A probabilistic paradigm for the parametric insurance of natural hazards

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ABSTRACT

There is a pressing need for simple and reliable risk transfer mechanisms that can pay out quickly after natural disasters without delays caused by loss estimation, and the need for long historical claims records. One such approach, known as parametric insurance, pays out when a key hazard variable exceeds a pre-determined threshold. However, this approach to catastrophe risk, based on making deterministic binary predictions of loss occurrence, is susceptible to basis risk (mismatch between payouts and realised losses).

A more defensible approach is to issue probabilistic predictions of loss occurrence, which then allows uncertainty to be properly quantified, communicated, and evaluated. This study proposes a generic probabilistic framework for parametric trigger modelling based on logistic regression, and idealised modelling of potential damage given knowledge of a hazard variable. We also propose various novel methods for evaluating the quality and utility of such a frameworkpredictions as well as more traditional trigger indices.

The methodology is demonstrated by application to flood-related disasters in Jamaica from 1998-2016 using gridded precipitation data as the hazard variable. A hydrologically-motivated transformation is proposed for calculating potential damage from daily rainfall data. Despite the simplicity of the approach, the model has substantial skill at predicting the probability of occurrence of loss days as demonstrated by traditional goodness-of-fit measures (i.e. pseudo-R^2 of 0.55) as well as probabilistic verification diagnostics such as Receiver Operating Characteristics. Using conceptual models of decision-maker expenses, we also demonstrate that the system can provide considerable utility to involved parties, e.g., insured parties, insurers, risk managers.

Keywords: parametric trigger, natural hazard risk, catastrophe risk transfer, flood, Caribbean islands

Risk Analysis

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1. INTRODUCTION

Economic losses due to natural hazards have shown an increasing trend since 1980, reaching an inflation-adjusted 10-year average of around $200 billion in 2014.[1,2] This trend is expected to continue, mostly due to more exposed assets in disaster-prone areas and the effects of climate change.[3–5] Recent years have seen greater worldwide commitment to reducing disaster losses, following the adoption of the Hyogo Framework for Action in 2005 and the Sendai Framework for Disaster Risk Reduction in 2015, the latter adopted by all UN member states.[6] Reducing disaster losses requires effective management of disaster risk at all levels, from mitigation and preparedness to response and recovery.[7,8] A crucial part of that task involves reducing financial vulnerability to disasters *ex-ante,* ensuring that necessary resources will be available following such events. This can be achieved through risk transfer instruments, ~~such as~~namely insurance and reinsurance cover~~,~~ or capital market instruments~~,~~ such as catastrophe bonds.[9]

Risk transfer instruments can be based on different types of trigger. These determine the conditions under which payouts are made after an event. This paper focuses on so-called *parametric triggers.* These make payouts when a key hazard variable is observed to exceed a pre-defined threshold. A payout could be triggered, for example, by a variable such as rainfall amount, wind speed, or earthquake magnitude being observed to exceed a threshold at a given location. Aggregate measures derived from several locations in a region can also be used as a trigger variable; e.g., the sum of rainfall over several sites.[10–13] The payouts issued by parametric risk transfer products can be either fixed or based on a certain index value calculated for each event. In either case, they are not meant to offset actual losses, but instead to cover short-term liquidity gaps following a disaster, and are typically used as a part of more comprehensive risk management strategies.[14]

Parametric triggers have several important strengths. Firstly, the product structure is simple and transparent, which ensures prompt payouts and timely access to funding after a disaster occurs. Secondly, they do not require explicit exposure and vulnerability models, which are often unavailable in many parts of the world. Thirdly, they avoid typical problems in regular insurance, such as moral hazard and adverse selection.[15–17] For these reasons, it is not surprising that in recent years the number of countries that have adopted such programmes has rapidly increased.[18]

The main drawback of parametric triggers is their susceptibility to basis risk,[19] which in this type of product occurs in natural hazards when there is a mismatch between payout and loss.[19] Basis risk arises in parametric triggers when is the risk that triggered payouts do not coincide with the occurrence of loss events.[20] This can lead toresult in situations where either a payout is issued when no loss event occurs (positive basis risk), or no payout is issued when a loss event does occur (negative basis risk), both having adverse consequences. The former leads to inefficient transactions as greater risk of overpayment brings higher product costs. The latter could result in a liquidity gap that overwhelms the capacity of the risk cedant to adequately respond to and recover from a disaster. Note that in parametric products where payouts are based on an index, basis risk may be considered to arise due to less than perfect correlation between the index value and the severity of the event. In this paper, which focuses on the prediction of event occurrence, the adopted definition is the one presented previously.

Basis risk is unavoidable in parametric products, as these are based on simple models, relying on the threshold exceedance of an environmental variable, which have limited ability to predict the occurrence of rare events. Resulting predictions are therefore highly uncertain. Simple binary outcomes of the type "event" / "no event", where this uncertainty remains unspecified, are inappropriate to describe such behaviour.[21] In this context, the use of probabilities can offer various advantages over traditional deterministic approaches, which are next described:

1. *Straightforward model construction*. Reducing basis risk requires that model predictive skill is improved, which can only be achieved if more robust triggers are employed. These may be based, for example, on a transformed environmental variable better able to explain loss occurrence, and/or on multiple environmental input variables. In this context, the use of probability to quantify uncertainty is advantageous, as it facilitates the construction of statistical models for capturing the occurrence of loss events using well-established techniques.[22]

2. *Transparent trigger optimization*. A statistical model is able to issue consistent predictions of loss event occurrence for any trigger condition, as well as to quantify sensitivity in occurrence to changes in the associated input variable(s) straightforwardly. Therefore, model construction can be disentangled from the definition of the event-triggering threshold, allowing this decision to be taken with the direct involvement of model users. This enables an objective and transparent trigger optimization procedure, where well-known issues with deterministic forecasts such as hedging and overforecasting are avoided.[21]

3. *Informative predictions*. Basis risk can be difficult to explain to end-users, which is a well-known problem in parametric risk transfer. This often results in unrealistic expectations towards the product.[14] During operational period, less technically informed users may be frustrated and perceive the product as ineffective when a destructive event occurs but the model simply issues a "no event" prediction, resulting in no payout, even though this is a plausible scenario.[23] This issue can be largely overcome by quantifying uncertainty through probabilities, which makes the predictions more informative and basis risk easier to understand.

