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Nonlinear electromagnetic propagation parameters of the ionosphere

Gyan Prakash, Ashutosh Sharma, Mahendra P Verma

DST Project and Ramanna Fellowship Programme,
Department of Education Building, Lucknow University, Lucknow - 226 007, Uttar Pradesh, India

and

Mahendra Singh Sodha*

Disha Academy of Research and Education, Disha Crown, Katchna Road, Shankar Nagar, Raipur - 492 007, Chattisgarh, India

E-mail : msodha@redifmail.com

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Abstract : For an appreciation of the nonlinear interaction of intense electromagnetic waves with the ionosphere, it is essential to have knowledge of the dependence of the nonlinear propagation parameters on the irradiance of the wave and the height of the ionosphere (at a given time). Using the contemporary theoretical models and the corresponding data base, the complex refractive index corresponding to the propagation of the two modes along the direction of earth's magnetic field, has been evaluated as a function of the irradiance of the wave, at different heights of the ionosphere (between 80 km and 200 km) on the basis of the mid-latitude ionospheric model of Gurevich. Both the cases *viz.* when the electron density is unaffected (pulse duration $< \tau_N$) and affected (pulse duration $> \tau_N$) by the wave have been considered; τ_N is the electron density relaxation time. The calculations correspond to the wave frequency $\omega = 10^7, 5\times10^7$ and $10^8 s^{-1}$, gyrofrequency of electrons $\omega_c = 4\times10^6 s^{-1}$ and day time ionosphere. Similar calculations can also be made for the night time ionosphere. It is seen that the refractive index and the absorption coefficient can in general, be expressed as third order polynomials of irradiance; the corresponding coefficients for different heights have been tabulated. To illustrate the importance of the results, the nonlinear wave equation corresponding to near horizontal propagation, has been solved in the geometrical optics approximation and numerical results have been presented for height of 80 km.

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1. Introduction

The study of nonlinear interaction of electromagnetic waves with the ionospheric plasma got initiated by the discovery of cross modulation in 1930. A phenomenological theory of

Corresponding Author

cross modulation was developed by Bailey and Martin [1]; the theory, based on Boltzmann's transfer equation was also subsequently published. Fejer [2] observed the interaction of short radio pulses in the ionosphere and proposed it as a technique for ionospheric diagnostics.

A host of nonlinear phenomena [3-5], associated with the interaction of high irradiance electromagnetic waves with the ionosphere, including demodulation, cross modulation, self action, mutual interaction [6], gyroresonance [7], generation of harmonics and combination frequencies [8-10] and phase perturbation [11] on reflection are caused by the enhancement of the electron temperature on account of Ohmic heating by the electric field of the waves. The enhancement of electron temperature leads to modification of the electron collision frequency and the electron density [12-15]; hence the propagation parameters (refractive index and absorption coefficient) also get modified. Other contemporary nonlinear phenomena include generation [16-18] of ELF/VLF waves by electrojet modulation, parametric coupling [19] of radio waves with ionospheric instabilities, self-focusing [3, 20-22] and instabilities [3, 23-28], associated with thermal self-focusing.

Since the sources of electromagnetic waves are mostly ground based, the ability to predict the path, irradiance and phase of a wave is a pre-requisite for communications and an understanding of the nonlinear phenomena in any region of the ionosphere and the integrated effect along the path. For prediction of the path, irradiance and phase of a wave in the ionosphere, a knowledge of the refractive index and absorption coefficient as a function of irradiance and height is necessary. Such a need constitutes the essential motivation for the present work. Another motivation is the relevance of this knowledge to HF over the horizon radar. This data is the basic input for the study of nonlinear processes. This communication presents a methodology for the evaluation of the propagation parameters, as functions of height and irradiance. Numerical results, corresponding to a specific ionospheric model [3] (with sufficient details) and propagation along the magnetic field have been given. With minor modifications, results corresponding to propagation at any angle to the magnetic field, can also be obtained. Mention should be made of the evaluation [29] of the effective collision frequency in the presence of a high power radio wave at four heights of the ionosphere, employing a relatively simple model.

Most of the formalisms for investigation of nonlinear phenomena in plasmas, presume thermal equilibrium which corresponds to identical temperatures of electrons, ions and neutral atoms/molecules. Such plasmas are indeed rare in nature; *e.g.* in ionospheric plasmas or laboratory plasmas, the temperatures of electrons, ions and neutral species are in general, different. Further, mostly the effect of increase in electron temperature is taken into account, only on the electron collision frequency. The effect of elevated electron temperature on ionization is usually ignored; in a few exceptions [3] (Section 3.2.3), *ad hoc* models have indeed been used for the purpose. The energy exchange between electrons and ions and between ions and neutral species, leading to a net heating of ions, has also been ignored in these studies. In view of the availability of a good and sufficient data base on the ionospheric parameters, collisions of the particles, ionization mechanisms and electron loss mechanisms, it is possible to systematically explore the dependence of the electromagnetic wave propagation parameters on the irradiance of the wave, at different heights of the ionosphere. The present paper is an exercise to this end. The novel features of the present work are:

(i) evaluation of the power received from the sun per electron at different heights from the data for undisturbed ionosphere [eq. (7b) of the present paper].

