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Letter

Quantum decoherence in the entanglement entropy of a composite particle and its relationship to coarse graining in the Husimi function

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I investigate quantum decoherence in a one-body density matrix of a composite particle consisting of two correlated particles. Because of the two-body correlation in the composite particle, quantum decoherence occurs in the one-body density matrix that has been reduced from the two-body density matrix. As the delocalization of the distribution of the composite particle grows, the entanglement entropy increases, and the system can be well described by a semi-classical approximation, wherein the center position of the composite particle can be regarded as a classical coordinate. I connect the quantum decoherence in the one-body density matrix of a composite particle to the coarse graining in a phase space distribution function of a single particle and associate it with the Husimi function.

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Subject Index   D10

In recent decades, quantum entanglement has attracted a great deal of interest in various fields. To estimate correlations in quantum systems, entanglement measures such as the entanglement entropy (EE) have been intensively studied [1–9]. Many entanglement measures are defined by reduced density matrices which describe the structure of the Schmidt decomposition and contain information about entanglements in quantum systems. In entangled states, the EE is produced by quantum decoherence caused by a reduction in the number of degrees of freedom (DOF).

In nuclear systems, many-body correlations are essential features in various phenomena. In light stable nuclei, α clusters are formed by four species of nucleons, spin-up and -down protons and neutrons, because of the four-body correlation as known in ^{8}Be and ^{20}Ne having 2α and ^{16}O $+ \alpha$ cluster structures. Concerning two-body correlations, deuteron and dineutron correlations, which are the two-nucleon pairing in the isospin $T = 0$ channel of a proton and a neutron and that in the $T = 1$ channel of two neutrons with strong spatial correlations, are recent hot subjects in unstable nuclei. To measure many-body correlations, a one-body density matrix $\hat{\rho}^{(1)}$ can be useful. Uncorrelated (non-entangled) states satisfy $\hat{\rho}^{(1)} = \left\{ \hat{\rho}^{(1)} \right\}^2$, and therefore correlations can be measured by analysis of $\hat{\rho}^{(1)}$. In this sense, the EE for the one-body density matrix is a promising measure to probe many-body correlations. In my previous papers, I calculated entanglement measures of the one-body density matrix in nuclear systems [10,11] and showed that the EE is enhanced by the delocalization of the distribution of clusters, which are composite particles of spatially correlated nucleons. The enhancement of the EE is consistent with the entanglement in composite particles described

by deformed oscillators in Ref. [12]. My first aim in the present paper is to understand how quantum decoherence occurs and how the EE is produced in the one-body density matrix of correlating particles.

Quantum decoherence—that is, the quantum entropy—has also been investigated with the coarse graining of distribution functions in a phase space. The Husimi function [13–15] is known to have finite Wehrl and Rényi–Wehrl entropies [16,17] because of the coarse graining by a Gaussian smearing of the Wigner function. It has been shown that the Wehrl and Rényi–Wehrl entropies are increased by delocalization of the distributions in quantum systems [18–21]. Campos et al. have discussed a correlation between the EE and the Rényi–Wehrl entropy in entangled states [21]. One of the fundamental questions in quantum physics is how the quantum decoherence in the reduced density matrix of entangled states can be connected to the coarse graining of distribution functions. My second aim in this paper is to understand the correspondence between the quantum decoherence in the one-body density matrix of correlating particles and the coarse graining in the Husimi function of a single-particle state.

In this paper, I investigate the EE of the one-body density matrix of a two-body system in which two particles are strongly correlated to form a composite particle, and I discuss how quantum decoherence occurs in the reduction of the DOF. To describe two-body wave functions, I adopt a cluster wave function in the generator coordinate method in nuclear physics [22,23]. Let us consider a system where two particles (c1 and c2) with masses m and um form a bound state with an attractive inter-particle force. I assume that the bound state is described by the lowest state of a harmonic oscillator (ho) potential and can be approximately treated as an inert composite particle, where intrinsic excitations cost a relatively high amount of energy compared with the center of mass (cm) motion of the composite particle. In this approximation, a total two-body wave function is given as

