Indian J. Phys. **81** (5 & 6), 585-595 (2007)

The dual gluon propagator and confinement potential in SU (2) QCD

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Received 31 October 2006, accepted 8 May 2007

Abstract In order to study the response of dual QCD vacuum in non-perturbative regime, the dual gluon propagator is evaluated by calculating a non-local quark current-current correlation relation in the dynamically broken phase of the magnetic symmetry The linearly rising potential is then shown to be responsible for the absolute colour confinement in the low energy regime with a constant restoring colour force between a quark-pair of a flux tube in *SU*(2) dual QCD vacuum

Keywords Monopoles, QCD, dual gluon propagator and confinement

PACS Nos. 11 15 Tk, 12 38 Aw, 12 38 Lg , 14 80 Hv

1. Introduction

The Quantum Chromodynamics (QCD) is the most successful non-Abelian gauge field theory of the strong interactions [1] which describes the properties and underlying structure of the hadrons in terms of the quarks and gluons [2, 3]. Since the quarks have never been seen as free states in nature, their confinement mechanism in the interior of hadrons deserves careful attention. The problem is then to explain the dynamics of confinement with in the framework of QCD [4]. On the other hand, it is worth noticing that the detailed investigations of QCD in infrared (IR) regime (*i.e.* at large distances) encounter various difficulties due to the failure of perturbation theory which only makes sense for the short distance processes [5, 6]. In past few years, promising evidences have emerged to solve the issue of quark confinement in a non-perturbative way by regarding the QCD vacuum as a dual version of the conventional superconductivity [7-10]. A firm link of confinement mechanism to the monopole condensation is, therefore, of crucial interest and in this

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respect especially, the 't Hooft's Abelian projection technique is worth to mention [11]. There, the gauge degrees of freedom are fixed (by a suitable choice of gauge fixing) in a way that the QCD reduces to an Abelian gauge theory with the appearance of the colour electric charges (quarks) and colour magnetic charges (monopoles - as topological objects). The facets of the colour confinement mechanism may then be described with the monopoles as essential built in ingredients of the dual QCD vacuum. In dual (magnetic) superconductor scenario of QCD vacuum, the colour magnetic monopoles get condensed and as a result of which the colour electric flux is excluded from the QCD vacuum with the formation of thin flux tubes between the colour electric sources [12-14]. Moreover, the introduction of such topological objects (*viz.* monopoles and dyons) in QCD vacuum and the dual dynamics between the colour iso-charges (quarks) and topological charges (monopoles) along with the qualitative picture of a flux tube can best be described by the magnetic symmetry structure of the non-Abelian gauge theories [15]. The magnetic symmetry, in fact, restricts some of the dynamical degrees of freedom of a non-Abelian gauge theory while keeping the full degrees of freedom intact, and as a result, the gauge potential corresponding to a non-Abelian SU(2) gauge group can be expressed in terms of the electric and magnetic potentials which are Abelian in nature. In particular, the purpose of this study is to extend our previous work [16] by deriving the dual gluon propagator in view of the magnetic symmetry structure of the local version of the dual SU(2) QCD vacuum in order to further investigate the nature of confinement potential in the IR regime. It is explicitly shown that how the quarks are absolutely confined by a linearly rising confinement potential at large distances in the background of the magnetic condensation in QCD vacuum.

2. SU(2) QCD and the dual gluon propagator

The topological properties of magnetic symmetry as an additional isometry express the dynamics of QCD explicitly at the level of the gauge potential. The entire formulation for QCD vacuum may then be derived in terms of two Abelian components A_μ (electric potential) and B_μ (magnetic potential) [15]. But the monopoles in such formalism appear as point like objects with the singular behaviour of the magnetic potential and to get rid of these undesirable features, one can introduce the dual magnetic potential ($\tilde{B}_\mu$) with the associated gauge field strength tensor $\left(\tilde{B}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}B^{\sigma\rho}\right)$ and a complex scalar field (ϕ) simultaneously for the monopole field [15]. The dual magnetic potential, therefore, describes the magnetic field of a monopole with a regular time-like potential. Since the monopoles for the present case appear as the point-like objects and have no obvious spin structure, their correct field-theoretical description requires to describe them by a complex scalar field in a way analogous to the scalar fields used in the formulations of QCD as first suggested by Mandelstam [17] and 't Hooft [18]. The monopole field plays the role of Ginzburg-Landau (GL) order parameter like the macroscopic Cooper-pair wave function in the conventional superconductivity. Such considerations then leads the correct physical description of the monopoles in the present dual formulation of the QCD vacuum and the dual QCD Lagrangian derived from the naive SU(2) QCD Lagrangian may then be re-expressed in the following form [15, 16],

