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Stability of the lower hybrid drift wave in a multi-ion plasma

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Abstract : We have investigated the stability of the lower hybrid drift (LHD) wave in a plasma containing hydrogen and positively and negatively charged oxygen ions. In our model, for the length and time scales of the fluctuations the electrons are treated as magnetized, while all the ions are treated as unmagnetized. The instability is driven by the equilibrium diamagnetic current associated with the hydrogen ions. The growth/damping rate is modified by a factor dependent directly on the number densities, square of the charges and masses of the oxygen ions. In the presence of the positive oxygen ions, the wave tends to stabilize due to the increased damping by the electrons. On the other hand, the negative oxygen ions tends to aid the instability due to the decreased damping by the electrons.

Keywords : Multi-ion cometary plasma, lower hybrid drift wave, stability

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1. Introduction

The frequent observations of low frequency wave activity in space plasma environments climaxed with satellite missions to comets Halley and Giacobini-Ziner. These cometary missions spurred intense research activity aimed at understanding the excitation mechanism and free energy sources of wave activity. Theoretical stability analyses have considered background hydrogen plasmas permeated by hydrogen (or heavier) ion beams with Maxwellian [1] or drifting ring [2–4] velocity distributions. Oblique propagation of waves have also been considered [5–8].

Many of the above studies either modeled cometary environments or were easily adaptable to them. Later, the consideration of co-existence of new born ion species fuelled yet another burst of intense theoretical activity aimed at understanding the characteristics of hydromagnetic [9,10] and electromagnetic [11,12] waves associated

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with these ions. These studies generally modeled the plasma with two (hydrogen and oxygen) ions; the effect of water ions have also been considered. Wave instability has thus been well investigated in cometary plasma environments containing more than one species of positive ion.

However, cometary plasmas also contain negative ions in abundant number Giotto observations of the inner coma of comet Halley revealed a composition of cometary neutral gas and dust, thermal ions and electrons, fast cometary pick-up ions, *etc.* Of the many species present, negatively charged oxygen ions were unambiguously identified [13].

Cometary plasma wave measurements have detected waves extending from the lower hybrid resonance frequency to the electron plasma frequency both at comets Halley and Giacobini-Ziner. *In situ* measurements at Halley have also revealed the existence of lower hybrid turbulence inside of the cometary bow shock. A large flux of energetic electrons were also observed which while penetrating the cometary atmosphere produces soft X-rays by a combination of bremsstrahlung and line radiation of oxygen atoms [14,15].

Cometary plasmas are a loosely bound collection of charged and neutral particles and hence inhomogeneities are likely to occur as the comet orbits the Sun A source of energy that can excite the lower hybrid drift instability (LHDI) is the diamagnetic currents that can exist in the plasma.

We have therefore, studied the stability of the lower hybrid drift (LHD) wave in a plasma consisting of hydrogen (H^+), positively charged oxygen ions (O^+), negatively charged oxygen ions (O^-) with the electrons and the hydrogen ions having diamagnetic drifts in opposite directions. We find that the positively charged oxygen ions reduce the growth rate of the wave, while the negatively charged oxygen ions enhance the growth of the wave.

2. The dispersion formula

We consider a four-component plasma consisting of electrons (e), hydrogen ions (H^+) and positively and negatively charged oxygen ions (O^+ and O^- respectively) plasma confined by a uniform magnetic field along the z-direction. The time and scale lengths of the fluctuations are such that the electrons are treated as magnetized while all the ions are treated as unmagnetised and traverse straight line orbits. Allowing for an equilibrium density homogeneity for the electrons and the H^+ ions, these species have diamagnetic drifts in opposite directions. On the other hand, the heavier oxygen ions are considered to have a uniform density. The LHD wave is characterized by a frequency ω satisfying $\Omega_H < \omega < \Omega_e$, with $\boldsymbol{k}$ perpendicular to $\boldsymbol{B}_0$ and $kL_n >> 1$; where Ω_e (Ω_H) are the electron (hydrogen) gyro-frequencies; $\boldsymbol{k}$ is the wave vector, $\boldsymbol{B}_0$ is the magnetic field in the z-direction and L_n is the scale length of the density gradient. The dispersion relation for the LHD wave in such a plasma, can be written as a generalization of the dispersion relation in [16]