$$\left|\Psi^{(2)}\right\rangle = \int ds\, F(s)|s;b\rangle_1 |s;b_2\rangle_2, \tag{1}$$

$$\langle r_i|s;b\rangle_i = \exp\left[-\frac{1}{2b^2}(r_i-s)^2\right]\bigg/\left(b^2\pi\right)^{1/4}, \tag{2}$$

where $b_2 = b/\sqrt{u}$, and $\left\langle\Psi^{(2)}|\Psi^{(2)}\right\rangle = 1$. r_1 (r_2) is the coordinate of c1 (c2). Here, I describe the one-dimensional case, but the present model can also be extended to the three-dimensional case. $|s;b\rangle_1 |s;b_2\rangle_2$ indicates the composite particle localized around the mean position s, and $\Psi^{(2)}$ is given by the superposition of different s states with the weight factor $F(s)$. We should comment that the present model for the two-body wave function gives the one-body density matrix of c1 equivalent to the one-body density matrix for a cluster composed of $(1 + u)$ particles with the equal mass m. $\left|\Psi^{(2)}\right\rangle$ can be expressed by the cm motion and the intrinsic wave functions as

$$\left|\Psi^{(2)}\right\rangle = |\Phi_G(R)\rangle|\phi_{\text{int}}(r)\rangle, \tag{3}$$

$$\langle R|\Phi_G(R)\rangle = \int ds\, \frac{F(s)}{\left(b_G^2\pi\right)^{1/4}} \exp\left[-\frac{1}{2b_G^2}(R-s)^2\right], \tag{4}$$

with the cm coordinate $R = u_1 r_1 + u_2 r_2$ ($u_1 = 1/(u+1)$ and $u_2 = u/(u+1)$), the relative coordinate $r = r_1 - r_2$, and $b_G = \sqrt{u_1}b$. Here, $\langle r|\phi_{\text{int}}(r)\rangle = \exp\left[-\frac{1}{2b_r^2}r^2\right]/\left(b_r^2\pi\right)^{1/4}$ with $b_r = b/\sqrt{u_2}$ is the lowest intrinsic state for the ho potential, $U_{\text{ho}}(\mu, b_r; r) = -\hbar^2 r^2/2\mu b_r^4$, with $\mu = u_2 m$. Thus, general low momentum states of the inert composite particle can be expressed by the form (1), in which the cm motion $\Phi_G(R)$ is expressed by the shifted Gaussian expansion as given in Eq. (4).

The one-body density matrix $\hat{\rho}^{(1)}_{\Psi^{(2)}}$ for c1 is defined by the matrix reduced from the many-body density matrix $\hat{\rho}^{(2)}_{\Psi^{(2)}} = |\Psi^{(2)}\rangle\langle\Psi^{(2)}|$ as $\hat{\rho}^{(1)}_{\Psi^{(2)}} = \text{Tr}_2\left[\hat{\rho}^{(2)}_{\Psi^{(2)}}\right]$ and is given as

$$\hat{\rho}^{(1)}_{\Psi^{(2)}} = \int ds\, ds'\, F^*(s')F(s))_2\langle s'; b_2|s; b_2\rangle_2 |s; b\rangle_{11}\langle s'; b|, \tag{5}$$

$$\rho^{(1)}_{\Psi^{(2)}}(q_1, q_1') = \left\langle q_1\left|\hat{\rho}^{(1)}_{\Psi^{(2)}}\right|q_1'\right\rangle = \int ds\, ds'\, \frac{F^*(s')F(s)}{(b^2\pi)^{1/2}}$$

$$\times \exp\left[-\frac{u}{4b^2}(s - s')^2\right]\exp\left[-\frac{1}{2b^2}(q_1 - s)^2 - \frac{1}{2b^2}(q_1' - s')^2\right], \tag{6}$$

