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General Relativity solutions in modified gravity

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ABSTRACT

Recent gravitational wave observations of binary black hole mergers and a binary neutron star merger by LIGO and Virgo Collaborations associated with its optical counterpart constrain deviation from General Relativity (GR) both on strong-field regime and cosmological scales with high accuracy, and further strong constraints are expected by near-future observations. Thus, it is important to identify theories of modified gravity that intrinsically possess the same solutions as in GR among a huge number of theories. We clarify the three conditions for theories of modified gravity to allow GR solutions, i.e., solutions with the metric satisfying the Einstein equations in GR and the constant profile of the scalar fields. Our analysis is quite general, as it applies a wide class of single-/multi-field scalar–tensor theories of modified gravity in the presence of matter component, and any spacetime geometry including cosmological background as well as spacetime around black hole and neutron star, for the latter of which these conditions provide a necessary condition for no-hair theorem. The three conditions will be useful for further constraints on modified gravity theories as they classify general theories of modified gravity into three classes, each of which possesses i) unique GR solutions (i.e., no-hair cases), ii) only hairy solutions (except the cases that GR solutions are realized by cancellation between singular coupling functions in the Euler–Lagrange equations), and iii) both GR and hairy solutions, for the last of which one of the two solutions may be selected dynamically.

1. Introduction

Recent measurements of gravitational waves (GWs) from binary black hole (BH) mergers by LIGO and Virgo Collaborations [1,2] clarified that the observed GWs are consistent with the prediction of General Relativity (GR) for binary coalescence waveforms. Moreover, the almost simultaneous detection of GWs from a neutron star (NS) merger [3], and the short gamma-ray burst [4] has significantly constrained a deviation of propagation speed of GWs over cosmological distance from the speed of light down order 10^{-15} [5]. The future measurements of GWs with unprecedented accuracies will make it possible to test modified gravity from completely different aspects.

Various gravitational theories alternative to GR have been proposed to explain inflation and/or late-time acceleration of the Universe [6]. Scalar-tensor theories of gravitation involve the representative frameworks for modification of GR such as Horndeski theory [7] (or generalized Galileon [8–12]), and even today sensible construction of scalar–tensor theories have been extensively investigated [13–22]. The possible deviations from astrophysical and cosmological predictions in GR have been explored as smoking guns of these theories [6,23,24].

The situation changes abruptly by the recent GW observations. The constraint on the propagation speed of GWs severely restricts theories of modified gravity for the late-time accelerated expansion [25–30] and those with the screening mechanism [31–34]. Moreover, the worldwide network of GW interferometer will include KAGRA [35], and further improve these tests of gravity both on strong-field regime and cosmological scales. Within next few years, it is plausible that no deviation from predictions in GR would be detected. If it is the case, GR or modified gravity theories sharing the same background solutions and perturbation dynamics with GR would be observationally preferred.[1]

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[1] It should be emphasized that even if GR and modified gravity theories share the same background solution, it is not necessarily true that the perturbation dynamics is also the same in both theories, as firstly addressed in Ref. [36] for specific theories. Nevertheless, our point is that if the observational data agree with the predictions of the perturbations in GR, it would suggest that the background solution is given by a GR solution.

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It is then important to note that no detection of deviation from GR predictions does not immediately exclude modified gravity theories especially in strong-field regime, as many theories could share the same solutions with GR. In GR, there is the no-hair theorem which states that the BH spacetime is solely determined by three conserved quantities or "hairs"; mass, angular momentum, and electric charge [37–39]. In general, scalar–tensor theories may possess BH solutions with nontrivial scalar hair [40–58] which are different from GR BH solutions with the constant profile of the scalar fields. Interestingly, however, there exist some class of modified gravity theories allowing only the BH metric solutions in GR with constant scalar field as the unique solutions [59–68]. This is the extension of no-hair theorems, and implies that these classes evade constraints on deviation of BH spacetime from GR one. Moreover, even in a case where GR and non-GR BH solutions exist simultaneously and the GR BH solution is not the unique solution, if it is the late-time attractor, the theory dynamically selects the GR BH solution and still evades the constraints. Therefore, taking into account the rapidly expanding frontier of the modified gravity theories and the remarkable progress of their constraints from GW observations, it is important to identify which class of the most general scalar–tensor theories could admit GR BH solutions.

