



THE UNIVERSITY *of* EDINBURGH

Edinburgh Research Explorer

Volcanic eruption forecasts from accelerating rates of drumbeat long-period earthquakes

Citation for published version:

Bell, AF, Naylor, M, Hernandez, S, Main, IG, Gaunt, HE, Mothes, P & Ruiz, M 2018, 'Volcanic eruption forecasts from accelerating rates of drumbeat long-period earthquakes' *Geophysical Research Letters*, vol 45, no. 3, pp. 1339-1348. DOI: 10.1002/2017GL076429

Digital Object Identifier (DOI):

[10.1002/2017GL076429](https://doi.org/10.1002/2017GL076429)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Peer reviewed version

Published In:

Geophysical Research Letters

General rights

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact openaccess@ed.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.



1 **Volcanic eruption forecasts from accelerating rates of drumbeat long-**
2 **period earthquakes**

3 **Andrew F. Bell¹, Mark Naylor¹, Stephen Hernandez², Ian G. Main¹, H. Elizabeth**
4 **Gaunt², Patricia Mothes², and Mario Ruiz²**

5 ¹School of GeoSciences, University of Edinburgh, Edinburgh, U.K.

6 ²Instituto Geofísico, Escuela Politécnica Nacional, Quito, Ecuador

7

8 Corresponding author: Andrew Bell (a.bell@ed.ac.uk)

9

10 **Key Points:**

- 11 • Power-law acceleration in rates of long period earthquakes observed before large
12 explosive eruption at Tungurahua volcano, Ecuador
- 13 • Earthquake source characteristics indicate repeated quasi-periodic activation of single
14 source driven by accelerated loading
- 15 • New Bayesian gamma point process methodology applied to analyze dataset and
16 provide probabilistic eruption forecasts
17

18 **Abstract**

19 Accelerating rates of quasi-periodic ‘drumbeat’ long period earthquakes (LPs) are commonly
 20 reported before eruptions at andesite and dacite volcanoes, and promise insights into the
 21 nature of fundamental pre-eruptive processes and improved eruption forecasts. Here we apply
 22 a new Bayesian MCMC gamma point process methodology to investigate an exceptionally
 23 well-developed sequence of drumbeat LPs preceding a recent large vulcanian explosion at
 24 Tungurahua volcano, Ecuador. For more than 24 hours, LP rates increased according to the
 25 inverse power-law trend predicted by material failure theory, and with a retrospectively
 26 forecast failure time that agrees with the eruption onset within error. LPs resulted from
 27 repeated activation of a single characteristic source driven by accelerating loading, rather
 28 than a distributed failure process, showing that similar precursory trends can emerge from
 29 quite different underlying physics. Nevertheless such sequences have clear potential for
 30 improving forecasts of eruptions at Tungurahua and analogous volcanoes.

31 **1. Introduction**

32 Accelerating rates of geophysical signals, such as seismicity (Kilburn & Voight, 1998; De
 33 La Cruz-Reyna & Reyes-Dávila, 2001; Neuberg et al., 2000; Ramos et al., 1999; Salvage &
 34 Neuberg, 2016; Voight, 1988; Voight & Cornelius, 1991) or ground deformation (McGuire &
 35 Kilburn, 1997), have been reported before a wide range of eruption styles. Such sequences
 36 evolve over timescales of minutes (Linde et al., 1993) to years (Robertson & Kilburn, 2016),
 37 and provide an opportunity for both improved understanding of the physical processes that
 38 control the approach to eruption, and more reliable, quantitative, eruption forecasts (Bell et
 39 al., 2013; Boué et al., 2015, 2016).

40 Here we apply a new Bayesian gamma point process model to analyse LP earthquakes
 41 preceding the July 2013 eruption at Tungurahua. We find rates increase over 24 hours
 42 according to Eq. (2), but with quasi-periodic inter-event times. Earthquake amplitudes also
 43 increase towards the eruption, despite the decreasing inter-event times. ‘Pseudo-prospective’
 44 forecasts illustrate the predictability of the process, including the effect of catalogue
 45 completeness close to eruption. First we summarize the earthquake data and statistical
 46 methods used. We then apply the new model in retrospective and simulated forecasting
 47 modes to evaluate model fit and parameter values, and determine likely forecasting
 48 performance. We then discuss the implications of our findings for understanding of volcanic
 49 processes, LP source mechanisms, and eruption forecasting.

50

51 **1.1. Material failure and volcanic earthquakes**

52 Similarities between accelerating pre-eruptive trends and those associated with material
 53 failure phenomena (Main, 1999; Vasseur et al., 2017) mean that they are often analysed
 54 within this conceptual framework (Kilburn, 2003; Main, 1999; Voight, 1988). Failure of all
 55 or part of the volcanic system (in response to elevated magma and gas pressure) is associated
 56 with a fundamental empirical relation between the acceleration in a geophysical precursor Ω
 57 (such as strain or number of earthquakes) and its rate:

$$58 \quad \frac{d^2\Omega}{dt^2} = K \left(\frac{d\Omega}{dt} \right)^\alpha \quad (1)$$

59 where α and K are constants. In the common case that $\alpha > 1$, solutions to Equation (1)
 60 take the form of an inverse power-law increase in the mean rate of precursory signals with
 61 time (Kilburn, 2003):

$$\frac{d\Omega}{dt} = k(t_f - t)^{-p} \quad (2)$$

62 where the power-law exponent, $p = 1/(\alpha - 1)$ describes the non-linearity of the
 63 acceleration, and k reflects the absolute amplitude (Bell & Kilburn, 2013). Equation (2)
 64 involves a singularity at a finite time, t_f , corresponding to an infinite precursor rate,
 65 realization of a system-wide fracture and the percolation threshold, and often equated to the
 66 initiation of the eruption process (Voight, 1988).
 67

68
 69 The material failure paradigm has most commonly been considered in the context of high
 70 frequency (5-15 Hz) volcano-tectonic earthquakes (VTs) (Bell & Kilburn, 2012; Kilburn &
 71 Voight, 1998). VTs result from brittle stick-slip and fracture events within the edifice, and are
 72 prevalent at volcanoes re-awakening after long repose intervals (Kilburn & Voight, 1998), or
 73 at systems strongly influenced by edifice deformation (Bell & Kilburn, 2012; Collombet,
 74 2003) or tectonic processes (Sigmundsson et al., 2014). The temporal occurrence of VTs is
 75 generally consistent with an inhomogeneous Poisson or clustered point process (Bell et al.,
 76 2011). Their magnitudes follow a Gutenberg-Richter distribution (Roberts et al., 2015), and
 77 sources are commonly distributed across many locations in the deforming system. As such,
 78 these characteristics share many fundamental similarities with generic failure phenomena
 79 (Kilburn, 2012; Main, 1999; Vasseur et al., 2017)

80 VTs are less common before eruptions at open-system andesitic and dacitic volcanoes,
 81 providing limited forecasting information. Instead, low frequency (1-5 Hz), long period
 82 earthquakes ('LPs') dominate seismicity before explosive or effusive events (McNutt, 2005).
 83 Their waveform properties involve emergent onsets and extended (often harmonic) coda, and
 84 require a strong resonance or scattering effect (Chouet & Matoza, 2013). LPs are potentially
 85 excited by a diverse range of source mechanisms, including hydrothermal fluid movement
 86 (Lipovsky & Dunham, 2015), brittle magma failure (De Angelis & Henton, 2011; Lavallée et
 87 al., 2008; Neuberg et al., 2006; Tuffen et al., 2008), incremental plug ascent (Iverson et al.,
 88 2006; Johnson et al., 2008), slow rupture of a poorly consolidated shallow edifice (Bean et
 89 al., 2013), or gas depressurization (Gil-Cruz & Chouet, 1997). For LPs at Tungurahua,
 90 suggested source mechanisms include gas depressurization (Molina, 2004), magma stick-slip
 91 or failure (Neuberg et al., 2018), and coupled magma ascent and gas depressurization (Bell et
 92 al., 2017). The statistical properties of LPs often include a restricted range or 'characteristic'
 93 distribution of magnitudes (Bell et al., 2017), repeating waveforms indicating multiple
 94 reactivation of a small number of source locations (Green & Neuberg, 2006), and periodic
 95 (anti-clustered) inter-event times, sometimes referred to as 'drumbeat' earthquakes (Bell et
 96 al., 2017; Iverson et al., 2006; White et al., 1998). Although accelerating rates of LPs have
 97 been reported before eruptions at several volcanoes (Boué et al., 2015; Neuberg et al., 2000;
 98 Salvage & Neuberg, 2016), it is not clear if these characteristics are consistent with the
 99 physics of a material failure process, and how such data and their patterns may be best used
 100 for forecasting.