where $\text{Tr}\hat{\rho}^{(1)}_{\Psi^{(2)}} = 1$. Note that $\hat{\rho}^{(1)}_{\Psi^{(2)}}$ for $u = 3$ equals the one-body density matrix of an α cluster composed of four nucleons with an equal mass investigated in previous papers [10,11]. The Wigner transformation (Wigner function) of $\hat{\rho}^{(1)}_{\Psi^{(2)}}$ is

$$\rho^W\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right) = \int d\eta \left\langle q_1 + \frac{\eta}{2}\left|\hat{\rho}^{(1)}_{\Psi^{(2)}}\right|q_1 - \frac{\eta}{2}\right\rangle \exp\left[-\frac{ip_1\eta}{\hbar}\right]$$

$$= 2\int ds\, ds'\, \frac{F^*(s')F(s)}{(b^2\pi)^{1/2}}\exp\left[-\frac{u}{4b^2}(s - s')^2\right]$$

$$\times \exp\left[-\frac{1}{2b^2}(q_1 - s)^2 - \frac{1}{2b^2}(q_1 - s')^2 - \frac{b^2}{\hbar^2}\left\{p_1 - \frac{i\hbar}{2b^2}(s - s')\right\}^2\right]. \tag{7}$$

The Rényi EE of order 2 (Rényi-2 EE) and von Neumann EE for $\Psi^{(2)}$ with the one-body density matrix $\hat{\rho}^{(1)}_{\Psi^{(2)}}$ are given as

$$S^{R2}\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}\right) = -\ln\left(\text{Tr}\left[\left\{\hat{\rho}^{(1)}_{\Psi^{(2)}}\right\}^2\right]\right) = -\ln\left(\int dq_1 dp_1\left\{\rho^W\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right)\right\}^2\right), \tag{8}$$

$$S^{vN}\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}\right) = -\text{Tr}\left[\hat{\rho}^{(1)}_{\Psi^{(2)}}\ln\hat{\rho}^{(1)}_{\Psi^{(2)}}\right]. \tag{9}$$

The latter equality in Eq. (8) generally holds because of the definition of the Wigner transformation. If $\rho^W\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right) \geq 0$ is satisfied in the entire phase space, I can consider the phase-space Shannon entropy $S^{Sh}\left(\rho(q, p)\right) = -\int \frac{dq\,dp}{2\pi\hbar}\rho(q, p)\ln\rho(q, p)$ for $\rho^W\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right)$ as,

$$S^{W-Sh}\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}\right) = S^{Sh}\left(\rho^W\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right)\right), \tag{10}$$

which I call the "Wigner–Shannon EE."

In the one-body density matrix $\hat{\rho}^{(1)}_{\Psi^{(2)}}$ and its Wigner transformation, quantum decoherence occurs and produces the EEs because of the factor $_2\langle s'; b_2|s; b_2\rangle_2 = \exp\left[-\frac{u}{4b^2}(s - s')^2\right]$, which originates in the reduction of the DOF of c2. Indeed, in the case of $u = 0$, without this factor, $\hat{\rho}^{(1)} = \left\{\hat{\rho}^{(1)}\right\}^2$ and the Rényi-2 and von Neumann EEs are zero, corresponding to a pure single-particle state.

Let us consider a semi-classical approximation of $\hat{\rho}^{(1)}_{\Psi^{(2)}}$. The factor $\exp\left[-\frac{u}{4b^2}(s - s')^2\right]$, which is the source of the quantum decoherence, has a sharp peak around $s' \approx s$ with a width $2b/\sqrt{u}$. I assume that the function $F(s)$ is a slowly varying function compared with $\exp\left[-\frac{u}{4b^2}(s - s')^2\right]$,

and it can be approximated as $F(s') \approx F(s)$. Then I obtain a semi-classical approximation,

$$\rho_{\Psi^{(2)}}^{(1)}(q_1, q_1') \stackrel{\text{cl}}{\approx} \rho_{\Psi^{(2)}}^{(1),\text{cl}}(q_1, q_1') = \int ds \frac{|f(s)|^2}{(b_r^2 \pi)^{1/2}} \exp\left[-\frac{1}{2b_r^2}(q_1 - s)^2 - \frac{1}{2b_r^2}(q_1' - s)^2 \right] \quad (11)$$

