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# Decision criteria in management using probabilities Kryteria podejmowania decyzji w zarzadzaniu z wykorzystaniem prawdopodobieństwa zdarzeń 


#### Abstract

Streszczenie: W artykule zilustrowano, jak pewne techniki czy kryteria wykorzystujące prawdopodobieństwo zdarzeń a dotyczące podejmowania decyzji pozwalają określić optymalne działania odnośnie magazynowania nietrwałych towarów, jak również innych decyzji w zarządzaniu. Porównywane są różne metody, a także pokazano proste, interesujące przykłady ukazujące, jak te kryteria mogą działać w praktyce. W większości przypadków menedżerowie muszą zdecydować, ile produktów zmagazynować, żeby sprostać niepewnemu popytowi. Jeżeli zmagazynuja zbyt dużo, część może nie zostać sprzedana, a z powodu nietrwałości zgromadzonych zapasów mogą one zostać wyrzucone i spowodować stratę. Z drugiej strony jeżeli nie można zaspokoić popytu z powodu zbyt małych zapasów, także powoduje to stratę. W tej sytuacji menedżer jest zawsze zainteresowany podjęciem optymalnej decyzji dotyczącej zapasów, której rezultatem jest najwyższy spodziewany zysk w długim okresie czasu.


Słowa kluczowe: analiza decyzji, zarządzanie adaptacyjne, analiza ryzyka


#### Abstract

In this paper, I will illustrate how some decision making techniques or criteria, using probabilities, will allow us to determine optimal courses of action not only for perishable-goods stocking problems, but also for a number of other managerial decision making applications.Various methods for decision making with probabilities are discussed and compared. Some simple interesting numerical examples show how these criteria can work in practice, and demonstrate their differences. Most of the time, managers must decide how much produce to stock to meet an uncertain demand. If produce is excessively stocked, some of it may go unsold, and because of its perishability, it may have to be thrown away at a loss. On the other hand, if sufficient stock is unavailable to meet the potential demand in the selling period, a situation of lost profit is incurred. In such cases, the manager is interested in deciding an optimal stocking level, one which results in the largest expected profit in the long term.


Keywords:. decision analysis, adaptive management, risk analysis

## Introduction

Some of the most complex managerial decisions are made with some uncertainty. Decision makers have to choose between alternative actions
each day. Frequently the alternatives and supporting information presented is inadequate to support or explain the recommended action. I am going to explain how the use of some of the probability concepts provides decision makers with a rational method for making choices.

Harris (1998) said "Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on."

According to Baker et al. (2001), decision making should start with the identification of the decision maker(s) and stakeholder(s) in the decision, reducing the possible disagreement about problem definition, requirements, goals and criteria.

## 1. Steps in decision making

A general decision making process can be generally divided into three steps. These steps will help enhance your (decision making) skills for different types of decision making. I shall introduce these by using the example of the oral health care products manufacturing company considering several alternative methods of expanding its production to accommodate an increasing demand for its products.

### 1.1. Step 1 - list all the viable alternatives

The first step a decision maker must take is to list all the possible alternatives that must be considered in the decision. In the case of our oral health care products manufacturer, company planners indicate that only three viable options are available to the company:

1. Expand the present factory.
2. Build a new factory.
3. Subcontract out extra production to other oral health care products manufacturers.

### 1.2. Step 2 - identify the future events that may occur

After having identified all the possible alternatives, the decision maker must list the future events affecting demand that may arise. These future events (not under the control of the decision maker) are called states of nature in decision theory. In the case of our oral health care products manufacturer, the most significant future events concern demand for the product. These are listed as:

1. High demand (resulting from high product acceptance).
2. Moderate demand (resulting from reasonable product acceptance but heavy competitive response).
3. Low demand (resulting from low product acceptance)
4. Failure (no product acceptance).

### 1.3. Step 3 - construct a payoff table

In this step the decision maker must construct a payoff table (a table which shows the payoffs, expressed in profits or any other measure of benefit which is appropriate to the situation) which would result from each possible combination of decision alternative and state of nature. Table 1 illustrates the 12 possible payoffs in the oral health care products manufacturing company's expansion decision.

Table 1. Payoff table for oral health care products company expansion decision (payoffs expressed in profits over the next 5 years)

| Decision maker's alternatives |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| States <br> of nature <br> (demand) | High | Expand | Build | Subcontract |  |
|  | Moderate | $€ 600000$ | $€ 800000$ | $€ 400000$ |  |
|  | Low | $€ 300000$ | $€ 350000$ | $€ 200000$ |  |
|  | Failure | $-€ 300000$ | $-€ 450000$ | $-€ 15000$ |  |

## 2. The environments where decisions are made

Decision makers must function in three styles of environments. In each of these environments, knowledge about the states of nature is different.
The decision theory can be made under three types of conditions, certainty, risk or uncertainty.