101 **1.2. Tungurahua volcano and precursors to the 14 July 2013 explosion**

102 Tungurahua is an active andesitic stratovolcano in the Eastern Cordillera of the
 103 Ecuadorian Andes (Arellano et al., 2008), monitored by the network of the Instituto
 104 Geofísico of the Escuela Politécnica Nacional (IGEPN). The ongoing eruption began in 1999,
 105 involving episodes of vulcanian and strombolian activity, some with large paroxysmal
 106 explosions, interspersed by periods of quiescence of a few months (Hidalgo et al., 2015).
 107 Seismicity is dominated by low frequency signals, with few VT or hybrid earthquakes.
 108 Eruptive episodes are commonly preceded by a few hours or days of elevated rates of LPs. At

109 11:47 UTC on 14 July 2013, Tungurahua experienced the largest paroxysmal explosion of
110 the current eruption, with the highest amplitude acoustic energy recorded at Tungurahua,
111 accompanied by a large gas plume, and sending ash to a height of 8.3 km above the vent
112 (Hall et al., 2015). The eruptive products primarily consisted of very low permeability ‘plug’
113 material, with relatively little juvenile pumaceous content (Hall et al., 2015).

114 **2. Data and methods**

115 **2.1. Monitoring data**

116 Seismic data associated with the eruption were recorded by the monitoring network of the
117 IGEPN. The seismicity was best recorded at the nearest 1 Hz short-period vertical component
118 seismometer located at station ‘RETU’, at 3900 m elevation (Bell et al., 2017). Primary
119 seismic data manipulation was undertaken using the Obspy python library (Krischer et al.,
120 2015). The highly similar earthquake waveforms indicates closely located sources, meaning
121 that the amplitude recorded at RETU is a reasonable approximation of relative earthquake
122 energy release. Peak amplitudes and 15 second RMS amplitudes yield similar results. Data
123 from RETU were manually picked to provide an earthquake catalogue for several days before
124 and after the paroxysmal explosion, and used to provide five minute relative seismic
125 amplitude (RSAM). 960 events were picked in the 24 hours before the explosion, of which
126 427 were recorded in the unlocated IGEPN catalogue for Tungurahua. None of the
127 earthquakes were of sufficiently high amplitude to be detected on the broader IGEPN seismic
128 network, and so no locations are available, although typical horizontal and vertical
129 location uncertainties for LP earthquakes at Tungurahua are on the order of a few km (Bell et
130 al., 2017). As the earthquakes are only well-recorded at RETU, they are likely to be located
131 at shallow levels in the edifice, and most probably in or close to the conduit.

132 **2.2. Periodicity**

133 We define periodicity as the ratio between the mean and standard deviation of the inter-event
134 times (Bell et al., 2017). For earthquakes that are randomly distributed in time (i.e. a Poisson
135 process), with average rate λ , the inter-event times follow an exponential distribution with
136 mean $\mu = 1/\lambda$ and variance $\sigma^2 = 1/\lambda^2$. Therefore, the periodicity, $\mu/\sigma = 1$. The
137 periodicity is equivalent to the coefficient of variation for the earthquake rate. The variance
138 of inter-event times for earthquakes that are clustered in time will be relatively high, giving
139 values of periodicity less than 1. The variance of highly periodic (anti-clustered) earthquakes
140 will be relatively small, resulting in periodicity values greater than 1. For a gamma
141 distribution the periodicity is $\sqrt{\gamma}$ (see supporting information).

143 **2.3. Gamma point process models**

144 For quasi-periodic earthquake processes, Poisson process models will incorrectly estimate
145 parameters and their uncertainties. We model earthquake occurrence times as an
146 inhomogeneous gamma process (Barbieri et al., 2001), with a mean rate evolving according
147 to Equation (2). A gamma process is a generalized form of Poisson process for quasi-periodic
148 data, and has been used to analyse biomedical data, such as neuron spiking (Barbieri et al.,
149 2001) and heartbeats (Barbieri et al., 2005). For clustered data, the gamma distribution has
150 previously been shown to be an emergent property of the superposition of independent
151 earthquakes and triggered earthquakes (Touati et al., 2009) or a non-homogeneous Poisson

152 process of independent earthquakes with an underlying rate change (Shcherbakov et al.,
 153 2005). However to our knowledge the gamma distribution has not previously been applied to
 154 quasi-periodic volcanic earthquake point process data. We use a Bayesian approach to
 155 estimate model parameters (Boué et al., 2015), but here applying Markov Chain Monte Carlo
 156 (MCMC) to the point process model likelihood function rather than binned event rates.
 157 MCMC is implemented through PyMC3 (Salvatier et al., 2016).
 158

159 **3. Results**

160 **3.1. Precursors to eruption**

161 Average LP rates, amplitudes, and RSAM increased systematically in the 24 hr lead up to
 162 the 14 July explosion (Fig. 1a; Fig. 3, Fig. S1). Individual LP earthquakes have peak
 163 frequencies of 2-3 Hz (Fig. 1b-d), an emergent onset, and coda of 20-30 seconds duration.
 164 Many earthquake waveforms are highly similar, indicating the repeated activation of fixed-
 165 location sources (Fig. 2a and b). Cross-correlation analysis, using a two-stage clustering
 166 method (Bell et al., 2017; Green & Neuberg, 2006; Rodgers et al., 2013) with cross-
 167 correlation thresholds of 0.7 and 0.8, finds only one dominant family of earthquakes with
 168 highly correlated waveforms, suggesting repeated activation of a single source location, but
 169 where the source progressively evolves through time (either due to a small change in
 170 location, or small change in source mechanism; Fig. 2a-c). Earthquake inter-event times are
 171 quasi-periodic, approximating a gamma distribution when the systematic rate increase is
 172 accounted for (Fig. 2d), meaning that they are not independent. Average inter-event times
 173 decrease from greater than 10 minutes early in the sequence to less than 10 s close to the
 174 explosion. At 200 minutes before the explosion, inter-event times decrease to equal to or less
 175 than the coda duration, so individual waveforms merge into continuous (non-harmonic)
 176 tremor (Fig. 1d), as seen at other volcanoes including Redoubt and Soufriere Hills (Hotovec
 177 et al., 2013; Neuberg et al., 2000). The merger results in masking of individual earthquakes,
 178 and hence an incomplete earthquake catalogue close to the eruption, despite efforts to pick
 179 earthquakes in the frequency domain, and using a template matching approach. Earthquake
 180 amplitudes have a restricted range of values that approximate a lognormal distribution
 181 (inconsistent with a power-law Gutenberg-Richter distribution of amplitudes). Despite hours
 182 of accelerating LP rates and amplitudes, the final onset of the paroxysmal explosion was
 183 effectively instantaneous (Fig. 1d).