where $f(s) \propto F(s)$ whose normalization is determined by $\int dq_1 \rho_{\Psi^{(2)}}^{(1),\text{cl}}(q_1, q_1) = \int ds |f(s)|^2 = 1$. This corresponds to

$$\hat{\rho}_{\Psi^{(2)}}^{(1)} \stackrel{\text{cl}}{\approx} \hat{\rho}_{\Psi^{(2)}}^{(1),\text{cl}} \equiv \int ds |f(s)|^2 |s; b_r\rangle_{11}\langle s; b_r|. \quad (12)$$

In the large u limit—that is, the large c2 mass limit— $\rho(q_1, q_1') \to \rho^{\text{cl}}(q_1, q_1')$ and $b_r \to b$, and the parameter s and the squared amplitude $|f(s)|^2$ are regarded as a classical coordinate and a classical distribution of the second particle (c2), respectively. Note that, even in the large u limit, $|s\rangle_1$ and $|s'\rangle_1$, which are states of the first particle (c1) parametrized by s and s', are not orthogonal to each other for $s \neq s'$ because of the quantum fluctuations of the c1 position around the second particle. Therefore, we call Eq. (12) a "semi-classical approximation."

We should comment that in the large m limit, that is, the large mass limit of the two particles c1 and c2, $|s\rangle_1$ and $|s'\rangle_1$ become approximately orthogonal to each other for $s \neq s'$, and Eq. (12) corresponds to the Schmidt decomposition meaning that the system corresponds to a classical system with the distribution probability $\mathcal{P}(s) \equiv |f(s)|^2$. In this classical limit, the von Neumann EE is given just by the Shannon entropy in the coordinate space for the classical value s as $S^{\text{vN}} \to -\int ds \mathcal{P}(s) \ln \mathcal{P}(s)$. This relation of the von Neumann EE to the coordinate-space Shannon entropy in the classical limit corresponds well to the work discussed for Ising models in Ref. [26].

In the semi-classical approximation given by Eq. (12), the Wigner function is approximated as

$$\rho^{\text{W}}\left(\hat{\rho}_{\Psi^{(2)}}^{(1)}; q_1, p_1\right) \stackrel{\text{cl}}{\approx} 2 \int ds |f(s)|^2 \exp\left[-\frac{1}{b_r^2}(s - q_1)^2 - \frac{b_r^2}{\hbar^2}p_1^2 \right], \quad (13)$$

which is positive semidefinite. Using $\hat{\rho}_{\Psi^{(2)}}^{(1),\text{cl}}$ I also define the EEs in the semi-classical approximation: $S^{\text{R2,cl}} = S^{\text{R2}}\left(\hat{\rho}_{\Psi^{(2)}}^{(1),\text{cl}}\right)$ and $S^{\text{W}-\text{Sh,cl}} = S^{\text{W}-\text{Sh}}\left(\hat{\rho}_{\Psi^{(2)}}^{(1),\text{cl}}\right)$.

As a simple example, I first consider the zero-momentum state of the composite particle in a finite volume V described by a constant $F(s)$. I assume that $V \gg b$ and the contribution of the box boundary can be ignored, and obtain

$$\rho_{\Psi^{(2)}}^{(1)}(q_1, q_1') = \frac{1}{V} \exp\left[-\frac{1}{4b_r^2}(q_1 - q_1')^2 \right], \quad (14)$$

$$\rho^{\text{W}}\left(\hat{\rho}_{\Psi^{(2)}}^{(1)}; q_1, p_1\right) = \frac{2b_r \pi^{1/2}}{V} \exp\left[-\frac{b_r^2}{\hbar^2}p_1^2 \right]. \quad (15)$$

In this case, $\rho_{\Psi^{(2)}}^{(1)}(q_1, q_1') = \rho_{\Psi^{(2)}}^{(1),\text{cl}}(q_1, q_1')$ is satisfied. The Rényi-2, von Neumann, and Wigner–Shannon EEs are

$$S^{\text{R2}} = \ln V_{\text{eff}} - \tfrac{1}{2}\ln(2\pi), \quad (16)$$

$$S^{\text{vN}} = S^{\text{W}-\text{Sh}} = S^{\text{R2}} + \tfrac{1}{2}(1 - \ln 2), \quad (17)$$

where $V_{\text{eff}} = V/b_r$ denotes the effective volume size for the cm motion. These results are not valid for a small V_{eff} because of the box boundary.