### 2.1. Decisions under certainty

In this situation, only one state of nature exists, that is, there is complete certainty about the future. People are reasonably sure about what will happen when they make a decision. Even though this environment sometimes exists, it is usually associated with very predictable decisions involving fairly inconsequential matters; even here it is generally impossible to ensure complete certainty about the future.

### 2.2. Decisions under uncertainty

In this environment exists more than one state of nature, but the decision maker has no knowledge about the several states, not even enough knowledge to permit the assignment of probabilities to the states of nature.

### 2.3. Decisions under risk

Here more than one state of nature exists, but the decision maker has knowledge about the several states which will support the assignment of probability values to each of the possible states.

It is easy to analyse the situation and make good decisions when we are under conditions of complete certainty.
In table 1, for instance, if the decision maker of the company knew that demand would be moderate, he would choose the alternative "build", since that yields him the highest payoff. In the same way, if he knew that demand would be low, he would choose the alternative "subcontract", even though that generates a loss, it is still his greatest alternative given that state of nature.

## 3. Criteria for decision making under uncertainty

In this situation, the decision maker knows that there is more than one possible state of nature, but which one is the true state is not known. In this case, there are four criteria he can use to make decisions, which we shall study briefly.

### 3.1. Maximax criterion

Is an optimistic criterion, the decision maker will select the decision alternative which will maximize the maximum payoff. In our problem illustrated in table 1, the decision maker first selects the maximum payoff possible for each decision alternative and then chooses the alternative that provides him with the maximum payoff within this group. In table 2, the maximum payoff possible for each of the three decision alternatives is circled. The alternative within this group of three which provides the maximum payoff is "build", with an associated payoff over the next 5 years of $€ 800000$.

Table 2. Payoff table for oral health care products company expansion decision (payoffs expressed in profits over the next 5 years)

| Decision maker's alternatives |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| States of nature (demand) |  | Expand | Build | Subcontract |
|  | High | $€ 600000$ | $€ 800000$ | €400000 |
|  | Moderate | $€ 300000$ | €350 000 | €200 000 |
|  | Low | -€300 000 | -€450 000 | -€15000 |
|  | Failure | -€500 000 | -€900 000 | -€200 000 |

### 3.2. Maximin Criterion

Is a pessimistic criterion, the decision maker will select the decision alternative which will maximize the minimum possible payoff. In our problem illustrated in table 1, the decision maker first selects the minimum payoff pos-
sible for each decision alternative and then chooses the alternative that provides him with the maximum payoff within this group. In table 3, the minimum payoff possible for each of the three decision alternatives is circled.

Table 3. Payoff table for oral health care products company expansion decision (payoffs expressed in profits over the next 5 years)

|  | Decision maker's alternatives |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
| States <br> of nature <br> (demand) | High | Expand | Build | Subcontract |
|  | Moderate | $€ 600000$ | $€ 800000$ | $€ 400000$ |
|  | Low | $€ 300000$ | $€ 350000$ | $€ 200000$ |
|  | Failure | $-€ 300000$ | $-€ 450000$ | $-€ 15000$ |

The alternative within this group of three which provides the maximum payoff is "subcontract", with an associated payoff over the next 5 years of €200 000.

### 3.3. Minimax regret criterion

This criterion departs from the focus on optimism versus pessimism. Instead, its focus is on making a decision that minimizes the regret that can be felt afterwards if the decision does not turn out well.
Let us look at the calculation of one regret value. Suppose he had chosen the alternative "subcontract" and it turns out that demand is high. The profit he will make from subcontracting with high demand is €400000, but had the decision maker known the demand was going to be high, he would not have subcontracted but would have chosen instead to "build" with a profit of $€ 800000$. The difference between $€ 800000$ (the optimal payoff "had he known") and €400 000(the payoff he actually realized from subcontracting) is $€ 400000$ and is known as the regret resulting from his decision.

In table 4, we can see the regret values obtained by subtracting each entry in the payoff table (table 1) from the largest entry in its row. The maximum regret for each of the three decision alternatives is circled. Finally we choose the minimum of these three regret values, in this case, €300 000 is the minimum regret value, and is associated with deciding to "expand".

Table 4. Regret values for oral health care products company

|  | Decision maker's alternatives |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
| States <br> of nature <br> (demand) | High | $€ x p a n d$ | Build | Subcontract |
|  | Moderate | $€ 50000$ | 0 | $€ 400000$ |
|  | Low | $€ 285000$ | $€ 435000$ | $€ 150000$ |
|  | Failure | $€ 300000$ | $€ 700000$ | 0 |

### 3.4. Criterion of realism

Is a middle ground criterion between maximax and maximin criterion, that is, between optimistic and pessimistic criterion.