(ii) taking into account the precise dependence of electron collision frequency and of electron density on the electron temperature.

(iii) incorporation of (i) and (ii) in the evaluation of nonlinear propagation parameters.

(iv) using the nonlinear propagation parameters to solve the nonlinear wave equation in a specific case and

(v) an unusual technique for solution of the energy balance equation (getting an explicit relationship between the electron temperature and the irradiance).

For propagation along the magnetic field in plasma, the refractive index $n_{\mp}$ and absorption coefficient $k_{\mp}$ corresponding to the two modes $E_x - iE_y$ and $E_x + iE_y$ are given by Ginzburg [30] as

$$\epsilon_{r\mp} - i\,\epsilon_{i\mp} = \left(n_{\mp} - ik_{\mp}\right)^2 = 1 - \frac{\left(\omega_p^2 / \omega^2\right)\left(N_e / N_{eo}\right)}{1 - i\left(v_e / \omega\right) \pm \left(\omega_c \omega\right)}, \tag{1a}$$

where ω_p is the plasma frequency in the undisturbed state (no electromagnetic wave), ω is the wave frequency, v_e is the electron collision frequency, ω_c is the gyrofrequency of the electrons, N_e and N_{e0} are the electron densities in the presence and absence of the wave and ϵ_r and ϵ_i are the real and imaginary parts of the effective dielectric constant.

For convenience, the CGS system of units has been used.

The ohmic loss per unit volume *i.e.* power lost per unit volume by the electric field to the electrons, is given by [31]

$$\underline{J}\cdot\underline{E} = \frac{e^2 N_e}{4m} v_e \left\{ \frac{1}{v_e^2 + (\omega - \omega_c)^2} + \frac{1}{v_e^2 + (\omega + \omega_c)^2} \right\} \left(E_{xo}^2 + E_{yo}^2\right)$$

$$= \frac{e^2 N_e}{8m} v_e \left(A_- A_-^* + A_+ A_+^*\right) \left\{ \frac{1}{v_e^2 + (\omega - \omega_c)^2} + \frac{1}{v_e^2 + (\omega + \omega_c)^2} \right\}, \tag{1b}$$

where E_{xo} and E_{yo} are the components of the amplitude of the electric vector along the x and y axes and A_- and A_+ are complex amplitudes of the modes $E_x - iE_y$ and $E_x + iE_y$, respectively.

It is well known that the electrons in the field of a high irradiance electromagnetic wave get heated to the steady state temperature in duration of the order of $1/\delta v_e$, where δ is the fraction of excess energy exchanged in a collision $\left(\sim 10^{-3}\right)$. However as a result of increase in electron temperature, the electron density attains an enhanced steady state value in periods of the order of the electron density relaxation time τ_N. Hence for pulse durations less that $\tau_N, N_e / N_{e0} \approx 1$, while the steady state value of N_e / N_{e0} may be used for $t > \tau_N$. The value of τ_N as a function of ionospheric height have been tabulated (Table 13) by Gurevich [3]. In day time ionosphere, τ_N lies between 32 s to 52 s for the range of heights - 100 km to 200 km.

This investigation is limited to day times and heights, below 200 km for which the electron dissipation is predominantly on account of recombination, rather than transport; in this case an explicit expression for the steady state electron density as a function of electron temperature is available.

Thus in this communication, we have evaluated $n_\pm$ and $k_\pm$ (corresponding to propagation during day time, along the direction of the magnetic field in day time ionosphere) at different heights of the ionosphere (80 km to 200 km) as a function of the irradiance of the wave for the two cases:

(i) Pulse duration much less than τ_N; in this case, the electron density remains unaltered ($N_e / N_{e0} \approx 1$).

(ii) Pulse duration much greater than τ_N; in this case, the electron density is a known function of electron temperature see eq.(5).

The data corresponds to propagation along the magnetic field with wave frequency $\omega = 10^7, 5\times10^7, 10^8 s^{-1}$ and electron gyrofrequency $\omega_c = 4\times10^6 s^{-1}$. It is seen that at different heights of the ionosphere $n_\mp$ and $k_\mp$ can in general be expressed by a third order polynomial in irradiance. As an illustration of the importance of the present results, the self action of a high irradiance electromagnetic wave, propagating horizontally in the day time ionosphere at a height of 80 km, has been considered; the appropriate nonlinear wave equation has been solved in the geometrical optics approximation.

This analysis does not take into account filamentation instabilities, which occur for large power beams in the upper ionosphere and hence is applicable to the region, where the instabilities do not occur.

2. Available data base on ionosphere and related phenomena

2.1 Structure of ionosphere :

For the purpose of this investigation, a typical model for the structure (during day time) has been used. Tables 1 and 2 of the book by Gurevich [3] detail this model.