$$1.0 + \chi_e + \chi_{H^+} + \chi_{O^+} + \chi_{O^-} = 0, \tag{1}$$

where the susceptibilities are given by

$$\chi_e = \frac{1}{k^2\lambda_{de}^2}\left[1 + \frac{\omega - kV_{de}}{\omega}\Gamma_0(b_e)\right]. \tag{2}$$

$$\chi_{H^+} = \frac{1}{k^2\lambda_{dH^+}^2}\left[1 + \frac{\omega - kV_{dH^+}}{\sqrt{2}kV_{H^+}} Z\left(\frac{\omega - kV_{dH^+}}{\sqrt{2}kV_{H^+}}\right)\right] \tag{3}$$

and

$$\chi_{O^+,O^-} = \frac{1}{k^2\lambda_{do^+,O^-}^2}\left[1 + \frac{\omega}{\sqrt{2}kV_{O^+,O^-}} Z\left(\frac{\omega}{\sqrt{2}kV_{O',O}}\right)\right]. \tag{4}$$

In the above, $V_j = (2T_j/m_j)^{½}$ is the thermal speed with T_j and m_j being the temperature and mass of the j-th species of particles, (j = H$^+$, O$^+$, O$^-$ or e). $\Gamma_0 = I_0(b_e)\exp(-b_e)$, where I_0 is the modified Bessel function of zeroth order with $b_e = \left(k^2V_e^2/\Omega_e^2\right)$, $\lambda_{dj} = \left(T_j/(4\pi n_{0j}c_j^2e^2)\right)^{1/2}$ (c_j is the charge number and n_{0j} is the equilibrium density) is the associated Debye length while Z denotes the plasma dispersion function. $V_{de(H^+)} = \pm V^2_{e(H^+)}/\left(L_n\Omega_{e(H^+)}\right)$ is the diamagnetic drift associated with the electrons (hydrogen).

2.1 Dispersion relations :

Case (i) : Both oxygen species are cold :

In this sub-section we derive a dispersion relation for the LHD wave in which both the heavy ion species are cold. We thus consider waves in the regime $V_{O^+}, V_O << (\omega/k) << V_{H^+}$.

We thus need the small parameter expansion of the Z function for the H$^+$ ions and the asymptotic expansion for the O$^+$ and O$^-$ ions. Substituting these expansions into (1) and after a lengthy simplification we get the expression for ω_r, the real part of the frequency and the expression γ for the growth/damping rate as

$$\omega_r = \frac{\Gamma_0 kV_{dH^+}\eta_{eH}}{1.0 + k^2\lambda_{dH^+}^2 + \tau_{He}\eta_{eH}(1.0 - \Gamma_0)} + \frac{k^2q}{\eta_{eH}\Gamma_0 kV_{dH^+}}, \tag{5}$$

where

$$q = \left(\tau_{HO^+}c_{O^+}^2\eta_{O^+H}V_{O^+}^2 + \tau_{HO^-}c_{O^-}^2\eta_{O^-H}V_{O^-}^2\right) \tag{6}$$

and

$$\gamma = -\sqrt{\frac{\pi}{2}}\frac{\left(\omega_r - kV_{dH^+}\right)}{kV_{H^+}}\frac{\omega_r^3}{2k^2q + \eta_{eH}\Gamma_0 kV_{dH^+}\omega_r} \tag{7}$$

with

$$\eta_{eH} = \frac{n_{0e}}{n_{oH}}, \quad \eta_{O^+H} = \frac{n_{OO^+}}{n_{oH}}, \quad \eta_{O^-H} = \frac{n_{OO}}{n_{oH}}$$

and

$$\tau_{He} = \frac{T_H}{T_e}, \quad \tau_{HO^+} = \frac{T_H}{T_{O^+}}, \quad \tau_{HO^-} = \frac{T_H}{T_{O^-}}.$$

The lower hybrid drift wave can be driven unstable if the diamagnetic drift velocity of the hydrogen ions $V_{dH^+} > \omega_r / k$; the growth rate of the wave, however, now being dependent on the densities, degree of ionization and temperatures of the heavier species of ions from (6) and (7). As a check on (5) and (7), we note that when the heavier ion densities are equal to zero, they reduce to the corresponding expressions in [16].