184 **3.2. Bayesian MCMC application of FFM to quasi-periodic data**

185 The observed increases in LP earthquake rates, amplitudes, and RSAM towards the
 186 explosion all closely follow the trend described by Equation (2) (Fig. 3). Retrospective
 187 modelling (i.e. with known fixed eruption time) finds distinctly different values of p for the
 188 different metrics (Fig. 3). The mean of the posterior distribution of p is 1.05 for earthquake
 189 rate, (excluding incomplete data within 200 minutes of the eruption time), whereas it takes
 190 values of 0.23 and 0.38 for amplitudes and RSAM, respectively. Retrospective ‘forecasts’
 191 (estimating the eruption time alongside other model parameters) also closely approximate the
 192 data (Fig 3 a-c) and highlight the marked difference between p for different data types, but
 193 show greater variance in the posterior distributions due to the covariance between p and t_f
 194 when the eruption time is not fixed. For both cases, observed earthquake rates fall below
 195 model predictions closer than 200 minutes before the eruption whereas average earthquake

196 amplitudes and RSAM more closely follow equation (2) up to the eruption onset (Fig 3 b &
197 c).

198 **3.3. Retrospective forecasts and posterior parameter distributions**

199 Repeated retrospective forecasts reveal the evolution of parameter posterior distributions,
200 including the failure time, as the sequence progresses (Fig. 4). The means of posterior
201 distributions for t_f (Fig. 4a) and p (Fig. 4b) based on earthquake times are stable until 90%
202 of the sequence is complete. The uncertainty in these parameters (including the eruption
203 time) decrease towards eruption as depicted by the width of the 5% and 95% credibility
204 intervals, and the indicative posterior probability density distributions. The mean t_f is one
205 hour later than the actual eruption time, though within the estimated uncertainty. The eruption
206 occurred whilst earthquake rates, amplitudes, and RSAM were still increasing (at the time of
207 onset, the mean inter-event times were 8-9 s). After 90% of the sequence, catalogue
208 incompleteness becomes important and results in an increasingly biased (late) estimate. The
209 degree of periodicity increases systematically through the sequence (Fig. 4c), and likely
210 increases even further in the final 10% of the sequence, but is partly masked by
211 incompleteness. Similar analysis based on earthquake amplitudes and RSAM show much
212 greater variance in model parameters until very close to the eruption, as a result of lower
213 values of p and greater non-linearity of the acceleration. Close to the eruption, these metrics
214 continue to increase according to a power-law (Fig. 3 b & c), and so provide more reliable
215 information about eruption timing.

216
217 Synthetic datasets can reproduce many of the characteristics of the real data (Fig. S3), and
218 their analysis provides further constraints on the nature of pre-eruptive sequences. Waveform
219 superposition results in an apparent power-law increase in earthquake amplitude at inter-
220 event times less than the earthquake coda duration, even for synthetic sequences generated
221 with constant input amplitude. Synthetic simulations show that the apparent power-law trends
222 and low exponent values for amplitude and RSAM are most easily explained by an
223 underlying linear increase in amplitude with time (Fig. S4). These properties are therefore
224 emergent consequences of the power-law increase in earthquake rate, a linear increase in the
225 ‘true’ earthquake size, and waveform superposition, and so may not hold much physical
226 significance in themselves. Residual discrepancies between simulations and observations
227 suggest that the acceleration in earthquake rate slows slightly in the final hour before the
228 eruption, though incompleteness means that it is not possible to resolve this effect in the real
229 data. Simulations also show that forecasting error (i.e. the variance of the posterior
230 distribution of t_f) decreases with increasing periodicity (e.g. Fig. S5), and so eruption
231 forecasts based on quasi-periodic drumbeat signals are expected to be more precise than those
232 for equivalent sequences with Poisson or clustered in inter-event times.

233

234 **4. Discussion**

235 Quasi-periodic inter-event times, a systematically increasing restricted range of
236 amplitudes, and highly-similar waveforms suggest that LPs in this sequence involve repeated
237 energy release from a single source location, most likely within or close to the conduit. Short
238 inter-event times close to the eruption imply rapid (<8-9 s) source re-activation is required,
239 including loading (e.g. shear stress or gas pressure increase) and renewal (e.g. fault or magma
240 healing, magma ascent). These characteristics are not consistent with a process underpinned

241 by material failure distributed through a large volume, even though the observed power-law
242 increase in mean earthquake rate with $p \approx 1$ are the same as those predicted by that model
243 for VT earthquakes at reawakening volcanoes (Kilburn, 2003). Rather, we suggest that these
244 similarities result as emergent properties of different complex, non-linear physical systems.
245

246 LP amplitudes are reported to decrease with increasing earthquake rate before some
247 explosions at Soufriere Hills (Neuberg et al., 2000) and Redoubt (Buurman et al., 2013).
248 Such behaviour might indicate that progressive weakening of the seismic source controls the
249 approach to eruption, but is inconsistent with observations here. Increasing amplitudes with
250 increasing earthquake rates have been reported for precursory sequences before large tectonic
251 earthquakes (Bouchon et al., 2011) and landslides (Poli, 2017), and attributed to increasing
252 slip or size of a repeatedly failing asperity within a zone of accelerating aseismic slip. For this
253 model to translate to the pre-eruptive sequence at Tungurahua would require accelerating
254 aseismic ascent of the magma column to drive repeated failure of a growing local asperity
255 such as a patch undergoing frictional stick-slip (Iverson et al., 2006) or shear failure of
256 magma (Neuberg et al., 2006; Tuffen & Dingwell, 2005), whilst maintaining co-located
257 sources to produce similar earthquake waveforms and sufficiently high gas pressure to drive
258 the ensuing explosion. The short inter-event times close to failure are difficult to explain with
259 a magma failure and healing model (Chouet & Matoza, 2013; Tuffen et al., 2003), unless new
260 magma is continually ascending into a seismogenic window (Neuberg et al., 2006). Existing
261 models for LP generation at Tungurahua suggest that the excitation mechanism might involve
262 local gas depressurization (Bell et al., 2017; Molina, 2004). A ‘two-phase’ model (Bell et al.,
263 2017; Holland et al., 2011), where excitation results from gas flux and depressurization, but
264 earthquake timing is determined by a transient breach of a low permeability barrier through
265 shear failure near the column margins driven by magma ascent, allows greater independence
266 between inter-event times and amplitudes. This model might provide an explanation for the
267 broader range of LP signals observed at Tungurahua (Bell et al., 2017), and an alternative
268 interpretation of the processes underlying this sequence.

269 In either scenario, accelerating earthquake rates and increasing amplitudes and RSAM are
270 likely to be driven by steadily accelerating magma ascent and high gas pressures, implying
271 both magma flow from depth and gas exsolution from supersaturated melt (Lensky et al.,
272 2008). The eruption onset occurs slightly before the predicted failure time, when the system
273 reaches some hidden critical threshold. The highly explosive, gas-driven vulcanian nature of
274 the eruption suggests that this is when the gas pressure exceeds the failure strength of the
275 plug, or the inter-event time reduces to either the finite duration of the source mechanism or
276 recovery time. The different characteristics of trends in LP earthquake rates, amplitudes, and
277 RSAM suggest the most informative forecasts of explosion timing would be initially based
278 on LP occurrence times, with additional information close to the eruption provided by RSAM
279 and earthquake amplitudes. Future work will see these different metrics combined into an
280 integrated Bayesian forecasting tool.