2.2. Electron collision frequency (ν_e) :

2.2.1 Electron-neutral species collisions (ν_{em}) :

It may be seen from Tables 6 and 7 of the book by Gurevich [3] that N_2, O_2 and O are the only neutral species, which make non-negligible contribution to ν_{em} between 80 km and 200 km of height; thus

$$\nu_{em} = \nu_{eN_2} + \nu_{eO_2} + \nu_{eO}, \tag{2}$$

where the suffix m refers to neutral species.

The electron collision frequency $\nu_{em}(T_e)$ and the average fraction $\delta(T_e)$ of energy, lost by an electron in collision with neutral species can be evaluated from the expressions and data, collated by Gurevich ([3], Section 2.5.2) and his model ([3], Tables 1 and 2) of the structure of the ionosphere.

2.2.2. Electron-ion collisions :

The values of collision frequency ν_{ei} of electrons with ions at different heights in the undisturbed ionosphere have been tabulated by Gurevich ([3] Section 2.5.2) The values ν_{ei}, corresponding to the ionosphere with electron temperature T_e (due to heating by the electric field of the wave) are given by

$$\nu_{ei} = \nu_{ei0}\left[N_e(T_e)/N_e(T_{e0})\right]\cdot\left[T_e/T_{e0}\right]^{-3/2} \tag{3}$$

where $N_e(T_e)$ is the electron (or ion) density corresponding to electron temperature T_e and T_{e0} is the electron temperature in the undisturbed ionosphere.

The fraction δ_{ei} of excess energy lost by an electron after collision (elastic) with an ion is $2m/M_i$ where M_i is the mass of the ion; hence the weighted mean value of δ_{ei} is

$$\delta_{ei} = \frac{2m}{M_H}\left\{\left(n_{NO^+}/30\right) + \left| n_{O_2^+}/32 \right| + \left(n_{O^+}/16\right)\right\}, \tag{4}$$

where $n_{NO^+}, n_{O_2^+}$. and n_{O^+} are the fractional concentrations of NO^+, O_2^+ and O^+ (tabulated by Gurevich [3] - Table 2).

Molecular/atomic weights of NO, O_2 and O are 30, 32 and 16,

and M_H is the mass of a hydrogen atom.

2.2.3. Ion-ion and electron - electron collisions :

Such collisions play ([3], Section 2.5.2) a negligible role in the ionospheric phenomena.

2.2.4. Ion - neutral species collisions :

The collision frequency, corresponding to ion – neutral species can be evaluated by using the data, given by Gurevich ([3], Section 2.5.2). In the relevant expressions for the collision frequency, the change in the ion temperature by the wave can in general be ignored.

2.3 Electron density :

For heights below 200 km in the ionosphere, the dominant mechanism of electron dissipation is the recombination with ions (not transport). Considering the essentials of chemical, ionization and recombination kinetics, the steady state electron density $\left(t > 10^3 s\right)$, as a function of electron temperature is given by ([3], eq. 2.248).

$$\frac{N_e(T_e)}{N_e(T_{e0})}=\frac{1}{2}\left\{n_{O^+}+\left[n_{O^+}^2+4n_{O_2^+}(T_e/T_{e0})^{0.7}+4n_{NO^+}(T_e/T_{e0})^{1.2}\right]^{1/2}\right\} \tag{5}$$

3. Steady state energy balance in ionosphere : relationship between electron temperature and wave irradiance

Using eq. (1b), the steady energy balance for the electrons in the ionosphere, in the presence a high irradiance electromagnetic wave (both modes present), may following Gurevich ([3], eq. 2.245), be expressed as

$$Q+\overline{J.E}=+\frac{N_e e^2}{8m}\nu_e\left\{\frac{1}{\nu_e^2+(\omega-\omega_c)^2}+\frac{1}{\nu_e^2+(\omega+\omega_c)^2}\right\}\left(A_-A_-^*+A_+A_+^*\right),$$

$$=N_e\sum\nu_{em}\delta_{em}\left(\frac{3}{2}kT_e-\frac{3}{2}kT\right)+N_e\nu_{ei}\bar{\delta}_{ei}\left(\frac{3}{2}kT_e-\frac{3}{2}kT_i\right), \tag{6a}$$

where Q is the net power gained by the electrons from the solar radiation on account of the excess energy of electrons, produced by photoionization, above the thermal energy of electrons (in the undisturbed ionosphere *i.e.* without the electromagnetic wave), $\underline{J}$ is the current density,

$\overline{J.E}$ is the time averaged Ohmic loss to electrons per unit volume (eq. (1b)),

k is Boltzmann's constant,

other symbols have been defined earlier.

The Ohmic heating terms in the above equation has been modified to take account of the magnetic field and the dependence of ν_e and δ_e on T_e, (different for different species).

On account of slow reaction rates, for periods much less than τ_N, the electron density remains practically unaltered [3] ($N_e \approx N_{eo}$) by the application of the field of the wave and consequent increase in the temperature of electrons. However for period exceeding τ_N, the electron density acquires a steady state value [3] (given by Eq. (5)), which is higher than that in the absence of the wave.

In Eq.(6a), the right hand side represents the power lost by electrons in collisions per unit volume, while the left hand side, corresponds to power gained from the sun and the field of the wave (Ohmic heating).

