + D_0 \cdot (Y_{2,t} - Y_{1,t}) \right) dt + \sigma_{1,t} dw \tag{9}$$

This formulation, while a stretch from the original physics, has been shown to capture the essential dynamics of flow-front propagation [6] and are in a form suitable for online flow-front estimation.

Parameter Estimation

The parameters, $c_{0,n}$ and D_0, of the coupled SDEs are estimated from the data obtained by numerically simulating the PDE in (1) as described earlier. A maximum likelihood method in combination with an extended Kalman filter method described in [9] is used to estimate the parameters of the proposed estimator. The likelihood function L is formulated using the one-step prediction errors, ϵ_k, and the associated variances, $R_{k|K-1}$:

$$L(\theta; \mathcal{Z}_N) = p(\mathcal{Z}_N | \theta) \tag{10}$$

$$= \left(\prod_{k=1}^{N} \frac{\exp\left(-\frac{1}{2} \epsilon_k^T R_{k|k-1}^{-1} \epsilon_k \right)}{\sqrt{\det(R_{k|k-1})} (\sqrt{2\pi})^{\mathcal{L}}} \right) p(z_0 | \theta) \tag{11}$$

where θ is a set of parameters, $\mathcal{Z}_N$ is the set of observations, $\mathcal{L}$ is the dimension of the observation space, and z_0 is the initial condition. For a given set of parameters and initial states, ϵ_k and $R_{k|k-1}$ are computed by a continuous-discrete extended Kalman filter [9]. The estimation of the parameters is done by maximizing the log-likelihood:

$$\hat{\theta} = \text{argmax}\{\log(L(\theta; \mathcal{Z}_N) | z_0)\}. \tag{12}$$

All computations were done using the free statistical software, R (version 3.3.2), and the *CTSM-R-package* (Continuous Time Stochastic Modelling in R version 0.6.8-5, [10]).

Results and Discussion

In previous sections we discussed the importance capturing the effect of variations in the permeability coefficient and increasing viscosity in the coupled SDEs based model of the flow-front progression as described by (8) and (9). Here we briefly describe the preliminary results. Figure 1(a) shows an example of a simulated and estimated flow-front where random perturbations were added to the permeability term. As seen in the figure it is possible to estimate the parameters of the model described by (8) and (9) such that variations in the flow-front are captured well by the estimation model. Figure 1(b) shows the simulated and estimated flow-front where the viscosity is increasing w.r.t time. It is seen that the time dependency of the viscosity in simulated flow is not captured properly in the estimated SDE model with eight states. The way around this problem is to increase the number of states (possibly introducing an extra state for changing viscosity explicitly) and through the noise term. Hence, further investigations are needed in this case. In Figure 1(c), the result of the case with both random perturbations on the permeability and a linearly increasing viscosity w.r.t. time is shown. It can be seen that coupled SDEs based model with eight states is able to capture the effect of the random perturbations on the permeability and the linearly increasing viscosity to some extent.

(a) Varying permeability   (b) Varying viscosity   (c) Varying permeability and viscosity
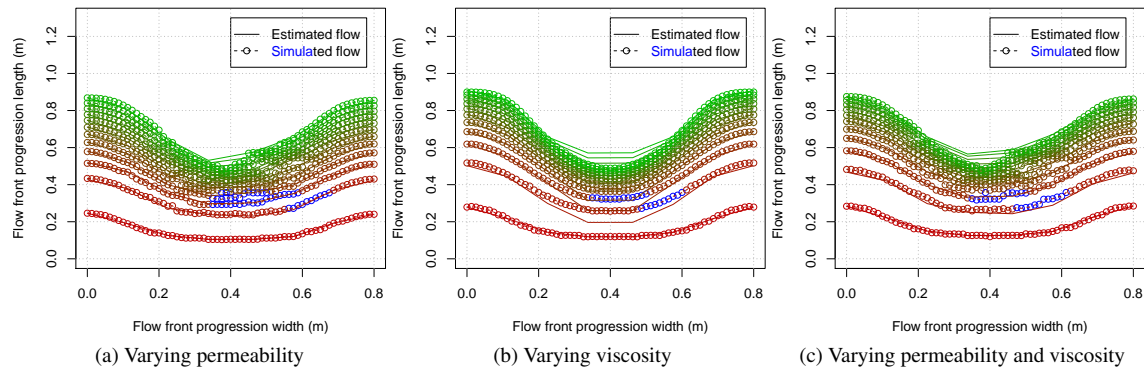
FIGURE 1: Figure 1(a), 1(b) and 1(c) show the result of estimated model with different parameter variations. The color scale describes the initial (red) simulated/estimated flow-front and the flow-front at the end (green).

Conclusions and Future Research

To avoid incipient faults and potential failures, an accurate estimation of the flow-front dynamics is essential for online monitoring of the production process of the wind turbine blades. In this paper, we have analyzed the effect of parameters variations on the performance of coupled SDEs based estimator of the flow-front dynamics. It is shown that SDE based estimator can handle insufficient knowledge of the system, noise characteristics and any variation in the parameters using the diffusion term in SDEs. Even-though the results of the investigations were encouraging, further investigations are required to fully establish the potential of using SDEs based estimator for modelling the flow-front dynamics. In our future research we will investigate the validity of SDEs based estimator for real experimental data.

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