281 **5. Conclusions**

282 This work outlines a new approach for reliable and informative retrospective analyses of
283 pre-eruptive LP seismicity, and verifiable and testable Bayesian forecasts. When applied to
284 the July 2013 eruption, the methods reveal a remarkable sequence, resembling a theoretical
285 ideal to a degree not reported before. The powerful paroxysmal explosion, involving
286 relatively little juvenile magma, is indicative of an eruption driven by high gas pressures and
287 a very low permeability barrier to gas ascent. Unusually, the resulting quasi-periodic
288 oscillating system, likely involving coupled incremental magma ascent and increasing gas

289 pressure, remained stable until a late stage. The methods offer quantitative, probabilistic
290 forecasts of the timing of explosions when pre-eruptive seismicity follow a quasi-periodic
291 power-law acceleration. Understanding of when and how these conditions might arise, and
292 how to reliably identify them in noisy data, is key for improved eruption forecasting in future.
293
294

295 **Acknowledgments**

296 A.F.B. was partly funded by a Carnegie trust research incentive grant. We thank S. De
297 Angelis and an anonymous review for insightful and constructive comments that helped
298 improve the manuscript. We also thank the staff of the IGEPN for their considerable efforts
299 in maintaining the monitoring network on Tungurahua volcano. Seismic data for Tungurahua
300 is curated by the IGEPN and available on request from [http://www.igepn.edu.ec/solicitud-de-](http://www.igepn.edu.ec/solicitud-de-datos)
301 [datos](http://www.igepn.edu.ec/solicitud-de-datos)
302

303

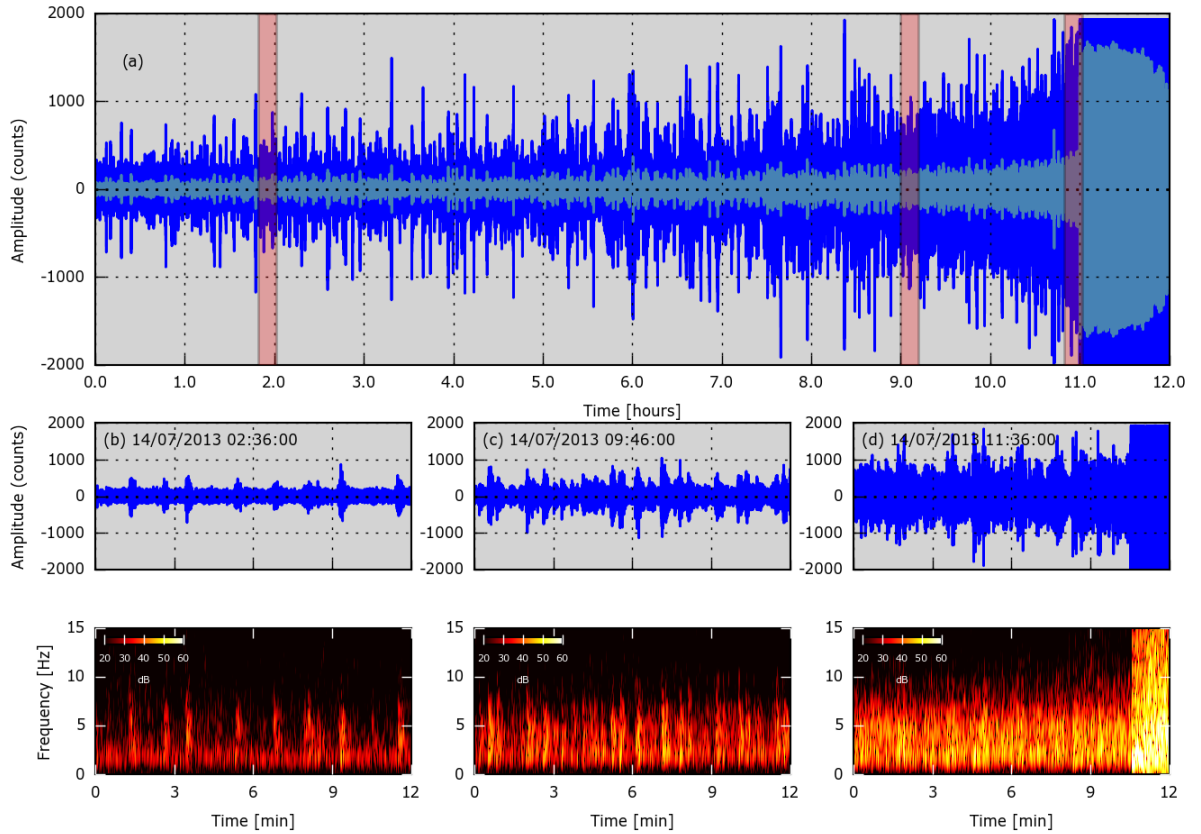
304 **References**

- 305 De Angelis, S., & Henton, S. M. (2011). On the feasibility of magma fracture within volcanic
 306 conduits: Constraints from earthquake data and empirical modelling of magma viscosity.
 307 *Geophysical Research Letters*, 38(19), n/a-n/a. <https://doi.org/10.1029/2011GL049297>
- 308 Arellano, S. R., Hall, M. L., Samaniego, P., Le Pennec, J.-L., Ruiz, A., Molina, I., & Yepes,
 309 H. A. (2008). Degassing patterns of Tungurahua volcano (Ecuador) during the 1999-
 310 2006 eruptive period, inferred from remote spectroscopic measurements of SO₂
 311 emissions. *Journal of Volcanology and Geothermal Research*, 176(1), 151–162.
 312 <https://doi.org/10.1016/j.jvolgeores.2008.07.007>
- 313 Barbieri, R., Quirk, M. C., Frank, L. M., Wilson, M. A., & Brown, E. N. (2001). Construction
 314 and analysis of non-Poisson stimulus-response models of neural spiking activity.
 315 *Journal of Neuroscience Methods*, 105(1), 25–37. [https://doi.org/10.1016/S0165-](https://doi.org/10.1016/S0165-0270(00)00344-7)
 316 0270(00)00344-7
- 317 Barbieri, R., Matten, E. C., Alabi, A. a, & Brown, E. N. (2005). A point-process model of
 318 human heartbeat intervals: new de nitions of heart rate and heart rate variability. *Am J*
 319 *Physiol Heart Circ Physiol*, 288, 424–435.
 320 <https://doi.org/10.1152/ajpheart.00482.2003.A>
- 321 Bean, C. J., De Barros, L., Lokmer, I., Métaxian, J.-P., O’Brien, G. S., & Murphy, S. (2013).
 322 Long-period seismicity in the shallow volcanic edifice formed from slow-rupture
 323 earthquakes. *Nature Geoscience*, 7(1), 71–75. <https://doi.org/10.1038/ngeo2027>
- 324 Bell, A. F., Hernandez, S., Gaunt, H. E., Mothes, P. A., Ruiz, M., Sierra, D., & Aguaiza, S.
 325 (2017). The rise and fall of periodic “drumbeat” seismicity at Tungurahua volcano,
 326 Ecuador. *Earth and Planetary Science Letters*, 475, 58–70.
 327 <https://doi.org/10.1016/j.epsl.2017.07.030>
- 328 Bell, A. F., & Kilburn, C. R. J. (2012). Precursors to dyke-fed eruptions at basaltic volcanoes:
 329 Insights from patterns of volcano-tectonic seismicity at Kilauea volcano, Hawaii.
 330 *Bulletin of Volcanology*, 74(2), 325–339. <https://doi.org/10.1007/s00445-011-0519-3>
- 331 Bell, A. F., & Kilburn, C. R. J. (2013). Trends in the aggregated rate of pre-eruptive volcano-
 332 tectonic seismicity at Kilauea volcano, Hawaii. *Bulletin of Volcanology*, 75(1), 1–10.
 333 <https://doi.org/10.1007/s00445-012-0677-y>
- 334 Bell, A. F., Naylor, M., & Main, I. G. (2013). The limits of predictability of volcanic
 335 eruptions from accelerating rates of earthquakes. *Geophysical Journal International*,
 336 194(3), 1541–1553. <https://doi.org/10.1093/gji/ggt191>
- 337 Bell, A. F., Greenhough, J., Heap, M. J., & Main, I. G. (2011). Challenges for forecasting
 338 based on accelerating rates of earthquakes at volcanoes and laboratory analogues.
 339 *Geophysical Journal International*, 185(2), 718–723. [https://doi.org/10.1111/j.1365-](https://doi.org/10.1111/j.1365-246X.2011.04982.x)
 340 246X.2011.04982.x
- 341 Bouchon, M., Karabulut, H., Aktar, M., Ozalaybey, S., Schmittbuhl, J., & Bouin, M.-P.
 342 (2011). Extended Nucleation of the 1999 Mw 7.6 Izmit Earthquake. *Science*, 331(6019),
 343 877–880. <https://doi.org/10.1126/science.1197341>
- 344 Boué, A., Lesage, P., Cortés, G., Valette, B., Reyes-Dávila, G. A., Arámbula-Mendoza, R., &
 345 Budi-Santoso, A. (2016). Performance of the “material Failure Forecast Method” in
 346 real-time situations: A Bayesian approach applied on effusive and explosive eruptions.
 347 *Journal of Volcanology and Geothermal Research*, 327, 622–633.
 348 <https://doi.org/10.1016/j.jvolgeores.2016.10.002>
- 349 Boué, A., Lesage, P., Cortés, G., Valette, B., & Reyes-Dávila, G. A. (2015). Real-time
 350 eruption forecasting using the material Failure Forecast Method with a Bayesian
 351 approach. *Journal of Geophysical Research: Solid Earth*, 120(4), 2143–2161.