The one-body density matrix is diagonalized in the momentum space with a Gaussian distribution, $\exp\left[-\frac{b_r^2}{\hbar^2}p_1^2\right]$. This indicates that the one-body density matrix of a free composite particle is equivalent to the thermal state of a single particle at finite temperature $kT = \hbar^2/2mb_r^2$. The temperature is

of the same order as the mean kinetic energy, $\hbar^2/2mb^2$, of constituent particles confined in the composite particle. Strictly speaking EEs are not thermodynamic entropies; however, by associating the one-body density matrix of the free composite particle with a quantum mixed state of a single particle, I can propose an interpretation of the entropy production and thermalization as follows: when the DOF of c2 are reduced, the quantum decoherence occurs, producing the entropy, and simultaneously, the intrinsic kinetic energy of the composite particle is converted into heat.

Next, I consider a composite particle moving in an external ho potential, where the lowest state of the composite particle is given by the Gaussian distribution $F(s) = \exp\left[-\frac{s^2}{2B^2}\right]\Big/\left(B^2\pi\right)^{1/4}$. This gives the exact solution to the two-body wave function, $\langle r_1, r_2|\Psi^{(2)}\rangle = \langle R, r|\Psi^{(2)}\rangle$, for ho potentials $U_{\mathrm{ho}}(M, \beta; R) + U_{\mathrm{ho}}(\mu, b_r; r)$, with $M = (u + 1)m$ and $\beta = \sqrt{B^2 + u_1 b^2}$. Note that, in the case of $u = 3$, it corresponds to the one-dimensional Tohsaki–Horiuchi–Schuck–Röpke wave function [24,25] for an α cluster confined in an external field [10], whereas, in the case of a large u, it corresponds to the model of a composite boson of a proton and an electron discussed in Ref. [27]. In the $B = 0$ limit, $\Psi^{(2)}$ describes a localized composite particle that corresponds to a non-entangled (uncorrelated) state of two constituent particles and has zero Renyi-2 and von Neumann EEs. As B enlarges and the delocalization of the cm of the composite particle grows, the EEs increase. The Wigner function and EEs for $\hat{\rho}^{(1)}_{\Psi^{(2)}}$ are

$$\rho^{\mathrm{W}}\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right) = \frac{2\sqrt{\gamma}b}{\sqrt{b^2 + B^2}} \exp\left[-\frac{1}{b^2 + B^2}q_1^2 - \frac{\gamma b^2}{\hbar^2}p_1^2\right], \tag{18}$$

$$S^{\mathrm{R2}} = \frac{1}{2}\ln\left(1 + v_{\mathrm{eff}}^2\right) - \frac{1}{2}\ln\gamma, \tag{19}$$

$$S^{\mathrm{W-Sh}} = S^{\mathrm{R2}} + 1 - \ln 2, \tag{20}$$

where $\gamma = \left(1 + (u + 1)v_{\mathrm{eff}}^2\right)/\left(1 + uv_{\mathrm{eff}}^2\right)$, and $v_{\mathrm{eff}} = B/b$ denotes the effective volume size. The EEs increase as v_{eff} enlarges and approach $\ln v_{\mathrm{eff}}$ in the large v_{eff} limit. In the semi-classical approximation, the Wigner function and EEs are

$$\rho^{\mathrm{W}}\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right) \overset{\mathrm{cl}}{\approx} \frac{2b_r}{\sqrt{b_r^2 + B^2}} \exp\left[-\frac{1}{b_r^2 + B^2}q_1^2 - \frac{b_r^2}{\hbar^2}p_1^2\right], \tag{21}$$