Case (ii) : One of the heavier species is hot :

We now consider, in this sub-section, the dispersion relation and the growth/damping rate when one of the heavier ionic species is hot. Specifically, we consider the case where the O^- ions are cold and the O^+ ions are hot. We thus consider waves in the regime

$$V_O << (\omega / k) << V_{H^+}, V_{O^+}.$$

We thus need the asymptotic expansion of the plasma dispersion function for the O ions and the small parameter expansion of the same for the O^+ and H^+ ions Substituting the appropriate expansions in (1) and simplifying we get the expressions for the real part of the frequency and the growth/damping rate as

$$\omega_r = \frac{\Gamma_0 k V_{dH^+} \eta_{eH}}{1.0 + k^2 \lambda_{dH^+}^2 + \tau_{He}\eta_{eH}(1.0 - \Gamma_0) + \tau_{HO^+}\eta_{O^+H}c_{O^+}^2} + \frac{\tau_{HO}\,\eta_{O^-H}c_{O^-}^2 k^2 V_O^2}{\eta_{eH}\Gamma_0 k V_{dH^+}}. \qquad (8)$$

The expression for the growth/damping can be derived similar to (7) as

$$\gamma = -\sqrt{\frac{\pi}{2}}\left\{\frac{\omega_r}{kV_{H^+}}Q - \frac{V_{dH}}{V_{H^+}}\right\}\frac{\omega_r^3}{2k^2V_{O^-}^2\tau_{HO^-}c_{O^-}^2\eta_{O^-H} + \eta_{eH}\Gamma_0 kV_{dH^+}\omega_r}. \qquad (9)$$

where

$$Q = 1 + c_{O^+}^2 \eta_{O^+H}(\tau_{HO^+})^{3/2}\left(\frac{m_{O^+}}{m_{H^+}}\right)^{1/2}.$$

Though the instability condition remains the same namely, $V_{dH} > \omega_r / k$, we find that the growth/damping rate, is also dependent on the number densities, masses, temperatures and degree of ionization of the heavier species. Besides, a higher diamagnetic drift for the hydrogen ions is now required as Q is always greater than 1.

Eqs. (8) and (9) are the new expressions derived and do not have a counterpart in [16]. However, from (8), we find that since O^+ is also now hot, its contribution is similar to that of hydrogen in (5), while the contribution of O^- remains the same. Similarly, inspecting (7) and (9) we find there is no change in the contributions of hydrogen and O^-, while the contribution of O^+ now appears in the numerator.

3. Results

We plot relations (7) and (9), the expressions for the growth/damping rate for typical parameters observed near the coma of comet Halley by the Giotto spacecraft. Negative ions, in three broad mass peaks at 7–19, 22–65 and 85–110 amu with densities ≥1, 5×10^{-2} and 4×10^{-2} cm^{-3} respectively were observed. Their energies ranged between 0.03 eV to 3.0 keV with a background of 1.0 keV. Of the many ionic species, O^- was unambiguously identified [13]. We therefore assigned a temperature range of 300 eV to the oxygen ions when they are cold and a temperature of 1.0 keV for the hydrogen ions. The densities of the oxygen ions were fixed at 0.25 cm^{-3} while that of hydrogen was set at 3.0 cm^{-3}. These assignments of temperatures and densities are in reasonably good agreement with the observed values. The temperature ratio of T_e / T_{H^+} was set = 20.0 while the normalized drift velocity V_{dH^+} / V_{H^+} was given a value of 0.8. The background magnetic field B_0 = 6.0 gammas. We first consider the case where both species of oxygen ions are cold.

Figure 1 is a plot of the growth rate (7) versus x_e as a function of the heavy ion densities with V_{dH^+} / V_{H^+} 0.8 and $n_{H^+} = 3.0\,\text{cm}^{-3}$. Curve (a), indicated by ◊, is for a plasma containing only H^+ and electrons. We find that the growth rate initially increases rapidly and then tends to flatten with increasing x_e. Curve (b), marked by □, is for a plasma in which one species of the heavy ion namely n_{o^+} is also present with $n_{o^+} = 0.25$ cm^{-3} and $c_{o^+} = 1$. The curve is similar to curve (a), but with a decrease in the magnitude of the growth rate; in fact the wave is damped from x_e = 0.1 to 0.35. Curve (c), indicated by Δ, is for the case where the heavier ion species is now negatively charged oxygen with $n_{o^-} = 0.25$ cm^{-3}, while the degree of ionization is the