- 352 <https://doi.org/10.1002/2014JB011637>
- 353 Buurman, H., West, M. E., & Thompson, G. (2013). The seismicity of the 2009 Redoubt
354 eruption. *Journal of Volcanology and Geothermal Research*.
355 <https://doi.org/10.1016/j.jvolgeores.2012.04.024>
- 356 Chouet, B. A., & Matoza, R. S. (2013). A multi-decadal view of seismic methods for
357 detecting precursors of magma movement and eruption. *Journal of Volcanology and*
358 *Geothermal Research*. <https://doi.org/10.1016/j.jvolgeores.2012.11.013>
- 359 Collombet, M. (2003). Seismicity rate before eruptions on Piton de la Fournaise volcano:
360 Implications for eruption dynamics. *Geophysical Research Letters*, *30*(21), 1–5.
361 <https://doi.org/10.1029/2003GL017494>
- 362 Gil-Cruz, F., & Chouet, B. a. (1997). Long-period events, the most characteristic seismicity
363 accompanying the emplacement and extrusion of a lava dome in Galeras Volcano,
364 Colombia, in 1991. *Journal of Volcanology and Geothermal Research*, *77*(1–4), 121–
365 158. [https://doi.org/10.1016/S0377-0273\(96\)00091-1](https://doi.org/10.1016/S0377-0273(96)00091-1)
- 366 Green, D. N., & Neuberg, J. (2006). Waveform classification of volcanic low-frequency
367 earthquake swarms and its implication at Soufrière Hills Volcano, Montserrat. *Journal*
368 *of Volcanology and Geothermal Research*, *153*(1–2), 51–63.
369 <https://doi.org/10.1016/j.jvolgeores.2005.08.003>
- 370 Hall, M. L., Steele, A. L., Bernard, B., Mothes, P. A., Vallejo, S. X., Douillet, G. A., ... Ruiz,
371 M. C. (2015). Sequential plug formation, disintegration by Vulcanian explosions, and
372 the generation of granular Pyroclastic Density Currents at Tungurahua volcano (2013–
373 2014), Ecuador. *Journal of Volcanology and Geothermal Research*, *306*, 90–103.
374 <https://doi.org/10.1016/j.jvolgeores.2015.09.009>
- 375 Hidalgo, S., Battaglia, J., Arellano, S., Steele, A. L., Bernard, B., Bourquin, J., ... Vásconez,
376 F. (2015). SO₂ degassing at Tungurahua volcano (Ecuador) between 2007 and 2013:
377 Transition from continuous to episodic activity. *Journal of Volcanology and Geothermal*
378 *Research*, *298*, 1–14. <https://doi.org/10.1016/j.jvolgeores.2015.03.022>
- 379 Holland, A., Watson, I. M., Phillips, J. C., Caricchi, L., & Dalton, M. P. (2011). Degassing
380 processes during lava dome growth: Insights from Santiaguito lava dome, Guatemala.
381 *Journal of Volcanology and Geothermal Research*, *202*(1–2), 153–166.
382 <https://doi.org/10.1016/j.jvolgeores.2011.02.004>
- 383 Hotovec, A. J., Prejean, S. G., Vidale, J. E., & Gomberg, J. (2013). Strongly gliding
384 harmonic tremor during the 2009 eruption of Redoubt Volcano. *Journal of Volcanology*
385 *and Geothermal Research*, *259*, 89–99. <https://doi.org/10.1016/j.jvolgeores.2012.01.001>
- 386 Iverson, R. M., Dzurisin, D., Gardner, C. A., Gerlach, T. M., LaHusen, R. G., Lisowski, M.,
387 ... Vallance, J. W. (2006). Dynamics of seismogenic volcanic extrusion at Mount St
388 Helens in 2004–05. *Nature*, *444*(1), 439–443. <https://doi.org/10.1038/nature05322>
- 389 Johnson, J. B., Lees, J. M., Gerst, A., Sahagian, D., & Varley, N. (2008). Long-period
390 earthquakes and co-eruptive dome inflation seen with particle image velocimetry.
391 *Nature*, *456*(7220), 377–381. <https://doi.org/10.1038/nature07429>
- 392 Kilburn, C. R. J. (2003). Multiscale fracturing as a key to forecasting volcanic eruptions.
393 *Journal of Volcanology and Geothermal Research*, *125*(3–4), 271–289.
394 [https://doi.org/10.1016/S0377-0273\(03\)00117-3](https://doi.org/10.1016/S0377-0273(03)00117-3)
- 395 Kilburn, C. R. J. (2012). Precursory deformation and fracture before brittle rock failure and
396 potential application to volcanic unrest. *Journal of Geophysical Research: Solid Earth*,
397 *117*(B2), n/a-n/a. <https://doi.org/10.1029/2011JB008703>
- 398 Kilburn, C. R. J., & Voight, B. (1998). Slow rock fracture as eruption precursor at Soufriere
399 Hills Volcano, Montserrat. *Geophysical Research Letters*, *25*(19), 3665.
400 <https://doi.org/10.1029/98GL01609>
- 401 Krischer, L., Megies, T., Barsch, R., Beyreuther, M., Lecocq, T., Caudron, C., &