In this Letter, we clarify the conditions for the existence of GR solutions in a quite general scalar–tensor theory defined by (1) below, where by "GR solution" we mean a solution with a metric satisfying the Einstein equations in GR and a constant profile of the scalar fields. Our analysis will expand that in Ref. [69] which showed that different gravitational theories share the Kerr solution same as in GR. Ref. [70] constructed the higher-order Ricci polynomial gravity theories that admit the same vacuum static solutions as GR. We will cover modified gravity theories which can be described by any class of single-/multi-field scalar–tensor theories. Our analysis solely exploits the covariant equations of motion without assuming any symmetry and ansatz for the metric and scalar fields, and hence any GR solution is within our subject. Note that "GR solution" here represents not only static or stationary BH solutions such as Schwarzschild, Kerr, and Schwarzschild–de Sitter solutions, but also any solution in GR in astrophysical or cosmological situation with/without the existence of matter. Our analysis will also apply higher dimensional spacetime, in which a caveat is that vacuum GR solutions include not only spherical BHs, but also black objects with nonspherical horizon topology [71,72], and hence the uniqueness of black objects does not hold.

It should be emphasized that our analysis focuses on GR solutions with the constant profile of the scalar fields, and there are several theories that do not fit our analysis, e.g., theories with self-gravitating media such as Lorentz-violating massive gravity [73–78], and theories where the small-scale behavior such as breaking of the Vainshtein screening is sensitive to the asymptotic time-dependence of the scalar fields [79–81]. Correspondingly, there are also several examples of BH solutions with the metric of GR in modified gravity theories that are not captured by the constant scalar field ansatz, e.g., the Schwarzschild–de Sitter BHs in the shift-symmetric Horndeski theories [50] and in the massive gravity theories [82–86], and the Kerr solution in the purely quartic Horndeski theory [58].

2. The model

We consider a wide class of single-/multi-field scalar–tensor theories in D-dimensional spacetime described by the action

$$S=\int d^Dx\sqrt{-g}\big[G_2(\phi^I,X^{JK})+G_4(\phi^I,X^{JK})R \\ +\phi^I_{;\mu_1}C^{\mu_1}_{1I}+\phi^I_{;\mu_1\mu_2}C^{\mu_1\mu_2}_{2I}+\phi^I_{;\mu_1\mu_2\mu_3}C^{\mu_1\mu_2\mu_3}_{3I}+\cdots \\ +L_m(g_{\mu\nu},\psi)\big], \tag{1}$$

where the Greek indices $\mu,\nu,\cdots$ run the D-dimensional spacetime, the capital Latin indices $I,J,\cdots$ label the multiple scalar fields, and semicolons denote the covariant derivative with respect to the metric $g_{\mu\nu}$. In addition to the Ricci curvature R and the matter Lagrangian $L_m(g_{\mu\nu},\psi)$ minimally coupled to gravity, the action involves arbitrary functions: G_2, G_4 are functions of the multiple scalar fields ϕ^I and the kinetic terms $X^{IJ}\equiv -g^{\mu\nu}\phi^I_{;\mu}\phi^J_{;\nu}/2$, and $C^{\mu_1}_{1I}, C^{\mu_1\mu_2}_{2I}, C^{\mu_1\mu_2\mu_3}_{3I},\cdots$ are functions of $(g_{\alpha\beta}, g_{\alpha\beta,\gamma}, g_{\alpha\beta,\gamma\delta},\cdots;\phi^I,\phi^I_{;\alpha},\phi^I_{;\alpha\beta},\cdots;\epsilon_{\mu\nu\rho\sigma})$ with $\epsilon_{\mu\nu\rho\sigma}$ being the Levi-Civita tensor. The dots in (1) contain contractions between arbitrary higher-order covariant derivatives of a scalar field and its corresponding C-function, $\phi^I_{;\mu_1\cdots\mu_n}C^{\mu_1\cdots\mu_n}_{nI}$. In order for Eq. (1) to be covariant with respect to $g_{\mu\nu}$, the dependence of $C^{\mu_1}_{1I}, C^{\mu_1\mu_2}_{2I}, C^{\mu_1\mu_2\mu_3}_{3I},\cdots$ on the metric should be through metric itself, curvature tensors associated with it, and their covariant derivatives.

This action is very generic and covers a lot of single-/multi-field models of scalar–tensor theories. Indeed, the term $\phi_{;\mu\nu}C^{\mu\nu}_2$ includes Ostrogradsky ghost-free single-field scalar–tensor theories such as Horndeski [7] (generalized Galileon [8–12]), Gleyzes–Langlois–Piazza–Vernizzi (GLPV) [14,15], and degenerate higher-order scalar–tensor (DHOST) theories [17,20] as a subclass. Specifically, the Horndeski action in the four-dimensional spacetime is described by $C^{\mu\nu}_2=C^{\mu\nu}_{\rm H}$ with