Similarly, the energy balance for the ions requires ([3], eq. 2.245)

$$N_e\nu_{ei}\bar{\delta}_{ei}\left(\frac{3}{2}kT_e-\frac{3}{2}kT_i\right)=N_e\bar{\nu}_{im}\delta_{im}\left(\frac{3}{2}kT_i-\frac{3}{2}kT\right)$$

$$\text{or }\frac{T_e-T_i}{\bar{\nu}_{im}\delta_{im}}=\frac{T_i-T}{\nu_{ei}\bar{\delta}_{ei}}=\frac{T_e-T}{\bar{\nu}_{im}\delta_{im}+\nu_{ei}\bar{\delta}_{ei}}. \tag{6b}$$

Substituting for (T_e-T_i) from eq. (6b) in eq. (6a), and dividing by an arbitrary normalizing parameter $(3/2)kT_0$ (where T_0 can be conveniently chosen as 500K) one obtains

$$\frac{2Q}{3kT_o}+\frac{e^2N_e}{12mkT_o}\nu_e\left\{\frac{\left(A_-A_-^*+A_+A_+^*\right)}{\nu_e^2+(\omega-\omega_c)^2}+\frac{\left(A_-A_-^*+A_+A_+^*\right)}{\nu_e^2+(\omega+\omega_c)^2}\right\}$$

$$=N_e\left(\frac{T_e-T}{T_o}\right)\left\{\sum\nu_{em}\delta_{em}+\frac{\nu_{ei}\delta_{ei}.\overline{\nu_{im}\delta_{im}}}{\nu_{ei}\delta_{ei}+\overline{\nu_{im}\delta_{im}}}\right\}, \tag{7a}$$

For undisturbed ionosphere, $EE^*=0$ and $T_e=T_{e0}$.

$$\frac{2Q}{3kT_0}=N_e(T_{e0}).\frac{(T_{e0}-T)}{T_0}\left\{\sum\nu_{em0}\delta_{em0}+\frac{\nu_{ei0}\delta_{ei0}\,\overline{\nu_{im}\delta_{im}}}{\nu_{ei0}\delta_{ei0}+\overline{\nu_{im}\delta_{im}}}\right\}, \tag{7b}$$

where the suffix 0 refers to the case $T_e=T_{e0}$.

For a chosen height, the values of all parameters (as functions of the electron temperature) on the R.H.S. of eq.(7b) are available in literature (see Section 2); this enables the evaluation of Q. As a typical example, $Q=4\times10^{-8}$ergs/cm^3.s at a height of 110 km in day time ionosphere. The dependence of $N_e(T_e)/N_e(T_{e0})$ and all ν's and δ's on electron temperature is available in literature (see Section 2); this makes the evaluation of T_e/T_{e0} as a function of irradiance very difficult. However the nonlinear parameter

$$\Lambda=\left(2000e^2/12m\omega_0^2kT\right)\left(A_-A_-^*+A_+A_+^*\right) \tag{8}$$

can be evaluated for various values of T_e with a knowledge of $2Q/3kT_0$ from Eq.(7b) by using the data base of Gurevich [3]. The factor 2000 (~M/m) has been introduced for numerical convenience.

4. Numerical results and discussion

The dependence of the propagation parameters $n_\mp$ and $k_\mp$ on the dimensionless irradiance has been numerically studied for ionospheric heights, between 80 km and 200 km for the cases when the electron density is influenced $(t>\tau_N)$ and uninfluenced $(t<\tau_N)$ by the wave. Calculations have been made, corresponding to the day as well as night time ionospheric structure (Table 1 of Gurevich [3]) and the values for the various parameters, given by Gurevich [3]. However in this paper, the results have been presented only for the daytime. It may be noted that the analytical expression for the electron density N_e in terms of electron temperature T_e *viz.* Eq.(5) is valid only for heights below 200 km, where the electron dissipation is mainly governed by generation - recombination mechanisms. At larger heights, the electron density is mainly governed by the transport processes and no closed form expression for N_e in terms of T_e is available.

For chosen heights (between 80km and 200km), the parameters $\nu_{em},\delta_{em},\nu_{ei},\delta_{ei},\overline{\nu_{im}},\delta_{im},\nu_e,N_e(T_e),n_\mp$, $k_\mp$ and Λ have been evaluated for different values of T_e/T_{e0} using the relations given before; both the cases $N_e/N_{e0}\approx1(t<\tau_N)$ and N_e/N_{e0}, given by eq.(5) $(t>\tau_N)$ have been taken into account. From this data base, $n_\mp$, $k_\mp$ at any height can be determined as a function of dimensionless irradiance Λ.