$$S^{\mathrm{R2}} \overset{\mathrm{cl}}{\approx} S^{\mathrm{R2,cl}} = \frac{1}{2}\ln\left(1 + v_{c,\mathrm{eff}}^2\right), \tag{22}$$

$$S^{\mathrm{W-Sh}} \overset{\mathrm{cl}}{\approx} S^{\mathrm{W-Sh,cl}} = S^{\mathrm{R2,cl}} + 1 - \ln 2, \tag{23}$$

where $v_{c,\mathrm{eff}} = B/b_r$. The EEs for $u = 1$ and $u = 8$ are shown in Fig. 1. S^{vN} is calculated numerically, as was done in the previous paper [10]. $S^{\mathrm{R2,cl}}$ for the semi-classical approximation agrees well with S^{R2} in the $v_{\mathrm{eff}} \geq 2$ case to within 10% error for $u = 1$, and the agreement is better for the larger mass ratio, $u = 8$. $S^{\mathrm{W-Sh}}$ has a constant shift $1 - \ln 2$ (a constant scaling $e/2$ in the e^S plot in Fig. 1) from S^{R2}, and it is finite even at $v_{\mathrm{eff}} = 0$. S^{vN} starts from zero at $v_{\mathrm{eff}} = 0$ and approaches $S^{\mathrm{W-Sh}}$ as v_{eff} increases. As the mass ratio u increases, the EEs converge on values in the large u limit.

Finally, I connect the quantum decoherence in the one-body density matrix of the composite particle to coarse graining in the phase space distribution function of a single particle and associate it with the Husimi function. Let us start from the Wigner transformation of the full two-body
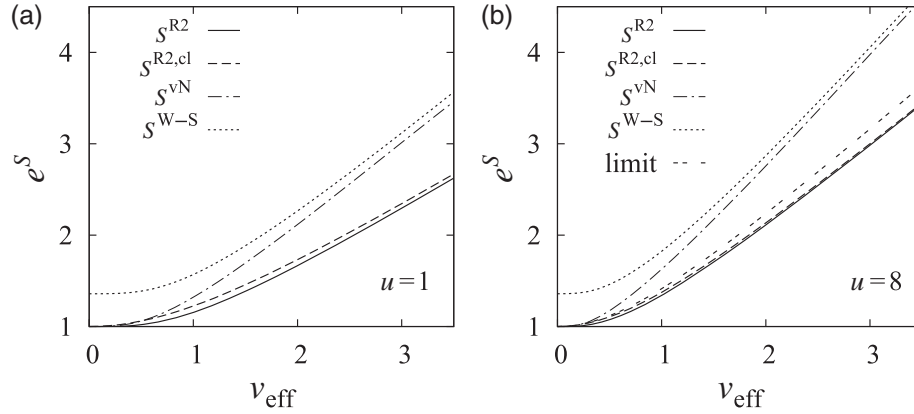
Fig. 1. EE of a composite particle with a Gaussian distribution for (a) $u = 1$ and (b) $u = 8$. e^S for the Rényi-2 (S^{R2} and $S^{R2,cl}$), von Neumann (S^{vN}), and Wigner–Shannon (S^{W-Sh}) EEs are plotted as functions of the effective volume size, v_{eff}. The Rényi-2 EE in the large u limit is also shown in panel (b).

density matrix $\hat{\rho}^{(2)}_{\Psi^{(2)}}$,

$$\rho^W\left(\hat{\rho}^{(2)}_{\Psi^{(2)}}; q_1, p_1, q_2, p_2\right) = \int d\eta d\xi \left\langle q_1 + \frac{\eta}{2}, q_2 + \frac{\xi}{2}\left|\hat{\rho}^{(2)}_{\Psi^{(2)}}\right|q_1 - \frac{\eta}{2}, q_2 + \frac{\xi}{2}\right\rangle$$

$$\times \exp\left[-\frac{i}{\hbar}p_1\eta - \frac{i}{\hbar}p_2\xi\right]. \tag{24}$$

