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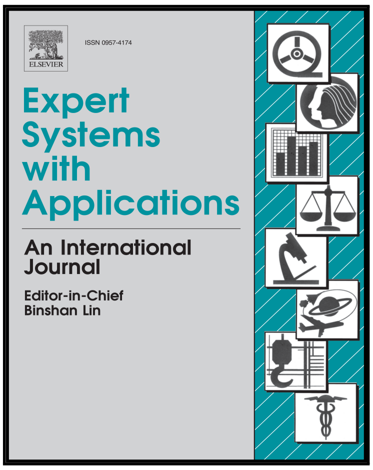

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Highlights

- To introduce an Integrated Fuzzy Clustering Cooperative Game DEA
- To provide a clustering technique to deal with lack of homogeneity among DMUs
- To provide a framework for measuring hospitals in different provinces
- Use of Core and Shapley values for ranking efficient DMUs in DEA

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Abstract

Hospitals are the main sub-section of health care systems and evaluation of hospitals is one of the most important issue for health policy makers. Data Envelopment Analysis (DEA) is a nonparametric method that has recently been used for measuring efficiency and productivity of Decision Making Units (DMUs) and commonly applied for comparison of hospitals. However, one of the important assumption in DEA is that DMUs must be homogenous. The crucial issue in hospital efficiency is that hospitals are providing different services and so may not be comparable. In this paper, we propose an integrated fuzzy clustering cooperative game DEA approach. In fact, due to the lack of homogeneity among DMUs, we first propose to use a fuzzy C-means technique to cluster the DMUs. Then we apply DEA combined with the game theory where each DMU is considered as a player, using Core and Shapley value approaches within each cluster. The procedure has successfully been applied for performances measurement of 288 hospitals in 31 provinces of Iran. Finally, since the classical DEA model is not capable to distinguish between efficient DMUs, efficient hospitals within each cluster, are ranked using combined DEA model and cooperative game approach. The results show that the Core and Shapley values are suitable for fully ranking of efficient hospitals in the healthcare systems.

Keywords: Data Envelopment Analysis; Fuzzy C-means; Core and Shapley value; Hospital efficiency

1. Introduction

Health is one of the most important issue in every society, hence providing good health care services is the center for well-being of people in the society. On the other hand, considering the extent and services that are offered in health

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care section, any mistake, even the small ones, can be irrecoverable. Hospitals are considered as a huge part of health care systems which consume the great health care resources. Hence, the improvement of hospitals performance is vital to consume available resources more efficiently while reducing health expenses for patients.

In recent years, Data Envelopment Analysis (DEA) has been recognized as one of the most frequent approaches for measuring efficiency of Decision Making Units (DMUs) such as hospitals. DEA has been proposed by Charnes et al. (1978) and then, extended by Banker et al. (1984) as a method to evaluate the performance of homogenous units. Literature reported many studies on efficiency estimating of hospitals (Emrouznejad and Yang, 2018). In previous studies, different versions of DEA models have been applied by researchers for efficiency evaluation of hospitals. Hollingsworth et al. (1999) reviewed the use of DEA for efficiency and productivity measurement of health cares. Also, Worthington (2004) examined the measurement of efficiency in range of healthcare services by using both econometric and mathematical programming frontier techniques. Ersoy et al. (1997) used DEA model to examine the technical efficiency of 573 Turkish hospitals. They found that less than ten percent of Turkish hospitals are efficient compared to their counterparts. The inputs considered in Ersoy et al. (1997) were number of beds, number of primary care physicians and number of specialists. The chosen outputs were inpatient discharges, outpatient visits and surgical operations. Hajialiafzali et al. (2007) calculated the efficiency of hospitals belong to Iranian Social Security Organization (SSO) in 2002 by using DEA and found that 26 out of 53 hospitals were efficient. They applied a super efficiency DEA model proposed by Andersen and Peterson (1993) (DEA-AP) for fully ranking of efficient hospitals. They selected four input variables number of full time equivalent (FTE) medical doctors, total number of FTE nurses, total number of other personal in FTE and average number of staff beds. The number of major surgeries and total number of medical intervention were selected as outputs. Dotoli et al. (2015) selected similar inputs as in Hajialiafzali et al. (2007) but days of hospitalization and number of surgeries were considered as output variables. Lee et al. (2009) investigated the relationship between hospital ownership and technical efficiency. The technical efficiency score was measured by DEA model. They used four inputs as service complexity, hospital size, number of labor and expenses for medical supplier and three outputs as the Medicare case mix adjusted number of discharges, number of outpatient and number of FTE trainees. This study was performed in Florida for four years. They found that non-profit hospitals were more efficient than profit, also the teaching hospitals were efficient than non-teaching hospitals. Caballer-Tarazona et al. (2010) designed a system to evaluate healthcare performance. They used a DEA model with efficiency indexes for calculating the performance of three healthcare service units of 22 hospitals in East Spain. Their method was useful for both health administration controlling hospitals performance and hospitals management. The results showed that the efficiency of the services were above the mean. Shahhoseini et al. (2011) measured technical efficiency of 28 similar types of hospitals (public and private) in all provinces of Iran. They collected the inputs data as the number of active beds, number of other professionals, number of nurses and number of physicians. Also, operations, outpatients visit, bed occupancy rate, average length of stay and inpatient bed days were outputs. The results indicated that 60 percent of hospitals are technically efficient and there are excess number of inputs (specifically in their non-clinical human resources) that should be attended by the managers. Rezaee and Karimdadi (2015) proposed a multi-group DEA model for considering the geographical location in efficiency evaluation. They selected inputs such as number of medical equipment, total number of personnel and number of operational beds. Also, the outputs were number of inpatients, number of outpatients, number of special patients, bed-day and bed occupancy rate. Lindlbauer and Schreyogg (2014) analyzed the association between hospital specialization and technical efficiency using a dataset for 11 consecutive years. Their results showed that the efficiency has negatively associated with Casemix specialization but positively associated with medical specialization. Fragkiadakis et al. (2014) evaluated the operational and economic efficiency of 87 Greek public hospitals using DEA over the period 2005-2009. They have also explored the efficiency trends over time and investigated the factors that can explain the efficiency results.

Gholami et al. (2015) examined the influence of IT investment on efficiency and quality of 187 US hospitals. They used two-stage double bootstrap DEA model and found a U-shaped relationship between IT investments and operational efficiency of hospitals. Sulku (2012) proposed a model based on DEA and the Malmquist index on the multiple inputs and multiple outputs of the ministry of health hospitals in 81 provincial markets in years 2001 and 2006 in Turkey. He offered inputs as the number of beds, number of primary care physicians, number of specialists and the produced outputs were inpatient discharges, outpatient visits and surgical operation are considered. He compared performance of hospitals and confirmed that the expected benefits from the health reforms in Turkey had been partially achieved in the short run. Using the Malmquist index, Anthun et al. (2017) investigated the productivity growth and optimal size of hospitals in Norway. They collected data of 16-years, 1994-2014 and indicated that the mean productivity increased by 24.6% with annual change 1.5%. They also concluded that estimated optimal size was smaller than the actual size of most hospitals.

Several researchers have used advanced DEA models to evaluate the hospitals. Ancarani et al. (2009) introduced a two-stage analysis for measuring hospital wards' efficiency. In the first stage, DEA was used to calculate technical efficiency scores of large Italian hospitals and in the second step, the variables affecting on DEA scores were considered. They presented the indicators for inputs as number of beds, surgery room utilization, number of physicians, units of non-medical personnel and maintenance costs for equipment. The outputs were number of cases multiplied by average diagnosis related group (DRG) weights, day-hospital or/and day-surgery cases and ambulatory cases. The results showed that both exogenous re-organization processes and decisions internal affected the ward's efficiency. Du et al. (2014) developed a slack-based additive super-efficiency DEA model to evaluate 119 general acute care hospitals in Pennsylvania. In their study, the inputs were both physical and financial and produced outputs were health services and health outcomes. They considered the quality and quantity indicators for both inputs and outputs. Kawaguchi et al. (2014) presented a dynamic network DEA to evaluate both the efficiencies of separate hospitals and the dynamic changes of efficiencies. The purpose of their study was to evaluate the policy effects of the reform for municipal hospitals from 2007-2009 in Japan. Kao et al. (2011) presented a two-stage approach of integrating independent component analysis and DEA to efficiency measurement of 21 hospitals of Taiwan in 2005. They compared the DEA and principal component analysis-DEA models. The results showed that the proposed model could improve the discriminatory capability of DEA efficiency.