It is therefore important that the underlying basis risk is well understood and that unrealistic expectations are not created regarding the risk transfer programme. These, however, are difficult tasks in the of case parametric triggers.[20]

Basis risk is unavoidable in parametric products based on deterministic models in which sources of uncertainty are neglected.[21] Furthermore, deterministic predictions of rare events do not provide uncertainty information required for optimal decision making, and are also easy to hedge rather than reflect true beliefs about losses.[22] These problems can be avoided by representing loss beliefs as probabilities, which then allow uncertainty to be quantified. It is necessary to construct a probabilistic framework that is able to address the following questions:Therefore, in this paper we propose a probabilistic framework for parametric catastrophe risk transfer and demonstrate how it can be used. The framework aims to address the following questions:

- How best to construct probabilities of losses from hazard data?

- How best to evaluate the performance of the resulting probabilities in predicting loss?

- How best to choose decision thresholds on probabilities so as to maximize value for different end users?

This paper proposes such a framework and demonstrates how it can be used. Our framework comprises a logistic regression model that can issue probabilities of occurrence of loss events, based on potential damage variables obtained from transformed environmental variables, and methods from the field of forecast verification, which allow the quality and utility of the predictive system to be evaluated. Parametric triggers are conceptually very similar to forecasts of binary events, which enables us to take advantage of the considerable vast body of literature on weather and climate forecast verification in the development of a novel evaluation procedure for application in the field of parametric catastrophe insurance.

~~The difference lies in that parametric triggers are based on observations or model estimations made available *a posteriori*, which are then compared with previously defined policy conditions that dictate when a payout is due. However, because actions triggered by the model are not influenced by what has happened in reality (i.e. whether a loss event has actually taken place), model results can be seen as a retrospective forecast.~~

The ~~paradigm~~ proposed framework~~in this study~~ recognizes uncertainties and allows~~minimizes~~ basis risk to be minimized while maintaining a simple and transparent procedure, which is fundamental in parametric programmes.[13] Its structure also allows users to better understand basis risk and its underlying causes, and to take part in the decision-making process that leads to the maximization of utility that can be obtained from the system in a scientifically sound and objective manner.

The following section presents a motivating example for the development of the framework. Section 3 describes the methodology for model construction and evaluation. Section 4 depicts its application to the case study. Section 5 provides a summary of the framework presented and discusses possible extensions.

2. MOTIVATING EXAMPLE – PARAMETRIC INSURANCE FOR JAMAICAN FLOODING

We demonstrate the methodology by applying it to flood-related loss events caused by rainfall in Jamaica. Even if the methodology is, in principle, applicable to any hazard or region in the world, flooding in Jamaica is selected for various reasons. Jamaica is located in the Caribbean, a region where countries are particularly exposed to extreme rainfall and resulting floods. Such events are expected to increase in the future due to anthropogenic climate change.[24] There is therefore a pressing need to improve resilience for floods, especially in the developing world, and parametric risk transfer programmes can be instrumental in this context.[14,25] This is

confirmed by the fact that the Caribbean have a multi-country risk pool in place based on parametric insurance.[11] Jamaica is one of the largest islands in that region, it is highly vulnerable to natural hazards,[26] and availability of disaster data is reasonably good.

Our methodology requires historical samples of concurrent environmental and loss event data. To suit a parametric risk transfer programme, the environmental variable(s) should meet three basic requirements: a) span a sufficiently long historical period; b) be obtainable in near real-time; c) be based on a dataset and methodology that is homogeneous throughout the entire period, i.e. both in the historical and the operational period. In this example, the environmental variable of interest is rainfall, for which we adopt CMORPH Version 1.0 data. CMORPH is a method that produces global precipitation estimates from passive microwave and infrared data at high spatial (~8 km) and temporal (30 minute) resolution.[27] Homogeneous precipitation estimates over time are available from January 1998 to the present[28] and new data can be obtained with just an 18-hour delay.[29] All the above requirements are thus met. CMORPH is widely used in meteorology, hydrology and other fields[30]; an example is CCRIF SPC's excess rainfall parametric insurance product, which utilizes CMORPH data as one of the model's input variables.[11] Hence, we consider it a suitable option for use as the environmental variable in the construction of the model. Data from 01/01/1998 to 31/01/2016 are used, comprising $n = 6605$ days. Figure 1 illustrates daily rainfall obtained from CMORPH for the 177 cells with a ~8km resolution in Jamaica, on two days of particularly intense rainfall over the country.
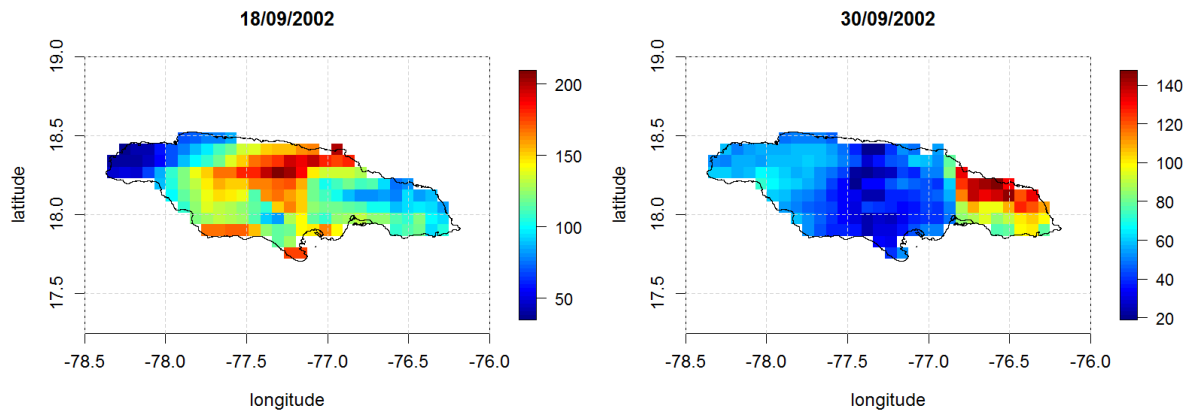
Figure 1: CMORPH daily rainfall (mm) in Jamaica on two illustrative days of intense rainfall.

EM-DAT is adopted as the source for raw historical disaster data. It is one of the main public disaster databases, maintained by the Centre of Research on Epidemiology of Disasters (CRED) and compiled from various sources, such as the UN, governmental and non-governmental agencies, insurance companies, research institutes and press agencies. Disaster data are collected at a country-aggregated level.[31]

The historical loss event catalogue used for model construction needs to contain data on past events that caused losses similar to or higher than those the risk cedant is interested in being covered against. In this study, events are selected assuming that the criteria for payout that the country of Jamaica is interested in matches EM-DAT's inclusion criteria, which ~~are~~require that one or more of the following occur: a) 10 or more fatalities; b) 100 or more people affected; c) the declaration of a state of emergency; d) a call for international assistance.