It is rewritten by a separable form in the phase space for the cm and relative coordinates as

$$\rho^W\left(\hat{\rho}^{(2)}_{\Psi^{(2)}}; q_1, p_1, q_2, p_2\right) = \rho^W\left(\hat{\rho}^{(G)}_{\Phi_G}; Q, P\right)\rho^W\left(\hat{\rho}^{(r)}_{\phi_{int}}; q, p\right), \tag{25}$$

where $\hat{\rho}^{(G)}_{\Phi_G} = |\Phi_G\rangle\langle\Phi_G|$ and $\hat{\rho}^{(r)}_{\phi_{int}} = |\phi_{int}\rangle\langle\phi_{int}|$. The Wigner function of the one-body density matrix can be written as

$$\rho^W\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right) = \int \frac{dq_2 dp_2}{2\pi\hbar}\rho^W\left(\hat{\rho}^{(2)}_{\Psi^{(2)}}; q_1, p_1, q_2, p_2\right)$$

$$= \frac{1}{u_2}\int \frac{dQ dP}{2\pi\hbar}\rho^W\left(\hat{\rho}^{(G)}_{\Phi_G}; Q, P\right)\rho^W\left(\hat{\rho}^{(r)}_{\phi_{int}}; \frac{q_1 - Q}{u_2}, p_1 - u_1 P\right) \tag{26}$$

$$= \frac{1}{u_2}\int \frac{dQ dP}{\pi\hbar}\rho^W\left(\hat{\rho}^{(G)}_{\Phi_G}; Q, P\right)\exp\left[-\frac{1}{b_r^2 u_2^2}(q_1 - Q)^2 - \frac{b_r^2}{\hbar^2}(p_1 - u_1 P)^2\right]. \tag{27}$$

Here I use the relations $q = (q_1 - Q)/u_2$, $p = p_1 - u_1 P$ and the transformation $dq_2 dp_2 = |J|dQ dP$ with the determinant of Jacobian $|J| = 1/u_2$. This means that $\rho^W\left(\hat{\rho}^{(1)}_{\Psi^{(2)}}; q_1, p_1\right)$ is regarded as a coarse-grained distribution function of $\rho^W\left(\hat{\rho}^{(G)}_{\Phi_G}; Q, P\right)$ with a Gaussian smearing. In other words, the quantum decoherence caused by the reduction of the DOF of c2 can be interpreted as the coarse graining in the phase space distribution of a single-particle state. It is important that, if the internal DOF are decoupled from the cm motion of the composite particle, Eq. (26) describes a general form of the coarse-grained distribution function that corresponds to the Wigner function of $\hat{\rho}^{(1)}_{\Psi^{(2)}}$.

Let us consider the $u = 1$ case and associate the coarse graining in Eq. (27) with the Husimi function. Eq. (27) for $u = 1$ is rewritten as

$$\rho^{\text{W}}\left(\hat{\rho}_{\Psi^{(2)}}^{(1)}; q_1, p_1\right) = 2\rho_{\hbar/2}^{\text{H}}\left(\hat{\rho}_{\Phi_G}^{(G)}; b_G^2; q_1, p_1\right), \tag{28}$$

$$\rho_{\hbar/2}^{\text{H}}\left(\hat{\rho}^{(1)}; \alpha; q, p\right) \int \frac{dQ\,dP}{\pi(\hbar/2)} \int d\eta \left\langle Q + \frac{\eta}{2} \middle| \hat{\rho}^{(1)} \middle| Q - \frac{\eta}{2} \right\rangle \exp\left[-\frac{iP\eta}{\hbar/2}\right]$$

$$\times \exp\left[-\frac{1}{\alpha}(q - Q)^2 - \frac{\alpha}{(\hbar/2)^2}(p - P)^2\right], \tag{29}$$