Cross-efficiency DEA has been applied in many studies. For example, Costantino et al. (2013) evaluated hospitals in a region of Southern Italy using fuzzy cross-efficiency DEA model. They used triangular fuzzy numbers to deal with uncertain data and estimated a fuzzy triangular efficiency for each hospital through a cross-evaluation by a compromise between objectives. Finally, results were defuzzified to obtain the ranking. Dotoli et al (2015) presented a novel cross-efficiency fuzzy DEA technique to evaluate the performance of DMUs under uncertainty and applied the proposed technique to performance evaluation of healthcare systems in an Italian region. Ruiz and Sirvent (2017) developed a fuzzy cross-efficiency evaluation based on possibility approach. This method was presented for fuzzy inputs and convex outputs. They also extended benevolent and aggressive fuzzy formulations in order to deal with the alternate optimal for the weights. In the previous works, some papers focused on the generating weights in cross-efficiency DEA model. As shown in the literature, the cross-efficiency DEA approach has some drawbacks. For instance, it produces the weights which may not acceptable for all DMUs (Wu et al., 2009; Lam, 2010). To overcome this problem and produce an acceptable and fair weights, different models have been introduced by researchers. Ramon et al. (2010) focused on the choice of the weights profiles to be used in the calculation of the cross-efficiency scores. Their approach allows the inefficient DMUs to make a choice of weights that prevent them from using unrealistic weighting schemes. Lam (2010) developed a novel methodology based on applying discriminant analysis, super-efficiency DEA model and mixed-integer linear programming to choose suitable weight sets to be used in computing cross-evaluation. Wu et al. (2011) reviewed the cross-efficiency DEA models and eliminated the assumption of average cross-efficiency scores. They utilized the Shannon entropy to determine the weights for ultimate cross-efficiency scores. Wu et al. (2015) developed a cross-efficiency DEA model for target setting of all DMUs. In their study, several secondary goal models have been proposed for weights selection considering both desirable and undesirable cross-efficiency targets of all the DMUs. Their results showed that the cross-efficiency targets were improved and reachable for the DMUs. Also, Wu et al. (2016) proposed a cross-efficiency DEA model based on Pareto improvement by integrating Pareto optimality estimation model and cross-efficiency Pareto improvement model. Their approach is suitable for generating a common set of weights for inputs and outputs and calculating efficiency of all DMUs based on them. Lin et al. (2016) used an iterative method for determining a unique weight set for positive input and output data and reducing the number of zero weights in cross-efficiency evaluation.

One of the powerful techniques for producing a set of fair weights is game theory approach. Liang et al. (2008) presented a new method based on cross-efficiency and non-cooperative game. Wu and Liang (2012) proposed a game cross-efficiency DEA model in which each DMU was viewed as a player who seeks to maximize its own score under the condition that the cross-evaluation scores of each of other DMUs does not deteriorate. Tavana and Khalili-Damghani (2014) proposed an efficient two-stage fuzzy DEA model with uncertain inputs and outputs to evaluate the efficiency scores of a DMU and its sub-divisions. They decomposed the efficiency score of two-stage DMU and used the Stackelberg game to calculate the efficiency scores of sub-divisions. Finally, they used the Monte Carlo simulation procedure to discriminately rank the efficient DMUs and sub-divisions. Liu et al. (2017) used cross-efficiency evaluation in concept of aggressive game cross-efficiency and proposed an aggressive secondary model to minimize the efficiencies of other DMUs under the constraints that the aggressive game cross-efficiency of the evaluated DMU in guaranteed. Zuo and Guan (2017) introduced the Nash equilibrium point with cross-efficiency concept into the parallel DEA model to measure the R&D efficiency of 30 provinces of China while taking the inter-DMU competition and inter sub-processes competition into account. Their model indeed takes the bargaining power of DMUs and the algorithm converges to a unique cross-efficiency. Hinojosa et al. (2017) used the Shapley value of two different cooperative games applied to dual cooperative transfer utility games and DEA model to perform the ranking of efficient DMUs. In their study, players were the efficient DMUs and the characteristic function of the cooperative game was defined as the change in the efficiency scores of the inefficient DMUs that occurs when a given coalition of efficient DMUs are the only efficient DMUs.

Some researchers have used fuzzy C-means (FCM) algorithm for clustering DMUs in DEA context. FCM algorithm has initially been introduced by Bezdek (1973, 1981) for clustering data. Ben-Arieh and Gullipalli (2012) used FCM clustering method for utilizing DEA with sparse input and output data. They applied optimal completion strategy algorithm to estimate the missing values and investigate data recovery effects on DEA results. Amin et al. (2011) clarified the role of alternative optimal solutions for the DEA clustering approach. They showed that different optimal solutions may conclude different clusters with different sizes and different production functions. Samoilenko and Osei-Bryson (2008) increased the discriminatory power of DEA model in a heterogeneity situation. They used cluster analysis to inquire into the differences between the DMUs in the sample. Then, they applied DEA to calculate the relative efficiencies of the DMUs in each subset of the sample. Azadeh et al. (2010) composed the integrated fuzzy DEA model with fuzzy C-means and used the model for cellular manufacturing system. Each of clusters indicated a degree of desirability for operator allocation. Herrera-Restrepo et al. (2016) used an integrated principal component analysis (PCA), DEA and clustering approach for Bank branch operational performance. They detected influential branches by PCA and then, clustered branches based on operating characteristics. Finally, they applied DEA to study branch efficiency performance from meta-frontier and cluster-frontier perspectives.

This paper evaluates 288 hospitals in 31 provinces of Iran. The provinces of Iran are different in term of economic growth, population, gross domestic product (GDP) and etc. It is clear that the characteristics of each province have impact on performance of hospitals. Therefore, in this study, first the provinces are clustered using a FCM algorithm to increase the homogeneity among hospitals. After dividing the provinces to different clusters, DEA has been applied for efficiency estimating of hospitals within each cluster. Although the DEA model determines the efficiency score for hospitals, but it is not able to distinct between efficient units. In recent years, many studies have focused on ranking efficient DMUs. Perhaps super efficiency model of Anderson and Peterson (1993) is one of the most common approach used for ranking efficient DMUs. However, as suggested by Banker and Chang (2006), the Andersen–Petersen super-efficiency procedure may not produce correct ranking, since it is based on different frontiers for different efficient DMUs, hence the efficiency scores generated may not be fair. To overcome this problem, we propose to combine DEA model with cooperative game approach to produce the fair efficiency scores using Shapley value. In addition to the Shapley value, the Core is applied to evaluate the efficient DMUs and the results of Shapley value and Core are compared.

The rest of this paper is organized as follows: in Section 2, the cross-efficiency DEA with Core and Shapley value approaches is described. In section 3, two numerical examples from the literature are compared with the proposed model. Section 4 discusses the proposed fuzzy C-mean clustering algorithm as well as the selection of input and output variables. In section 5, the applicability of the proposed integrated DEA and cooperative game approach has been shown by applying it to the real dataset of hospitals in Iran. Finally, conclusion and direction for future research have been drawn in Section 6.

2. Methodology

The methodology of this paper is based on fuzzy C-means for clustering provinces, cross-efficiency DEA for estimating of hospitals in each cluster and Core and Shapley value for fully ranking of efficient hospitals. Hence, this section describes foundations for the above methods.

2.1. Fuzzy C-Means

Fuzzy C-means (FCM) algorithm developed by Dunn (1973) is one of the common clustering techniques for allocating data points to two or more clusters (Zhang et al., 2016). It is used for pattern recognition and clustering tasks. Clustering is the process in which the samples are divided into the categories with similar members. These categories are called clusters. A cluster is a collection of similar objects that are different from objects in other clusters. Clustering is heterogeneous population distribution into a number of homogeneous sub-categories or clusters. For being similarity, the various criteria can be considered. In this paper, FCM technique is applied to classified Iranian provinces into several clusters. In Iran, some provinces are larger and more developed than others, hence we have classified them based on population and GDP per capita. To classify data based on similar property, Bezdek (1981) presented FCM algorithm for clustering n measured DMUs (objects, hospitals, etc.) into C clusters. The algorithms clusters data to two or more clusters using minimization of an objective function $J(U,V)$ defined as follow:

$$J(U,V)=\sum_{i=1}^{C}\sum_{j=1}^{n}u_{ij}^{m}\left\|x_j-v_i\right\|^2 \qquad (1)$$

Where m is any real number between one and infinity $(1 \le m \le \infty)$ is the controller of fuzziness, x_j is the jth measured data and v_i is the center of cluster i and u_{ij} $(0 \le u_{ij} \le 1)$ is the degree of membership of x_j in the cluster i. v_i is the center of each cluster, so phrase $\|\ldots\|$ delivering the concept of similarity between each data and the center of each cluster with respect to a fuzzy partition matrix U and a set of prototype V. By minimizing the above objective function and put zero with constraint $\sum_{i=1}^{C}u_{ij}=1,\ \forall j=1,\ldots,n$, the matrix U can be gained as Formula (2):

$$u_{ij}=\frac{1}{\sum_{k=1}^{C}\left(\left\|x_j-v_i\right\| \big/ \left\|x_j-v_k\right\|\right)^{2/m-1}} \qquad (2)$$

Now, a new set of prototype V is defined as follow:

$$v_i=\frac{\sum_{j=1}^{n}u_{ij}^{m}x_j}{\sum_{j=1}^{n}u_{ij}^{m}} \quad 1 \le i \le C \qquad (3)$$

Using the formulas (1), (2) and (3), data can be classified based on similar characteristic. The steps of the FCM algorithm are summarized as follows:

Step 1: Randomly select the set of c and centers V and initialize matrix U by using Formula (2).