$$C^{\mu\nu}_{\rm H}=G_3g^{\mu\nu}+G_{4X}(g^{\mu\nu}\Box\phi-\phi^{;\mu\nu})+G_5G^{\mu\nu} \\ -\frac{1}{6}G_{5X}[g^{\mu\nu}(\Box\phi)^2-3\Box\phi\phi^{;\mu\nu}+2\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}], \tag{2}$$

and GLPV action is given by $C^{\mu\nu}_2=C^{\mu\nu}_{\rm H}+C^{\mu\nu}_{\rm bH}$ with

$$C^{\mu\nu}_{\rm bH}=F_4\epsilon^{\alpha\beta\mu}{}_{\gamma}\epsilon^{\tilde\alpha\tilde\beta\nu\gamma}\phi_{;\alpha}\phi_{;\tilde\alpha}\phi_{;\beta\tilde\beta} \\ +F_5\epsilon^{\alpha\beta\gamma\mu}\epsilon^{\tilde\alpha\tilde\beta\tilde\gamma\nu}\phi_{;\alpha}\phi_{;\tilde\alpha}\phi_{;\beta\tilde\beta}\phi_{;\gamma\tilde\gamma}, \tag{3}$$

where G_n, F_n are functions of ϕ, $X=-g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}/2$, and $G_{nX}\equiv\partial G_n/\partial X$. Likewise, it is also clear that quadratic- and cubic-order DHOST theories are a subclass and described by the $\phi_{;\mu\nu}C^{\mu\nu}_2$ term. It also includes parity-violating theories with Chern–Simons term or Pontryagin density $\epsilon_{\alpha\beta\gamma\delta}R^{\alpha\beta}{}_{\mu\nu}R^{\gamma\delta\mu\nu}/2$ [87–96], the multi-Galileon theories [97–105], those with complex scalar fields, and even more general higher-order theories involving derivatives higher than second order, which can be free from the Ostrogradsky ghost by imposing a certain set of ghost-free conditions [16,21,22]. Note that in this paper we will focus only on the conditions for obtaining the GR solutions and actually it does not matter whether the theory (1) contains the Ostrogradsky ghost or not. Hence, the following analysis for (1) to allow GR solutions is powerful and exhausts almost all the known scalar–tensor theories of modified gravity.

3. Conditions for GR solutions

We focus on a solution in GR with a given value of cosmological constant Λ for $\Phi^I\equiv(\phi^I,\phi^I_{;\alpha},\phi^I_{;\alpha\beta},\cdots)=\Phi^I_0$, where $\Phi^I_0\equiv(\phi^I_0,0,0,\cdots)$ and ϕ^I_0 is constant, which satisfies the Einstein equation

$$G^{\mu\nu}=8\pi GT^{\mu\nu}-\Lambda g^{\mu\nu}, \tag{4}$$

where $T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g_{\mu\nu}}$ is the stress energy tensor for the matter component, which is further decomposed into the classical and constant parts $T^{\mu\nu}=T^{\mu\nu}_m-(8\pi G)^{-1}\Lambda_m g^{\mu\nu}$, where the latter denotes the contribution of matter vacuum fluctuations. We elucidate that the theory (1) possesses GR solutions if the following conditions on the functional forms at $\Phi^I = \Phi_0^I$ are satisfied:

1. $G_2, G_4, C_{1I}^{\mu_1}, C_{2I}^{\mu_1\mu_2}, C_{3I}^{\mu_1\mu_2\mu_3}, \cdots$ and their derivatives appearing in the Euler–Lagrange equations are regular at $\Phi^I = \Phi_0^I$.
2. If $T_m^{\mu\nu} = 0$, $8\pi G(G_2 + 2\Lambda G_4) = -(16\pi G G_4 - 1)\Lambda_m$ and $(D-2)G_{2\phi^I} = -2D(\Lambda + \Lambda_m)G_{4\phi^I}$ for $\Phi^I = \Phi_0^I$. If $T_m^{\mu\nu} \neq 0$, $G_4 = (16\pi G)^{-1}$, $G_2 = -\Lambda/(8\pi G)$, and $G_{2\phi^I} = G_{4\phi^I} = 0$ for $\Phi^I = \Phi_0^I$.
3. $C_{1I}^{\mu_1}{}_{;\mu_1} = C_{2I}^{\mu_1\mu_2}{}_{;\mu_1\mu_2} = \cdots = 0$ for $\Phi^I = \Phi_0^I$.