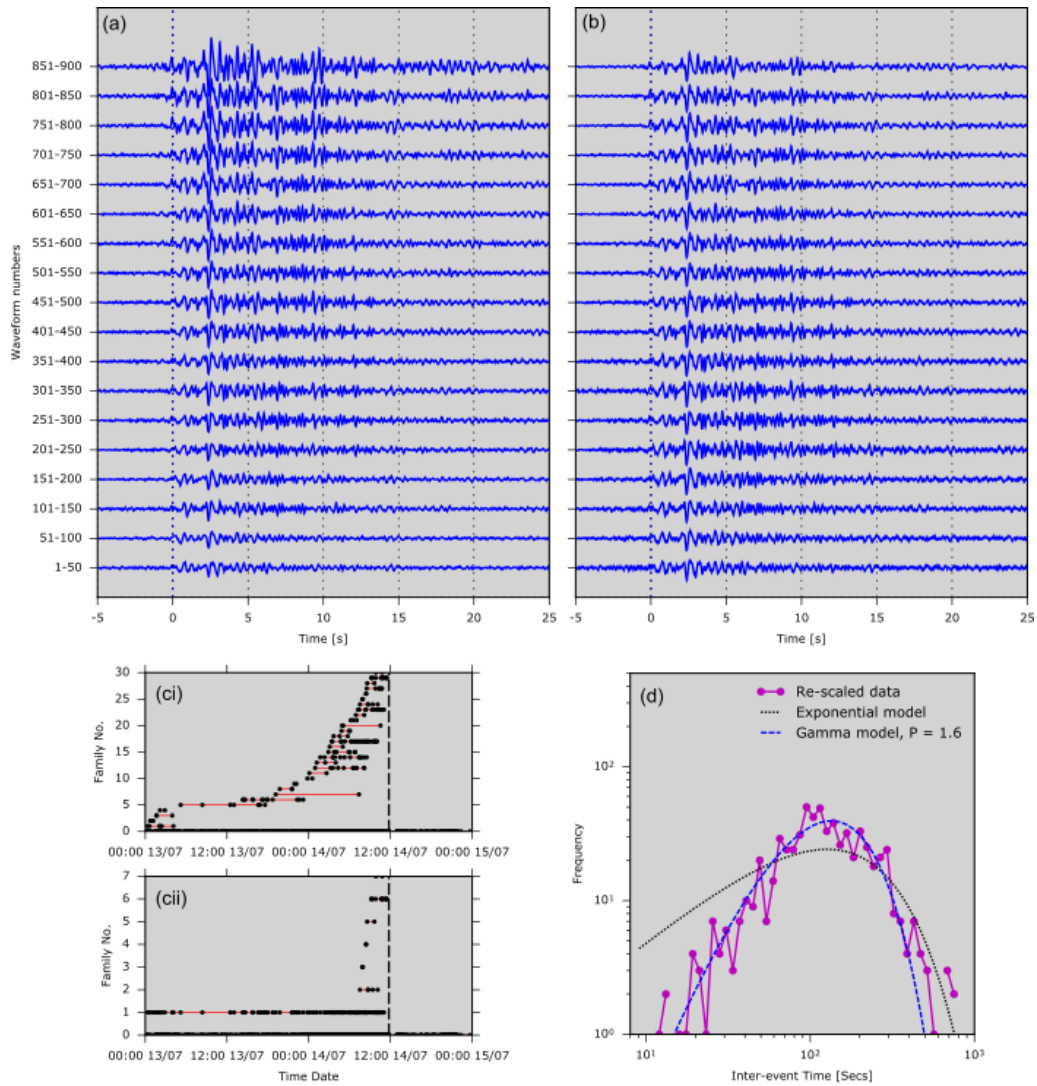
- 402 Wassermann, J. (2015). ObsPy: a bridge for seismology into the scientific Python
 403 ecosystem. *Computational Science & Discovery*, 8(1), 14003.
 404 <https://doi.org/10.1088/1749-4699/8/1/014003>
- 405 De La Cruz-Reyna, S., & Reyes-Dávila, G. A. (2001). A model to describe precursory
 406 material-failure phenomena: Applications to short-term forecasting at Colima volcano,
 407 Mexico. *Bulletin of Volcanology*, 63(5), 297–308.
 408 <https://doi.org/10.1007/s004450100152>
- 409 Lavallée, Y., Meredith, P. G., Dingwell, D. B., Hess, K.-U., Wassermann, J., Cordonnier, B.,
 410 ... Kruhl, J. H. (2008). Seismogenic lavas and explosive eruption forecasting. *Nature*,
 411 453(7194), 507–510. <https://doi.org/10.1038/nature06980>
- 412 Lensky, N. G., Sparks, R. S. J., Navon, O., & Lyakhovskiy, V. (2008). Cyclic activity at
 413 Soufrière Hills Volcano, Montserrat: degassing-induced pressurization and stick-slip
 414 extrusion. *Geological Society, London, Special Publications*, 307(1), 169–188.
 415 <https://doi.org/10.1144/SP307.10>
- 416 Linde, A. T., Agustsson, K., Sacks, I. S., & Stefansson, R. (1993). Mechanism of the 1991
 417 eruption of Hekla from continuous borehole strain monitoring. *Nature*, 365(6448), 737–
 418 740. <https://doi.org/10.1038/365737a0>
- 419 Lipovsky, B. P., & Dunham, E. M. (2015). Vibrational modes of hydraulic fractures:
 420 Inference of fracture geometry from resonant frequencies and attenuation. *Journal of*
 421 *Geophysical Research: Solid Earth*, 120(2), 1080–1107.
 422 <https://doi.org/10.1002/2014JB011286>
- 423 Main, I. G. (1999). Applicability of time-to-failure analysis to accelerated strain before
 424 earthquakes and volcanic eruptions. *Geophysical Journal International*, 139(3), F1–F6.
 425 <https://doi.org/10.1046/j.1365-246x.1999.00004.x>
- 426 McGuire, W., & Kilburn, C. R. J. (1997). Forecasting volcanic events: some contemporary
 427 issues. *Geologische Rundschau*, 86, 439–445. <https://doi.org/10.1007/s005310050152>
- 428 McNutt, S. R. (2005). Volcanic Seismology. *Annual Review of Earth and Planetary Sciences*,
 429 33(1), 461–491. <https://doi.org/10.1146/annurev.earth.33.092203.122459>
- 430 Molina, I. (2004). Resonances of a volcanic conduit triggered by repetitive injections of an
 431 ash-laden gas. *Geophysical Research Letters*, 31(3), L03603.
 432 <https://doi.org/10.1029/2003GL018934>
- 433 Neuberg, J., Collinson, A. S. D., Mothes, P. A., C. Ruiz, M., & Aguaiza, S. (2018).
 434 Understanding cyclic seismicity and ground deformation patterns at volcanoes:
 435 Intriguing lessons from Tungurahua volcano, Ecuador. *Earth and Planetary Science*
 436 *Letters*, 482, 193–200. <https://doi.org/10.1016/j.epsl.2017.10.050>
- 437 Neuberg, J., Tuffen, H., Collier, L., Green, D., Powell, T., & Dingwell, D. (2006). The trigger
 438 mechanism of low-frequency earthquakes on Montserrat. *Journal of Volcanology and*
 439 *Geothermal Research*, 153(1–2), 37–50.
 440 <https://doi.org/10.1016/j.jvolgeores.2005.08.008>
- 441 Neuberg, J., Luckett, R., Baptie, B., & Olsen, K. (2000). Models of tremor and low-
 442 frequency earthquake swarms on Montserrat. *Journal of Volcanology and Geothermal*
 443 *Research*, 101(1–2), 83–104. [https://doi.org/10.1016/S0377-0273\(00\)00169-4](https://doi.org/10.1016/S0377-0273(00)00169-4)
- 444 Poli, P. (2017). Creep and slip: Seismic precursors to the Nuugaatsiaq landslide (Greenland).
 445 *Geophysical Research Letters*, 44(17), 8832–8836.
 446 <https://doi.org/10.1002/2017GL075039>
- 447 Ramos, E. G., Hamburger, M. W., Pavlis, G. L., & Laguerta, E. P. (1999). The low-frequency
 448 earthquake swarms at Mount Pinatubo, Philippines: Implications for magma dynamics.
 449 *Journal of Volcanology and Geothermal Research*. [https://doi.org/10.1016/S0377-](https://doi.org/10.1016/S0377-0273(99)00091-8)
 450 [0273\(99\)00091-8](https://doi.org/10.1016/S0377-0273(99)00091-8)
- 451 Roberts, N. S., Bell, A. F., & Main, I. G. (2015). Are volcanic seismic b-values high, and if