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}\tilde{B}_{\mu\nu}^{2} - \frac{1}{2}F_{\mu\nu}B^{\mu\nu} + \bar{\psi}_1 i\gamma^\mu D_\mu \psi_1 + \bar{\psi}_2 i\gamma^\mu D_\mu^{*} \psi_2$$

$$+ m(\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2) + \left|\mathcal{D}_\mu\phi\right|^2 - V(\phi\phi^*) \tag{1}$$

where, $D_\mu \equiv \left[\partial_\mu + g_s(A_\mu + B_\mu)/2i\right]$ and $\mathcal{D}_\mu \equiv \left[\partial_\mu + i4\pi(\tilde{A}_\mu + \tilde{B}_\mu)/g_s\right]$. However ψ_1 and ψ_2 are the components of the $SU(2)$ iso-doublet spinor source for the quarks and $V(\phi\phi^*)$ is an effective potential. The field strength tensors corresponding to the electric and magnetic counterparts are given as follows,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \qquad \tilde{B}_{\mu\nu} = \partial_\mu \tilde{B}_\nu - \partial_\nu \tilde{B}_\mu\,. \tag{2}$$

However, the presence of space-like electric potential ($\tilde{A}_\mu$) preserves the dual structure of QCD vacuum by including the electric coupling to the monopoles. The dynamical breaking of the magnetic symmetry by an effective potential enforces the magnetic condensation in QCD vacuum and for this purpose the effective potential from phenomenological view point is then given by,

$$V(\phi\phi^*) = \Omega\left(\phi\phi^* - \phi_0^2\right)^2, \tag{3}$$

where, $\Omega = 3\lambda/\alpha_s^2$ and ϕ_0 is the vacuum expectation value (VEV) of the complex scalar monopole field (ϕ) and $\alpha_s = g_s^2/4\pi$ is the strong coupling constant. The magnetic condensation as a result of the dynamical breakdown of the magnetic symmetry in QCD vacuum ultimately reflects itself in terms of two mass modes (scalar and vector) of the condensed vacuum. In order to visualise the confinement mechanism in such condensed vacuum, let us try to derive the structure of the dual gluon propagator and confinement potential (in view of the condensation of monopole configurations their in vacuum) by calculating the action associated with the Lagrangian (1). In the static limit, where the quarks get decoupled with the magnetic potential along with the negligibly weak coupling between $\tilde{A}_\mu$ and the complex scalar monopole field (ϕ) [15], an effective action corresponding to the Lagrangian (1) in the quenched approximation may then be expressed in the following form,

$$S = \int d^4x \left\{\frac{1}{2}\left(\tilde{B}^\mu\left(\eta_{\eta\nu}\partial^2 - \partial_\mu\partial_\nu\right)\tilde{B}^\nu + \tilde{m}^2\tilde{B}_\mu\tilde{B}^\mu\right) + \left(\frac{1}{2}\left(A^\mu\left(\eta_{\eta\nu}\partial^2 - \partial_\mu\partial_\nu\right)A^\nu - J_\mu A^\mu\right)\right. + \ldots, \tag{4}$$