Since the values of $C_{1I}^{\mu_1}, C_{2I}^{\mu_1\mu_2}, C_{3I}^{\mu_1\mu_2\mu_3}, \cdots$ depend on a choice of the coordinate, in the condition 1 we require that as functions of Φ^I they are regular. If the condition 1 is not satisfied and some of the functions in the Euler–Lagrange equations diverge at $\Phi^I = \Phi_0^I$ which occurs e.g., for $G_2 \propto \sqrt{X}$ in the cuscuton [106] and more generally $G_n \propto X^{(3-n)/2}$ in the cuscuta-Galileon [107], the divergence of the Euler–Lagrange equations should be avoided either by constraining dynamics with nonzero velocity and/or gradient of the scalar field, or by cancellation by other divergence with the opposite sign through the entire time evolution. An example where the cancellation between singular coupling functions identically holds is the Einstein-scalar–Gauss–Bonnet theory defined by $L = (M_{\rm Pl}^2/2)R + X + f(\phi)\mathcal{R}_{\rm GB}$ that is equivalent to the Horndeski theory with $G_5 = -4f'(\phi)\ln X$, $G_4 = M_{\rm Pl}^2/2 + 4f''(\phi)X(2-\ln X)$, $G_3 = -4f^{(3)}(\phi)X(7-3\ln X)$, and $G_2 = X + 8f^{(4)}(\phi)X^2(3-\ln X)$ [12] (with the notation $G_2 \to K$ and $G_3 \to -G_3$). Since G_{2XX}, G_{4X}, and $C_H^{\mu\nu}$ [see Eq. (2)] and their derivatives appearing in the Euler–Lagrange equations become singular for $X = 0$, the condition 1 is violated.

The condition 2 depends on the existence of nonzero $T^{\mu\nu}$ and/or Λ; for instance, for vacuum solutions in GR with $T_m^{\mu\nu} = \Lambda_m = \Lambda = 0$, the condition 2 reduces to $G_2 = G_{2\phi^I} = 0$ at $\Phi^I = \Phi_0^I$.

The condition 3 is satisfied by the Horndeski (2), GLPV (3), and DHOST theories so long as their functions $G_3, F_4, \cdots$ and their derivatives are regular at $\phi = \phi_0$ and $X = 0$. In contrast, e.g., $C_2^{\mu\nu} \supset R^{\mu\nu}$ violates the condition 3 as $C_2^{\mu\nu}{}_{;\mu\nu} \supset g^{\mu\nu}R_{;\mu\nu}/2$ from the Bianchi identity, which is nonvanishing in the presence of nonzero matter component. Furthermore, if $C_2^{\mu\nu} \supset R^{\mu\lambda\rho\sigma}R^\nu{}_{\lambda\rho\sigma}$ the condition 3 is violated even without matter. The condition 3 excludes such possibilities.

Since our "GR solutions" may be the solutions in GR with the cosmological constant Λ (4), it is manifest that our conditions 1–3 are not relevant for the solution of the cosmological constant problem. If we require it, in addition to the conditions 1–3, further fine-tunings for the mass scales would be requested, which are beyond the scope of our analysis.

Before closing this section, it should be emphasized that the conditions 1–3 are different from those for a no-hair theorem. In establishing a no-hair theorem (mostly for BH solutions), the scalar fields ϕ^I are assumed to be general functions of the spacetime coordinates (e.g., functions of the radial coordinate for static and spherically symmetric BH solutions), and then the conditions that $\phi^I = $ const. (and the spacetime metric of a GR solution) is a unique solution are deduced. For instance, the no-hair theorem for shift-symmetric Horndeski theory considered in [66] adopted a condition for finite Noether current, which amounts to the condition 1. Under the shift symmetry and the conditions for the Ricci-flat solutions with $R^{\mu\nu} = 0$, i.e., $T_m^{\mu\nu} = \Lambda_m = \Lambda = 0$ from Eq. (4), our condition 2 reads $G_2 = 0$, which is consistent with the assumption of the asymptotic flatness in the no-hair theorem. The condition 3 is also identically satisfied. The uniqueness of GR solution is guaranteed by the additional conditions of the staticity, spherical symmetry, and asymptotic flatness of the metric. In our case the conditions 1–3 obtained from the assumptions that solutions with the metric satisfying the Einstein equations in GR and $\phi^I = $ const. exist still allow the non-GR solutions where the metric is different from any solution in GR and the scalar fields ϕ^I have nontrivial profiles, and the solutions with the metric of GR but $\phi^I \neq$ const., as we did not require the uniqueness of the solution. Thus, our conditions 1–3 should be regarded as the necessary conditions for establishing a no-hair theorem when the symmetries of the spacetime and the ansatz of the scalar fields are more specified, e.g., ϕ^I are functions of the radial coordinate for static and spherically symmetric BH spacetimes.