Step 2: update the centers of each cluster by using Formula (3).

Step 3: calculated a new objective function by using Formula (1).

Step 4: if $\left|J_{new}-J_{old}\right| \le \varepsilon$ stop, otherwise returns to step 2.

By using the above FCM algorithm, the provinces of Iran are classified in different clusters. Then, in each cluster, the following methodology is run separately.

2.2. Cross-efficiency DEA

DEA model is used to estimate the efficiency score of hospitals in each cluster. Let in each cluster we have *n* independent hospitals (*H*) and each $H_j\,(j=1...n)$ consumes *m* inputs as $x_{ij}\,(i=1,..,m)$ which have the weights w_i to produce *s* outputs as $y_{rj}\,(r=1,...,s)$ which have the weights μ_r. DEA calculates the efficiency of hospital *d*th (H_d) in each cluster by following model:

$$\begin{aligned}
&\max \sum_{r=1}^{s} \mu_r y_{rd} \\
&s.t:\ \sum_{i=1}^{m} w_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0 \quad j=1,...,n \\
&\sum_{i=1}^{m} w_i x_{id} = 1 \\
&w_i \geq 0 \quad i=1,...,m \\
&\mu_r \geq 0 \quad r=1,...,s
\end{aligned} \tag{4}$$

As mentioned before, in above DEA model, inefficient hospitals can be fully ranked, while the scores of all efficient hospitals are equal to unit. In other words, DEA is not able to fully rank of the efficient hospitals. Several researchers introduced different approaches for ranking efficient units including the cross-efficiency. The advantage of cross-efficiency is using peer evaluation instead of self-evaluation. The cross-efficiency matrix $E=(\mathrm{E}_{dj})\in R_{+}^{n\times n}$ is shown in Table (1). Each element of matrix shows the efficiency of H_j by using weights of H_d which are produced by model (4). By solving the model (4), for hospital H_d under evaluation, the weights $(w_{1d}^{*},...,w_{md}^{*},\mu_{1d}^{*},...,\mu_{sd}^{*})$ for inputs and outputs are produced. The cross-efficiency for each $H_j\,(j=1...n)$ in each cluster can be computed by the following equation which is shown in Table (1):

$$E_{dj} = \frac{\sum_{r=1}^{s} \mu_{rd}^{*} y_{rj}}{\sum_{i=1}^{m} w_{id}^{*} x_{ij}} \quad d,j=1,...,n \tag{5}$$

[Table 1 here]

As shown in Table (1), the mean of efficiencies for each $H_j\,(j = 1...n)$ can be considered as the cross-efficiency score.

2.3. Core and Shapley value

In cooperative game theory, Core and Shapley value are used to divide the pay-offs gained by coalition between members. For using cooperative game, first the pay-off of each coalition should be calculated. According to Nakabayashi and Tone (2006) and Wu et al. (2009), first, the cross-efficiency matrix is normalized by dividing each value in a row by the sum of all values $\sum_{j=1}^{n} E_{dj}\ \ (d = 1,...,n)$. Each element of *d*th row after row-wise normalizing is shown as $(E_{d1}^{'},...,E_{dn}^{'})$ which $\sum_{j=1}^{n} E_{dj}' = 1\ (d = 1,...,n)$. Then, each hospital is considered as a player and the players can organize a coalition with each other. To calculate the pay-off gained by each coalition, a suitable characteristic function should be defined. The characteristic functions are used in Core and Shapley value to evaluate and rank players. In this paper, the characteristic function *C(S)* which *S* is the subset of *N* (the number of players) for the coalition $S\,(S \subset N\,)\ \ N = 1,...,n$ is introduced as follows:

$$C(S) = \min_{d=1,...,n}\{E_d^{'}(S)\} \tag{6}$$

where $E_d^{'}(S)$ is computed by equation (7).

$$E_d^{'}(S) = \sum_{j \in S} E_{dj}^{'}(S)\ \ (d = 1,...,n) \tag{7}$$

To prove the equation (6), Nakabayashi and Tone (2006) considered the model (8) as the characteristic function for game *(N, C)*, where *N* is the number of players and *C* is the characteristic function.

$$
\begin{aligned}
& C(S) = \min \sum_{d=1}^{n} w_d E_d^{'}(S) \\
& s.t: \sum_{d=1}^{n} w_d = 1 \\
& w_d \geq 0 \quad d = 1, \ldots, n
\end{aligned} \tag{8}
$$

Nakabayashi and Tone (2006) proved that the game *(N, C)* in model (8) is supper-additive $C(S \cup T) \geq C(S) + C(T)$ for any $S \subset N$ and $T \subset N$ with $S \cap T = \phi$. The dual program of model (8) is presented as follows:

$$
\begin{aligned}
& C(S) = \max y \\
& s.t: y \leq E_d^{'}(S) \quad d = 1, \ldots, \mathrm{n} \\
& y \text{ is free}
\end{aligned} \tag{9}
$$

One can easily find out that the optimal solution of the model (9) is as equation (6). So, the equation (6) can be considered as characteristic function instead of the model (8). After calculating the pay-offs of each coalition, the pay-off for each player in coalition can be calculated by Core and Shapley value approaches.

The Core concept was introduced by Gillies (1959) as a solution in cooperative game. The Core is set of feasible solutions and set of imputation that are not dominated by other imputations. In game *(N, C)* this method leads to the following definition:

A vector $x \in R^N$ is a Core allocation of the cooperative game if *x* satisfy the efficiency requirement (10) and (11):

$$\sum_{i=1}^{n} x_i = C(N) \tag{10}$$

and for every coalition $S \subset N$:

$$\sum_{i \in S} x_i \geq C(S) \tag{11}$$

Indeed, x_i is the pay-off of *i*th player in the coalition *S*. For more details, reader can refer to Gillies (1959). Actually, there is not a single vector *x* that satisfies (10) and (11). To create a vector *x* which belongs to the least Core, the model (12) is introduced. The model (12) does not guarantee satisfying (10) and (11), only when the optimal solution to model (12) is non-negative the solution found belongs to the Core and thus satisfies (10) and (11).

$$\begin{aligned}
&\max \theta \\
&s.t: \\
&\sum_{i=1}^{n} x_i = C(N) \\
&\sum_{i \in S} x_i - \theta \geq C(S) \\
&x_i \geq 0 \quad i = 1, \ldots, n
\end{aligned} \tag{12}$$

Model (12) maximizes θ and finds the maximum value x_i for player *i*.

Another solution method is Shapley value in cooperative game which was introduced by Shapley (1953). This method can calculate the score of each player in coalitions. The Shapley value of *j*th, player in the game *(N, C)* is defined as Formula (13):

$$x_i = \frac{(s-1)!(n-s)!}{n!} \{C(S) - C\{S - i\} \tag{13}$$

where *s* is the number of players in coalition. The phase $\{C(S) - \mathrm{C(S-i)}\}$ means that if player *i*th joins to the coalition *S*, how much value can be increased.

This paper finds out the fair weights for fully ranking of efficient hospitals in each cluster. To calculate a set of fair common weights, the model (14) was presented by Nakabayashi and Tone (2006). Although Nakabayashi and Tone (2006) only used the Shapley value for obtained the final common weights, this paper applied Core approach and compares the results of two methods. In model (14), the weights $w = (w_1, \ldots, w_n) \in R^n$ are associated with imputations $x = (x_1, \ldots, x_n) \in R^n$ where x_i is calculated using model (12) or equation (13).

$$
\begin{aligned}
& \min p \\
& s.t: \\
& wE_j^{'} + s_j^{+} - s_j^{-} = x_j \quad (j = 1,...,n) \\
& w_1 + ... + w_n = 1 \\
& s_j^{+} \le p \ , \ s_j^{-} \le p \quad (j = 1,...,n) \\
& \mathrm{w}_i \ge 0 \quad (i = 1,...,\mathrm{n}) \\
& \mathrm{s}_j^{+} \ge 0 \ , \ s_j^{-} \ge 0 \quad (j = 1,...,n)
\end{aligned}
\tag{14}
$$

Where $wE_j^{'}$ multiplies by *j*th column of matrix $E^{'}$ (the normalized cross-efficiency matrix). Finally, the final DEA-Game efficiency score of *j*th hospital is obtained from equation (15).

$$E_j^{DEA-Game} = \sum_{d=1}^{n} w_d^{*} E_{dj} \quad , j = 1,...,n \tag{15}$$

Where w_d^{*} is the optimal weights calculated by model (14).

We should have pointed out that if there are many number of efficient DMUs, for example 30, it is necessary to calculate $2^{30} - 1 =\sim 10^{9}$ different coalitions for calculating the Shapley values (Wu et al., 2009; Castro et al., 2009; Van Campen et al., 2018) and this is very time consuming. In such cases, e.g. if there are more than 25 efficient DMUs, we propose to use the super efficiency approach of Anderson & Peterson (1993) or cross-efficiency DEA and select the top 25 DMUs first, then run the proposed algorithm on the selected 25 DMUs (25 players).

3. Numerical examples: comparison with state-of-the-art

In this section, the proposed approach is compared to some previous studies. The first example considers the dataset used in in Wu and Liang (2012). As seen in Table (2), the dataset has 1 input and 4 output variables.