In order to minimize basis risk it is crucial to perform the model fitting using historical event data that are as accurate as possible. Data quality control supported by independent sources is therefore carried out. This aims to ensure that the catalogue start and end dates refer to the event that the model is intended to identify, which in this case is the occurrence of loss due to flooding, as it is unlikely that the reported dates on any one database reflect this specific definition. As

an example, if a flood was caused by heavy rainfall due to a tropical cyclone, the reported start

and end dates may refer to the days during which it passed over the country, rather than the

days during which damage took place due to resulting floods, or to the days during which a

state of emergency was in place.[32] Sources used to perform data quality control included

situation reports and press releases issued by the Government of Jamaica during the events

(available on ReliefWeb), reports from reputable sources such as the Economic Commission

for Latin America and the Caribbean (ECLAC) or the National Oceanic and Atmospheric

Administration (NOAA), local news articles, and research works. The historical event

catalogue is shown in Table I.

Table I: Historical event catalogue. Original EM-DAT disaster numbers, number of fatalities, people
affected and loss are included for reference.

EM-DAT Disaster No.	Start date	End date	Duration (days)	Fatalities	People affected	Loss (10^3 USD)
2001-0615	29/10/2001	05/11/2001	8	1	200	55 487
2002-0325	23/05/2002	02/06/2002	11	9	25 000	20 000
2002-0656	18/09/2002	20/09/2002	3	4	1 500	30
2002-0627	28/09/2002	30/09/2002	3			1 000
2004-0415	11/08/2004	13/08/2004	3	1	126	300 000
2004-0462	10/09/2004	12/09/2004	3	15	350 000	595 000
2005-0351	07/07/2005	09/07/2005	3	1	8 000	30 000
2005-0382	16/07/2005	18/07/2005	3	4	2 296	1 000
2005-0585	16/10/2005	21/10/2005	6	1	100	3 500
2006-0656	23/11/2006	24/11/2006	2	1	5 000	
2007-0360	19/08/2007	20/08/2007	2	4	33 188	300 000
2007-0523	29/10/2007	04/11/2007	7	1		
2008-0352	28/08/2008	29/08/2008	2	12	4 000	66 198
2010-0501	29/09/2010	30/09/2010	2	15	2 506	150 000
2012-0410	24/10/2012	24/10/2012	1	1	215 850	16 542

3. METHODS

This section describes the proposed probabilistic framework, which involves model

construction and evaluation. The objective of this work is first to develop a model that can issue

probabilities of occurrence of loss events given certain environmental variables. The model is

then evaluated and its value to users quantified, and ultimately provides a simple framework

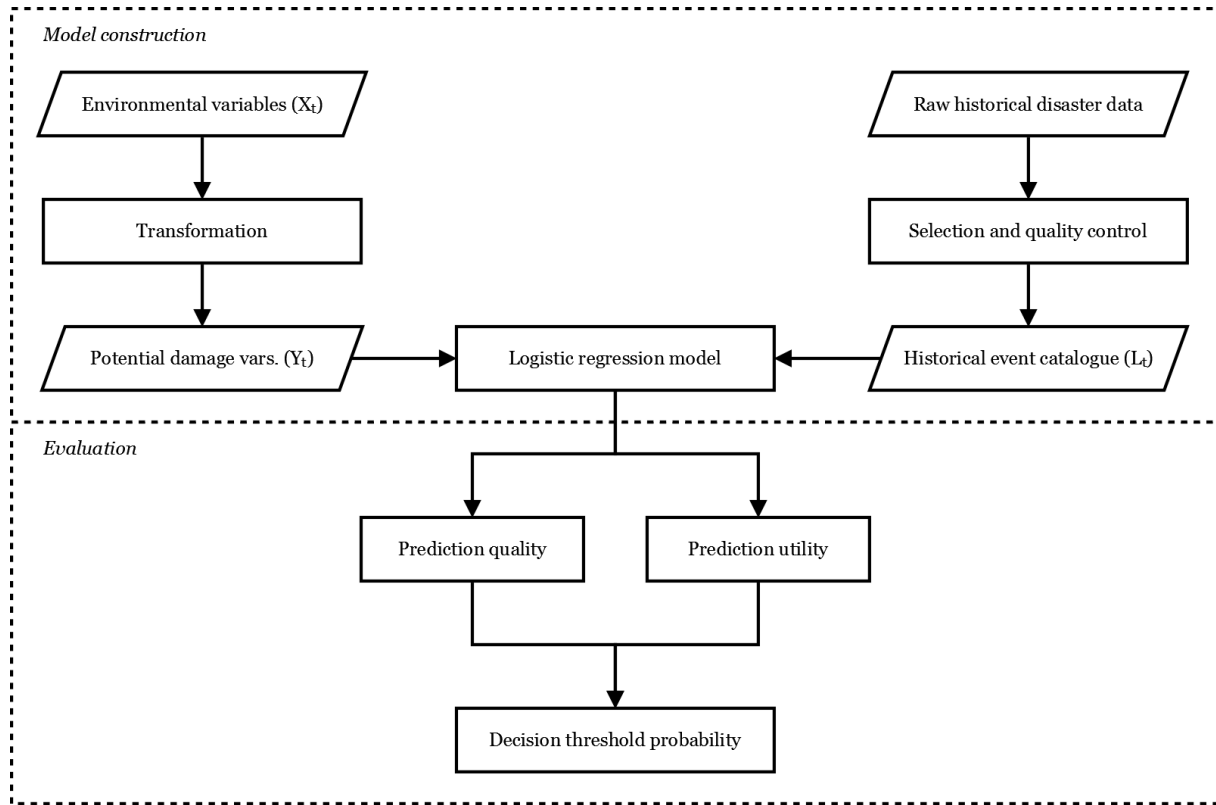for decision-making. The workflow is presented in Figure 2.

Figure 2: Workflow.

3.1.   Model construction

We start by proposing a generic probabilistic modelling framework, which is readily adaptable

to different natural hazards.

Consider the occurrence of loss caused by a natural hazard on each day $t = 1, \dots, T$ over some

region G and let L_t be a binary variable defined as

$$L_t = \begin{cases} 0 & \text{if loss occurs on day } t \text{ in } G, \\ 1 & \text{if loss doesn't occur on day } t \text{ in } G. \end{cases} \qquad (3.1)$$

More precisely "if loss occurs on day t" corresponds to "if day t is within a loss event's start and end dates".

The aim is to predict the occurrence of loss based on a potential damage variable Y_t defined for time t. The following logistic regression model gives a natural representation of the occurrence of loss on day t:

$$L_t \sim Bernoulli(p_t), \tag{3.2}$$

with

$$log\left(\frac{p_t}{1 - p_t}\right) = \beta_0 + \beta_1 Y_t, \tag{3.3}$$

where coefficient β_1 corresponds to the variable Y_t. The parameters β_0 and β_1 can be estimated by fitting the model to historical samples of concurrent potential damage and loss data. This model is readily ~~expanded~~ <u>expandable</u> to include additional explanatory variables.