3.1. Proof

While our statement holds for the wide class of theories (1), as we will see below, the proof is very simple. We mainly discuss the single-field case of (1), as the extension to the multi-field case is straightforward. We denote the Euler–Lagrange equations for (1) with respect to the metric and the scalar field as

$$\mathcal{E}^{\mu\nu} \equiv \frac{1}{\sqrt{-g}}\left[\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} - \partial_\alpha\left(\frac{\partial \mathcal{L}}{\partial g_{\mu\nu,\alpha}}\right) + \cdots\right] = 0,$$
$$\mathcal{E}_\phi \equiv \frac{1}{\sqrt{-g}}\left[\frac{\partial \mathcal{L}}{\partial \phi} - \nabla_\alpha\left(\frac{\partial \mathcal{L}}{\partial \phi_{;\alpha}}\right) + \cdots\right] = 0, \tag{5}$$

which correspond to Einstein equation and Klein–Gordon equation, and show that for the theories to allow GR solutions, the above three conditions should be satisfied.

First, let us focus on the G_2, G_4 terms and the matter component L_m in the action (1) and set all the C-functions zero. The Euler–Lagrange equations (5) are then given by

$$\mathcal{E}^{\mu\nu} = \frac{1}{2}g^{\mu\nu}G_2 - G^{\mu\nu}G_4 + \frac{1}{2}T^{\mu\nu}$$
$$- \frac{1}{2}(G_{2X} + RG_{4X})\phi^{;\mu}\phi^{;\nu} + (\nabla^\mu\nabla^\nu - g^{\mu\nu}\Box)G_4,$$
$$\mathcal{E}_\phi = G_{2\phi} + RG_{4\phi}$$
$$+ \frac{1}{2}\nabla_\mu(G_{2X}\phi^{;\mu}) + \frac{1}{2}R\nabla_\mu(G_{4X}\phi^{;\mu}). \tag{6}$$

Substituting $\phi = $ const., the second lines of $\mathcal{E}^{\mu\nu}$ and $\mathcal{E}_\phi$ vanish so long as we assume the condition 1, namely, G_2, G_4, and their derivatives involved in (6) do not diverge at $\phi = $ const.

Furthermore, substituting (4) and its trace $(2-D)R/2 = 8\pi G T_m{}^\mu{}_\mu - D(\Lambda + \Lambda_m)$ to (6) yields

$$g^{\mu\nu}\left(G_2 + 2\Lambda G_4 + \frac{16\pi G G_4 - 1}{8\pi G}\Lambda_m\right) = T_m^{\mu\nu}(16\pi G G_4 - 1),$$
$$(D-2)G_{2\phi} + 2D(\Lambda + \Lambda_m)G_{4\phi} = 16\pi G G_{4\phi} T_m{}^\mu{}_\mu. \tag{7}$$

For vacuum solutions, $T_m^{\mu\nu} = 0$, EOMs (7) match the former case of the condition 2. On the other hand, with $T_m^{\mu\nu} \neq 0$, each side of (7) has to vanish separately as $T_m^{\mu\nu}$ varies with x^μ, leading to the latter case of the condition 2. Thus, the action (1) without C-functions allows GR solutions if the condition 1 and the condition 2 are satisfied.

Next, we consider the remaining C-terms of the action (1) and clarify that their contribution to the Euler–Lagrange equations vanish for $\phi = $ const. under the condition 1 and the condition 3. The contribution of $\mathcal{L}_n \equiv \sqrt{-g}\phi_{;\rho_1\cdots\rho_n}C_n^{\rho_1\cdots\rho_n}$ to the Euler–Lagrange equations (5) is given by

$$\mathcal{E}^{\mu\nu} \supset \left(\frac{1}{2}g^{\mu\nu}C_n^{\rho_1\cdots\rho_n} + \frac{\partial C_n^{\rho_1\cdots\rho_n}}{\partial g_{\mu\nu}}\right)\phi_{;\rho_1\cdots\rho_n} + \sum_{k=1}^{\infty}\frac{(-1)^k}{\sqrt{-g}}\partial_{\alpha_k}\cdots\partial_{\alpha_1}\left(\sqrt{-g}\frac{\partial(\phi_{;\rho_1\cdots\rho_n}C_n^{\rho_1\cdots\rho_n})}{\partial g_{\mu\nu,\alpha_1\cdots\alpha_k}}\right),$$

$$\mathcal{E}_\phi \supset \frac{\partial C_n^{\rho_1\cdots\rho_n}}{\partial\phi}\phi_{;\rho_1\cdots\rho_n} + (-1)^n C_n^{\rho_1\cdots\rho_n}{}_{;\rho_1\cdots\rho_n} + \sum_{k=1}^{\infty}(-1)^k\nabla_{\alpha_k}\cdots\nabla_{\alpha_1}\left(\phi_{;\rho_1\cdots\rho_n}\frac{\partial C_n^{\rho_1\cdots\rho_n}}{\partial\phi_{;\alpha_1\cdots\alpha_k}}\right). \tag{8}$$

The $C_n^{\rho_1\cdots\rho_n}{}_{;\rho_1\cdots\rho_n}$ term in $\mathcal{E}_\phi$ vanishes from the condition 3. All other terms in the right-hand sides of (8) are multiplied by derivatives of ϕ, and thus vanish for $\phi =$ const. so long as the C-functions and their derivatives are regular.