[Table 2 here]

The results of DEA, arbitrary cross-efficiency DEA (traditional cross-efficiency DEA), aggressive cross-efficiency DEA (proposed by Sexton et al., 1986), cross-efficiency DEA-Game (Wu and Liang, 2012) and the proposed cross-efficiency DEA-Game in this paper are shown in Table (3).

[Table 3 here]

The rankings produced by each model are also shown in Table 3. The Spearman's Rank-Order Correlation between the proposed DEA-Game model and DEA-Game suggested of Wu and Liang (2012) is 0.829 which is significant at the 95% level.

We further, compare our results with the DEA-Game approaches suggested by Li et al. (2016) and Hinojosa et al. (2017). For this purpose, consider the data and the results of the different models are reported in Table (4).

[Table 4 here]

As seen in Table (4), DEA cannot distinguish between five DMUs A, B, C, D and E, since all of them are efficient. The ranking produced by three DEA-Game models proposed by Li et al. (2016), Hinojosa et al. (2017) and this paper are reported in Table (4). Our proposed model produced the same ranking model Li et al. (2016). The Spearman's Rank-Order Correlation between our proposed model and the DEA-Game model of Hinojosa et al. (2017) is also significant.

4. An application in hospital efficiency

The data in this study are gathered from 288 hospitals in 31 provinces of Iran. First, the FCM is applied for clustering the provinces. In this paper, the criteria considered for clustering of provinces are gross domestic product (GDP) per capita and population. One of the most important criteria for examining the amount of attention to the health sector is the index of GDP. GDP is the monetary value of all the finished goods and services produced within a country's borders in a specific time period including industry, agriculture and services. Healthcare is also one of the sub services and hospitals are the most important medical center of health systems. The low share of healthcare in GDP causes the reduction of quality of medical services. Lack of sufficient attention to share of healthcare in GDP and not allocating sufficient funds to this sector has the negative effects on people health. Besides this, GDP per capita is the proxy of income and income has impact on the rate of going to the hospitals. So, the first index for clustering the province is GDP per capita which has undeniable impact on hospitals performance in each province. In other words, the hospitals in provinces with similar GDP per capita should be compared with each other.

Other index in data clustering is the population of the province. Hospitals are associated with a large portion of society, therefore, in most countries access to health services is known as a basic and essential right for citizens. On the other hand, health improvement and expansion of health services have a significant impact on major factors such as population, fertility, mortality, immigration, family and so on. Also, in provinces with high population, entering and leaving patients are more, so they need more beds and equipment. Therefore, population is an important factor for estimating hospitals performance. The GDP per capita and population of each province are shown in Table (5). Also, Figure (1) shows the results of FCM method for clustering the provinces based on GDP and population indicators.

[Table 5 here]

[Figure 1 here]

One of the most important steps in evaluation of hospitals is selection the suitable input and output variables. For selection of suitable inputs and outputs, different researchers have selected different variables. According to the previous studies and available data, this paper considers the input variables as the total number of personnel, number of medical equipment in each hospital and number of active beds that means the beds which are available for use. The personnel are staffs, permanent staffs, contract workers and other staffs. The selected outputs are the number of inpatients, outpatients and special patients separately and the fourth output is bed-days. The variable number of bed-days is non-discretionary, and it should be considered as non-discretionary in output-oriented DEA models. Since this paper apply an input-oriented DEA model, so non-discretionary of the output variable such as bed-days would not affect the results. If one run an output-oriented version then it requires to consider number of bed-day as a non-discretionary variable. Also, in this paper, similar to the most studies, number of active beds has been considered as a proxy for capital in hospital (Csakvari et al. 2014; Rezaee and Karimdadi, 2015; Lobo et al. 2016). When running input-orientation, obviously the lower the number of active beds means the higher efficiency. Minimum, mean and maximum value of the selected inputs and outputs for each cluster are shown in Table (6).

[Table 6 here]

5. Results and discussion

In this section, the results of efficiency estimating of 288 hospitals in Iran are evaluated. First, FCM algorithm is applied for clustering the provinces based on GDP per capita and population. In this paper, FCM is configured as follows: number of clusters *C* and fuzziness parameter *m* are set to 5 and 2, respectively. We assumed convergence criterion ε and maximum number of iterations as 10^{-5} and 100, respectively. Due to the existence small number of provinces, number of clusters are set to five clusters. Indeed, we would like to have about six provinces in each cluster. Although there are several studies for determining parameters *m* and *C*, but they are generally suitable for large dataset. In fact, different researchers have introduced different approaches for selecting number of clusters (*C*). Based on Bezdek's suggestion, the value of *C* should be selected between 2 and $\sqrt{n}$ (Bezdek, 1998). In our case study, *C* is between 2 and $\sqrt{31}$. In Table (7), the results of clustering for C=2, 3, 4 and 5 are shown. For C= 3, 4 and 5, Tehran province is a separate cluster. Also, big provinces Esfahan, Khorasan Razavi, Khuzestan, and Fars have been clustered together for *C*=4 and 5. Figure (2) shows the mean efficiency of hospitals in the provinces for different *C* values. As shown, Semnan have four hospital and the mean efficiency of these hospitals for *C*=2, 3, 4 and 5 is equal to one. Also, the lowest efficiency score is 0.580 (for *C*= 4 and 5) which is related to hospitals of Sistan-o-Baluchestan province. For more details about setting FCM parameters, readers can refer to Chiu (1994), Azadeh et al. (2010) and Schwämmle and Jensen (2010).

[Table 7 here]

[Figure 2 here]

As mentioned before, we analyze the results for *C*=5. Clusters 1, 2, 3, 4 and 5 include 57, 36, 47, 72 and 76 hospitals, respectively. The provinces in the different clusters include developed and developing provinces. While the third cluster has the most province, the fifth cluster only has the province of Tehran. Cluster 3 includes many provinces. The provinces in the clusters are partly similar in terms of area, population or general development. Some of the provinces are developed than others, but in general, all of them are in the same classification in Iran and they are comparable. Cluster 5 consists of 76 hospitals in Tehran province. Tehran, as the capital of Iran, is the most densely populated province which has many hospitals with good facilities, so this province is located in cluster 5, alone. Generally, in each cluster, there are similar provinces in terms of GDP per capita and population. The results of clustering are shown in Table (5) and Figure (1). The objective function values of FCM for different values of *m* and *C* are shown in Figure (3).

[Figure 3 here]

Figure (3) shows that Tehran and Bushehr have the most population and GDP per capita, respectively. Tehran, with population of around 9 million in the city and 13 million in the wider metropolitan area (Statistical center of Iran‡), is the most populous city of Iran. Bushehr lies in a vast plain running along the Persian Gulf coast of south-western Iran. The GDP per capita of Bushehr is high due to the existence of some oil and gas industries. For example, South Pars / North Dome field, which is a natural gas condensate field located in the Persian Gulf, is near of Bushehr.

We have used an input-oriented DEA model (4) since hospital managers could control the resources used. Indeed, hospitals managers can set the value of personnel or number of equipment (proxy of capitals) for reaching the efficient frontier. The results of clustering show that the similar provinces are clustered together. There is no a major difference among the size of hospitals within each cluster. Indeed, the hospitals in each cluster have high level homogeneity in terms of the size and so we assume constant returns to scale assumption. It is notable that one can use DEA under variable returns to scale assumption, however, in this case many of DMUs will show efficiency score equal to 1. Beside this, in game theory, if the number of players (efficient DMUs here) increase, the calculation of Shapley value would be very difficult. As it can be seen in the results, reported in Table (8), in cluster 1, the most efficient hospitals are in East Azarbaijan province. Among 14 hospitals of East Azarbaijan, six of them received rank 1. Alborz, as a large province, has the hospitals with high efficiency score. The minimum efficiency for Alborz hospitals is 0.62. In West Azarbaijan, just one hospital is efficient. None of the hospitals in Kerman and Sistan-o- Baluchestan are efficient and in Mazandaran and Gilan, three hospitals are efficient.

In cluster 2, Kurdistan includes four hospitals that two of them are fully efficient, and the efficiency score the other two are close to 1. Although Golestan is a small province, four of seven hospitals that had been checked in this province are efficient. All hospitals in Kermanshah and Lorestan are inefficient. In addition, Hamedan, Hormozgan and Markazi have four efficient hospitals all together.

‡ www.amar.org.ir/english

As mentioned before, Cluster 3 includes many provinces. The results show that this cluster has the most efficient hospitals. Three of them are located in Yazd, three are in Semnan and other provinces have one or two efficient hospitals. Cluster 4 has 18 efficient hospitals which six of them are in Khorasan Razavi while Khouzestan and Esfehan have four efficient hospitals and Fars has three. Finally, cluster 5 has 76 hospitals that 14 of them are efficient.