where, $\tilde{m} = \sqrt{2}\left(g_s\,\alpha_s^{-1}\right)\phi_0$ is the mass acquired by the dual gauge field ($\tilde{B}_\mu$) for the case $\lambda = 1$ as a result of the dynamical breaking of the magnetic symmetry in the strong coupling limit through the effective potential (3) and imparts the superconducting features to the QCD vacuum. This mass acquisition by dual gauge field, in turn, guarantees that the colour electric field penetrates the vacuum up to a finite depth. The colour electric flux then effectively screens out which leads the dual Meissner effect in the QCD vacuum. Such Meissner effect may clearly visualise through the presence of a Meissner-like term $\tilde{m}^2 \tilde{B}_i^2$ in the free energy which arises because of the interaction of the dual magnetic gauge field ($\tilde{B}_\mu$) to the complex scalar field ϕ [19] in the Lagrangian given by equation (1). The flux tube structure between a quark-pair, therefore, emerges as an immediate consequence of it. The penetration depth defined as $\tilde{\lambda} = \tilde{m}^{-1}$, gives an effective measurement of the strength of the dual Meissner effect and is, therefore, an important guiding parameter to discuss confinement in the condensed phase of QCD vacuum. However, the current J_μ in equation (4) is due to the dynamical quark part in the Lagrangian (1) as given below,

$$J_\mu = \frac{g_s}{2}\left(\bar{\psi}_2 \gamma_\mu \psi_2 - \bar{\psi}_1 \gamma_\mu \psi_1\right), \tag{5}$$

may be replaced by the current constituted by a flux tube consisting of a quark and anti-quark sitting at its opposite ends as an external source in the following form,

$$J_\mu = \eta_\mu 0 \left\{\frac{g_s}{2}\delta(r - r_1) - \frac{g_s}{2}\delta(r - r_2)\right\}. \tag{6}$$

Further, with above considerations, the solution of the field strength tensor $\left(F_{\mu\nu}\right)$ corresponding to the electric potential [20] can be written in terms of the current given by equation (6) as follows,

$$F_{\mu\nu} = (n.\partial)^{-1}\left(n_\mu J_\nu - n_\nu J_\mu\right), \tag{7}$$

where, $n_\mu = (0, -\boldsymbol{n})$ is a fixed space-like vector along the direction of the line joining by the quark-pair in the flux tube. Here, J_μ is the source term for the gluon field A_μ. The action in its simplest form can be written by eliminating $\tilde{B}_\mu$ in the static limit (where the quark field decouple to the magnetic potential) with the help of equation of motion for $\tilde{B}_\mu$ which is derived from the Lagrangian given by equation (1) in the following form,

$$\left(\tilde{B}_{\mu\nu}^{,\nu} + \tilde{F}_{\mu\nu}^{,\nu}\right) + \tilde{m}^2 \tilde{B}_\mu = i\, 4\pi (g_s)^{-1} \left(\phi^* \overleftrightarrow{\partial_\mu} \phi\right), \tag{8}$$

where, $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho} F^{\sigma\rho}$. In the equation (8), the derivatives of the complex scalar monopole field vanish as it has a definite value at each space-time point (i.e. $\phi^* \overleftrightarrow{\partial_\mu} \phi = 0$). The equation (8) in the Lorentz gauge, therefore, leads $\tilde{B}_\mu$ in the following form,

$$\tilde{B}_\mu = -\left(\partial^2 + \tilde{m}^2\right)^{-1} \tilde{F}_{\mu\nu}^{,\nu}. \tag{9}$$

Here, the longitudinal component of the dual magnetic gauge field is strongly coupled to the longitudinal excitations of the complex scalar monopole field. However in the quenched approximation with the naive dual Ginzburg-Landau type Lagrangian [16], the Meissner effect seems to generate only the Yukawa potential associated to the massive dual gauge field ($\tilde{B}_\mu$) which can easily be seen in terms of the propagator of the corresponding Klein-Gordan (KG) equation. It is therefore necessary to investigate the present situation in the presence of colour electric sources to obtain the full and effective dual gluon propagator and consequently the confinement potential. Now using the equation (9), the action (4) (where the quantum effects of the mass gained by the gauge field $\tilde{A}_\mu$ are suppressed) may then be re-structured in its simplest form as given below,

$$S = -\frac{1}{2}\int d^4x \left\{ \tilde{F}^{\cdot\nu}_{\mu\nu}\left(\partial^2 + \tilde{m}^2\right)^{-1} \tilde{F}^{\cdot\nu}_{\mu\nu} + \tilde{F}^{2}_{\mu\nu} \right\} + \dots \tag{10}$$