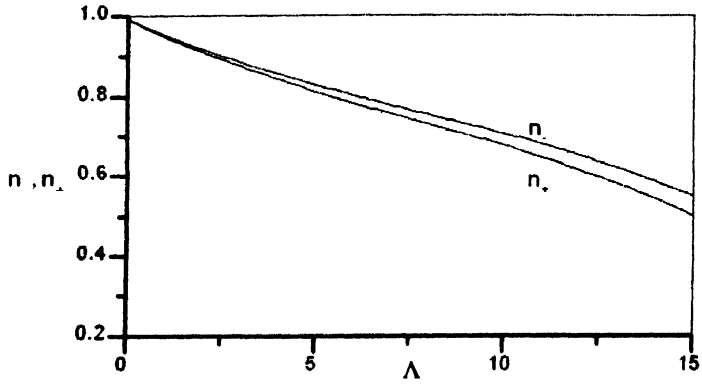

Figure 1. Day time dependence of refractive index n_+ and n_- of the extraordinary and ordinary modes on dimensionless irradiance Λ at a height of 80 km (day time) for $\omega = 5\times10^7 s^{-1}$ and steady state $(t > \tau_N)$. On account of the low value of ω_p/ω, the difference in the values of n_+ and n_- is too small, to reflect in the figure.

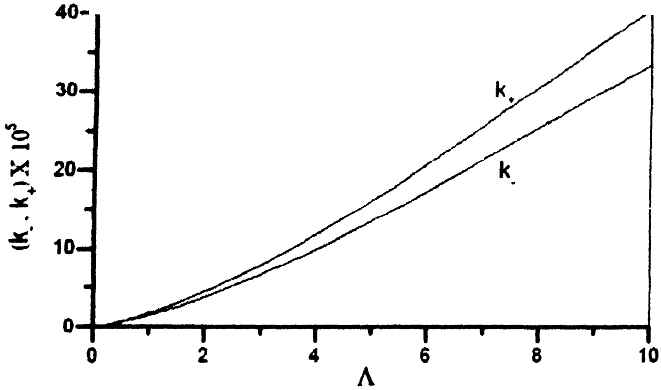

Figure 2. Day time dependence of absorption coefficient k_+ and k_- . of the extraordinary and ordinary modes on dimensionless irradiance Λ at a height of 80 km (day time) for $\omega = 5\times10^7 s^{-1}$ and steady state $(t > \tau_N)$. The values of k_+ and k_- at low values of irradiance are very close on account of a low value of ω_p/ω.

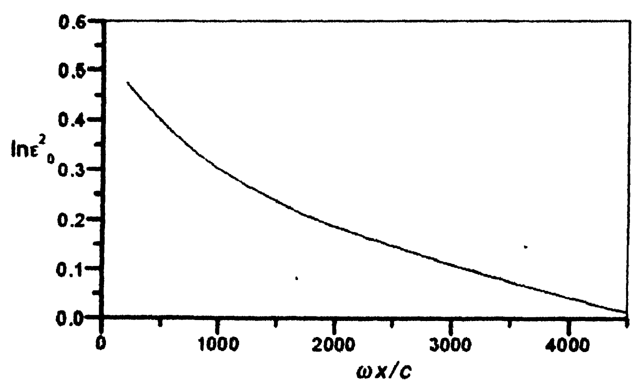

Figure 3. Day time dependence of $\ln\varepsilon_0^2$ on $\omega x/c$ for horizontal propagation of the extraordinary mode $(E_x + iE_y)$ of frequency $\omega = 5\times10^8 s^{-1}$ along the magnetic field at a height of 80 km, the departure from the straight line indicates self action.

It is seen that at all heights, $n_{\mp}$, $k_{\mp}$ can be expressed as a third order polynomial in the dimensionless irradiance Λ; thus

$$n_{-} = n_{-0}(z) + n_{-1}(z)\Lambda + n_{-2}(z)\Lambda^2 + n_{-3}(z)\Lambda^3, \quad (9a)$$

$$n_{+} = n_{+0}(z) + n_{+1}(z)\Lambda + n_{+2}(z)\Lambda^2 + n_{+3}(z)\Lambda^3, \quad (9b)$$

$$k_{-} = k_{-0}(z) + k_{-1}(z)\Lambda + k_{-2}(z)\Lambda^2 + k_{-3}(z)\Lambda^3, \quad (9c)$$

and

$$k_{+} = k_{+0}(z) + k_{+1}(z)\Lambda + k_{+2}(z)\Lambda^2 + k_{+3}(z)\Lambda^3, \quad (9d)$$

Tables 1 to 8 display the z dependence of the coefficients, occurring in eqs. (9a) to (9d); sample expressions for the dependence of the coefficients on z have not been found. The coefficients in a few cases (where the polynomial fit is not good) have not been given in the tables.

Table 1. Coefficients of polynomial expansion of n_{-} in Λ (steady state $t > \tau_N$).