The potential damage variable Y_t is obtained through the transformation of an environmental variable X_t, which represents the intensity of a certain natural hazard, so that

$$Y_t = D(X_t, \Phi), \tag{3.4}$$

where $D(.)$ is a non-linear operator designed to capture some of the physical processes of how the hazard creates damage, and Φ is the set of parameters of the transformation function. Within this framework, the predictive ability of the system can be improved without using explicit hazard, exposure and vulnerability models, which are instead emulated. The flexibility of such a framework also allows the environmental variable X_t to be directly used, which corresponds to the special case of $D(X_t, \Phi) = X_t$. An example application for floods is presented in Section 4.

The models of (3.3) and (3.4) assume stationarity. This is to aid clarity and because the flooding study of Section 4 uses a relatively short data series for which stationarity is a reasonable assumption. However, the regression framework means that extensions to allow nonstationarity, such as trends or annual variations in environmental variables, or changes in loss occurrence due to changes in vulnerability, are relatively straightforward.

3.2.   Evaluation

A logistic regression model constructed as described in Section 3.1 is able to produce predictive probabilities p_t for the occurrence of loss events caused by any natural hazard. However, parametric programmes require an unambiguous definition of when payouts are due or not, meaning that . ~~This means that~~ a decision threshold probability q above which a loss event is considered to occur must be set~~, effectively converting the issued probabilities into binary outcomes~~. The evaluation procedure described in this section consists of quantifying the quality and utility of the binary predictive systems obtained with the different possible threshold probabilities, ultimately enabling users of the system to define the optimal one. The verification measures proposed in this section may also be used in the evaluation of traditional parametric triggers.

3.2.1. Quality

A large number of different verification measures are available in the literature and, in most cases, more than one is necessary to obtain an informed picture about the quality of a predictive system.[33,34] The measures anticipated to be most relevant to the proposed probabilistic framework for parametric catastrophe risk transfer are now presented. They are formulated as a function of the number of hits (a), false alarms (b), misses (c) and correct rejections (d), which represent the four possible outcomes or contingencies for an event, as shown in Table II.

We adopt a simple daily event definition to aid clarity, as this allows unambiguous comparison between observed and modelled events. It should be noted that in practice, clusters of daily events are usually considered single disaster events, and it may be desirable to adopt this definition instead. However, because such events persist for varying durations, counting observed or modelled *non-events* in that case is not straightforward. There is not yet a natural solution to this issue, which warrants further research.[35]

Table II: Schematic contingency table for *n* binary events.

Event predicted	Event observed		
	Yes	No	Total
Yes	*a* (Hits)	*b* (False alarms)	*a* + *b*
No	*c* (Misses)	*d* (Correct rejections)	*c* + *d*
Total	*a* + *c*	*b* + *d*	*a* + *b* + *c* + *d* = *n*

Frequency bias, *B*, is the ratio between the number of predictions of occurrence and the number of actual occurrences:

$$B = \frac{a + b}{a + c}. \tag{3.5}$$

In general, a bias of 1 is desirable, meaning that events are predicted at the same rate at which they occur; in such cases, predictions are said to be unbiased. It should be noted that bias and skill are not necessarily related. A predictive system may be unbiased but have no skill, or vice-versa; analysing both is therefore necessary. In the case of parametric programmes, for which no model is perfect, a risk cedant may be more tolerant to false alarms than to missed events, for example. Then a bias greater than 1 would be preferred, which corresponds to a lower decision threshold probability. In practice, this could result in higher insurance premiums, but also reduces the probability that no payout would be issued following an event. This could

maximize the value of the system to its users, which is the main objective of the probabilistic framework.

In terms of skill, we first calculate the hit and false alarm rates. The hit rate, H, is the proportion of correctly predicted event occurrences, and is given by

$$H = \frac{a}{a + c}.$$
(3.6)

The false alarm rate, F, is the proportion of incorrectly predicted non-occurrences, given by

$$F = \frac{b}{b + d}.$$
(3.7)

By calculating these two measures for different decision threshold probabilities over the range 0 to 1, and plotting them against one another, a Receiver Operating Characteristic (ROC) curve is obtained.[36] A curve above the diagonal $H = F$ represents presence of skill, i.e., a better than random predictive system. However, caution needs to be exercised in its interpretation. While hit and false alarm rates are useful for understanding predictive performance, they are unsuitable as performance measures on their own. One reason is that they are degenerate for vanishingly rare events.[34] In other words, when the base rate $s = (a + c)/n$ decreases toward 0, so do H and F. This is likely to affect modelling of triggering events for parametric programmes due to the inherent rare nature of disasters caused by natural hazards. For this reason, to complete the analysis of model performance, we adopt the extremal dependence index, or EDI, which is given by

$$EDI = \frac{logF - logH}{logF + logH}.$$
(3.8)

Due to its properties, which include non-degeneracy, base-rate independency and asymptotical equitability, the EDI is particularly suited for the verification of predictions of rare binary

events.[37] It takes values in the interval [-1, 1], where zero distinguishes better- and worse-than-random predictions.

3.2.2. Utility

In Section 1, we defined basis risk as the risk associated with the mismatch between payout and loss, which, in the case of parametric triggers, arises when triggered payouts do not coincide with the occurrence of loss events. Basis risk should first be quantified using a suitable measure before it is minimized.

At first glance, a measure of prediction quality appears reasonable. Prediction quality can be defined as the degree of correspondence between predictions and observations, which directly relates with the definition of basis risk. However, the goodness of any forecast system is related not only with its predictive quality, but also to its utility, which is the economic value that it brings to its users. In fact, for users, a measure of value is generally more important than a measure of quality, as they are primarily concerned with the expected benefit that such a system will bring in the context of their respective decision-making problems.[33] Even though quality and utility are related, predictions with greater accuracy or skill may not necessarily be the most valuable to end users.[38] Therefore, defining the optimal decision threshold can only be achieved by maximizing utility, which goes beyond the standard definition of basis risk and leads to an objective maximization of the economic benefit that users can obtain from the system.

The general framework that allows users of a binary predictive system to quantify the value that they can obtain from it is now described. Table II shows the four possible of combinations of event prediction and occurrence. Each outcome has an associated expense, which can be expressed in the form of an expense matrix (Table III).

Table III: Schematic expense matrix.