In order to calculate the dual gluon propagator for the present formulation, the action (10) may then be simplified and approximated in terms of a non-local quark current-current correlation relation through the dual gluon propagator as follows,

$$\left\langle J_\mu J_\nu \right\rangle = -\frac{1}{2}\int d^4x\, J_\mu \left\{ \frac{\eta^{\mu\nu}}{\partial^2 + \tilde{m}^2} + \frac{n^2}{(n\cdot\partial)^2}\left(\frac{\tilde{m}^2}{\partial^2 + \tilde{m}^2} \right) n^{\mu\nu} \right\} J_\nu + \dots, \tag{11}$$

considered due to the requirement of the conservation of current and $D^{\mu\nu}$ can be now identified as an effective dual gluon propagator in the following form,

$$D^{\mu\nu}(k, \tilde{m}) = -\left\{ \frac{\eta^{\mu\nu}}{\left(k^2 - \tilde{m}^2\right)} + \frac{\tilde{m}^2 n^{\mu\nu}}{(k^2 - \tilde{m}^2} \right\}, \tag{12}$$

where, $n^{\mu\nu}$ is defined in the following form,

$$n^{\mu\nu}(k, n) = \frac{n^2}{(n\cdot k)^2}\left(\eta^{\mu\nu} - \frac{n^\mu n^\nu}{n^2} \right). \tag{13}$$

The first term in the gluon propagator (12) is of short-range while the second term contains the long-range correlation factor $(n\cdot k)^{-2}$ which manifests itself in terms of the absolute confinement of the quark-pair [21] and its consequences are discussed in the forthcoming section in view of the string tension and linear confinement potential at large distances.

3. The string tension and confinement scenario

In order to calculate the exact shape of the static confinement potential [8], the energy contents from the action (4) with the definition of current given by equation (6) may be derived in the form as given below,

$$K_C = \frac{1}{2}\int d^3r_1 \int d^3r_2 \left\{\rho(r_1)\, U_C(r_2 - r_1)\, \rho(r_2)\right\}, \tag{14}$$

where $U_C(r_2 - r_1)$ is the potential term and has the following form,

$$U_C(\boldsymbol{r}) = -\frac{g_s}{4} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-ik.r}}{\left(\boldsymbol{k}^2 + \tilde{m}^2\right)} + \int \frac{d^3k}{(2\pi)^3} \frac{\tilde{m}^2}{\left(\boldsymbol{k}^2 + \tilde{m}^2\right)} \frac{r^2 e^{-ik.r}}{(\boldsymbol{r}\cdot\boldsymbol{k})^2} \tag{15}$$

where, $\boldsymbol{n} = \boldsymbol{r}_1 - \boldsymbol{r}_2 = \boldsymbol{r}$. The first term in the equation (15) leads the usual short-ranged static Yukawa potential as follows,

$$U_Y(r) = -\frac{g_s^2}{4}\int \frac{d^3k}{(2\pi)^3} \frac{e^{-ik.r}}{\left(\boldsymbol{k}^2 + \tilde{m}^2\right)} = -\frac{\alpha_s}{4}\frac{e^{-r/\tilde{\lambda}}}{r}, \tag{16}$$

where, $d^3k = k^2 \sin\theta\, dk\, d\theta\, d\phi$. It is very difficult to make a direct calculation for the second term is equation (15) which can be rewritten as follows,

$$U_L(\boldsymbol{r}) = -\frac{g_s^2}{4}\int \frac{d^3k}{(2\pi)^3} \frac{\tilde{m}^2}{\left(\boldsymbol{k}^2 + \tilde{m}^2\right)} \frac{r^2 e^{-ik.r}}{(\boldsymbol{r}\cdot\boldsymbol{k})^2}$$

$$= \frac{\alpha_s}{2\pi}\int_0^\infty dk \frac{1}{\left(1 + k^2\tilde{\lambda}^2\right)}\int_0^1 dz\left[\frac{1}{z^2} - \frac{2}{z^2}\sin\left(\frac{krz}{2}\right)\right]. \tag{17}$$

