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Baryogenesis from inverted hierarchical mass models with tribimaximal mixings

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Abstract : We estimate the baryon asymmetry for two specific structures of Inverted Hierarchical mass models: bimaximal mixings(BM) and tribimaximal mixings (TBM), both with opposite CP-parity. Starting from the light neutrino mass matrices, the heavy right handed Majorana neutrino mass matrices are constructed *via* inverse seesaw relation. The estimated baryon asymmetry for tribimaximal mixing mass model with down quark mass matrix taken as Dirac neutrino mass matrix is found to be consistent with the experimental value. Through the estimation of baryon asymmetry, we establish the validity of tribimaximal mixings of inverted hierarchical neutrino mass model. The present calculation also discriminates the three possible choices of Dirac neutrino mass matrix.

Keywords : Seesaw mechanism, inverted hierarchy, lepton and baryon asymmetry.

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1. Introduction

The inverted hierarchical pattern of light left-handed neutrinos ($m_1 \simeq m_2 > m_3$) are best understood in terms of two mass models[1]: Inverted hierarchical type-2A (InvT2A) and Inverted Hierarchical type-2B(InvT2B) based on the relative CP-parity between m_1 and m_2. For InvT2B, the mass pattern is (m_1,-m_2, m_3). The inverted hierarchical model of neutrinos with odd CP parity (m_1, -m_2, m_3) generally predicts nearly bimaximal mixings in the diagonal basis of the charged lepton mass matrix. A familiar mass matrix of InvT2B is generally given by

$$m_{LL} = \begin{pmatrix} \delta_1 & 1 & 1 \\ 1 & \delta_2 & \delta_3 \\ 1 & \delta_3 & \delta_2 \end{pmatrix} m_0, \qquad (1)$$

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where $\delta_{1,2,3} < 1$. The diagonalisation of mass matrix (1) gives following mass eigenvalues :

$$m_{1,2} = \frac{m_0}{2}\left[(\delta_1 + \delta_2 + \delta_3) \pm x\right], m_3 = m_0(\delta_2 - \delta_3);$$

$$x^2 = 8 + \left(\delta_1^2 + \delta_2^2 + \delta_3^2\right) - 2\delta_1\delta_2 - 2\delta_1\delta_3 + 2\delta_2\delta_3$$

and the three mixing angles are calculated as

$$\tan^2\theta_{23} = 1, \sin\theta_{13} = 0, \ \tan 2\theta_{12} = \frac{2\sqrt{2}}{(\delta_1 - \delta_2 - \delta_3)}.$$

Such a simplest form (1) is found to be realised within seesaw framework [2] $m_{LL} = -m_{LR}M_{RR}^{-1}m_{LR}^T$ using diagonal form of Dirac neutrino mass matrix $m_{LR} = \mathrm{diag}\left(\lambda^m, \lambda^n, 1\right)v$ and a suitable non-diagonal texture of heavy Majorana mass matrix M_{RR}. For each pair of (m,n), the corresponding texture of M_{RR} are different, and the light neutrino mass matrices, m_{LL} are left unaffected. For example, we use the following form of m_{LL} from Ref.[1]:

$$m_{LL} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -\left(\lambda^3 - \lambda^4\right)/2 & -\left(\lambda^3 + \lambda^4\right)/2 \\ 1 & -\left(\lambda^3 + \lambda^4\right)/2 & -\left(\lambda^3 - \lambda^4\right)/2 \end{pmatrix} m_0 \quad . \tag{2}$$