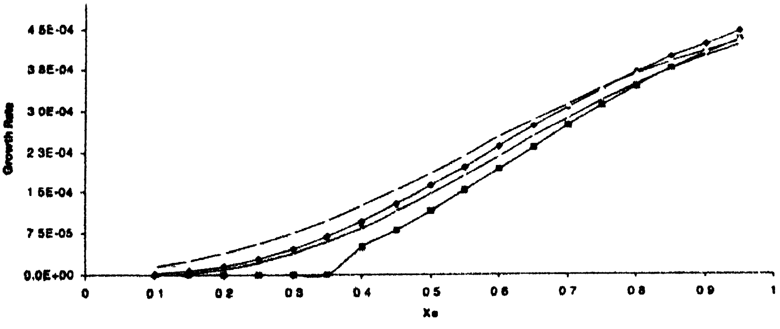

Figure 1. Plot of the growth rate (7) *versus* x_e as a function of the heavy ion densities. Parameters common to all the curves are V_{dH^+} / V_{H^+} = 0.8 and $n_{H^+} = 3.0$ cm^{-3}. Curve (a) indicated by ◊, is for a plasma containing only H^+ and electrons. Curve (b) marked by □, is for a plasma which has $n_{o^+} = 0.25$ cm^{-3} and $c_{o^+} = 1$. Curve (c) indicated by Δ, has $n_{o^-} = 0.25$ cm^{-3} and $c_{o^-} = 1$. Curve (d) marked by x, is for $n_{o^+} = n_{o^-} = 0.25\,\text{cm}^{-3}$, $c_{o^+} = c_{o^-} = 1$.

same, namely $c_o = 1$. A comparison of curves (a) and (b) with (c) shows that the growth rate has increased. Finally, curve (d) marked by x, is for the case where both species of oxygen ions are present. Thus the parameters of the oxygen ions for this curve are as follows : $n_{o^+} = n_{o^-} = 0.25\,\mathrm{cm}^{-3}, c_{o^+} = c_{o^-} = 1$. The curve is similar to curve (a), for a single ion plasma. This is to be expected since the product of the density of the oxygen ions and charges on these ions now cancel.

We next consider the case where one of the heavier ion species, namely O^+, is hot. Figure 2 is thus a plot of the growth rate (9) *versus* x_e as a function of the heavy ion densities with $V_{dH^+}/V_{H^+} = 0.8$, $n_{H^+} = 3.0\,\mathrm{cm}^{-3}$ and $\tau_{HO^+} = 0.2$. Curve (a), indicated by ◊, is for a plasma containing only H^+ and electrons and is the same as curve (a) of Figure 1. Curve (b), marked by □, is for a plasma in which n_{o^+} is also present with $n_{o^+} = 0.25$ cm^{-3} and $c_{o^+} = 1$. The curve is again similar to curve (a), with

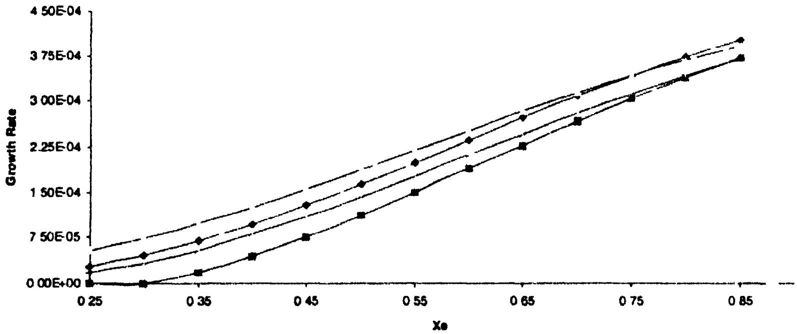

Figure 2. Plot of the growth rate (9) versus x_e as a function of the heavy ion densities. Parameters common to all the curves are $V_{dH^+}/V_{H^+} = 0.8$, $n_{H^+} = 3.0\,\mathrm{cm}^{-3}$ and $\tau_{HO^+} = 0.2$ Curve (a) indicated by ◊ is for a plasma containing only H^+ and electrons. Curve (b) marked by □, is for a plasma in which n_{o^+} is also present with $n_{o^+} = 0.25$ cm^{-3} and $c_{o^+} = 1$. Curve (c) indicated by Δ, is for the case where $n_o = 0.25$ cm^{-3} and $c_o = 1$ Curve (d) marked by x, has $n_{o^+} = n_o = 0.25$ cm^{-3}, $c_{o^+} = c_o = 1$