Table 5. Minimum and maximum payoffs for each decision alternative for oral health care products company

| Decision maker's alternatives |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| States of nature (demand) |  | Expand | Build | Subcontract |
|  | High | €600000 | €800 000 | $€ 400000$ |
|  | Moderate | $€ 300000$ | $€ 350000$ | €200 000 |
|  | Low | -€300 000 | -€450 000 | -€15000 |
|  | Failure | -€500 000 | - $€ 900000$ | €200 000 |

This criterion requires the decision maker to specify a coefficient of optimism, represented by $\alpha$, where $\alpha$ is between 0 and 1 ( 0 expresses pessimism about nature, 1 expresses optimism about nature). First we must determine both the maximum and the minimum payoff for each decision alternative. In table 5, the maximum payoff for each decision alternative is circled and the minimum payoff is inside a box.

Then for each decision alternative, compute:
Measure of realism $=\alpha$ (maximum payoff) $+(1-\alpha)$ (minimum payoff)
Presume that in our example the decision maker feels fairly optimistic and assigns a value of 0.8 to $\alpha$. With these conditions, the measures of realism are:

Table 6. Measures of realism

| Expand | $0.8 \times(€ 600000)+0.2 \times(€-500000)=€ 380000$ |
| :--- | :--- |
| Build | $0.8 \times(€ 800000)+0.2 \times(€-900000)=€ 460000$ |
| Subcontract | $0.8 \times(€ 400000)+0.2 \times(€-200000)=€ 280000$ |

The use of the realism criterion in this case advises to choose the alternative "build".

## 4. Decision making under conditions of risk

In this case, the decision maker faces several states of nature, but he is supposed to have believable evidential information and knowledge about the various states which will support the assignment of probability values to each of the possible states. In this case, there are three criteria he can use to make decisions which we shall examine briefly: the expected value, the criterion of rationality and the criterion of maximum likelihood. I shall introduce these by using the example of John a seller of cherries in a market en-
vironment where "tomorrow's demand for cherries" is a discrete random variable.

Table 7. Cases demanded during 100 days

| Daily demand | No. days demanded | Probability of each level <br> of demand |
| :---: | :---: | :---: |
| 12 | 20 | 0.2 |
| 13 | 40 | 0.4 |
| 14 | 30 | 0.3 |
| 15 | 10 | 0.1 |
| Total | 100 | 1 |

John purchases cherries for $€ 6$ a case and sells them for $€ 16$ a case. The product has no value after the first day it is offered for sale. John faces the problem of how many cases to order today for tomorrow's business.
A 100-day observation of past demand gives the information shown in table 7 . The probabilities are obtained by normalizing the distribution. Demand was for 12 cases on 20 of the 100 days, that is, $20 / 100=0.2$ of the time.

### 4.1. The expected value criterion

In this criterion, the decision maker has to calculate the expected value for each decision alternative (the sum of the weighed payoffs for that alternative, where the weights are the probability values assigned by the decision maker to the states of nature that can happen).

Let us assume that John has no reason to believe that demand will behave in a different way in the future. His problem is to determine how many cases he should buy today for tomorrow's business. If buyers tomorrow request more cases than the number in stock, John's profits suffer by $€ 10$. On the other hand, there are costs which result from stocking excess units on any day. Suppose that on a certain day John has 15 cases in stock but sells only 12 . He makes a profit of $€ 120, € 10$ per case on 12 cases. But this must be reduced by $€ 18$, the cost of the three cases not sold and of no value.

One way of illustrating John's problem is to construct a table showing the results in euros of all possible combinations of purchases and demand. In table 8, we can see the profit resulting from any possible combination of supply and demand.

Table 8. Conditional profit table

| Market size, | Possible stock action |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| cases | $\mathbf{1 2}$ cases | $\mathbf{1 3}$ cases | 14 cases | 15 cases |
| 12 | $€ 120$ | $€ 114$ | $€ 108$ | $€ 102$ |
| 13 | 120 | 130 | 124 | 118 |
| 14 | 120 | 130 | 140 | 134 |
| 15 | 120 | 130 | 140 | 150 |

This table reflects the losses which occur when stock remains unsold at the end of the day, but does not tell John which number of cases he should stock each day in order to maximize profits.

Under conditions of risk, he does not know in advance the size of any day's market, but he must decide how many cases, stocked consistently, will maximize profits over a long period of time. The best number of cases to stock is to assign probabilities to the possible profits. Using the probabilities contained in table 7 and the information contained in table 8, John can compute the expected profit of each possible stock action.