Using the input values λ=0.3, m_0=0.035eV, the corresponding oscillation parameters are calculated as $\Delta m_{21}^2 = 9.30 \times 10^{-5} eV^2, \Delta m_{23}^2 = 2.50 \times 10^{-3} eV^2, \tan^2\theta_{12} = 0.98, \sin^2 2\theta_{23} = 1.0, \sin\theta_{13} = 0$. The predicted solar angle is nearly maximal. Several attempts have been made to tone down solar mixing angle but the effect is not so satisfactory to the experimentally acceptable level. It has been pointed out [3] that there is also a possibility to realise the tribimaximal mixings [4] within the inverted hierarchical mass matrix in eq.(1), provided the numerical values of $\delta_{1,2,3}$ are comparatively larger but smaller than 1. As a specific example, we follow the ref [3] and use the following values : $\delta_1 = 0.56855627$, $\delta_2 = 0.71572329$, $\delta_3 = 0.12477548$, $m_0 = 0.035 eV$ in eq.(1). The predictions on neutrino oscillation parameters are : $\Delta m_{21}^2 = 8.34 \times 10^{-5} eV^2, \Delta m_{23}^2 = 1.95 \times 10^{-3} eV^2, \tan^2\theta_{12} = 0.45,\ \tan^2\theta_{23} = 1.0, \sin\theta_{13} = 0$. The solar angle is slightly smaller than tribimaximal mixing and agrees with recent data. The above prediction does not require any fine tuning of the solar angle from charged lepton sector or from renormalization effects.

The inverted hierarchical neutrino mass matrix in eq.(1) with bimaximal mixings as well as tribimaximal mixings is independent of pair of (m,n) appeared in m_{LR} within the framework of seesaw formula, and this can however, be fixed in the estimation of baryon asymmetry[5] *via* lepton asymmetry [6-8], which depends on the texture of M_{RR}. For our interest, we take up three different cases [9] of (m,n) dependence on m_{LR} and M_{RR} as allowed by $SO(10)$ grand unified theory :

Case 1 : $m_{LR} \equiv$ charge lepton mass matrix with (m,n) = (6,2);

Case 2 : $m_{LR} \equiv$ up quark mass matrix with (m,n) = (8,4);

Case 3 : $m_{LR} \equiv$ down quark mass matrix with (m,n) = (4,2).

For a particular choice of m_{LL}, one can have three possible structure of M_{RR} depending on the above choice of (m,n). The choice of non-diagonal m_{LR} in the basis of diagonal M_{RR}, plays crucial role in the calculation of baryon asymmetry *via* lepton asymmetry produced by the decay of lightest of heavy Majorana neutrino M_1[10].

In this work, we start with light Majorana neutrino mass matrix m_{LL} and translate this matrix to M_{RR} *via* the inversion of seesaw formula, $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. Using the right-handed Majorana mass M_{RR}, we estimate the baryon asymmetry for bimaximal and tribimaxima cases for three different choices of Dirac neutrino mass matrix m_{LR}. In Section 2, we briefly mention and expression for lepton and baryon asymmetry and in Section 3, we present one representative example of numerical calculation and results. Finally in Section 4, we conclude with summary and discussion.

2. Expression for lepton and baryon asymmetry

For hierarchical mass structure of heavy right handed Majorana neutrinos, considering the out of equilibrium and CP-violating decay of physical Majorana neutrino M_1, the CP asymmetry is expressed as [11] :

$$\epsilon_1 = \frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left[\left(h^* m_{LL} h^\dagger\right)_{11}\right]}{\left(hh^\dagger\right)_{11}}, \tag{3}$$

where $h = m'_{LR}/v$ is the 3x3 Dirac neutrino Yukawa coupling matrix normalised to $h_{33}=1$ in the basis, where M_{RR} is diagonal with real and positive eigenvalues. For quasi-degenerate structure *i.e.*, for $M_1 \simeq M_2 < M_3$, one has to consider the resonance enhancement factor $R = M_1/2(|M_2|-|M_1|)$ and the expression (3) is modified to [12, 13]:

$$\epsilon_1 = \frac{M_2}{8\pi v^2}\frac{\mathrm{Im}\left[\left(h^* m_{LL} h^\dagger\right)_{11}\right]}{\left(hh^\dagger\right)_{11}} R\ . \tag{4}$$

Again the electoweak sphaleron interaction [6,14,15] partly converts the lepton asymmetry to baryon asymmetry. The baryon asymmetry of the universe Y_B^{SM} which is defined as the ratio of baryon number density (η_B) to photon number density (η_γ) in standard model (SM) case, is expressed in terms of washout factor κ_1 and ϵ_1 as [12]:

$$Y_B^{SM} = \frac{\eta_B}{\eta_\gamma} \simeq 0.0216\,\kappa_1\,\epsilon_1\,. \tag{5}$$