a decrease in the magnitude of the growth rate. However, the wave is damped only upto $x_e = 0.3$, unlike the previous case where it was damped upto $x_e = 0.35$. Curve (c), again indicated by Δ, is for the case where $n_{o^-} = 0.25$ cm^{-3} and $c_{o^-} = 1$. Similar to Figure 1, the growth rate has increased. Finally, curve (d) marked by x, is for the case where both species of oxygen ions are present. Thus the parameters of the oxygen ions for this curve are as : $n_{o^+} = n_o = 0.25$ cm^{-3}, $c_{o^+} = c_{o^-} = 1$. The curve is again similar to curve (a). And as mentioned in connection with relation (9), the growth rate is now marginally smaller for curves (b) and (d), when compared to the corresponding values for the same curves in Figure 1.

To verify our analytic results, the dispersion relation (1) was solved numerically retaining the full Z – function for all species of ions.

Figure 3 thus depicts of the growth rate versus x_e as a function of the heavy ion densities. The parameters common to all the curves are $V_{dH^+}/V_{H^+} = 0.8, n_{H^+} = 3.0$ cm^{-3}, $\tau_{He} = 0.05$, $\tau_{HO^+} = 3.33$ and $\tau_{HO^-} = 3.33$ wherever appropriate. Curve (a) indicated by

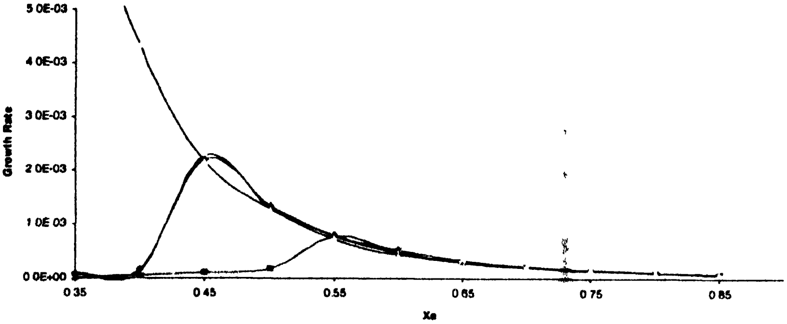

Figure 3. Plot of the growth rate from full numerical solution of (1) *versus* x_e. The parameters common to all the curves are $V_{dH^+}/V_{H^+} = 0.8$, $n_{H^+} = 3.0$ cm^{-3} $= 0.05$, $\tau_{HO^+} = 3.33$ and $\tau_{HO^-} = 3.33$ wherever appropriate. Curve (a) indicated by ◊, is for a plasma containing only H$^+$ and electrons. Curve (b) marked by □, is for a plasma in which n_{o^+} is also present, with $n_{o^+} = 0.25$ cm^{-3} and $c_{o^+} = 1$. Curve (c) indicated by Δ, is for the case where $n_o = 0.25$ cm^{-3} and $c_{o^-} = 1$. Curve (d) marked by *x*, is for the case where both species of oxygen ions are present with $n_{o^+} = n_{o^-} = 0.25$ cm^{-3}, $c_{o^+} = c_{o^-} = 1$.

◊, is for a plasma containing only H$^+$ and electrons. The growth rate which starts from low values, reaches a maximum at $x_e = 0.45$ and then decreases rapidly. Curve (b) marked by □, is for a plasma in which n_{o^+} is also present, with $n_{o^+} = 0.25$ cm^{-3} and $c_{o^+} = 1$. The curve is similar to curve (a). However, there is a considerable reduction in the growth rate and the maximum now occurs at $x_e = 0.55$. Curve (c) indicated by Δ, is for the case where $n_{o^-} = 0.25$ cm^{-3} and $c_{o^-} = 1$. The growth rate starts from a high value and then decreases. Curve (d) marked by *x*, is for the case where $n_{o^+} = n_{o^-} = 0.25$ cm^{-3}, $c_{o^+} = c_{o^-} = 1$. The curve is almost identical to curve (a) as the oxygen ions are of equal density and opposite charge.