The z-integral does not converge on (0, 1) and therefore it has typical divergences at small values of z as well as at large values of momentum k [21, 8]. The integral given by equation (17) leads the ultra-violet (UV) divergences at small z which contributes as an infinite term (independent of r) and the integral may then only be evaluated by calculating a difference $U_L(\boldsymbol{r}) - U_L(\bar{\boldsymbol{r}})$ with the help of the Frullani's integration technique [8, 22] in the following form,

$$U_L(\boldsymbol{r}) - U_L(\bar{\boldsymbol{r}}) = \frac{\alpha_s}{8}\frac{r}{\tilde{\lambda}^2}\ln\left[1 + (\bar{\Lambda}\,\tilde{\lambda})^2\right], \tag{18}$$

where, $\bar{\Lambda}$ is a sharp cut-off parameter which is introduced in order to make the k-integral converge at large k except in the region where $r \ll \tilde{\lambda}$. Further, $U_L(\bar{\boldsymbol{r}})$ may be regarded as a constant which is small enough because of the parameters of the present model [16]. The total integration of equation (15) then leads the exact shape of the confining potential in the following form,

$$U_C(r) = -\frac{\alpha_s}{4}\frac{e^{-r/\tilde{\lambda}}}{r} + \frac{\alpha_s}{8}\frac{r}{\tilde{\lambda}^2}\ln\left[1+X^2\right], \tag{19}$$

where, $X = \Lambda\tilde{\lambda}$ sets a typical scale for the strength of confinement at different couplings in the IR region of dual QCD vacuum. The potential given by equation (19) is composed of the well-known short ranged Yukawa potential and typical linear potential which is responsible for the confinement of colour electric sources. The confinement potential (19) is screened due to the dual gauge field mass ($\tilde{m}$) which is responsible for the colour flux screening [16, 23] and becomes equivalent to the Coulomb potential in the vanishing limit of such screening dual gauge field mass as $U_{Cou}(r) = -\alpha_s/4r$. The total confinement force between a quark-pair is also derivable from the potential (19) in the following form,

$$F_C(r) = -\frac{\alpha_s}{4r}\left[\frac{r+\tilde{\lambda}}{r\tilde{\lambda}}\right]e^{-r/\tilde{\lambda}} - \frac{\alpha_s}{8}\frac{1}{\tilde{\lambda}^2}\ln\left[1+X^2\right]. \tag{20}$$

It is worth mentioning that the term in addition to the Coulomb force in the first term of the equation (20) may consider as a typical non-perturbative effect due to the monopole condensation which, in turn, enhances the effective strength of the confinement force even at very short distances when compared to the only Coulomb force which arises from the well known phenomenological Cornell (funnel) potential $\left(-\alpha_s/r+\sigma r\right)$ in QCD [24]. The colour force given by equation (20) further reduces to a constant restoring force (*i.e.* confining force) at large distances $r \to \infty$ whose magnitude (in terms of the VEV of the monopole field) is given in the following form,

$$F_R = \pi\phi_0^2 \ln\left[1+X^2\right] = \sigma, \tag{21}$$

where, σ is the string tension per unit length for the resulting flux tube in the dual superconducting QCD vacuum which, in fact, characterizes the strength of the linear confinement potential in dual QCD vacuum. The string tension (or the energy per unit length of the flux tube) given by equation (21) is similar to the energy per unit length of an Abrikosov vortex in the type-II superconductors in the conventional superconductivity [25, 26]. However the superconducting type of QCD vacuum is decided by the GL parameter $\kappa = \sqrt{6}/g_s$ which has a unique value at any particular strong coupling constant. The linear term in equation (19) with the string tension given by equation (21) is responsible for the absolute confinement of any colour electric source in the dynamically broken phase of magnetic symmetry in the dual QCD vacuum. The numerical estimation of the potential $U_C(r)$ may be calculated by identifying X which changes for different values of strong coupling constant. For the purpose of the numerical estimation of the different parameters, it is worth to mention the linearly rising behaviour of the Regge trajectories for the hadrons which supports the flux tube model with a linear potential. The observed value of the Regge slope parameter (RSP) is $\alpha' \simeq 0.93$ GeV^{-2} which has a relationship with the string tension as $\alpha' = (2\pi\sigma)^{-1}$ [10]. This value of the RSP also comes from the measurements for the hadrons in the electron scattering experiments with the typical mass and radius about 1 GeV and 1 fm respectively [27]. The maximum value of the cut-off parameter then decreases with the increase in the strong coupling constant in order to maintain the string tension of the flux tube or RSP. The dimensionless fitting parameter X for a fixed strength of the colour force attraction is then given as,