Event predicted	Event observed	
	Yes	No
Yes	E_a	E_b
No	E_c	E_d

The mean expense of using a certain predictive system can be obtained by multiplying the expected relative frequencies as expressed in Table II by the corresponding expenses in Table III,[39] so that

$$E_{system} = \frac{a}{n}E_a + \frac{b}{n}E_b + \frac{c}{n}E_c + \frac{d}{n}E_d.$$ (3.9)

While Equation (3.9) allows calculating the mean expense, it is also helpful to calculate a measure of value, which corresponds to the economic benefit obtained by using the predictive system. To do so, let us first define a baseline for the definition of the value of the predictions. Although different possibilities could be chosen, here we assume that the baseline corresponds to a case where loss events are never predicted to occur (i.e. $H = 0$; $F = 0$). In this case, the average expense is given by

$$E_{baseline} = sE_c + (1-s)E_d.$$ (3.10)

Value can then be defined as

$$V = E_{baseline} - E_{system}.$$ (3.11)

The mean expense associated with a perfect predictive system, in which model predictions and observations always agree (i.e. $H = 1$; $F = 0$), can also be informative, and is given by

$$E_{perfect} = sE_a + (1-s)E_d,$$ (3.12)

18

which corresponds to the absolute upper bound on the value that can be obtained from the system.

When presented with predictions in the form of probabilities, users face the question of what is the decision threshold probability q that maximizes the value that they can obtain from it. Varying the threshold over the range 0 to 1 allows a sequence of values $V(q)$ to be calculated. This allows the maximum value to be found, which corresponds to the optimal decision.

We illustrate the framework for value evaluation through a simplified model of the decision process from the perspective of two of the users of a hypothetical parametric insurance product:

1. The insured party or risk cedant, which is interested in transferring part of its risk of sustaining losses due to a certain natural hazard;

2. A catastrophe risk manager, the technical expert responsible for setting up and running the model that triggers payouts based on the occurrence of a pre-defined condition.

We now define the expense matrices associated with the predictive system, starting with the insured party. Let E_A represent the payout that the country wants to receive from the insurer should a loss event occur. ~~For simplicity, w~~We consider that in case of correspondence between event prediction and occurrence, <u>an insurance payout takes place corresponding to post-disaster funding expectations from the country,</u> ~~the insurance payout perfectly offsets the losses~~, and that therefore there is no net gain or loss for the country. Now suppose that E_P defines the insurance premium, which is the amount of money that the country must pay for the insurance policy. This is given by

$$E_P(q) = \frac{a(q) + b(q)}{n} E_A m = B(q)sE_A m, \qquad (3.13)$$

where m corresponds to the relative margin of profit of the insurer ($m > 1$). <u>To aid clarity, this illustrative pricing model does include factors such as volatility.</u> Lastly, suppose that $E_N > E_A$

is constant and represents the losses that the country will sustain when a loss event occurs but no payout is issued. Thus E_N includes indirect economic costs that may arise as a consequence of lack of funding to finance post-disaster response and recovery.[40]

Let us now define the expenses for the second party, the catastrophe risk manager. Suppose that E_C is the operational cost related with administrative actions that need to be taken whenever the model triggers a payout, E_R is the cost associated with the reputational loss and model recalibration, which is incurred whenever the model triggers a payout that does not correspond to an actual loss event, and E_L is the cost associated with the reputational loss and potential loss of client, which may happen if the model fails to trigger a payout when a loss event occurs.

Table IV shows the expense matrices for the two parties. Substitution into Equation (3.9) allow mean expenses to be calculated.

Table IV: Expense matrices for different users of the system.

User	Event predicted	Event observed	
		Yes	No
Insured party	Yes	$E_P(q)$	$E_P(q) - E_A$
	No	$E_P(q) + E_N$	$E_P(q)$
Risk manager	Yes	E_C	$E_C + E_R$
	No	E_L	0

Note that in parametric risk transfer products only one threshold can be set in the policy conditions. It is possible that no single threshold will be optimal for users with different expense matrices. This means that the overall maximum value may not correspond to the maximum value for all individual users. Nevertheless, even in situations where this cannot be achieved, this framework provides a means to take decisions on probability thresholds that are acceptable and beneficial to all users.

4. RESULTS

This section illustrates the framework by applying it to flooding in Jamaica.

4.1. Model construction

Suppose that the study region, G, corresponds to the country of Jamaica and let $X_t(g_j)$ represent the rainfall amount accumulated over cell g_j on day t. Now suppose that L_t defines whether day t coincides with a flooding event.

Before proceeding with the model fitting, it is sensible to analyse the environmental variable *vis-à-vis* the historical disaster data. Even if the former is not expected to be the best predictor for the latter, as described below, in the case of floods one can reasonably expect that on loss event days, rainfall is higher than on most other days. Examining the conditional probability distributions of rainfall on event and non-event can serve as a useful sanity check. In Figure 3, quantile-quantile plots and kernel density plots of such distributions for CMORPH daily rainfall data above 5 mm are presented, showing that daily rainfall is in fact higher on loss event days. This suggests that rainfall is likely to be informative for loss.
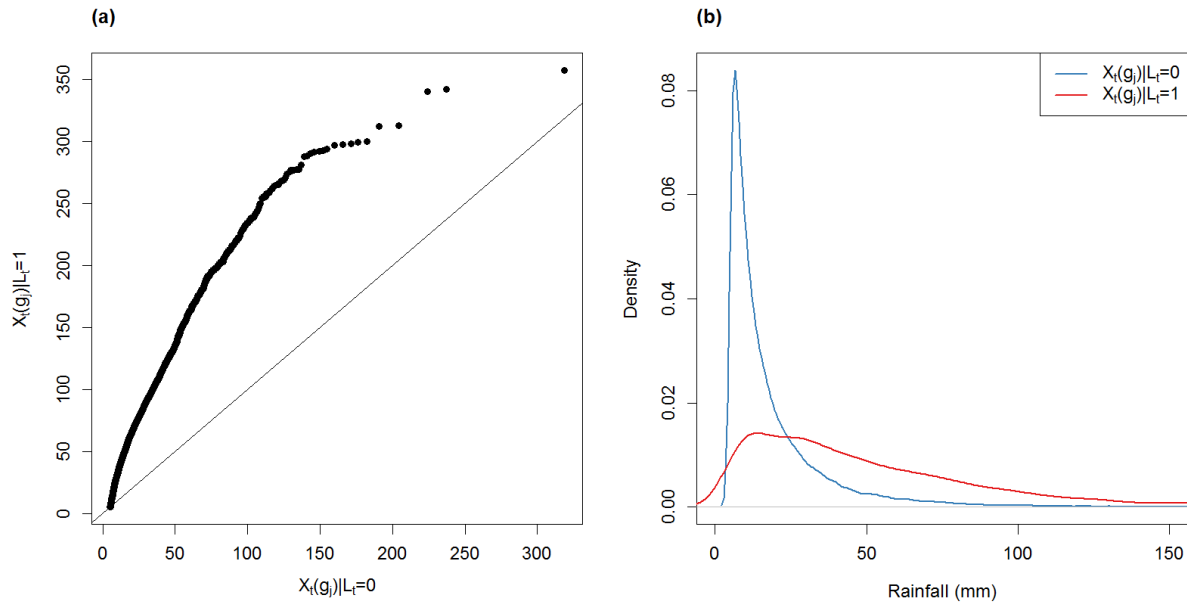
Figure 3: CMORPH daily rainfall above 5 mm. (a) Quantile-quantile plots. (b) Kernel density plots.