It should be emphasized that our analysis does not include the case that a GR solution with $\phi =$ const. is obtained via the cancellation of the independent contributions arising from singular coupling functions in the Euler–Lagrange equations (5). For example, as we mentioned earlier, the Einstein-scalar–Gauss–Bonnet theory $L = (M_{\rm Pl}^2/2)R + X + f(\phi)\mathcal{R}_{\rm GB}$ exhibits such cancellation and does not satisfy the conditions 1–3. Nevertheless, it has been shown that the Schwarzschild metric with $\phi = \phi_0 =$ const. is a solution if $f'(\phi_0) = 0$ [108–111]. This is an example of exceptional cases of our proof.

In summary, deriving Euler–Lagrange equations for the full action (1) and then plugging GR solution $\phi =$ const. and (4) lead to the equations (7). Generalization to the multi-field case is also straightforward. We thus conclude that the general theories (1) allow GR solutions if the conditions 1–3 are satisfied.

4. Examples

Now it is intriguing to consider specific examples. From the fact that GW observations are consistent with GR, we are interested in identifying a class of modified gravity that allows GR solutions by satisfying the conditions 1–3.

Let us consider single-field models in the four-dimensional spacetime. For the Horndeski theory with (2), the Euler–Lagrange equations for a static, spherically-symmetric spacetime were derived in [112]. In [113], the no-hair vacuum solution with $\Lambda = 0$ is considered by assuming asymptotic flatness of the spacetime and the scalar field profile $\phi = \phi(r)$, and it is clarified that the Euler–Lagrange equations allow the Schwarzschild solution if the condition 1 and the condition 2 are satisfied, where the condition 2 reduces to $G_2 = G_{2\phi} = 0$. Note that in this case the condition 3 is automatically satisfied as long as G_3, G_4, and G_5 are regular for $\phi =$ const. Note also that the argument in the present paper is fully performed in the covariant manner without any ansatz for the metric and the scalar field, and in general the assumptions such as the asymptotic flatness and $\phi = \phi(r)$ are not necessary.

In the same vein, we can consider GLPV and DHOST theories and check that our statement holds. We consider a static, spherically-symmetric metric ansatz

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{9}$$

and plug it to the action (1). While we fix the gauge as $g_{tr} = 0$ and $g_{\theta\theta} = r^2$ at the action level, we do not lose any independent EOMs, regardless of a choice of a specific theory (see Sec. V C of [114]). EOMs for GLPV theory are given by $\mathcal{E}_Q^{\rm H} + \mathcal{E}_Q^{\rm bH4} + \mathcal{E}_Q^{\rm bH5} = 0$ with $Q = A, B, \phi$, where the Horndeski terms $\mathcal{E}_Q^{\rm H}$ are given in [112, 113] (with the notation $G_2 \to K$ and $G_3 \to -G_3$), and $\mathcal{E}_Q^{\rm bH4}$ and $\mathcal{E}_Q^{\rm bH5}$ are contributions from the beyond Horndeski terms F_4 and F_5 in (3), respectively, and given by (A.1) and (A.2) in Appendix A. Clearly, assuming that F_4, F_5, and their derivatives in EOMs are not singular at $\phi =$ const., $\mathcal{E}_Q^{\rm bH4}$ and $\mathcal{E}_Q^{\rm bH5}$ vanish for $\phi =$ const. Likewise, we explicitly checked that EOMs for quadratic- and cubic-order DHOST theories vanish for $\phi =$ const. so long as we assume the regularity.

In summary, to guarantee the existence of GR solutions, the form of G_2 and G_4 are severely constrained by the condition 2 and the form of other functions in Horndeski, GLPV, and DHOST theories are not constrained so long as they are regular at $\phi =$ const. The condition is different from the condition $G_4 = G_4(\phi)$ and $G_5 =$ const. obtained in [27–30] by imposing the propagation speed of GWs to be the same as GR, as G_4 and G_5 are only terms that have nonminimal coupling to gravity and affects GW propagation speed. Note that our condition and the constraint on G_4 and G_5 from the propagation speed of GWs are independent from several aspects; the latter is observational, valid only on cosmological scales, and can be applied to the models where the scalar fields act as the source of dark energy. Thus, the models which satisfy the GW constraint on the large scales would contain non-GR solutions on small scales.