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	n_{-0}	$n_{-1}.10^6$	$n_{-2}.10^5$	$n_{-3}.10^6$	n_{-0}	$n_{-1}.10^4$	$n_{2}.10^4$	$n_{3}.10^5$	n_{-0}	$n_{-1}.10^4$	$n_{-2}.10^4$	$n_{-3}.10^4$
80	1.00	-4.70	-1.65	3.40	1.00	-1.20	-3.87	7.92	1.00	-5.73	-14	2.92
90	1.00	-95.7	-5.19	9.82	1.00	-23	-12.9	24.4	0.996	-90.0	-52.6	9.91
100	1.00	-1040	-16.2	29.1	0.990	-254	47.7	77.0	0.953	0638	-481	37.1
110	1.00	-1530	7.65	-1.94	0.993	-422	23.5	-10.1	0.968	-1141.2	-503.1	67.3
120	1.00	-1090	6.40	-2.26	0.981	-275.1	-14.4	-5.26	0.915	-653.6	-239.1	21.5
130	0.998	-484.1	-1.3	0.974	0.976	-242	10.4	-3.084	0.941	-1421.9	15.6	2.78
150	0.975	0.000	0.000	0.000	0.927	-215	3.60	-0.217	0.512	-581.5	15	-0.102
200	0.996	-15.3	0.002	0.000	0.896	-4.12	0.006	0.000	0.464	-40.3	0.08	0.000

It is useful to once again dwell on the application of these results [eqs. (9a) to (9d) and Tables 1 to 8] to the nonlinear propagation of electromagnetic waves in the ionosphere. An interesting observation is the fact that $n_{\mp}$ and $k_{\mp}$ depend on the sum of irradiance corresponding to both the ordinary and extraordinary modes; thus the two modes are nonlinearly coupled leading to mutual interaction and nonlinear Faraday effect, in addition to the self action effects.

Table 2. Coefficients of polynomial expansion of n_{+} in Λ (steady state $t > \tau_N$).

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	n_{+0}	$n_{+1}.10^5$	$n_{+2}.10^5$	$n_{+3}.10^5$	n_{+0}	$n_{-1}.10^4$	$n_{+2}.10^4$	$n_{+3}.10^5$	n_{+0}	$n_{+1}.10^4$	$n_{+2}.10^4$	$n_{+3}.10^4$
80	1	-0.497	-1.65	0.338	0.999	-1.29	-4.19	8.59	1	-6.81	-16.3	3.41
90	1	-9.66	-5.29	0.998	0.998	-25	-14	0.264	0.995	-105.4	-62.2	12
100	1	-10.6	-16.5	2.95	0.989	-274.7	-52.4	83.8	0.926	628.6	-	261.9
110	1	-156	7.77	-0.197	0.992	-458	25.0	-10.9	0.963	-819	50	-37.5
120	1	-111	6.50	-0.229	0.979	-299.0	15.4	-5.67	0.947	1790.4	-28.5	11.7
130	0.997	-49.2	-1.32	0.098	0.974	-263.0	11.1	-3.34	0.950	-0.2375	161.4	-3.32
150	0.974	0.000	0.000	0.000	0.921	-238.8	3.95	-0.244	0.434	-509	13.2	-0.090
200	0.995	-1.55	0.002	0.000	0.887	-4.52	0.076	-0.000	0.296	-28.6	0.058	0.000

Table 3. Coefficients of polynomial expansion of k_- in Λ (steady state $t > \tau_N$)

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	k_{-0}	$k_{-1}.10^8$	$k_{-2}.10^9$	$k_{-3}.10^{10}$	$k_{-0}10^7$	$k_{-1}.10^7$	$k_{-2}.10^7$	$k_{-3}.10^6$	$k_{-0}.10^5$	$k_{-1}.10^5$	$k_{-2}.10^4$	$k_{-3}.10^5$
80	4.06	-12.2	319.2	-466.0	4.85	-140.5	374	-5.49	0.287	-9.35	2.71	-4.08
90	3.85	-0.306	200.0	-230.4	4.24	-262.5	237	-2.68	0.316	-5.55	1.86	-2.07
100	16.8	11.9	313	-307	22.2	109	415	-3.59	-1790	10700	-864	168
110	-4.66	9.35	27.5	-17.8	-26.0	124.7	35.9	-0.152	873.3	-19667	798	-547
120	4.97	4.12	4.11	-1.96	4.59	50.2	6.45	-0.224	4069	-8064	214.4	-0.698
130	24.4	1.09	1.34	-0.474	4.51	19.9	1.30	-0.002	4535	-9444	245	-0.985
150	*324*	*0.000*	*0.000*	*0.000*	*4.634*	*12.6*	*-0.128*	*0.000*	*-1499*	*9446*	*19.3*	*1.20*
200	4.48	0.004	0.000	0.000	5.55	486	0.000	0.000	-395	210	-0.029	0.000

Table 4. Coefficients of polynomial expansion of k_+ in Λ (steady state $t > \tau_N$).