Flood damage is not directly caused by rainfall, but rather from different actions originated by water flowing and submerging assets located on land that is usually dry. Therefore, even if here we study flood damage caused by rainfall, rainfall itself – the environmental variable – is undoubtedly not the best predictor for the model. Within more traditional flood risk models, hydrologic and hydraulic models, which route rainwater to the exposed assets, are combined with exposure and vulnerability models, which represent the built environment and the damaging phenomena.[41–43] These models tend to be quite complex and so have various drawbacks. These include decision-makers potentially finding models difficult to interpret; development or implementation being arduous; large amounts of data being required in order to estimate models, which may not always be available; or that resulting estimates may still be accompanied by large uncertainties.[44,45] We therefore propose a variable transformation that aims to emulate the physical processes behind the occurrence of flood damage due to rainfall. It is divided into two steps: estimation of potential runoff based on daily rainfall, and of a potential damage index, given runoff. These are described below.

The rainfall-runoff mechanism is a key physical process that has been widely studied in hydrology[46,47] and depends on several geomorphological and climatic parameters. For simplicity, we only aim to capture what are considered its two dominant effects[48]: 1) infiltration of rainfall in the soil, which makes the relationship between rainfall and runoff strongly non-linear; 2) overland flow, which produces a spatial and temporal aggregation of the rainfall.

Regarding the first, not all the rainfall produces runoff, but part of it infiltrates into the soil according to its characteristics (e.g. porosity, hydraulic conductivity) and water content. The simplest approach to reproduce this effect is to adopt a constant parameter u, which represents the daily rate of the infiltration. The resulting potential runoff, or amount of rainwater estimated to remain over the surface, is

$$R_t(g_j) = max\{X_t(g_j) - u, 0\}. \qquad (4.14)$$

Concerning the second, overland flow accumulates the excess of rainfall over the surface of the hydrological catchment. In hydrology, this process is modelled by the convolution of the rainfall with a function representing the hydrological response of the catchment. We reproduce it through a weighted moving time average, which preserves the accumulation effect and allows the contribution of rainfall on previous days to be weighted according to transformation parameters. We restrict the moving average to a three-day period, which is reasonable for the size of the study area. Including additional days did not improve the model fit. The potential runoff volume accumulated over cell g_j over days $t, t-1, t-2$ is given by

$$R_t^*(g_j) = \theta_0 R_t(g_j) + \theta_1 R_{t-1}(g_j) + \theta_2 R_{t-2}(g_j), \qquad (4.15)$$

where $\theta_0, \theta_1, \theta_2 > 0$ and $\theta_0 + \theta_1 + \theta_2 = 1$.

Finally, let Y_t be an explanatory variable related to potential damage for day t, which is defined as

$$Y_t = \sum_{j=1}^{J} \frac{R_t^*(g_j)^\lambda - 1}{\lambda}, \tag{4.16}$$

where the Box-Cox transformation offers a flexible, non-linear approach to converting runoff to potential damage for each cell without requiring explicit exposure and vulnerability models. The summation in equation (4.16) is designed to capture the belief that damage accumulates over grid cells.

In order to obtain the Y_t variable that best describes potential flood losses due to rainfall, the transformation parameters u, θ_1, θ_2 and λ, defined in the previous subsection, are optimized to give the final logistic regression model. This is achieved ~~my~~ by maximizing the likelihood using a quasi-Newton algorithm. Pseudo-R^2 is calculated for a first assessment the goodness-of-fit of the model. The statistic proposed by Nagelkerke[49] is adopted, which gives $R^2 = 0.548$. This suggests that the model has good predictive skill. It is worth ~~nothing~~ noting that pseudo-R^2 values for logistic regression models cannot be interpreted in the same way as the non-pseudo-R^2 used for linear regression models, as they are normally lower.[50]

The computed parameters of both the variable transformation procedure and the logistic regression model are shown in Table V~~Table I~~. The logistic regression model is plotted in Figure 4. It can issue probabilities of occurrence of flooding loss events due to rainfall for any given day. In Figure 5, both the input data and the results obtained along the model construction process are shown in the form of a time series covering two events that took place in September of 2002 (Hurricane Isidore and Hurricane Lili). The figure illustrates all the steps presented in Section 3.1 in a simple way. The bottom panel shows the probabilities of loss that the model would have estimated for each day over the displayed period, including for those two events.

Table V: Parameters of both the variable transformation procedure and logistic regression model.

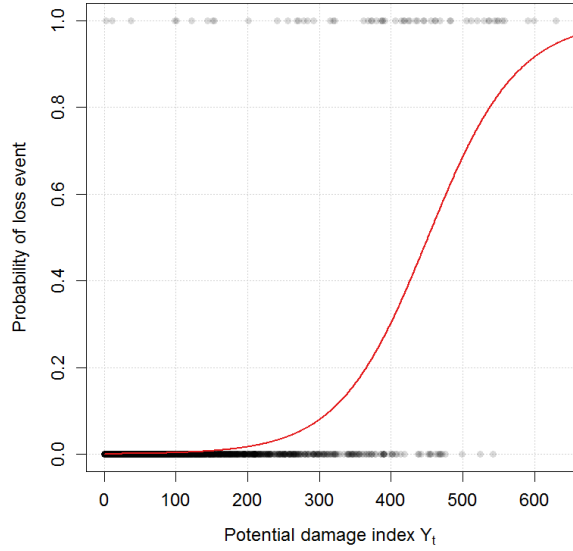| Variable transformation	
u	8.396
θ_0	0.554
θ_1	0.277
θ_2	0.169
λ	-0.132
Logistic regression model	
β_0	-7.277
β_1	0.016