[Table 8 here]

Note that the proposed methodology only is explained for cluster 1, similar discussion can be given for other clusters. As shown in Table (8), eleven hospitals in cluster 1 are efficient, hence the traditional DEA cannot rank these hospitals. We used the proposed game theory for ranking efficient DMUs. First, by using the weights of inputs and outputs from model (4), the cross-efficiency matrix, shown in Table (9), is constructed based on Formula (5). As seen in Table (9), some cross-efficiencies are very low, for instance, the efficiency score of East Azarbaijan 6 is 0.107 by using weights of East Azarbaijan 5. This is a common drawback for cross-efficiency DEA since not all DMUs would like to use the weights generated by one unit only. To overcome this problem and to produce fair and acceptable weights, in this paper, the game theory is combined with the cross-efficiency DEA.

[Table 9 here]

Table (9) should be row-normalized. Then, for estimating efficiency scores of efficient hospitals, cooperative game approach is applied. In cooperative game, each efficient hospital is considered as a player and players form a coalition with each other. Now we can use the Core and Shapley value methods with pay-off of coalitions. The Core score of each player is calculated using model (12) and the Shapley value of each player is calculated using formula (13). Figure (4) shows the Core and Shapley value of each hospital of cluster 1 before using common weights.

[Figure 4 here]

Since the weights of standard cross-efficiency DEA model are not fair, the common weights generated by model (14) are considered for fully ranking of efficient hospitals. After solving model (14), the final cross-efficiency DEA-

Game scores are calculated by formula (15) for Core and Shapley value approaches, separately. The results of final DEA-Game scores for the efficient hospitals in different clusters are shown in Table (10).

[Table 10 here]

As shown in Table (10), there is not much differences between ranks in Core and Shapley value methods. The Spearman correlation between Core and Shapley value ranks for clusters 1 to 5 are 0.961, 0.976, 0.912, 0.975 and 0.923, respectively. The Spearman correlations are significant at the 0.01 level for all clusters. Briefly, the ranks of two methods for cluster 1 are as follows:

In each of the two methods, hospitals of East Azarbaijan 14 and East Azarbaijan 5 have ranked first and second. In Shapely value, East Azarbaijan 4 and in Core, Gilan 6 have ranked third. Also, In Shapely value, Gilan 6 and in Core, West Azarbaijan 1 have ranked forth. In Table (10), for other clusters, final ranks of Shapley value and Core methods are shown. This shows that how one can rank efficient units in DEA using game theory within each cluster in a fair and acceptable way.

The results of Table (8) show that in Cluster 1, there are 11 efficient hospitals and 46 inefficient hospitals. That is, more than 80% of hospitals in cluster 1 are inefficient. In this cluster, the most efficient hospitals are located in East Azerbaijan which is more developed than other provinces. Also, among the efficient hospitals, according to Table (10), two hospitals in East Azarbaijan are ranked first and second. The results indicate that the policy makers should give priority to improve the performance of hospitals in other provinces of this cluster.

In cluster 2, according to the results of Table (8), among 36 hospitals, 10 hospitals are efficient and 26 hospitals are inefficient. In other words, only about 28% of hospitals in this cluster are located on the efficient frontier. There is no efficient hospital in Kermanshah province, which is one of the undeveloped provinces. According to the results of Table (8), one hospital in Hamedan and three hospitals in Golestan have been ranked 1 to 4. In this cluster, policy makers should also pay more attention to improve the performance of hospitals in underdeveloped provinces such as Kermanshah and Lorestan.

According to the results of Table (8), in the third cluster, 64 hospitals are inefficient. All three hospitals in Semnan province, which have been investigated, are efficient. Yazd province has the first and second ranks of the most efficient hospitals (see Table (10)) and the first and second ranks of the most inefficient hospitals (see Table (8)).

The cluster 4 includes the large and developed provinces. In this cluster, 17 of the 72 hospitals under evaluation are efficient. In the last cluster, only hospitals in Tehran province are located. Tehran is the capital and most developed province of Iran. The mean of efficiency scores for hospitals in the Tehran province is over 72%, which indicates that hospitals have had a good performance. The results reported in Tables (8) and (10) are very useful to policy makers as they can priorities to take steps to improve the performance of hospitals in undeveloped provinces.

We should also mention that the running times of the proposed DEA-Game model for different clusters are shown in Table (11). These results are implemented by MATLAB 2017 software in Intel Core i5 CPU processor (8GB RAM). As seen in this table, the running times for 10 efficient DMUs is only 19 second while the running time for 19 efficient DMUs is about 30 minutes. There is a clear indication that, the running time is increasing polynomial in the respect to number of DMUs.

[Table 11 here]

6. Conclusion

This paper assesses performance of 288 hospitals from 31 provinces of Iran by using fuzzy C-means for clustering similar provinces in terms of GDP and population. After clustering, DEA model is used to evaluate efficiency of each hospital within each clusters. Efficient hospitals determined by DEA model are considered as the players in cooperative game approach and then, final scores are calculated by combination of cross-efficiency DEA and Core or Shapley value methods. We showed that the cooperative game approach produces a set of fair weights for fully ranking of the efficient hospitals in each cluster. The conclusion of this paper indicates that the proposed approach is effective and suitable for fully ranking of decision making units. Although we used the proposed approach in hospital efficiency but it can easily be applied in any other DEA applications. One limitation of the proposed methodology is its computational complexity if there are many efficient DMUs. In other words, calculation of coalition and Shapley value in game theory would be difficult as the number of efficient hospitals increase. Assume *n* is the number of efficient hospital, in order to calculate Shapley values, we should investigate $2^n - 1$ different coalitions in order to rank all efficient hospitals. For such cases we proposed a two-stage procedure to run a super efficiency or cross-efficiency DEA model first to select the top 25 highest efficient DMUs within each cluster before estimating the Shapley values.

Author Contributions Section:

Hashem Omrani: Supervision, Conceptualization, Methodology, Software, Formal Analysis, Validation.

Khatereh Shafaat: Data curation, Writing- Original draft preparation, Software.

Ali Emrouznejad: Methodology, Writing- Reviewing and Editing, Supervision.

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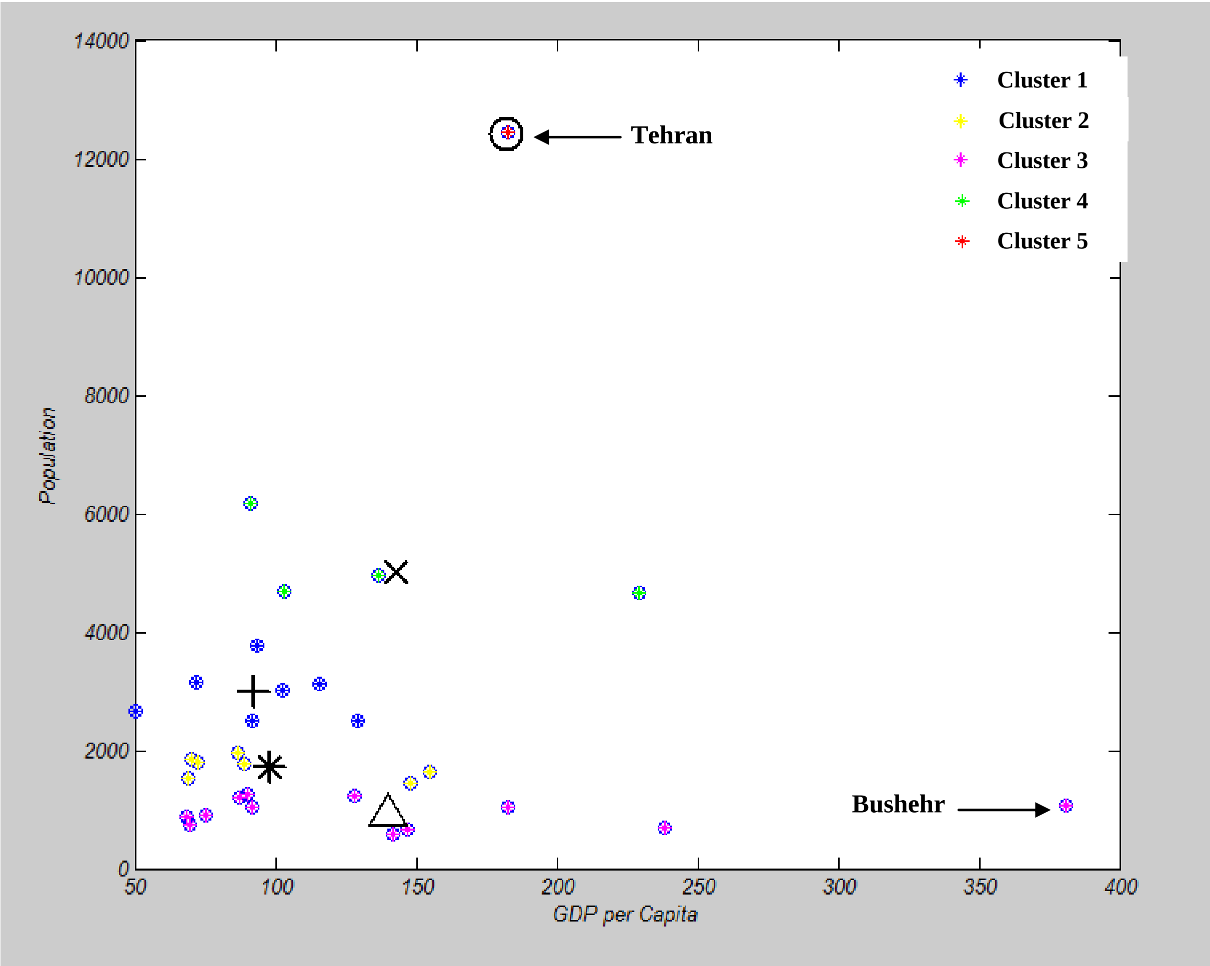

