Indian J. Phys. 81 (4) 485-493 (2007)

Analysis of the spacetime singularities arising in higher dimensional monopole-Vaidya Collapse

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Received 25 January 2006, accepted 13 February 2007

Abstract : We consider the end state of collapsing null radiation together with monopole field in higher dimensional spacetime The condition for the formation of a naked singularity as a result of the collapse has been obtained It is found that under certain restriction on the mass function, the collapse leads to a strong curvature naked singularity, violating CCH

Keywords : Cosmic censorship, gravitational collapse, naked singularity, black hole

PACS Nos. : 04 20 Dw, 04 20 Cv, 04 20 Jb

1. Introduction

In the framework of the Einstein Theory of relativity, spacetime singularities can be formed by gravitational collapse. If a singularity forms, it is crucial to consider physical phenomena around it, whether the singularity is covered by event horizon of the gravity or not. The Cosmic Censorship Hypothesis (CCH) [1] states that such singularities will always be covered by event horizons and hence can never be visible from the outside. Despite several attempts by many researchers, neither general proof nor precise mathematical formulation of this hypothesis has been available so far. On the contrary, several examples of naked singularities have been found. The examples of naked singularities known in spherical collapse arise from various forms of matters. These include dust [2–5], radiation [6–10], perfect fluid [11,12], null strange quark fluid [13,14] *etc.*

Vaidya solution is one of the most important solution among the above solutions. Papapetrou [15] first showed that this solution could yield a naked singularity. Since then,

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this solution is being used to analyse the scenario of gravitational collapse in general relativity Wang and Wu [16] generalized the Vaidya solution to a more general case which include most of the known solutions to the Einstein field equations, such as monopole solution, charged Vaidya solution, Husain solution Later on, Patil *et al* [17] have shown that under certain conditions on mass function, the gravitational collapse of generalized Vaidya solutions may yield naked singularities which appear in Vaidya and Tolman-bondi spacetimes are of the same nature Various important features such as the degree of in-homogeneity of the collapse necessary to produce a naked singularity the Cauchy horizon equation, the apparent horizon equation, and the strength of the singularity have a mutual correspondence in both metrics [*c f* 18] Recently, Sijie Gao and Lemos [19] have shown that it is possible to match the two solutions in one single spacetime the Tolman-Bondi-Vaidya spacetime

There is a considerable motivation provided in the past few years for considering the possibility for the spacetime to have extra dimensions String theory has created the interest in higher dimensional spacetimes Although the extra dimension is not directly observable, but TeV-scale theory [20,21] suggests that our universe, may have large extra dimensions Hence to prove or to disprove cosmic censorship hypothesis, it becomes necessary to study the gravitational collapse in the higher dimensions as well In this respect many research papers have been appeared on higher dimensional gravitational collapse [22–28], which show that either naked singularities or black holes may form in higher dimensional collapse depending upon the nature of the initial data

In the present paper, we discuss the gravitational collapse of null fluid around monopole field in the higher dimensional spacetime Our aim is to investigate the influence of a monopole field on Vaidya collapse and to study the naked singularities that may possibly form in this collapse

This work is organized as follows In the Section 2, we describe the five dimensional monopole-Vaidya spacetime In Section 3, we discuss the nature of the collapse, finally in Section 4 we conclude the paper

2. Monopole-Vaidya solution in five dimensional spacetime

Let us consider five dimensional spherically symmetric spacetime described by the metric [28,29]

$$ds^2 = -\left(1-\frac{m(v,r)}{r^2}\right)dv^2 + 2dv\,dr + r^2\left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2\right), \tag{1}$$

where v is an advanced Eddington time coordinate, r is the radial coordinate with $0 < r < \infty$, and $m(v,r)$ is the mass function giving gravitational mass inside the sphere of radial coordinate r

Non-vanishing components of the Einstein tensor for the above metric are given by

$$G^0_0 = G^1_1 = \frac{-3m'}{2r^3}, \quad G^1_0 = \frac{3m}{2r^3}, \quad G^2_2 = G^3_3 = G^4_4 = \frac{-m''}{2r^2}, \tag{2}$$

where $\{x^\mu\} = \{v, r, \theta_1, \theta_2, \theta_3\}$, $(\mu = 0,1,2,3,4)$ and ( ) and (′) represent partial derivatives with respect to v and r respectively