Figure 4, which is the numeric solution corresponding to (9) thus also depicts the variation of the growth rate *versus* x_e. The parameters common to all the curves are $V_{dH^+}/V_{H^+} = 0.8$, $n_{H^+} = 3.0$ cm^{-3} $\tau_{He} = 0.05$, $\tau_{HO^+} = 0.2$ and $\tau_{HO^-} = 3.33$ wherever

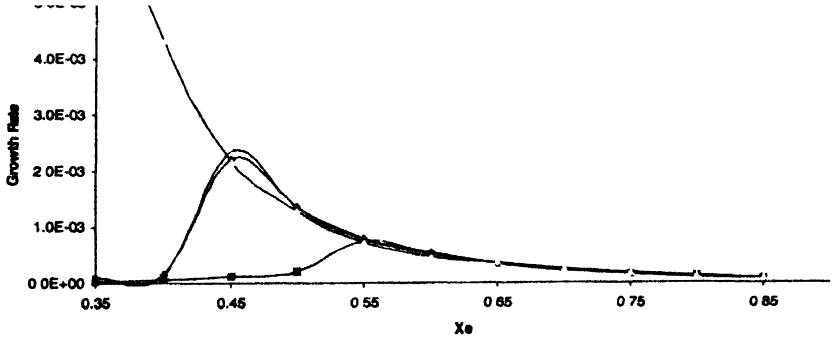

Figure 4. which is the numeric solution corresponding to (9) thus also depicts the variation of the growth rate *versus* x_e. The parameters common to all the curves are $V_{dH^+}/V_{H^+} = 0.8$, $n_{H^+} = 3.0$ cm^{-3}, $\tau_{He} = 0.05$, $\tau_{HO^+} = 0.2$ and $\tau_{HO^-} = 3.33$ wherever appropriate. Curve (a) indicated by ◊, is for a plasma containing only H$^+$ and electrons. Curve (b) marked by □, is for a plasma which has $n_{o^+} = 0.25$ cm^{-3} and $c_{o^+} = 1$. Curve (c) indicated by Δ, is for the case where $n_{o^-} = 0.25$ cm^{-3} and $c_{o^-} = 1$. Curve (d) marked by *x*, is for the case where both species of oxygen ions are present with $n_{o^+} = n_{o^-} = 0.25$ cm^{-3}, $c_{o^+} = c_{o^-} = 1$.

appropriate. Curve (a) indicated by ◊, is for a plasma containing only H^+ and electrons and is the same as curve (a) of Figure 3. Curve (b) marked by □, is for a plasma which has $n_{0^+} = 0.25$ cm^{-3} and $c_{0^+} = 1$. The growth rate has again now considerably reduced and is similar to the corresponding curve in Figure 3 in all other respects. Curve (c) indicated by Δ, is for the case where $n_{0^-} = 0.25$ cm^{-3} and $c_{0^-} = 1$. The growth rate again starts with a large value and decreases with increasing x_e. Finally curve (d) marked by x, is for the case where both species of oxygen ions are present with $n_{0^+} = n_{0^-} = 0.25$ cm^{-3}, $c_{0^+} = c_{0^-} = 1$. The curve is similar to curve (a). And as in Figure 2, the magnitude of growth rates for curves (b) and (d) are slightly lower than the corresponding values of the same curves in Figure 3.

The LHD wave is driven unstable when the drift velocity V_{dH^+} of the hydrogen ions is greater than the phase velocity of the wave as is evident from (7) and (9). The electrons, however, damp the wave. Curve (b) in all the figures, which corresponds to the presence of O^+ in the plasma, also results in more electrons (due to the charge neutrality condition) and hence the wave growth is reduced; it is even damped over a part of the wavelength range studied. On the other hand, the addition of O^- to the plasma (curve (c) in all the figures) reduces the number of electrons and hence, the corresponding damping. This explains the general increase in the growth rate when O^- is added. Finally, since we have chosen the density and charge in such a manner that their product cancels when O^+ and O^- are both present, curve (d) resembles curve (a) (which is for a single ion plasma) in all the figures.

4. Conclusions

We have in this paper, studied the stability of the lower hybrid drift wave in a plasma containing hydrogen and positively and negatively charged oxygen ions. Two cases were studied : one, in which both species of oxygen ions were cold and second when the O^+ ions were hot and the O^- ions were cold. For both the cases, the wave is driven unstable by the diamagnetic drift of the hydrogen ions which should exceed the phase velocity of the wave for the instability. We find that the growth/damping rate to be modified by a factor that depends on the number densities, the square of the charges on the oxygen ions and the masses of these ions. The electrons have a damping effect on the wave. Thus, when O^+ ions are present in the plasma with a corresponding increase in the electron density to preserve charge neutrality, the growth rate decreases; the wave is even damped over a part of the wavelength region studied. The attendant decrease in electron density when O^- is present increases the growth rate of the wave.

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