- 452 so when? *Journal of Volcanology and Geothermal Research*, 308, 127–141.
 453 <https://doi.org/10.1016/j.jvolgeores.2015.10.021>
- 454 Robertson, R. M., & Kilburn, C. R. J. (2016). Deformation regime and long-term precursors
 455 to eruption at large calderas: Rabaul, Papua New Guinea. *Earth and Planetary Science*
 456 *Letters*, 438(September 1994), 86–94. <https://doi.org/10.1016/j.epsl.2016.01.003>
- 457 Rodgers, M., Roman, D. C., Geirsson, H., LaFemina, P., Muñoz, A., Guzman, C., & Tenorio,
 458 V. (2013). Seismicity accompanying the 1999 eruptive episode at telica volcano,
 459 nicaragua. *Journal of Volcanology and Geothermal Research*, 265, 39–51.
 460 <https://doi.org/10.1016/j.jvolgeores.2013.08.010>
- 461 Salvage, R. O., & Neuberg, J. (2016). Using a cross correlation technique to refine the
 462 accuracy of the Failure Forecast Method: Application to Soufrière Hills volcano,
 463 Montserrat. *Journal of Volcanology and Geothermal Research*, 324, 118–133.
 464 <https://doi.org/10.1016/j.jvolgeores.2016.05.011>
- 465 Salvatier, J., Wiecki, T. V., & Fonnesbeck, C. (2016). Probabilistic programming in Python
 466 using PyMC3. *PeerJ Computer Science*, 2, e55. <https://doi.org/10.7717/peerj-cs.55>
- 467 Shcherbakov, R., Yakovlev, G., Turcotte, D. L., & Rundle, J. B. (2005). Model for the
 468 Distribution of Aftershock Interoccurrence Times. *Physical Review Letters*, 95(21),
 469 218501. <https://doi.org/10.1103/PhysRevLett.95.218501>
- 470 Sigmundsson, F., Hooper, A., Hreinsdóttir, S., Vogfjörð, K. S., Ófeigsson, B. G., Heimisson,
 471 E. R., ... Eibl, E. P. S. (2014). Segmented lateral dyke growth in a rifting event at
 472 Bárðarbunga volcanic system, Iceland. *Nature*, 517(7533), 191–195.
 473 <https://doi.org/10.1038/nature14111>
- 474 Touati, S., Naylor, M., & Main, I. G. (2009). Origin and Nonuniversality of the Earthquake
 475 Interevent Time Distribution. *Physical Review Letters*, 102(16), 168501.
 476 <https://doi.org/10.1103/PhysRevLett.102.168501>
- 477 Tuffen, H., & Dingwell, D. B. (2005). Fault textures in volcanic conduits: evidence for
 478 seismic trigger mechanisms during silicic eruptions. *Bulletin of Volcanology*, 67(4),
 479 370–387. <https://doi.org/10.1007/s00445-004-0383-5>
- 480 Tuffen, H., Dingwell, D. B., & Pinkerton, H. (2003). Repeated fracture and healing of silicic
 481 magma generate flow banding and earthquakes? *Geology*, 31(12), 1089–1092.
 482 <https://doi.org/10.1130/G19777.1>
- 483 Tuffen, H., Smith, R., & Sammonds, P. R. (2008). Evidence for seismogenic fracture of
 484 silicic magma. *Nature*, 453(7194), 511–514. <https://doi.org/10.1038/nature06989>
- 485 Vasseur, J., Wadsworth, F. B., Heap, M. J., Main, I. G., Lavallée, Y., & Dingwell, D. B.
 486 (2017). Does an inter-flaw length control the accuracy of rupture forecasting in
 487 geological materials? *Earth and Planetary Science Letters*, 475(August), 181–189.
 488 <https://doi.org/10.1016/j.epsl.2017.07.011>
- 489 Voight, B. (1988). A method for prediction of volcanic eruptions. *Nature*, 332(6160), 125–
 490 130. <https://doi.org/10.1038/332125a0>
- 491 Voight, B., & Cornelius, R. R. (1991). Prospects for eruption prediction in near real-time.
 492 *Nature*, 354(APRIL 1991), 56–58. <https://doi.org/10.1038/350695a0>
- 493 White, R. A., Miller, A. D., Lynch, L., & Power, J. A. (1998). Observations of hybrid seismic
 494 events at Soufriere Hills Volcano, Montserrat: July 1995 to September 1996.
 495 *Geophysical Research Letters*, 25(19), 3657–3660. <https://doi.org/10.1029/98gl02427>
- 496