Figure 1: The results of Clustering for 31 provinces of Iran

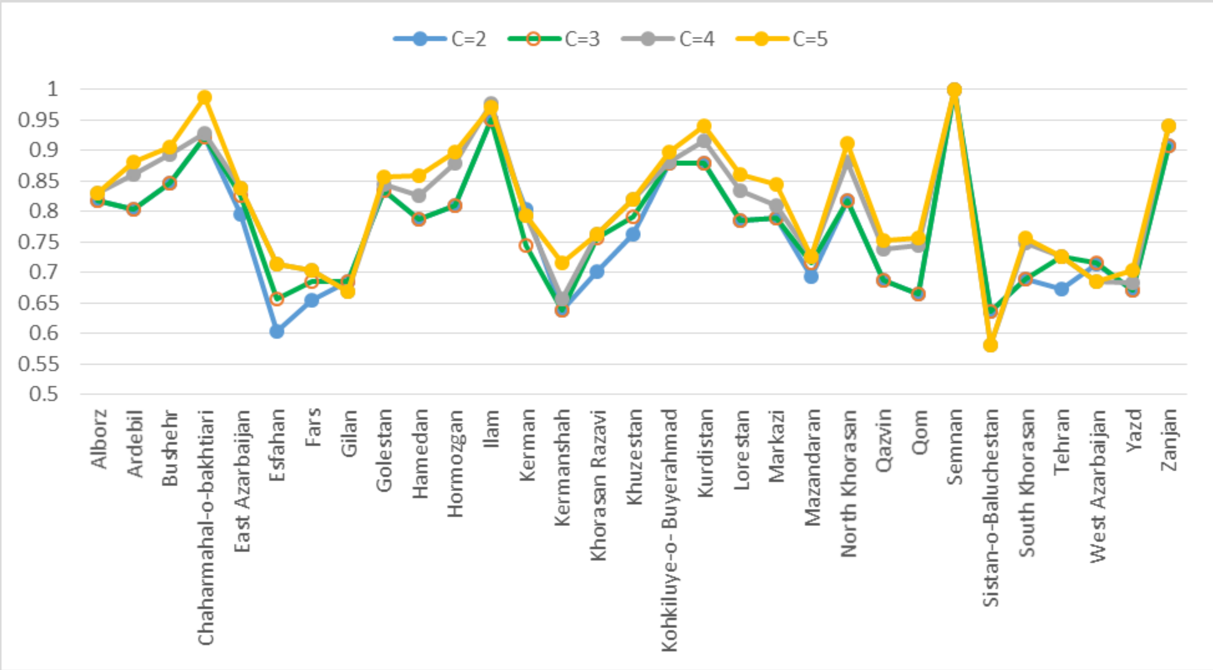

Figure 2: Mean efficiency of hospitals in different provinces

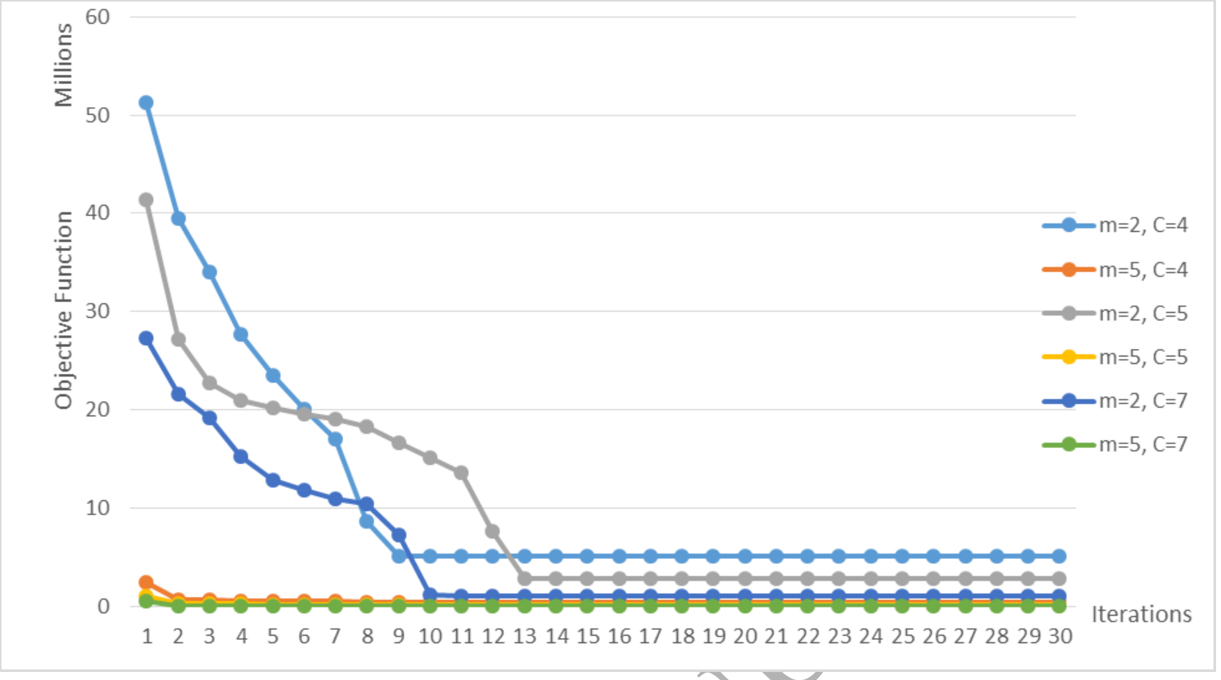

Figure 3: The value of objective function for different *m* and *C* parameters

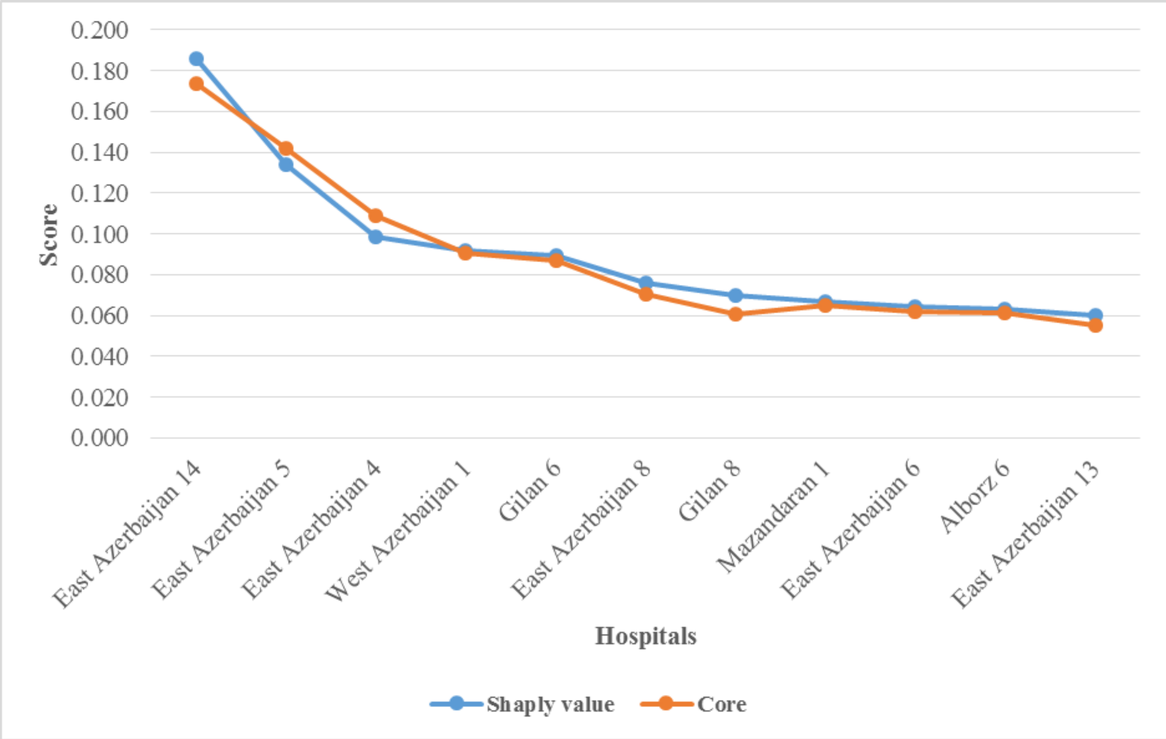

Figure 4: Ranking efficient hospitals in cluster 1 before using common weight

Table 1: Cross-efficiency DEA matrix

DMUs	DMUs				
	1	**2**	**3**	**…**	**n**
1	E_{11}	E_{12}	E_{13}	…	E_{1n}
2	E_{21}	E_{22}	E_{23}	…	E_{2n}
3	E_{31}	E_{32}	E_{33}	…	E_{3n}
…	…	…	…	…	…
n	E_{n1}	E_{n2}	E_{n3}	…	E_{nn}
Mean	$\overline{E_1}$	$\overline{E_2}$	$\overline{E_3}$	…	$\overline{E_n}$