Figure 4: Constructed logistic regression model.
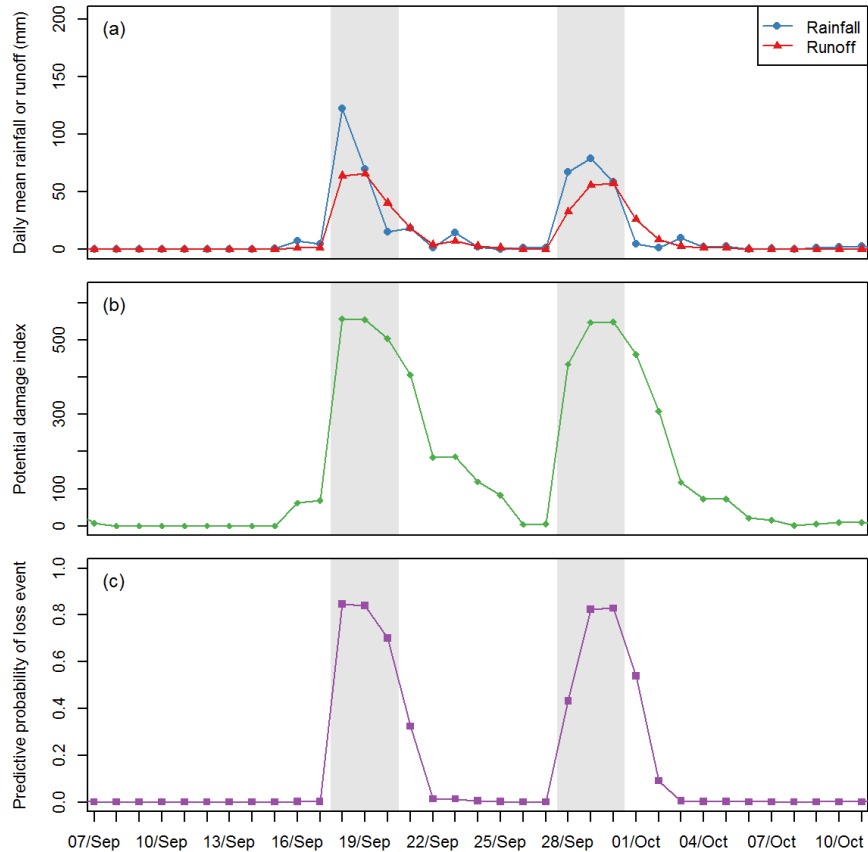
Figure 5: Time series showing two flooding loss events that took place in Jamaica in 2002: (a) Mean daily rainfall (blue) and mean daily runoff (red) over all the grid cells in the country. (b) Potential damage index. (c) Predictive probabilities of loss events produced by the logistic regression model. Grey areas represent event days.

4.2. Evaluation

We next evaluate the model's predictive quality as well as the utility it brings to users when different decision threshold probabilities q over the range of 0 to 1 are used. Figure 6 shows the frequency bias B over this range of probabilities. An unbiased predictive system, with $B = 1$, will not necessarily correspond to the optimal one, but can still serve as a useful reference. In this case, an unbiased system would be obtained for a threshold probability $q = 0.27$. Assuming

stationarity, a lower value would lead to a rate of event prediction higher than that of event
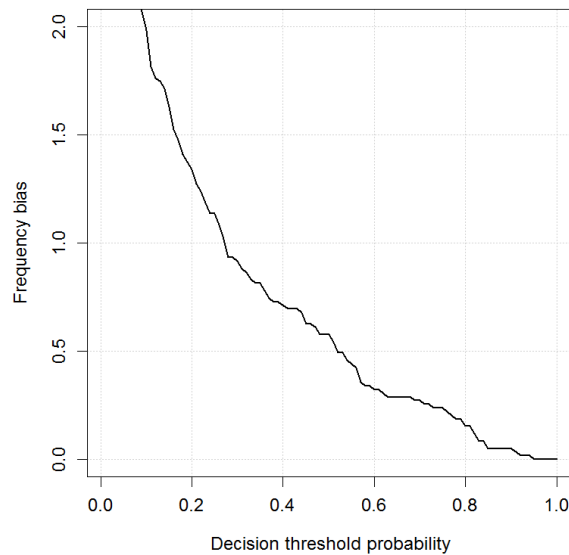
occurrence, and vice versa.

Figure 6: Frequency bias.

The ROC curve, shown in Figure 7a, is markedly above the diagonal $H = F$, strongly suggesting

that the constructed model has good predictive skill. However, the low value of the base rate,

which in the Jamaica case study is $s = 59/6605 \simeq 0.0089$, inevitably leads to low values of

H and F and potentially contributes to this behaviour. To complement the analysis, we also

calculate the Extremal Dependency Index EDI over the range of decision threshold

probabilities. As shown in Figure 7b, EDI is considerably higher than 0 over the entire range,

supporting the idea that the model has good predictive skill. Its value tends to decrease as the
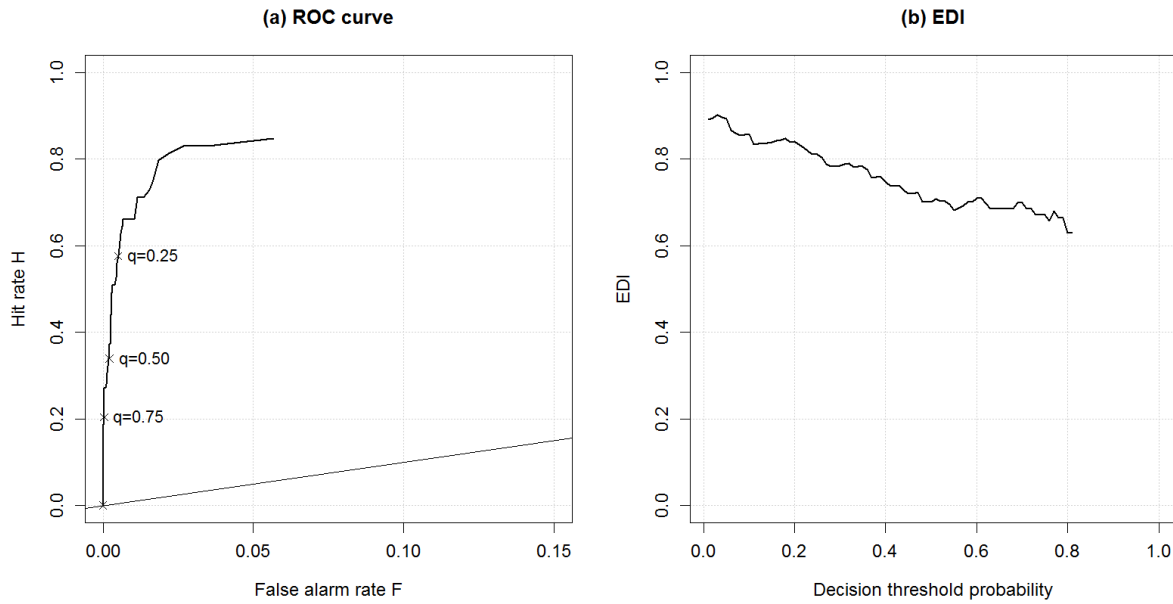
threshold probability increases.