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	$k_{+0}10^9$	$k_{+1}.10^8$	$k_{+2}.10^8$	$k_{+3}.10^9$	$k_{+0}10^7$	$k_{+1}.10^6$	$k_{+2}.10^6$	$k_{+3}.10^6$	$k_{+0}.10^7$	$k_{+1}.10^4$	$k_{+2}.10^4$	$k_{+3}.10^5$
80	4.68	-12.7	33	-48.2	5.88	-16.9	44.3	-6.57	3.99	-1.29	3.74	-5.67
90	4.28	-0.368	20.6	-23.8	4.82	-0.032	27.5	-3.08	44.6	-0.085	2.58	-2.84
100	17.3	12.3	32.3	-31.6	25.8	12.9	48.8	-4.09	97300	-78.9	-205	1052
110	-4.92	9.67	2.84	-1.84	-29.3	14.3	4.41	-0.183	-46.2	5.346	865	-65400
120	4.49	4.30	0.414	-0.196	5.02	5.98	0.747	-0.023	512200	-137	485.2	-272
130	25.2	1.124	0.139	-0.049	5.28	2.34	0.158	-0.002	74600	-615	258.7	-119
150	335	0.000	0.000	0.000	1.762	1.67	-0.022	0.000	914400	1035.8	-21.8	1.39
200	3.30	0.004	0.000	0.000	6.58	0.005	0.000	0.000	0.062	0.004	-0.067	0.000

Table 5. Coefficients of polynomial expansion of n_+ in Λ (electron density, unchanged by the wave, $t > \tau_N$).

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	n_{-0}	n_{-1}	n_{-2}	n_{-3}	n_{-0}	n_{-1}	n_{-2}	n_{-3}	n_{-0}	n_{-1}	n_{-2}	n_{-3}
80	1	0.000	0.000	0.000	0.999	0.000	0.000	0.000	0.999	0.000	0.000	0.000
90	1	0.000	0.000	0.000	0.998	0.000	0.000	0.000	0.994	0.000	0.000	0.000
100	1	0.000	0.000	0.000	0.986	0.000	0.000	0.000	0.943	0.000	0.000	0.000
110	1	0.000	0.000	0.000	0.979	0.000	0.000	0.000	0.913	0.000	0.000	0.000
120	1	0.000	0.000	0.000	0.978	0.000	0.000	0.000	0.905	0.000	0.000	0.000
130	1	0.000	0.000	0.000	0.974	0.000	0.000	0.000	0.784	0.000	0.000	0.000
150	0.998	0.000	0.000	0.000	0.949	0.000	0.000	0.000	0.765	0.000	0.000	0.000
200	0.996	0.000	0.000	0.000	0.913	0.000	0.000	0.000	0.555	0.000	0.00	0.000

Table 6. Coefficients of polynomial expansion of n_+ in Λ (electron density, unchanged by the wave, $t < \tau_N$).

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	n_{+0}	n_{+1}	n_{+2}	n_{+3}	n_{+0}	n_{+1}	n_{+2}	n_{+3}	n_{+0}	n_{+1}	n_{+2}	n_{+3}
80	1	0.000	0.000	0.000	0.999	0.000	0.000	0.000	0.999	0.000	0.000	0.000
90	1	0.000	0.000	0.000	0.998	0.000	0.000	0.000	0.994	0.000	0.000	0.000
100	1	0.000	0.000	0.000	0.987	0.000	0.000	0.000	0.943	0.000	0.000	0.000
110	1	0.000	0.000	0.000	0.981	0.000	0.000	0.000	0.913	0.000	0.000	0.000
120	1	0.000	0.000	0.000	0.979	0.000	0.000	0.000	0.905	0.000	0.000	0.000
130	1	0.000	0.000	0.000	0.976	0.000	0.000	0.000	0.784	0.000	0.000	0.000
150	0.998	0.000	0.000	0.000	0.953	0.000	0.000	0.000	0.765	0.000	0.000	0.000
200	0.996	0.000	0.000	0.000	0.920	0.000	0.000	0.000	0.555	0.000	0.00	0.000

Table 7. Coefficients of polynomial expansion of k_- in Λ (Electron density, unchanged by the wave $t < \tau_N$).

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	$k_{-0}10^8$	$k_{-1}.10^8$	$k_{-2}.10^8$	$k_{-3}.10^9$	$k_{-0}\,10^6$	$k_{-1}.10^6$	$k_{-2}.10^6$	$k_{-3}.10^7$	$k_{-0}.10^5$	$k_{-1}.10^5$	$k_{-2}.10^7$	$k_{-3}.10^7$
80	1.22	1.49	1.75	-4.0	1.46	1.75	2.043	-4.77	1.076	1.45	138	-33.1
90	1.45	2.93	0.631	-1.64	1.7	3.48	0.736	-1.94	1.27	2.6	58.4	-15.5
100	4.52	9.34	-0.830	-0.852	5.2	10.8	-0.101	-0.989	4	8.4	-5.1	-8.33
110	1.12	3.18	-0.337	0.172	1.75	3.54	-0.34	0.162	1.37	2.84	-28.6	1.43
120	1	1.1	-0.083	0.031	1.23	1.36	-0.11	0.041	0.972	1.095	-8.78	0.343
130	0.531	0.4	-0.024	0.0071	0.7	0.47	-0.028	0.008	0.558	0.38	-2.35	0.069
150	0.603	0.092	-0.001	0.000	0.82	0.104	-0.002	0.000	0.721	0.093	-0.181	0.001
200	0.317	0	0	0	0.414	0.001	0	0	0.44	0.001	0	0

Table 8. Coefficients of polynomial expansion of k_+ in Λ (Electron density, unchanged by the wave $t < \tau_N$).