$$X = \left[e^{\phi_0^{2}/(2\pi^2\alpha')} - 1 \right]^{1/2}. \tag{22}$$

For the string tension with its fixed value as an input parameter, the cut-off parameter ($\bar{\Lambda}$) re-adjusts itself in terms of X in accordance with the field penetration depth. The different parameters those are used for the graphical presentation of the confinement force and potential are then computed for three extreme cases of strong coupling constant in full IR sector of QCD [10, 16] and are given in the Table 1. In fact, the strong coupling constant $(\alpha_s) \sim 0.2$ up to ~ 1 leads to various crucial non-perturbative effects in the IR regime with a transition in its type-I ($\kappa < 1$) to type-II ($\kappa > 1$) superconducting nature at $\alpha_s \sim 0.47$ [16, 28] where the GL parameter acquires the unit value with the range of Yukawa force ~ 0.16 fm. We have also graphically presented the behaviour of the confinement force for these three cases in Figure 1. In order to have the response of the confinement potential, we have plotted confinement potential at large and short distances for these coupling regions as shown in Figure 2. Since X attains comparatively higher values at relatively lower couplings in IR sector i.e. it increases while approaching towards the UV regime and may therefore be subjected to the formation of quark-gluon plasma (QGP) at extremely short distances with the dominating Yukawa force (potential) which is completely a separate issue of discussion and beyond the scope of this paper. It can also be noticed that at short distances the Yukawa potential approaches towards the origin of the plots for the case of lower couplings and this is because of the decrease in the range of the Yukawa force in comparison to their strength at higher couplings. The confinement potential for the dual QCD vacuum given by equation (19) then indicates almost the same behaviour as that of the phenomenological Cornell potential [24, 29] at large distances (*i.e.* ≥ 0.2 fm) at different couplings in view of the parameters given in

Table 1. Numerical estimate of various confinement parameters for different couplings.

g_s	α_s	ϕ_0 (GeV)	X	$\sim \kappa$
1.66	0.22	0.156	2.89	1.5
2.42	0.47	0.170	2.36	1.0
3.47	0.96	0.183	2.02	0.7

Table 1. The confinement potential given by equation (19) can also reproduce the Cornell potential [29] and Lattice data [29, 30] with the appropriate value of the $U_L(\bar{r})$ in equation

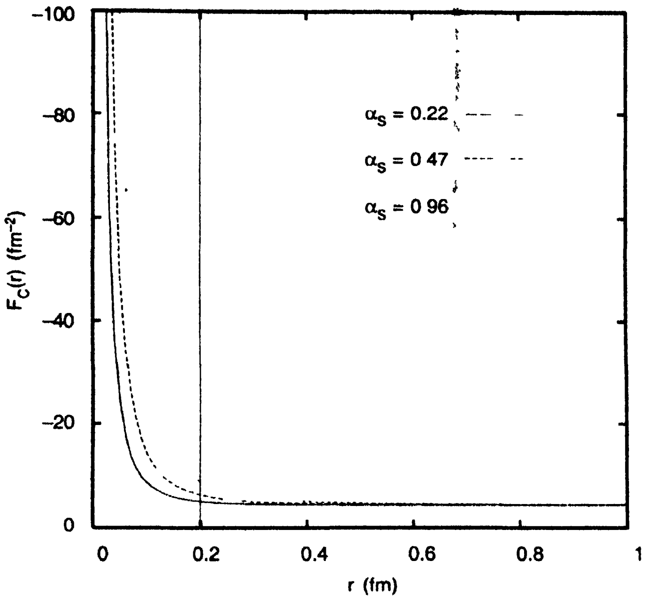

Figure 1. Schematic presentation of the colour confinement force in dual QCD vacuum.