Figure 7: Skill measures: (a) Receiver Operating Characteristic (ROC) curve. (b) Extremal Dependency Index (EDI).

The proposed verification procedures show that the logistic regression model illustrated in this section is skilful, and should be able to support a hypothetical parametric programme for Jamaica. The final step is to define the decision threshold probability. To do so, as previously discussed, analysing quality measures is insufficient: utility must also be quantified. However, based on quality measures alone, threshold probabilities between 0.10 and 0.30 appear reasonable due to relatively high skill (high H and low F, EDI close to the maximum) and low bias (around 1).

In order to quantify utility, consider the following hypothetical expenses from the perspective of two users, an insured party, in this case the government of Jamaica, and a risk manager, as described in Section 3.2.2: E_A = \$1 000 000; E_N = \$ 2 000 000; m = 1.15; E_C = \$5 000; E_R = \$20 000; E_L = \$350 000. Figure 8 shows the expenses for the two users over the range of decision threshold probabilities.
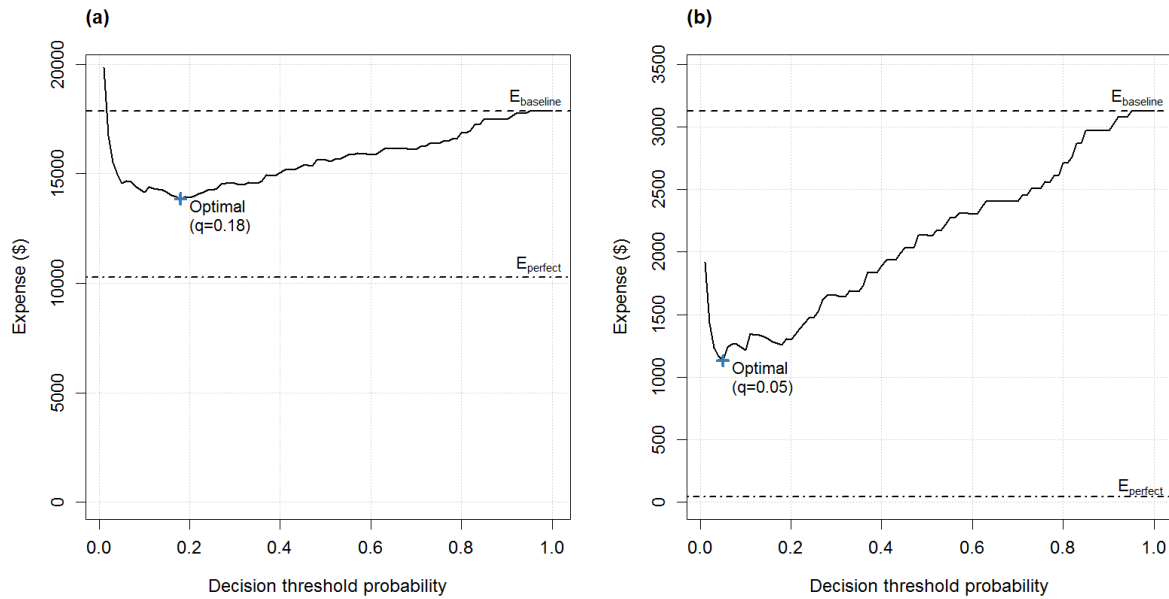
Figure 8: User expenses over the range of decision threshold probabilities. The plus sign in blue corresponds to the lowest expense for each user: (a) Insured party. (b) Risk manager.

In this case, the optimal decision threshold for the insured party is $q = 0.18$, corresponding to an expected expense $E = \$14\,170.0$ (Figure 8a). This means that the maximum possible benefit for the country is objectively achieved by defining that payouts should occur when the model issues a probability of occurrence of loss events q above 0.18. However the above threshold is not optimal for the risk manager. Instead their threshold probability that would maximize value is $q = 0.05$, which corresponds to an expected expense $E = \$1\,130.0$ (Figure 8b).

The above example illustrates how different users of an imperfect predictive system may have different optimal decision thresholds. However, how can this issue be addressed when only one threshold can be set, as in the context of parametric triggers? The answer is case dependent. A possible approach could be to select the threshold that maximizes the sum of value over all users. Yet, in some cases, this could be unfair to some of the users, possibly even resulting in negative value. On the other hand, in a hypothetical scenario where no single threshold can be agreed upon, users may take advantage of the information provided by the system in order to

adjust their expense matrices. In this example suppose that the risk manager is unable to persuade the country, who is the main client, into defining a threshold other than $q = 0.18$. In that case, the information provided by the system could be used by the risk manager to adjust their expense matrix, in order to align the optimal decision threshold with the insured party's. This could be achieved by promoting training actions that would improve understanding of the model, for example, thus reducing reputational losses associated with failure to correctly predict a loss event occurrence.

5. CONCLUSION

This framework has been designed to provide a probabilistic basis for a parametric insurance product. The framework quantifies natural hazard event occurrence using environmental variables. This is achieved using logistic regression to establish a relationship between the probability of an event occurring and the environmental variables. This relationship may be directly established or via potential damage variables constructed from the environmental variables. For example, when modelling flooding over Jamaica, the probability of a flooding event is related to rainfall run-off, which is derived from gridded rainfall data aggregated over Jamaica and over a three-day period.

The framework also includes an explicit approach for users to calculate mean expenses from predicted probabilities. Often this will require that users only specify expenses for scenarios that are relatively straightforward to elicit. This is demonstrated by considering optimal pay-out criteria for an insured party and risk manager in the case of a parametric insurance product covering Jamaican flooding. Within the framework we can also verify predictions and ensure that they are fit for purpose. Methods from the forecast verification literature are drawn upon to achieve this. These verify ~~both the accuracy and reliability of~~ prediction~~s~~ quality, which in turn ensures the ~~reliability~~ quality of subsequent loss calculations.

Various extensions may improve the proposed framework. A by-product of a logistic regression model is that uncertainty in the relationship between event occurrence and environmental or damage variables may be quantified. Such uncertainty is readily propagated through to loss occurrence estimates. This has been neglected here in favour of brevity and to aid clarity. Such uncertainty is likely to be largest with natural hazards that have low occurrence rates or are supported by relatively short data records. Due to the latter we assume stationarity in occurrence rates when modelling Jamaican flooding, which corresponds to an aggregate assumption of stationarity for environmental variables and vulnerabilities. However, fFor other natural hazards, or where data records are longer, capturing nonstationarity may improve loss estimate precision. Improved precision may also be achieved when event occurrence data are scarce by extending the framework so that data are pooled (for example, over multiple countries, or by allowing serial dependence over time) or deficiencies in data are recognised (which motivated rigorous quality control of Jamaican flood start and end dates in Section 2).

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