5. Classification

Let us describe further application of the conditions 1–3 to general theories of modified gravity on strong-field regime. The conditions 1–3 classify general theories of modified gravity into three classes. The classification clarifies the origin of differences between many known examples of no-hair theorem and hairy solutions, and helps us to explore GR and non-GR solutions in theories of modified gravity in various contexts explained below.

First, if a theory satisfies the conditions 1–3, it of course allows GR solutions, but may also allow other solutions with a nontrivial scalar field(s). Therefore, the conditions 1–3 are necessary conditions for a no-hair theorem, and theories satisfying the conditions 1–3 serves as a candidate in which a no-hair theorem can be established. If a GR solution is unique solution, such a theory intrinsically passes constraints on deviation from GR. To guarantee the uniqueness of GR solutions, one may need to impose some additional conditions, for instance, symmetries of spacetime, ansatz for scalar field, and/or internal symmetry of the theory. Indeed, theories satisfying the conditions 1–3 include Brans–Dicke theory, the shift-symmetric Horndeski theory, and the shift-symmetric GLPV theory as a subclass, for which no-hair theorems in the four-dimensional spacetime have been proven [64–66,115].

On the other hand, if a theory does not satisfy at least one of the conditions 1–3, it inevitably possesses only non-GR solutions,[2] except for the case of the cancellation discussed in the second paragraph from the last in Sec. 3. Therefore, focusing on the violation of the conditions 1–3, one can identify the candidate classes that possess analytic solutions of hairy BH. An example is the Einstein-scalar–Gauss–Bonnet theory discussed in [42–46] with $G_5 \sim \ln X$ in (2), which violates the condition 1, leading to hairy BH solutions.[3] Other examples of hairy BH solutions obtained by the violation of the condition 1 were explicitly constructed in [58] for

[2] As we emphasized at the end of Sec. 1, throughout the paper we state "GR solutions" as solutions satisfying the Einstein equation (4) and the constant scalar field profile $\Phi^I = \Phi_0^I$. Another type of solution satisfying the Einstein equation for a nonconstant profile of the scalar field, e.g., the stealth Schwarzschild solution [50], is included in "non-GR solutions" here.

[3] As we mentioned briefly at the end of Sec. 3, Refs. [108–111] showed that the Einstein-scalar–Gauss–Bonnet theories admit the Schwarzschild BH solution, which would be unstable due to the tachyonic instability triggered by coupling of the scalar field to the Gauss–Bonnet term. Since the theory does not satisfy the con-

the Horndeski and beyond-Horndeski theories in which G_i and F_i in (2) and (3) and their derivatives are not analytic at $X = 0$.

The last possibility is inbetween the above two: a class that possesses GR solutions by satisfying the conditions 1–3 and allows other hairy solutions at the same time, one of which may be attractor. If a GR solution is the attractor, it may be said that a no-hair theorem holds in a dynamical way, and it passes observational constraints on deviation from GR spacetime. In contrast, the opposite case that a non-GR BH solution is dynamically selected rather than a GR one could also happen, such as the spontaneous scalarization [116,117]. The key for the coexistence of GR and non-GR solutions is that the theory satisfies the conditions 1–3 to allow GR solutions, and also has some internal symmetry to allow hairy BH solutions without spoiling spacetime symmetry. Specific examples of this class include a hairy solution in a subclass of the shift-symmetric Horndeski theories with a nontrivial linear time dependence of the scalar field $\phi = qt + \psi(r)$ [50] (see also [118]), the Kerr-like hairy solution in Einstein-complex scalar theory with $U(1)$ symmetry with the complex scalar field profile $\Phi = \phi(r,\theta)e^{i(m\varphi-\omega t)}$ [51], the Bocharova–Bronnikov–Melnikov–Bekenstein (BBMB) solution [40,41] in a conformally coupled scalar field, and a similar solution in a two-field extension of the Horndeski theory [52]. Checking whether a GR or non-GR solution is the attractor requires further studies on a case-by-case basis.