Combining eq (2) with field equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{3}$$

we find that the corresponding energy momentum tensor can be written in the form [16,30]

$$T_{\mu\nu} = T^{(n)}_{\mu\nu} + T^{(m)}_{\mu\nu}, \tag{4}$$

where

$$T^{(n)}_{\mu\nu} = \sigma\, l_\mu l_\nu$$

and

$$T^{(m)}_{\mu\nu} = (\rho + P)(l_\mu \eta_\nu + l_\nu \eta_\mu) + P g_{\mu\nu} \tag{5}$$

Using eqs (2–5), we can write the expressions for σ, ρ and P as

$$\sigma = \frac{3\dot{m}(v,r)}{2\kappa r^3}, \quad \rho = \frac{3m'(v,r)}{2\kappa r^3}, \quad P = \frac{-m''(v,r)}{2\kappa r^2}, \tag{6}$$

where ρ, P are energy density and pressure, while σ is the energy density of the Vaidya null radiation

We have considered null vectors l_μ, η_μ such that

$$l_\mu = \delta^0_\mu, \; \eta_\mu = \frac{1}{2}\left[1 - \frac{m(v,r)}{r^2}\right]\delta^0_\mu - \delta^1_\mu,$$

$$l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0, \; l_\lambda \eta^\lambda = -1 \tag{7}$$

$T^{(n)}_{ij}$, can be considered as the component of the matter field that moves along the null hyper surface v = constant When $\rho = P = 0$, the solutions reduce to the higher dimensional Vaidya solution with $m = m(v)$ [31] This fluid in general belongs to the type II fluids and the energy conditions for this type of fluids are given by [28–30,32]

(i) The weak and strong energy condition

$$\sigma > 0, \; \rho \geq 0, \; P \geq 0 \tag{8}$$

(ii) The dominant energy condition

$$\sigma > 0, \; \rho \geq P \geq 0, \; P \geq 0 \tag{9}$$

Following Refs. [16,29], we define the monopole solution in five dimensional spacetime

$$g(v,r) = ar^2, \tag{10}$$

where a is an arbitrary positive constant

Monopoles are formed due to a gauge-symmetry breaking and have many properties of elementary particles Most of their energy is concentrated in a small region near monopole core [33]

Mass function in five dimensional Vaidya space is defined by [31, 34]

$$f(v) = \begin{cases} 0 & v < 0 \\ \lambda v^2 & 0 \leq v \leq T \\ m_0 & v > T \end{cases} , \tag{11}$$

where λ is a positive constant

Since the energy momentum tensor is linear in terms of the mass functions, a linear superposition of particular solutions is also a solution of the Einstein field equations [16] In particular, the combination

$$m(v,r) = \lambda v^2 + ar^2 , \tag{12}$$

would represent the monopole-Vaidya solution in five dimensional spacetime

For $v < 0$, the spacetime is higher dimensional monopole solution with $f(v) = 0$ The radiation is focussed into a central singularity at $r = 0$, $v = 0$, of growing mass $f(v)$ At $v = T$, say, the radiation is turned off For $v > T$, the exterior spacetime settles to the higher dimensional Schwarzschild field embedded in a monopole field Thus $v = 0$ to $v = T$, the metric is monopole-Vaidya, whereas for $v > T$ it is an Schwarzschild-monopole spacetime Note that presence of monopole breaks the asymptotic geometry of the spacetime

To investigate the structure of the collapse we need to consider the radial null geodesics defined by $ds^2 = 0$ taking $\theta_1 = \theta_2 = \theta_3 = 0$ into account

Radial null geodesic equations (taking the null condition $K^a K_a = 0$ into account where $K^a = dx^a/dk$ is the tangent vector of a geodesic) for the metric (1) are

$$\frac{dK^v}{dk} + \left(\frac{m}{r^3} - \frac{m}{2r^2} \right) \left(K^v \right)^2 = 0 , \tag{13}$$

$$\frac{dK^r}{dk} + \left(\frac{m}{2r^2} + \frac{m}{r^3} - \frac{m'}{2r^2} + \frac{mm'}{2r^4} - \frac{m^2}{r^5} \right) \left(K^v \right)^2 + \left(\frac{m'}{r^2} - \frac{2m}{r^3} \right) K^v K^r = 0 \tag{14}$$