How much the produced asymmetry is washed out is described by Boltzmann equation and its solution can be parametrized by a parameter κ known as dilution factor [16] :

$$\kappa_1 \simeq \begin{cases} \dfrac{0.3}{K(\ln K)^{0.6}} & \text{if } 10 \le K \le 10^6, \\[2mm] \dfrac{1}{2\sqrt{K^2+9}} & \text{if } 0 \le K \le 10, \end{cases} \tag{6}$$

where $K = \tilde{m}/m^*$, with $\tilde{m} = (hh^\dagger)_{11} v^2/M$ is the effective neutrino mass and $m^* = \frac{16\pi^{\frac{5}{2}}}{3\sqrt{5}} g^* \frac{1}{2} \frac{v^2}{M_{pl}}$ is the equilibrum neutrino mass [17]. In standard model scenario $g^* = 106.75$ is the value of the massless degree of freedom and $M_{pl} = 1{\cdot}2 \times 10^{19}$GeV is the Planck's constant. Out of equilibrium decay of M_1 is characterised by $K < 1$ condition.

3. Numerical calculation and results

For numerical calculation of baryon asymmetry we choose a basis U_R where $M_{RR}^{diag} = U_R^T M_{RR} U_R = \text{diag}(M_1, M_2, M_3)$ with real and positive eigenvalues [18,19]. We transform $m_{LR} = \text{diag}(\lambda^m, \lambda^n, 1)v$ to the U_R basis by $m_{LR} \to m'_{LR} = m_{LR} U_R$. In this prime basis the Dirac neutrino Yukawa coupling becomes $h = (m_{LR} U_R)/v$.

As an example, we consider tribimaximal pattern of m_{LL} discussed in Section 1. The light neutrino mass matrix is given by (in eV)

$$m_{LL} = \begin{pmatrix} -0.0198595 & -0.0349297 & -0.0349297 \\ -0.0349297 & 0.025 & -0.00435837 \\ -0.0349297 & -0.00435837 & 0.025 \end{pmatrix}.$$

Taking Dirac neutrino mass matrix as down quark mass matrix (case 3) *i.e.*, $m'_{LR} = \text{diag}(\lambda^4, \lambda^2, 1)v$, with $\lambda = 0.3$ and $v = 174$GeV we have the correponding M_{RR} (in GeV) as

$$M_{RR} = \begin{pmatrix} -1.44 \times 10^{10} & -2.70 \times 10^{11} & -3.01 \times 10^{12} \\ -2.70 \times 10^{11} & 5.03 \times 10^{12} & -3.69 \times 10^{13} \\ -3.01 \times 10^{12} & -3.69 \times 10^{13} & 6.21 \times 10^{14} \end{pmatrix}.$$

The mass eigenvalues are $M_{RR}^{diag} = \text{diag}(9.76 \times 10^{10}, 2.89 \times 10^{12}, 6.23 \times 10^{14})$. For this structure, we found $(hh^\dagger)_{11} = 6.56 \times 10^{-5}, \text{Im}(h^* m_{LL} h^\dagger)_{11} = = 7.09 \times 10^{-16}, \tilde{m}_1 = 2.03 \times 10^{-2} eV, = 1{\cdot}08 \times 10^{-3} eV, K = 18.73$, and $\kappa_1 = 0.008$. The lepton asymmetry is found to be $\epsilon_1 = 2.08 \times 10^{-6}$. Following eq (3), we found $Y_B^{SM} = 3.78 \times 10^{-10}$ which is consistent with the experimental bound [20] : $Y_B^{CMB} = (6.1^{+0.3}_{-0.2}) 10^{-10}$. We follow the same procedure to estimate lepton and baryon asymmetry for other cases. For quasi-degenerate structure which appears in bimaximal case, we use eq.(4) to calculate lepton asymmetry. Such degeneracy is found to be lifted in tribimaximal case and the expression in eq.(3) is used. For all the cases under consideration, the heavy mass eigenvalues are collected in Table1 the effective mass parameters and dilution factors are presented in Table 2. Finally, the estimated lepton and baryon asymmetry are collected in Table3.