(18) and the fitting parameter X in equation (22). In any case, the linearity in potential at large distances continues intact as evident by the Figure 2. Remarkably enough, the general behaviour of the confinement potential does not depend over the type of the

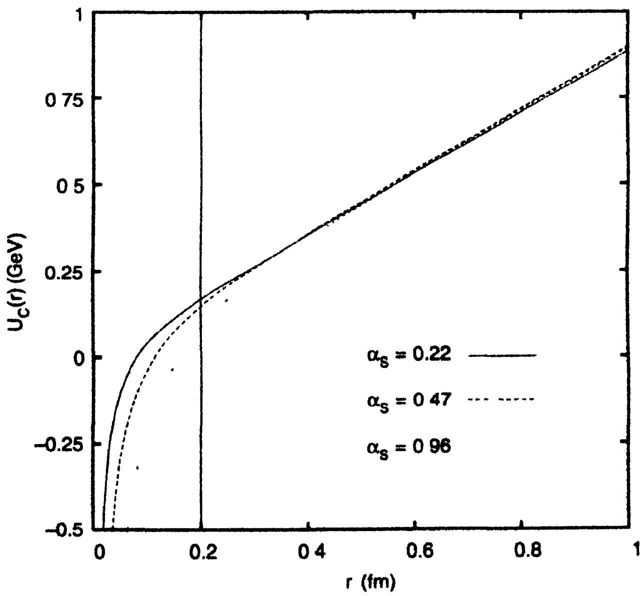

Figure 2. The confinement potential at various length scales in different coupling regimes.

superconducting nature of dual QCD vacuum (*i.e.* independent of its type-I or type-II nature) and varies with the same strength at all the couplings for the quite large (in the sense of hadronic scale) distances. Thus, for the present formulation the colour electric sources (i.e. quarks) are absolutely confined at large distances *via* a linearly rising potential *i.e.* $U_C(r) \sim \sigma r$ in the deep IR regime of QCD vacuum.

4. Epilogue

The dynamical breaking of the magnetic symmetry through the effective potential given by equation (3) subsequently induces the monopole condensation in the dual QCD vacuum. The action (4) associated to the dual *SU*(2) QCD Lagrangian (1) is derived in its simplest form given by equation (10) with the quark current as an external source and by using the equation of motion (8) for dual magnetic gauge field in the background of monopole condensation. The current configuration for the flux tube structure given by equation (6) then leads a non-local quark current-current correlation relation (11) where the quark-current given by equation (5) associated to the dynamical quark part in the Lagrangian (1) has been replaced by the equation (6). The dual gluon propagator (12) along with the quark current-current correlation (11) has been shown to lead the confinement potential (19) in the dynamically broken phase of dual QCD vacuum which is constituted of the usual Yukawa and linear terms given by equations (16) and (18) respectively. Using the confinement potential, the colour confinement force given by equation (20) is derived and graphically presented in Figure 1 for different couplings at various length scales. The transition from constant restoring force to the Yukawa one at short distances is shown to occur below 0.2 fm which is quite obvious in view of the maximum range ~ 0.21 fm of the Yukawa force at α_s = 0.96 for the present formulation. However the change in the behaviour of the confinement force from one state (short distance : UV region) to another (large distance : IR region) is apparent in manner and can be clearly seen in Figure 1. The graphical plot (Figure 2) of the confinement potential for various values of couplings demonstrates that the dual QCD vacuum changes the strength of the confinement potential at short distances with the change in the range of Yukawa force from ~ 0.12 fm - 0.21 fm. The string tension given by equation (21) is shown analogous to the energy of a vortex in a type-II superconductor in conventional superconductivity which conclusively leads a flux tube structure in the QCD vacuum in a parallel (but dual) way to the magnetic flux confinement. The graphical presentation of the confinement potential also clearly indicates that the absolute confinement at large distances is independent of the superconducting state of dual QCD vacuum. The linear potential and large distances are, therefore, inextricably locked together in the magnetically condensed QCD vacuum in the IR sector so that one can not be possibly evoked without other by any means. The present dual QCD model is thus strongly suggestive of a linearly confining potential for quarkonium systems at large distances and the results are in a quite close agreement with those obtained by various others [8, 21, 31, 32].

Acknowledgments

The author would like to thank Prof P K Raina for his critical comments and support.

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