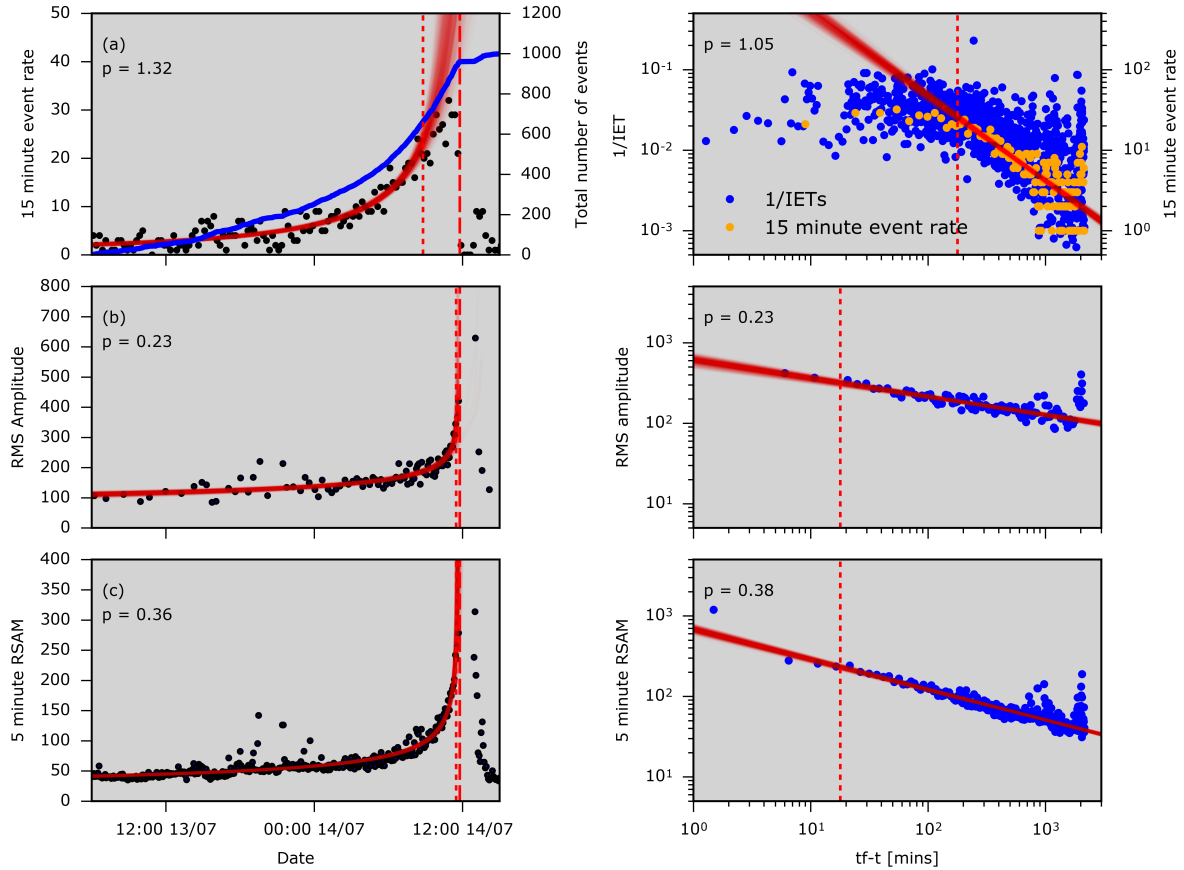


497
498
499
500
501
502
503
504
505
506

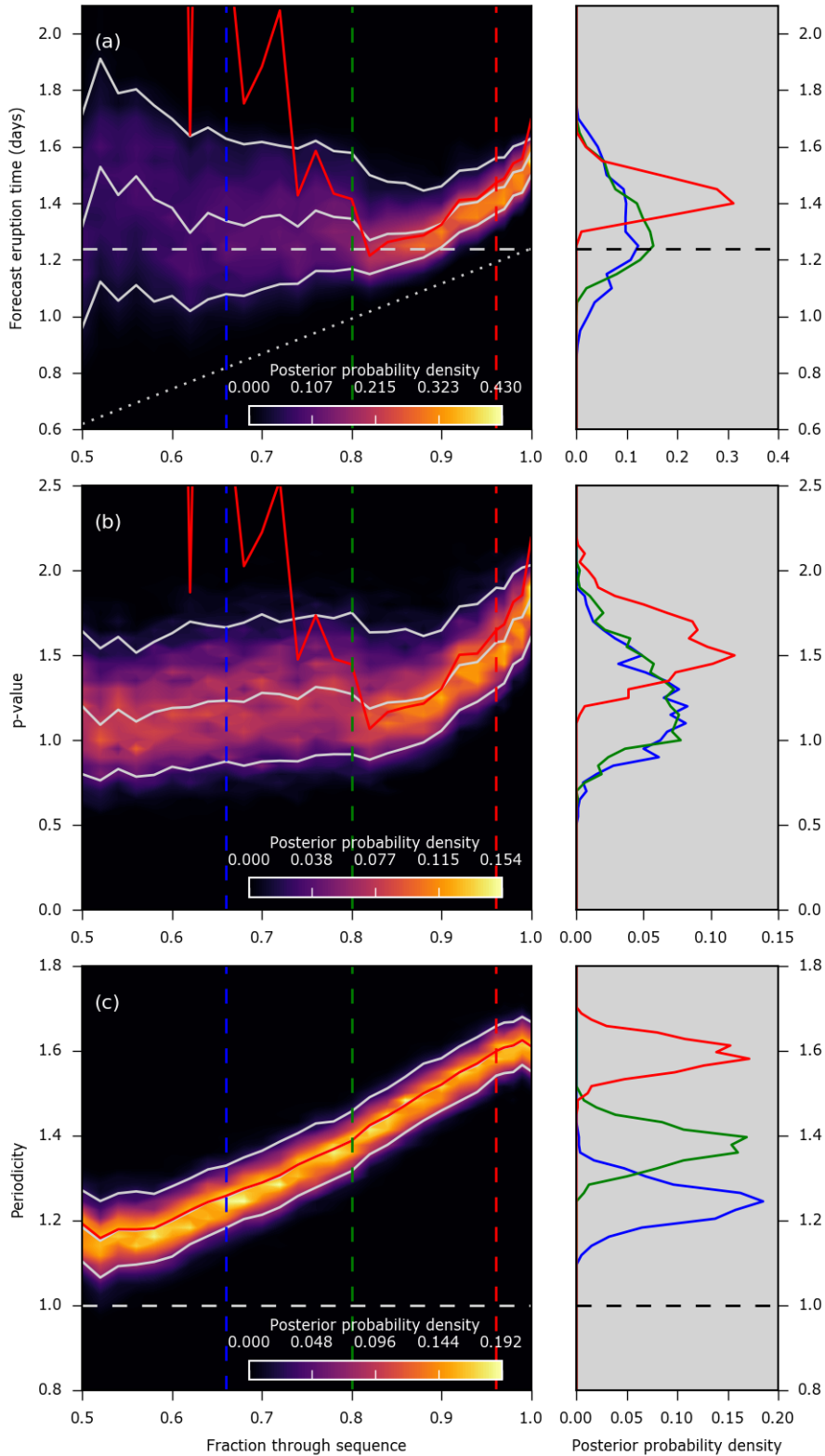
Figure 1: (a) Velocity time series for 12 hours of data recorded at RETU between 00:46 and 12:46 UTC on 14th July 2013, documenting the increase in earthquake rate and amplitude before the large explosion at 11:46. Dark blue line represents all data; light blue line represents data filtered between 0.1 and 8 Hz, and averaged over 10 seconds. (b)-(d) 12 minute times series and spectrograms for the intervals indicated by red bars in (a). Note quasi-periodic inter-event times and narrow range of earthquake amplitudes (b); progressive increase in earthquake rate and amplitude (c); and merger of earthquakes to form superficially continuous tremor immediately prior to eruption (d).



507
 508 **Figure 2:** Top: Average waveform ‘stacks’ for 50 consecutive LP earthquakes recorded at
 509 RETU on 13 and 14 July 2013; (a) with original relative amplitudes preserved; (b) each stack
 510 normalized by amplitude to highlight waveform similarity. Bottom left: Families of similar
 511 earthquakes recorded at station RETU, 13-14 July 2013, determined using; (ci) single
 512 clustering stage based on cross-correlation threshold of 0.7; (cii) second additional cluster
 513 coalescence stage using cross-correlation threshold of 0.8. Black circles depict occurrence
 514 time of earthquakes belonging to different families. Red lines indicate temporal extent of
 515 each family. Family 0 consists of ‘singleton’ earthquakes that have no cross-correlations with
 516 other earthquakes above the 0.7 threshold. (d) Inter-event time distribution and best fitting
 517 models for re-scaled data for July 2013 pre-eruptive sequence. Data has been re-scaled
 518 according to equation (2) and mean posterior parameter values to account for increasing
 519 mean earthquake rate with time. Circles and solid lines represent actual data. Dotted black
 520 line shows best-fitting exponential inter-event time distribution model for the null hypothesis
 521 of a Poisson process for each phase ($P=1$). Dashed lines represent maximum-likelihood
 522 Gamma distributions with periodicity given by P value.
 523



524
 525 **Figure 3:** Linear time (left panels) and log-time from failure (right panels, i.e. ‘time
 526 reversed’) plots for (a) 15 minute earthquake rates and 1/(inter-event times), (b) mean 15
 527 minute RMS earthquake amplitudes, and (c) mean 5 minute relative seismic amplitudes. In
 528 left panels, red curves show 500 ‘hind-cast’ power-law models with parameter values
 529 (including eruption time) drawn from posterior distribution and corresponding mean hind-
 530 cast p-values. In right panels, red lines show 500 ‘retrospective’ power-law posterior models,
 531 with eruption time known a priori, and corresponding mean retrospective p-values. In both
 532 instances, parameter posterior distributions are determined using data occurring before red
 533 dotted line. In (a) left panel, blue line represents total number of earthquakes. Note that the
 534 increase in earthquake rate slows towards eruption due to the merger of earthquakes (Fig.
 535 1d), leading to a mismatch between model prediction and observations (a), and different
 536 mean power-law exponents and non-linearity for earthquake rates, amplitudes, and energy
 537 release rates.
 538



539
 540 **Figure 4:** Left panels: Evolution of posterior distribution of (a) eruption time; (b) p-value;
 541 and (c) periodicity (mean inter-event time divided by its standard deviation, with horizontal
 542 dashed line representing a homogeneous Poisson process for reference), based on earthquake
 543 occurrence times. Colour scale indicated the posterior probability density for parameters in a
 544 hindcast model, as determined incrementally through the sequence. White lines indicate the
 545 mean and 5% and 95% credibility intervals of the posterior. Solid red lines indicate
 546 maximum-likelihood estimates of power-law parameter values for an inhomogenous gamma

547 process. In (a), horizontal white dashed line indicates true value of eruption time, and dotted
548 white line indicates actual time at which hindcast is made. Right panel: Posterior probability
549 density distributions at times indicated by correspondingly coloured dashed lines in left panel
550 (0.66, 0.8, and 0.96 of the sequence). Note increase in eruption time and p values estimates
551 after 90% of the sequences due to rate saturation, and non-stationary increasing value of
552 periodicity through the sequence.