Let

$$K^v = \frac{dv}{dk} = \frac{R(v,r)}{r} \tag{15}$$

Using the null condition, $K^a K_a = 0$, we get

$$K^r = \frac{R}{2r} \left(1 - \frac{m(v,r)}{r^2} \right) \tag{16}$$

where R satisfies the differential equation

$$\frac{dR}{dk}-\frac{R^2}{2r^2}\left(1-\frac{3m}{r^2}+\frac{m'}{r}\right)=0 \tag{17}$$

3. Nature of the collapse

To investigate the nature of the collapse, we follow the method described in Ref [10] Roughly speaking naked singularities are singularities that may be seen by physically allowed observer *i e* outgoing light rays starting from the singularity terminate on the singularity in the past. The central shell-focussing singularity (*i e* that occurring at $r = 0$) is naked, if the radial null geodesic equation admits one or more positive real roots [35]

With the choice of the mass function (12), the metric (1) admits a homothetic Killing field given by

$$\xi^a = v\frac{\partial}{\partial v}+r\frac{\partial}{\partial r}, \tag{18}$$

which satisfies the equation

$$L_\xi g_{ab}=\xi_{a;b}+\xi_{b;a}=2g_{ab}, \tag{19}$$

where L denotes the lie derivative

It can be seen that $\xi^a K_a$ is constant along radial null geodesics, that is

$$\xi^a K_a = vK_v + rK_r = S, \tag{20}$$

where S is constant

Eq (20), because of the eqs (12), (15) and (16) yields

$$R=\frac{2S}{\lambda X^3-(1-a)X+2}, \tag{21}$$

where we have put $X = v/r$ and is known as a self similarity variable

To investigate the nature of the singularity we need to consider the radial null geodesics defined by $ds^2 = 0$

Equation for the radial null geodesics for the metric (1) are given by

$$\frac{dv}{dr}=\frac{2}{1-m/r^2} \tag{22}$$

With the choice of the mass function (12), above equation becomes

$$\frac{dv}{dr}=\frac{2}{1-\lambda(v/r)^2-a}. \tag{23}$$

It can be observed that the above differential equation has a singularity at $r=0,\ v=0$

For the geodesic tangent to be uniquely defined and to exist at this point, we must have [35]

$$X_0=\lim_{\substack{v\to0\\ r\to0}}\frac{v}{r}=\lim_{\substack{v\to0\\ r\to0}}\frac{dv}{dr}=\frac{2}{1-\lambda X_0^2-a} \tag{24}$$

that is

$$\lambda X_0^3 - (1-a) X_0 + 2 = 0 . \tag{25}$$

The above algebraic equation decides the nature of the singularity. If the above equation has a real and positive root, then there is a future directed radial null geodesic originating from $r = 0$, $v = 0$; in this case, the singularity will be naked. If eq. (25) has no real and positive root, then the singularity will be covered and the collapse proceeds to form a black hole.

From the *theory of equations*, it can be checked that eq. (25) has two real and positive roots if

$$0 < a < 1 \text{ and } 0 < \lambda \leq \frac{(1-a)^3}{27} \tag{26}$$

Thus if the above condition is satisfied, then the higher dimensional monopole-Vaidya collapse leads to a naked singularity, otherwise the collapse ends into a black hole. In particular if we choose $\lambda = 0.01$, $a = 0.5$ then we get two positive real roots $X_1 = 6.1005$ and $X_2 = 3.4373$. Table 1 shows the positive roots corresponding to different values of a and λ.

Table 1. Roots of the eq (25) corresponding to different values of a and λ.

a	λ	Positive real roots	
		Root X_1	Root X_2
0.3	0.010000	6.1005	3.4373
	0.009000	6.6667	3.3333
	0.008000	7.2963	3.2492
0.5	0.004000	7.8078	5.0000
	0.003000	10.0000	4.5742
	0.002000	13.2001	4.3232
0.7	0.001000	10.0172	9.9892
	0.000900	12.4347	8.5259
	0.000800	14.0281	8.0661
0.9	0.000030	41.6047	24.3100
	0.000020	56.9593	22.1832
	0.000010	87.8885	20.9149

Four dimensional monopole-Vaidya collapse has been discussed in Ref. [17] and it has been shown that, the collapse leads to a naked singularity if

$$0 < \lambda \leq \frac{(1-a)^2}{8} \tag{27}$$

Thus comparison with the analogous 4D case shows that naked singularity occurs for a slightly smaller value of the parameter λ in 5D.