Table 2: The data for numerical example 1

	x	y1	y2	y3	y4
A	1	3	1	2	3
B	1	4	5	5	2
C	1	6	9	6	2
D	1	3	2	2	1
E	1	2	2	3	2
F	1	1	4	3	3

Table 3: The data for numerical example 1

	DEA		**Arbitrary cross-efficiency DEA (traditional)**		**Aggressive cross-efficiency DEA**		**Cross-efficiency DEA-Game (Wu and Liang, 2012)**		**Proposed approach in this paper**	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
A	1	1	0.778	3	0.714	3	0.995	2	0.650	3
B	0.917	4	0.804	2	0.729	2	0.906	4	0.739	2
C	1	1	0.981	1	0.889	1	1	1	0.980	1
D	0.500	6	0.417	6	0.370	6	0.474	6	0.408	6
E	0.750	5	0.602	5	0.559	5	0.743	5	0.501	5
F	1	1	0.777	4	0.694	4	0.986	3	0.621	4

Table 4: The data and results of numerical example 2

DMUs	Inputs		Outputs		DEA-CCR	DEA-Game proposed by Li et al. (2016)		DEA-Game proposed by Hinojosa et al. (2017)		Proposed approach in this paper		
	x_1	x_2	y_1	y_2		Shapley	rank	Shapley	rank	Shapley	Score	rank
A	2	7	3	1	1	2.1517	1	0.53601263	1	0.269747	0.827	1
B	2	12	4	1	1	1.5705	2	0.11725638	4	0.276153	0.765	2
C	5	5	2	1	1	0.7827	3	0.12878097	3	0.166447	0.628	3
D	10	4	2	1	1	0.6753	4	0.13878705	2	0.145575	0.566	4
E	3	6	1	1	1	0.3881	5	0.03355497	5	0.142079	0.535	5
F	10	6	1	1	0.75							
G	4	12	2.5	1	0.5625							
H	5	11	3.5	1	0.733108							

Table 5: GDP per capita and population for provinces of Iran

Row	Provinces	The number of hospitals	GDP per capita (million Rials)	Population (×1000)	In cluster
1	**Alborz**	6	129.2	2502	1
2	**Ardebil**	5	89.8	1267	3
3	**Bushehr**	1	381.2	1077	3
4	**Chaharmahal-o-bakhtiari**	2	75.4	914	3
5	**East Azarbaijan**	14	93.4	3780	1
6	**Esfahan**	20	136.9	4965	4
7	**Fars**	19	103	4688	4
8	**Gilan**	10	92	2511	1
9	**Golestan**	7	70.1	1839	2
10	**Hamedan**	7	88.8	1777	2
11	**Hormozgan**	2	154.8	1642	2
12	**Ilam**	4	142	569	3
13	**Kerman**	4	102.3	3026	1
14	**Kermanshah**	9	86.9	1962	2
15	**Khorasan Razavi**	21	91.2	6171	4
16	**Khuzestan**	12	229.3	4659	4
17	**Kohkiluye-o-Buyerahmad**	2	238.2	681	3
18	**Kurdistan**	4	69.3	1514	2
19	**Lorestan**	7	72.6	1785	2
20	**Markazi**	3	147.8	1442	2
21	**Mazandaran**	10	115.8	3127	1
22	**North Khorasan**	4	68.6	888	3
23	**Qazvin**	6	128.3	1226	3
24	**Qom**	8	87.4	1193	3
25	**Semnan**	3	146.7	651	3
26	**Sistan-o-Baluchestan**	4	50	2659	1
27	**South Khorasan**	1	69.5	750	3
28	**Tehran**	76	182.4	12433	5
29	**West Azarbaijan**	19	71.6	3160	1
30	**Yazd**	9	182.8	1046	3
31	**Zanjan**	2	91.6	1037	

Table 6: The summary of inputs and outputs for hospitals in different clusters

		Inputs			outputs			
		Personnel	**Equipment**	**Active beds**	**Inpatients**	**Outpatients**	**Special patients**	**Bed-day**
Cluster1	Min	36	14	32	505	48	6	520
	Mean	477.2	379.9	188.5	119,841.2	200,304.4	37,060.1	51,497.2
	Max	1,154	2,058	591	490,876	915,744	133,112	199,714
Cluster 2	Min	38	30	32	9,915	3,374	25	3,882
	Mean	427.2	254.9	170.5	103,705.3	165,295.8	42,908.7	45,845.4
	max	1,443	677	581	647,635	505,350	498,031	146,929
Cluster 3	Min	80	16	14	2,097	1,732	27	5,745
	Mean	417.9	332.4	149.1	103,795.3	162,967.9	49,048.1	40,192
	max	1,028	2,074	377	393,741	479,516	255,135	99,034
Cluster 4	Min	73	36	30	4,196	3,963	230	4,467
	Mean	490.3	352.5	167.5	107,312.3	202,120.6	31,682	44,285.7
	max	2,729	2,676	806	655,239	1,914,155	160,613	230,058
Cluster 5	Min	21	10	15	365	1,806	455	189
	Mean	493.3	288.3	161.3	102,874.5	190,893.4	35,185.7	42,712.7
	max	2,109	1,139	618	818,257	801,327	304,349	202,044

Table 7: The results of FCM for number of clusters C=2, 3, 4 and 5

Number of clusters	Cluster	Provinces in cluster	Number of hospitals in cluster
C=2	1	East Azerbaijan, Esfahan, Tehran, Khorasan Razavi, Khuzestan, Fars	162
	2	Other provinces	126
C=3	1	Tehran	76
	2	East Azerbaijan, West Azerbaijan, Esfahan, Khorasan Razavi, Khuzestan, Fars, Kerman, Mazandaran	109
	3	Other provinces	103
C=4	1	Tehran	76
	2	Esfahan, Khorasan Razavi, Khuzestan, Fars	72
	3	East Azerbaijan, West Azerbaijan, Alborz, Sistan-o-Baluchestan, Kerman, Kermanshah, Gilan, Mazandaran	66
	4	Other provinces	74
C=5	1	East Azerbaijan, West Azerbaijan, Alborz, Sistan-o-Baluchestan, Kerman, Gilan, Mazandaran	57
	2	Kermanshah, Hamedan, Golestan, Kurdistan, Lorestan, Markazi, Hormozgan	36
	3	Other provinces	47
	4	Esfahan, Khorasan Razavi, Khuzestan, Fars	72
	5	Tehran	76

Table 8: The results of DEA scores hospitals in different clusters

Cluster 1				Cluster 2				Cluster 3			
East Azarbaijan		**Gilan**		**Kermanshah**		**Lorestan**		**Yazd**		**Ilam**	
1	0.926	**1**	0.820	**1**	0.610	**1**	0.998	**1**	0.229	**1**	0.922
2	0.864	**2**	0.784	**2**	0.932	**2**	0.54861	**2**	0.932	**2**	1
3	0.997	**3**	0.562	**3**	0.85	**3**	0.994	**3**	0.700	**3**	1
4	1	**4**	0.336	**4**	0.663	**4**	0.900	**4**	0.385	**4**	0.992
5	1	**5**	0.738	**5**	0.723	**Markazi**		**5**	0.630	**North Khorasan**	
6	1	**6**	1	**6**	0.398	**1**	1	**6**	0.459	**1**	0.787
7	0.558	**7**	0.810	**7**	0.895	**2**	0.952	**7**	1	**2**	0.860
8	1	**8**	1	**8**	0.959	**3**	0.583	**8**	1	**3**	1
9	0.607	**9**	0.051	**9**	0.403	**Hormozgan**		**9**	1	**4**	1
10	0.66	**10**	0.576	**Hamedan**		**1**	0.795	**Qom**		**Semnan**	
11	0.691	**West Azarbaijan**		**1**	0.756	**2**	1	**1**	0.496	**1**	1
12	0.403	**1**	1	**2**	1			**2**	0.566	**2**	1
13	1	**2**	0.814	**3**	0.631			**3**	0.715	**3**	1
14	1	**3**	0.550	**4**	0.920			**4**	1	**Kohkiluye-o-Buyer ahmad**	
Mazandaran		**4**	0.672	**5**	1			**5**	0.599	**1**	0.794
1	1	**5**	0.377	**6**	0.78			**6**	1	**2**	1
2	0.572	**6**	0.621	**7**	0.916			**7**	0.871	**Zanjan**	
3	0.688	**7**	0.523	**Golestan**				**8**	0.794	**1**	1
4	0.808	**8**	0.679	**1**	0.951			**Qazvin**		**2**	0.881
5	0.830	**9**	0.931	**2**	1			**1**	1	**Chaharmahal-e-bakhtyari**	
6	0.492	**Alborz**		**3**	0.187			**2**	0.588	**1**	0.973
7	0.588	**1**	0.936	**4**	1			**3**	0.744	**2**	1
8	0.739	**2**	0.768	**5**	1			**4**	0.842	**South Khorasan**	
9	0.845	**3**	0.77	**6**	1			**5**	0.743	**1**	0.756
10	0.695	**4**	0.628	**7**	0.851			**6**	0.596	**Bushehr**	
		5	0.865	**Kurdistan**				**Ardebil**		**1**	0.905
Sinstan-o-Baluchestan		**6**	1	**1**	1			**1**	0.884		
1	0.513	**Kerman**		**2**	0.862			**2**	1		
2	0.439	**1**	0.755	**3**	0.900			**3**	0.937		
3	0.68	**2**	0.557	**4**	1			**4**	0.760		
4	0.686	**3**	0.890					**5**	0.820		
		4	0.967								