Table 1. The three right-handed Majorana neutrino masses in GeV for bimaximal case(BM) and tribimaximal case(TBM).

(m,n)	BM $\lvert M_i \rvert$	TBM $\lvert M_i \rvert$
(4,2)	6.2783×10^{11}, 6.2838×10^{1}, 5.38×10^{16}	9.67×10^{10}, 2.89×10^{12}, 6.23×10^{14}
(6,2)	5.6527×10^{10}, 5.6532×10^{10}, 5.38×10^{16}	8.10×10^{8}, 2.83×10^{12}, 6.23×10^{14}
(8,4)	4.5971×10^{8}, 4.5974×10^{8}, 5.34×10^{16}	6.56×10^{6}, 2.30×10^{10}, 6.21×10^{14}

Table 2. Contains the values of effective mass parameter $\tilde{m}_1$ in eV, decay parameter K and dilution factor κ_1 for bimaximal (BM) and tribimaximal (TBM) case.

	BM			TBM		
(m,n)	$\tilde{m}_1$	K	κ_1	$\tilde{m}_1$	K	κ_1
(4,2)	3.16×10^{-3}	2.19	0.2	2.03×10^{-2}	18.73	8.40×10^{-3}
(6,2)	2.85×10^{-4}	0.26	0.17	1.98×10^{-2}	18.37	8.66×10^{-3}
(8,4)	2.83×10^{-4}	0.26	0.17	1.98×10^{-2}	18.37	8.66×10^{-3}

Table 3. Calculation of lepton asymmetry ϵ_1 and baryon asymmetry Y_B for bimaximal case (BM) and tribimaximal case (TBM) for three choices of (m,n).

	BM		TBM	
(m,n)	ϵ_1	Y_B	ϵ_1	Y_B
(4,2)	1.64×10^{-2}	4.25×10^{-5}	2.08×10^{-6}	3.78×10^{-10}
(6,2)	1.47×10^{-2}	5.40×10^{-5}	1.55×10^{-9}	2.89×10^{-13}
(8,4)	1.62×10^{-4}	5.94×10^{-7}	1.28×10^{-11}	2.40×10^{-15}

4. Summary and discussion

We start with the bimaximal and tribimaximal mixing pattern of inverted hierarchical light neutrino mass matrix m_{LL}. Heavy right-handed Majorana neutrino mass matrices M_{RR} are constructed *via* inverse seesaw relation. In case of bimaximal mixing pattern of light neutrinos, we observe that the corresponding heavy right handed neutrino masses manifest quasi-degenerate structure *i.e.*, $M_1 \approx M_2 < M_3$. This peculiar structure of heavy masses enhance the produced asymmetry known as 'Resonance enhancement" by modifying the propagator. This scenario completely changes when we come to tribimaximal mixing (TBM) pattern, with hierarchical pattern of heavy neutrinos *i.e.*, $M_1 < M_2 < M_3$. This type of hierarchical structure will bypass the resonance enhancement effect and this is clearly seen in the produced asymmetry. For bimaximal case, the range of lepton asymmetry is found to be $10^{-4} < \epsilon_1 < 10^{-2}$ whereas in tribimaximal scenario, the range is $10^{-11} < \epsilon_1 < 10^{-6}$. Our estimated baryon asymmetry $Y_B = 3.78\times10^{-10}$ for tribimaximal mixing pattern with down quark mass matrix taken as Dirac neutrino mass matrix is consistent with the experimental value [20]. For hierarchical structure of heavy Majorana neutrino masses, the condition $M_1 > 4\times10^8$ GeV satifies the famous Davidson-Ibarra bound [21]. In the present calculation, we are able to establish the validity of tribimaximal mixing in inverted hierarchical mass model, and also to discriminate the three possible choices of Dirac neutrino mass matrix through the estimation of baryon asymmetry.

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