ω, s^{-1} / z, km	10^8				5.10^7				10^7			
	$k_{+0}10^8$	$k_{+1}.10^8$	$k_{+2}.10^8$	$k_{+3}.10^9$	$k_{+0}\,10^6$	$k_{+1}.10^6$	$k_{+2}.10^7$	$k_{+3}.10^7$	$k_{+0}.10^5$	$k_{+1}.10^5$	$k_{+2}.10^6$	$k_{-3}.10^7$
80	1.3	1.45	1.85	-4.3	1.71	2.054	24	-5.6	1.48	1.97	19.2	-4.65
90	1.53	2.99	0.671	-1.72	1.99	4.09	8.64	-2.27	1.75	3.6	8	-2.15
100	4.53	9.35	-0.083	-0.853	6.11	12.7	-1.19	-1.17	5.58	11.7	-0.623	-1.18
110	1.14	3.28	-0.346	0.176	2.06	4.16	-4.01	0.19	1.92	3.95	-3.92	0.192
120	1.02	1.2	-0.099	0.0392	1.44	1.6	-1.27	0.05	1.36	1.53	-1.23	0.048
130	0.672	0.365	-0.021	0.005	0.822	0.55	-33.6	0.009	0.784	0.533	-0.33	0.009
150	0.557	0.099	-0.019	0	0.967	0.123	-0.023	0	1.045	0.135	-0.0263	0
200	0.242	0.003	0	0	0.49	0.001	0	0	0.7	0.002	0	0

In what follows, the nonlinear wave equation, corresponding to near horizontal propagation of a single mode has been solved in the geometrical optics approximation. The propagation phenomena are governed by the wave equation

$$\nabla^2 A_{\pm} + \frac{\omega^2}{c^2}(n_{\pm} - ik_{\pm})^2 A_{\pm} = 0, \tag{10}$$

where $A_{\pm}$ denotes the amplitude of the wave $E_x \pm iE_y$ and $n_{\mp}$ and $k_{\mp}$ are functions of z and Λ. The solution of the above equation, with the dependence [eqs.(9a) to (9d) and Tables 1 to 8] of $n_{\mp}$ and $k_{\mp}$ on z and Λ describes the self action of the wave.

For near horizontal propagation of either mode, the z dependence of $n_{\mp}$ and $k_{\mp}$ can be ignored; the wave equation then reduces to the following in the geometrical optics approximation :

$$\frac{d\varepsilon_0}{dx} + \frac{\omega}{c} k_0(\varepsilon_0).\varepsilon_0 = 0,$$

where $\varepsilon_0 = \sqrt{\Lambda}$ is the dimensionless amplitude and $k_0 = k_{\pm}$ is the corresponding value of the absorption coefficients. Since k_0 can be expressed as a third order polynomial in $\Lambda = \varepsilon_0^2$ *i.e.*

$$k_0 = a_0 + a_1\varepsilon_0^2 + a_2\varepsilon_0^4 + a_3\varepsilon_0^6,$$

one gets

$$\int_{\varepsilon_{00}^2} \frac{d\varepsilon_0^2}{\varepsilon_0^2\left(a_0 + a_1\varepsilon_0^2 + a_2\varepsilon_0^4 + a_3\varepsilon_0^6\right)} = -\int_0 2\frac{\omega}{c}dx = -2\frac{\omega}{c}x,$$

where $\varepsilon_0 = \varepsilon_{00}$ at $x = 0$.

Substituting the values of a_0, a_1, a_2 and a_3 for k_+, corresponding to the mode $E_x + iE_y, (t > \tau_N)$ and $\omega = 10^8 s^{-1}$ at a height z=80km (day time), the integral has been evaluated numerically. The initial condition is $\varepsilon_0^2 = 3.5$ at x=0. Figure 3 illustrates the dependence of ln ε_0 on $\omega x / c$ for the steady state $(t > \tau_N)$; the departure of the curve from a straight line is a manifestation of the nonlinearity.

5. Conclusion

From a knowledge of the ionization, recombination and collision kinetics and energy balance in the ionosphere, the refractive index and absorption coefficient corresponding to both modes of the electromagnetic wave, propagating in the ionosphere along the magnetic field have been evaluated as a function of the irradiance of the wave. It is seen that these propagation parameters at a given height can be expressed as third order polynomials in dimensionless irradiance. The cases when the electron density is unaffected by the wave $(t < \tau_N)$ and is affected $(t > \tau_N)$ have been distinguished.

As an illustration of the importance of these results, the nonlinear wave equation for the near horizontal propagation of an electromagnetic wave along the magnetic field has been solved and numerical results, presented for a specific case; the results indicate significant self action.

For the prediction and interpretation of nonlinear processes, experiments in specific regions, data specific to the region of interaction time of the day has to be obtained, which can be done by using the protocol, developed herein. However, the present data can certainly be used for an understanding of the nonlinear interaction phenomena.

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