Table 8 (Continue)

Cluster 4

Khorasan Razavi		Esfahan		Fars	
1	0.569	**1**	0.833	**1**	0.911
2	0.634	**2**	1	**2**	0.711
3	1	**3**	0.289	**3**	0.565
4	0.75	**4**	0.365	**4**	0.328
5	1	**5**	1	**5**	0.527
6	1	**6**	0.977	**6**	1
7	0.589	**7**	0.671	**7**	1
8	0.698	**8**	0.642	**8**	0.499
9	0.731	**9**	1	**9**	0.596
10	0.769	**10**	0.928	**10**	0.751
11	1	**11**	0.855	**11**	0.723
12	0.780	**12**	0.45	**12**	0.678
13	0.306	**13**	0.549	**13**	0.575
14	0.598	**14**	0.663	**14**	0.610
15	1	**15**	1	**15**	0.809
16	0.610	**16**	0.434	**16**	0.82
17	0.686	**17**	0.706	**17**	0.399
18	0.861	**18**	0.740	**18**	1
19	1	**19**	0.510	**19**	0.835
20	0.890	**20**	0.627		
21	0.545				

Khuzestan	
1	1
2	0.885
3	0.735
4	1
5	0.492
6	0.449
7	1
8	0.839
9	0.740
10	1
11	0.829
12	0.865

Cluster 5

Tehran					
1	0.794	**33**	0.728	**65**	0.842
2	0.805	**34**	0.855	**66**	0.584
3	0.732	**35**	0.760	**67**	1
4	0.409	**36**	0.929	**68**	0.418
5	0.488	**37**	0.528	**69**	1
6	0.818	**38**	1	**70**	0.848
7	0.603	**39**	1	**71**	0.355
8	0.649	**40**	0.773	**72**	0.662
9	0.40	**41**	0.819	**73**	0.681
10	0.356	**42**	0.789	**74**	0.82
11	0.479	**43**	0.706	**75**	0.672
12	0.604	**44**	0.704	**76**	0.816
13	1	**45**	0.408		
14	0.756	**46**	0.301		
15	0.662	**47**	0.585		
16	0.923	**48**	0.805		
17	0.611	**49**	0.803		
18	0.384	**50**	1		
19	0.421	**51**	0.900		
20	0.827	**52**	1		
21	0.821	**53**	0.877		
22	1	**54**	1		
23	0.833	**55**	0.532		
24	0.659	**56**	0.959		
25	0.686	**57**	1		
26	0.864	**58**	1		
27	0.640	**59**	0.599		
28	0.453	**60**	1		
29	0.429	**61**	0.578		
30	0.614	**62**	0.781		
31	1	**63**	0.994		
32	0.720	**64**	0.259		

Table 9: Cross-efficiency matrix

	East Azarbaijan4	East Azarbaijan5	East Azarbaijan6	East Azarbaijan8	East Azarbaijan13	East Azarbaijan14	West Azarbaijan1	Alborz6	Gilan6	Gilan8	Mazandaran1
East Azerbaijan 4	1	0.670	0.215	0.423	0.271	0.430	0.597	0.445	0.494	0.176	0.376
East Azerbaijan 5	0.58	1	0.107	0.362	0.074	0.940	0.435	0.132	0.150	0.139	0.136
East Azerbaijan 6	0.379	0.628	1	0.662	0.187	0.733	0.431	0.304	0.335	0.755	0.301
East Azerbaijan 8	0.552	0.974	0.852	1	0.544	0.895	0.882	0.552	0.945	0.491	0.760
East Azerbaijan 13	0.427	0.217	0.153	0.268	1	0.336	0.788	0.800	0.969	0.070	0.936
East Azerbaijan 14	0.237	0.587	0.053	0.165	0.020	1	0.164	0.045	0.049	0.138	0.040
West Azerbaijan 1	0.731	0.693	0.264	0.500	0.395	0.877	1	0.419	0.498	0.177	0.580
Alborz 6	0.783	0.636	0.674	0.626	0.678	0.706	0.752	1	0.863	0.551	0.749
Gilan 6	0.853	0.553	0.354	0.459	0.199	0.276	0.355	0.332	1	0.178	0.272
Gilan 8	0.208	0.602	0.843	0.521	0.078	0.961	0.287	0.155	0.129	1	0.145
Mazandaran 1	0.506	0.391	0.268	0.438	0.880	0.541	0.999	0.777	0.986	0.149	1

Table 10: Final ranks of efficient hospitals in different clusters

Cluster 1			**Cluster 2**			**Cluster 3**			**Cluster 4**			**Cluster 5**		
Efficient hospitals	**Shapley**	**Core**	**Efficient hospitals**	**Shapley**	**Core**	**Efficient hospitals**	**Shapley**	**Core**	**Efficient hospitals**	**Shapley**	**Core**	**Efficient hospitals**	**Shapley**	**Core**
East Azerbaijan 14	0.69	0.65	**Hamedan 2**	0.648	0.607	**Yazd 8**	0.672	0.726	**Fars 18**	0.73	0.797	**Tehran 60**	0.667	0.721
East Azerbaijan 5	0.528	0.541	**Golestan 4**	0.606	0.62	**Yazd 7**	0.49	0.642	**Khorasan Razavi 11**	0.647	0.646	**Tehran 67**	0.632	0.668
Gilan 6	0.472	0.48	**Golestan 2**	0.561	0.64	**Semnan 1**	0.474	0.628	**Esfahan 9**	0.569	0.584	**Tehran 13**	0.608	0.691
West Azerbaijan 1	0.448	0.448	**Golestan 5**	0.455	0.505	**Ilam 3**	0.404	0.586	**Khorasan Razavi 15**	0.533	0.541	**Tehran 22**	0.466	0.554
East Azerbaijan 4	0.444	0.497	**Kurdistan 1**	0.366	0.36	**Zanjan 1**	0.396	0.443	**Esfahan 15**	0.497	0.515	**Tehran 69**	0.403	0.522
Mazandaran 1	0.364	0.364	**Hormozgan 2**	0.36	0.376	**Semnan 2**	0.356	0.403	**Khozestan 4**	0.484	0.539	**Tehran 54**	0.369	0.386
East Azerbaijan 8	0.347	0.339	**Golestan 6**	0.359	0.357	**North Khorasan 4**	0.331	0.452	**Khozestan 7**	0.471	0.445	**Tehran 57**	0.365	0.359
Alborz 6	0.338	0.35	**Kurdistan 4**	0.295	0.296	**Yazd 9**	0.33	0.467	**Esfahan 2**	0.468	0.452	**Tehran 50**	0.348	0.449
East Azerbaijan 13	0.334	0.334	**Hamedan 5**	0.282	0.291	**Qom 4**	0.328	0.484	**Fars 7**	0.44	0.496	**Tehran 52**	0.345	0.358
Gilan 8	0.316	0.272	**Markazi 1**	0.238	0.25	**Ilam 2**	0.322	0.406	**Khorasan Razavi 6**	0.439	0.475	**Tehran 58**	0.293	0.38
East Azerbaijan 6	0.31	0.272				**Semnan 3**	0.319	0.418	**Khorasan Razavi 3**	0.438	0.377	**Tehran 31**	0.289	0.374
						Qazvin 1	0.293	0.404	**Esfahan 5**	0.397	0.34	**Tehran 39**	0.211	0.352
						North Khorasan 3	0.291	0.372	**Fars 6**	0.35	0.327	**Tehran 38**	0.197	0.338
						Kohgeluye 2	0.291	0.388	**Khozestan 1**	0.304	0.315			
						Chahrmahal 2	0.237	0.37	**Khorasan Razavi 5**	0.286	0.29			
						Qom 6	0.212	0.34	**Khorasan Razavi 19**	0.278	0.233			
						Ardebil 2	0.155	0.219	**Khozestan 10**	0.216	0.198			
Spearman correlation	0.961(*)			0.976(*)			0.912(*)			0.975(*)			0.923(*)	

***Correlation is significant at the 0.01 level.**

Table 11: The running time of the proposed model for different clusters

Cluster	Number of DMUs	Number of efficient DMUs	Time (second)
1	57	11	34.852410
2	36	10	19.062698
3	47	17	1801.718352
4	72	17	1801.718352
5	76	13	117.938605