6. Conclusion

The recent and future GW observations allow us to place a stringent constraint on deviation from GR, and hence it is important to identify theories of modified gravity that can intrinsically share the same solutions with GR. We have investigated a quite general class (1) of single-/multi-field scalar–tensor theories with arbitrary higher-order derivatives in arbitrary spacetime dimension. We confirmed that GR solutions are allowed if the conditions 1–3 are satisfied. This approach yields an independent result from the one requiring the propagation speed of gravitational wave to be the speed of light as in GR [4]. Our analysis was fully covariant, and hence can be applied to any astrophysical or cosmological situation. The conditions 1–3 classify general theories of modified gravity into three classes, each of which possesses i) only GR solutions (i.e., no-hair cases), ii) only hairy solutions (except the cases that GR solutions are realized by cancellation between singular coupling functions in the Euler–Lagrange equations), and iii) both GR and hairy solutions, for the last of which one of the two solutions may be selected dynamically.

There will be several extensions of our analysis. One of them is the possibility of GR solutions with $X =$ const. The simplest example is the Schwarzschild solution with a nontrivial scalar field [50], and it would be important to see whether theories (1) also admit generic GR solutions with $X =$ const. and clarify characteristics in their dynamics.

Furthermore, while we have focused on the scalar–tensor theories, it would be definitively important to extend our analysis to the theories which include vector fields, fermions, and other tensor fields as in bigravity theories. In such higher-spin theories the conditions that they admit GR solutions would be able to be obtained by requesting that all components of vector, fermion and tensor fields vanish in the entire spacetime. More rigorous derivation of the conditions for each theory is out of scope of our paper and will be left for the future work.

ditions 1–3, the realization of the Schwarzschild solution exploits the cancellation of the terms absent in GR, which falls in the exception of our analysis as mentioned above.

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Appendix A. EOMs for beyond Horndeski Lagrangians

EOMs for the beyond Horndeski terms F_4 and F_5 in (3) for the static, spherically-symmetric metric ansatz (9) are respectively given by

$$\begin{aligned}
\mathcal{E}_A^{\rm bH4} &= -\frac{2B^3\phi'^3}{r^2}\bigg[\left(\phi' + 5\frac{B'}{B}r\phi' + 8r\phi''\right)F_4 \\
&\quad + 2r\phi'^2 F_{4\phi} - r\phi'(B\phi'^2)'F_{4X}\bigg], \\
\mathcal{E}_B^{\rm bH4} &= \frac{2AB^2\phi'^4}{r^2}\frac{(rA)'}{A}(5F_4 - B\phi'^2F_{4X}), \\
\mathcal{E}_\phi^{\rm bH4} &= \frac{2AB^3\phi'^2}{r^2}\bigg[4\left\{\left(\alpha - 5\frac{A'}{A}\frac{B'}{B}\right)r\phi' \right. \\
&\quad \left. - \left(3\frac{A'}{A} + 5\frac{B'}{B}\right)\phi' - 6\frac{(rA)'}{A}\phi''\right\}F_4 + \left\{18\frac{(rA)'}{A}\phi'' \right. \\
&\quad \left. + \left(-\alpha + 11\frac{A'}{A}\frac{B'}{B}\right)r\phi' + \left(3\frac{A'}{A} + 11\frac{B'}{B}\right)\phi'\right\}B\phi'^2F_{4X} \\
&\quad - \frac{(rA)'}{A}\phi'^2\left\{6F_{4\phi} - 2B\phi'^2F_{4\phi X} + (B\phi'^2)'B\phi'F_{4XX}\right\}\bigg],
\end{aligned} \tag{A.1}$$

and

$$\begin{aligned}
\mathcal{E}_A^{\rm bH5} &= -\frac{3B^4\phi'^4}{r^2}\bigg[\left(\frac{7B'}{B}\phi' + 10\phi''\right)F_5 \\
&\quad + 2\phi'^2F_{5\phi} - \phi'(B\phi'^2)'F_{5X}\bigg], \\
\mathcal{E}_B^{\rm bH5} &= \frac{3A'B^3\phi'^5}{r^2}(7F_5 - B\phi'^2F_{5X}), \\
\mathcal{E}_\phi^{\rm bH5} &= \frac{3AB^4\phi'^3}{r^2}\bigg[5\left\{\left(\alpha - 7\frac{A'}{A}\frac{B'}{B}\right)\phi' - 8\frac{A'}{A}\phi''\right\}F_5 \\
&\quad + \left\{\left(-\alpha + 14\frac{A'}{A}\frac{B'}{B}\right)\phi' + 22\frac{A'}{A}\phi''\right\}B\phi'^2F_{5X} \\
&\quad - \frac{A'}{A}\phi'^2\left\{8F_{5\phi} - 2B\phi'^2F_{5\phi X} + (B\phi'^2)'B\phi'F_{5XX}\right\}\bigg],
\end{aligned} \tag{A.2}$$

where $\alpha \equiv \frac{A'^2}{A^2} - 2\frac{A''}{A}$.

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