where $\rho_{\hbar/2}^{\text{H}}$ is normalized as $\int \frac{dq\,dp}{2\pi(\hbar/2)}\rho_{\hbar/2}^{\text{H}} = 1$. $\rho_{\hbar/2}^{\text{H}}$ eventually has the same form as the normal Husimi function, except for the scaling of Planck's constant $\hbar \to \hbar/2$. I call $\rho_{\hbar/2}^{\text{H}}$ the "$\hbar/2$-Husimi function." It is clear that $\rho^{\text{W}}\left(\hat{\rho}_{\Psi^{(2)}}^{(1)}; q_1, p_1\right)$ for the two-body state $\Psi^{(2)}$ is equivalent to twice the $\hbar/2$-Husimi function for the single-particle state, $|\Phi_G\rangle$. The Gaussian smearing in the coarse graining originates from the reduction of the DOF of c2 in $\rho^{\text{W}}\left(\hat{\rho}_{\phi_{\text{int}}}^{(r)}; (q, p)\right)$, as shown previously. Note that the $\hbar/2$-Husimi function is not a distribution function for a physical single-particle state, but is regarded as a "distribution" function defined in the down-scaled phase space, $\hbar \to \hbar/2$. The reason for the down scaling $\hbar \to \hbar/2$ is that the (q_2, p_2) phase space is scaled down in the transformation from (q_1, p_1, q_2, p_2) to (q_1, p_1, Q, P).

Considering the one-to-one correspondence between $\rho^{\text{W}}\left(\hat{\rho}_{\Psi^{(2)}}^{(1)}; q_1, p_1\right)$ and $\rho_{\hbar/2}^{\text{H}}\left(\hat{\rho}_{\Phi_G}^{(G)}; q_1, p_1\right)$, I can connect entropies defined by the $\hbar/2$-Husimi function to EEs as

$$S_{\hbar/2}^{\text{Wehrl}}\left(\hat{\rho}_{\Phi_G}^{(G)}\right) = S^{\text{W-Sh}}\left(\hat{\rho}_{\Psi^{(2)}}^{(2)}\right) + \ln 2, \tag{30}$$

$$S_{\hbar/2}^{\text{R2-Wehrl}}\left(\hat{\rho}_{\Phi_G}^{(G)}\right) = S^{\text{R2}}\left(\hat{\rho}_{\Psi^{(2)}}^{(2)}\right) + \ln 2. \tag{31}$$

Here I define the Wehrl entropy and the Rényi–Wehrl entropy of order 2 in the down-scaled phase space as

$$S_{\hbar/2}^{\text{Wehrl}}\left(\hat{\rho}^{(1)}\right) = -\int \frac{dq\,dp}{2\pi(\hbar/2)}\rho_{\hbar/2}^{\text{H}}\left(\hat{\rho}^{(1)}; q, p\right)\ln \rho_{\hbar/2}^{\text{H}}(q, p),$$

$$S_{\hbar/2}^{\text{R2-Wehrl}}\left(\hat{\rho}^{(1)}\right) = -\ln\left[\int \frac{dq\,dp}{2\pi(\hbar/2)}\left\{\rho_{\hbar/2}^{\text{H}}\left(\hat{\rho}^{(1)}; q, p\right)\right\}^2\right]$$

In summary, I investigated the quantum decoherence in the one-body density matrix of the composite particle that comprises two correlated particles in the inert composite particle approximation. Because of the two-body correlation in the composite particle, the quantum decoherence occurs by the reduction of the DOF of the second particle. As the delocalization of the distribution of the composite particle grows, the entanglement entropy increases. I found a one-to-one correspondence between the quantum decoherence in the reduced density matrix and the coarse graining in the phase space distribution, which is related to the Husimi-like function defined in the down-scaled phase space. In the present paper, the inert composite particle approximation is applied to static systems but it can also be extended to time-dependent systems if the energy scale of the internal DOF of the composite particle is decoupled from that of the external DOF. The present study may shed light on the fundamental problems of quantum decoherence and coarse graining which produces entropies in quantum systems. I discussed general features of quantum entanglement of a composite particle, and it is interesting to apply the present concept of entanglement to various systems such as hadron, nuclear, and atomic systems.

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