Then, if he stocks 14 cases each day under the conditions given, he will have average profits of $€ 127.2$ per day. This is the best he can do, because the choice of any one of the other three possible stock actions will result in a lower average daily profit.

Table 9. Expected profit from stocking 12, 13, 14 and 15 cases

| Market <br> size, <br> Cases | 12 cases <br> Conditional <br> profit | 12 cases <br> Expected <br> profit | 13 cases <br> Expected <br> profit | 14 cases <br> Expected <br> profit | 15 cases <br> Expected <br> profit |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | $€ 120$ | $120^{*} 0.2=24$ | $114^{*} 0.2=22.8$ | $108^{*} 0.2=21.6$ | $102^{*} 0.2=20.4$ |
| 13 | 120 | $120^{*} 0.4=48$ | $130^{*} 0.4=52$ | $124^{*} 0.4=49.6$ | $118^{*} 0.4=47.2$ |
| 14 | 120 | $120^{*} 0.3=36$ | $130^{*} 0.3=39$ | $140^{*} 0.3=42$ | $134^{*} 0.3=40.2$ |
| 15 | 120 | $120^{*} 0.1=12$ | $130^{*} 0.1=13$ | $140^{*} 0.1=14$ | $150^{*} 0.1=15$ |
| Total |  | $€ 120$ | $€ 126.8$ | $€ 127.2$ | $€ 122.8$ |

### 4.2. The Criterion of Rationality

This criterion is also known as the Principle of Insufficient Reason or Equal Probabilities Criterion. Since the probabilities of states of nature are not known, it is assumed that all states of nature will occur with equal probability. In table 10, I have repeated the conditional profit table first shown in table 8, added equal probability weight alternatives to the four states of nature, and computed the expected values of the four stocking alternatives.

Table 10. Expected value calculations using the criterion of rationality

| Possible <br> demand,cases | Possible stock action |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 12 cases | 13 cases | 14 cases | 15 cases |
| 12 | $€ 120^{*} .25=30$ | $€ 114^{*} .25=28.5$ | $€ 108^{*} .25=27$ | $€ 102^{*} .25=25.5$ |
| 13 | $120^{*} .25=30$ | $130^{*} .25=32.5$ | $124^{*} .25=31$ | $118^{*} .25=29.5$ |
| 14 | $120^{*} .25=30$ | $130^{*} .25=32.5$ | $140^{\star} .25=35$ | $134^{*} .25=33.5$ |
| 15 | $120^{*} .25=30$ | $130^{*} .25=32.5$ | $140^{*} .25=35$ | $150^{*} .25=37.5$ |
|  | $€ 120$ | $€ 126$ | $€ 128$ | $€ 126$ |

By this criterion, we find that the optimal stocking decision is 14 cases, with an expected profit of $€ 128$.

### 4.3. The Maximum LikelihoodCriterion

Using the Maximum Likelihood criterion John just selects the state of nature that has the highest probability of occurrence, then, having assumed that this state will occur, he selects the decision alternative which will yield the highest payoff. In table 11, we can see that "demand for 13 cases" with an assigned probability of 0.4 is the state of nature with the highest probability of occurrence and that the stock action "13 cases" has the highest payoff for the state of nature, $€ 130$.

Table 11. Determining the optimal stock action using the maximum likelihood criterion

| Possible <br> demand, <br> cases | Probability <br> of this demand | $\mathbf{1 2}$ cases | $\mathbf{1 3}$ cases | $\mathbf{1 4}$ cases | 15 cases |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $€ 120$ | $€ 114$ | $€ 108$ | $€ 102$ |  |
| 12 | 0.2 | 120 | 130 | 124 | 118 |  |
| 13 | 0.4 | 120 | 130 | 140 | 134 |  |
| 14 | 0.3 | 120 | 130 | 140 | 150 |  |
| 15 | 0.1 |  | Possible stock action |  |  |  |

If we use this criterion in a situation where a huge number of states of nature exist and each of them has a small, nearly equal probability of occurrence, it is probable to make some serious mistakes.

## Conclusion

Most managers are faced with making decisions in an uncertain or risky environment. The concept of uncertainty is so common in our life that it becomes hard to define it. Some people would call it "luck" and others would say that under uncertainty man is forced to take risk. Statistically speaking, we assign probability with the occurrence or non-occurrence of an event. Managers base their decisions on the basis of their past experience or on the basis of informed supposition, a better term for which will be the subjective probability, which provides a quantitative way to express one's beliefs and conviction about each outcome. Managers must choose the best decision alternative without having any control of the events that affect the profits resulting from the decision. When the probabilities of some events happening are known, we refer to these decision-making situations as decision makingunder risk or decision making with probabilities.

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