Strength of a naked singularity :

Strong curvature singularity is regarded as a more serious violation of cosmic censorship as compared to a weak one. It is believed that spacetime cannot be extended through a strong singularity, but is possibly extendible through a weak one [36]. In Tipler's sense [37], singularity is said to be a strong curvature singularity if collapsing volume elements crushed to zero at the singularity. According to Clarke and Krolak criteria [38], a singularity is strong curvature type if

$$\varphi = \lim_{k \to 0} k^2 R_{ab} K^a K^b > 0 \,, \tag{28}$$

where K^a is tangent to the radial null geodesic, R_{ab} is the Ricci tensor and k is an affine parameter.

In order to examine the strength of a singularity along radial null geodesics, we observe that

$$\frac{dX}{dk} = \frac{1}{r}\frac{dv}{dk} - \frac{v}{r^2}\frac{dr}{dk} \,. \tag{29}$$

Using eqs. (15) and (16), above equation becomes

$$\frac{dX}{dk} = \frac{R}{2r^2}\left[\lambda X^2 - (1-a)X + 2\right] = \frac{S}{r^2} \,. \tag{30}$$

From eqs. (15) and (16), we obtain

$$\varphi = \lim_{k \to 0} k^2 R_{ab} K^a K^b = \lim_{k \to 0} k^2 \left[\frac{3\dot{m}}{2r^3}\left(K^v\right)^2\right]$$

$$= 3\lambda X_0 \lim_{k \to 0}\left(\frac{kR}{r^2}\right) \tag{32}$$

At the singularity, $k \to 0$, $r \to 0$ and $X \to a_+$, where a_+ is a root of eq. (25). Hence applying L'Hospital's rule and using some algebra, we obtain

$$\lim_{k \to 0}\left(\frac{kR}{r^2}\right) = \frac{}{1 - \lambda X_0^2 - a} \tag{33}$$

Inserting the above equation into eq. (32), we get

$$\varphi = \frac{3\lambda X_0}{\left(1 - \lambda X_0^2 - a\right)^2} > 0 \,. \tag{34}$$

Thus Clarke and Krolak condition for the strong curvature singularity [38] is satisfied. Hence, the naked singularity arising in this spacetime is strong.

4. Conclusion

We have presented the scenario for the gravitational collapse of a monopole field together with Vaidya null radiation in higher dimensional spacetime. The possible occurrence of a naked singularity has been investigated and it has been shown that at least for a particular choice of the parameters, a naked singularity is formed.

We have estimated the strength of a naked singularity by Clarke and Krolak criteria [38] and it is found that these singularities are of strong curvature type.

Since in the absence of monopole field, spacetime becomes asymptotically flat, one can argue that the condition of asymptotically flatness does not play any significant role in the formation of the naked singularity.

Occurrence of a strong curvature singularity in the higher dimensional monopole-Vaidya spacetime suggests that this solution violates the CCH.

Similar solutions in four dimensions have been obtained earlier [17] and it has been shown that under certain conditions on the mass function $m(v,r)$, these solutions admit naked singularities. In fact, we can find a class of the functions $m(v,r)$ such that

$$m(v,r) = \lambda v + F(v,r)$$

where $F(v,r)/r$ is a polynomial in r(or in v), then the only contribution to the condition for the existence of the null geodesics emanating from the central singularity is from λv, the Vaidya mass. However, while choosing the function $m(v,r)$, one has to take the care that the energy conditions mentioned in eqs. (8) and (9) should not be violated. The same idea of choosing $m(v,r)$ can be extended to higher dimensions as well.

Acknowledgment

One of the authors (KDP) would like to thank IUCAA, Pune (India) for kind hospitality under Associate-ship program, where part of this work was done.

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