# Continuous Obstructed Detour Queries 

Rudra Ranajee Saha<br>Department of CSE, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh darklord.saha@gmail.com<br>\section*{Tanzima Hashem}<br>Department of CSE, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh tanzimahashem@cse.buet.ac.bd

## Tasmia Shahriar

Department of CSE, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh shahriartasmia@gmail.com

Lars Kulik
Dept of CIS, University of Melbourne, Melbourne, Australia
lkulik@unimelb.edu.au


#### Abstract

In this paper, we introduce Continuous Obstructed Detour (COD) Queries, a novel query type in spatial databases. COD queries continuously return the nearest point of interests (POIs) such as a restaurant, an ATM machine and a pharmacy with respect to the current location and the fixed destination of a moving pedestrian in presence of obstacles like a fence, a lake or a private building. The path towards a destination is typically not predetermined and the nearest POIs can change over time with the change of a pedestrian's current location towards a fixed destination. The distance to a POI is measured as the summation of the obstructed distance from the pedestrian's current location to the POI and the obstructed distance from the POI to the pedestrian's destination. Evaluating the query for every change of a pedestrian's location would incur extremely high processing overhead. We develop an efficient solution for COD queries and verify the effectiveness and efficiency of our solution in experiments.


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## 1 Introduction

Efficient processing of location-based queries in the presence obstacles like a river, a fence or a private property has become an important research area in recent years. Obstructed space is different from road networks and the Euclidean space, which ignore the obstacles in the space. It is not possible to adapt the query processing algorithms for the Euclidean space or road network settings to the obstructed space as the presence of obstacles brings new challenges for processing location-based queries in real time. Considering the importance of the applications of obstructed location-based queries for pedestrians, in the last few years, researchers have developed solutions $[1,5,16,20]$ for variant location-based queries in the obstructed space that were previously addressed in the Euclidean space or road networks.

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(a) Euclidean Space.

(b) Obstructed Space.

Figure 1 An Example of a Continuous Detour Query for $k=2$.

We introduce a Continuous Obstructed Detour (COD) query that allows a moving pedestrian continuously monitor the POI with the smallest obstructed detour distance, which is measured as the summation of distances from the user's current location to the POI, and from the POI to the user's destination by avoiding the obstacles. For example, a tourist enjoying a scenic view may not follow a predetermined walking path and instead want to visit a restaurant or a souvenir shop before arriving at the hotel. A pedestrian roaming around the city may want to buy a medicine from a pharmacy before she goes to her usual bus stop to home. In both scenarios, users have fixed destinations but do not have a predetermined path to reach the destination, and need to visit a POI before reaching the destinations.

A COD query can be extended to a COkD query that continuously returns $k$ POIs with the $k$ smallest detour distances for a moving user heading towards a fixed destination. Figure 1 shows an example of a continuous detour query for $k=2$ in both Euclidean and obstructed space. The Euclidean distance is measured as the length of the direct line connecting two locations. In Figure 1(a), when a user is at $l_{c}$, POIs $p_{1}$ and $p_{2}$ are the 1st and 2nd nearest detour POIs based on the Euclidean distances. When the user moves to $l_{c}{ }^{\prime}$, the answer changes, and $p_{3}$ and $p_{4}$ become the 1st and 2 nd nearest detour POIs based on the Euclidean distances. In Figure 1(b), the obstacles are shown using rectangles. The obstructed distance is the length of the shortest path between two locations without crossing any obstacle. Figure 1(b) shows that $p_{2}$ and $p_{1}$ are the 1st and 2 nd obstructed nearest detour POIs when the user is at $l_{c}$. When the user moves to $l_{c}{ }^{\prime}$, the answer changes, and $p_{15}$ and $p_{3}$ become the 1st and 2nd obstructed nearest detour POIs.

The COkD query cannot be modeled and processed as a continuous obstructed nearest neighbor (POI) query because of the presence of a destination. Although the obstructed distance of a POI to the destination is constant, it differs for multiple POIs, and the obstructed nearest detour POI is determined with respect to both current location and destination of the moving pedestrian. Hence, the solution $[10]$ for moving $k$ nearest neighbor ( $k \mathrm{NN}$ ) queries in the obstructed space is not applicable for $\mathrm{CO} k \mathrm{D}$ queries.

Since the path to reach the destination is not predefined, for a CO $k$ D query, the obstructed nearest detour POIs need to be re-evaluated in real time with respect to every changed location and the destination location of the moving user. Thus, a COkD query can be processed with the repeated evaluation of obstructed $k$ detour ( $\mathrm{O} k \mathrm{D}$ ) queries, where an $\mathrm{O} k \mathrm{D}$ query returns $k$ obstructed nearest detour POIs with respect to a user's current location and destination. Researchers have proposed obstructed $k$ group nearest neighbor (OkGNN) algorithms $[15,16]$ that return $k$ POIs having $k$ smallest obstructed aggregate distances with respect to multiple query locations. An OkGNN query is same as an obstructed $k$ detour ( $\mathrm{O} k \mathrm{D}$ ) query when the number of query location is two. However, the straightforward
application of the $\mathrm{O} k \mathrm{D}$ algorithm for processing a $\mathrm{CO} k \mathrm{D}$ query is not feasible as it would incur extremely high processing overhead, specially in the obstructed space as the computation of the number of obstructed distance increases with the increase of the number of the query re-evaluation. The search for the obstructed nearest detour POIs independently using $\mathrm{O} k \mathrm{D}$ queries accesses the same POIs and obstacles multiple times. Thus, the major challenges for processing a COkD query efficiently is to reduce the frequency of the query re-evaluation and the retrieval of the same POIs and obstacles from the database.

To address the challenges for a COkD query, we develop a safe region [10, 13] based solution that avoids the re-evaluation of the query as much as possible. The key idea of our safe region based approach is to retrieve the obstructed nearest detour POIs from a database with respect to a user's current location and destination, and then identify the regions, obstructed integrated safe region (OISR) and obstructed safe regions (OSRs) with respect to the retrieved POIs. We exploit geometric properties to compute such regions. If a user resides in the OISR, the user's movement does not change the order of already retrieved $k$ obstructed nearest detour POIs. Thus, the computation of an OISR avoids the re-computation of the query answer. If a user leaves an OISR, we compute obstructed safe regions (OSRs) with respect to the retrieved POIs to check whether new POIs are required to be retrieved from the database. Computation of OSRs allows us to avoid the retrieval of the same POIs multiple times, which in turn decreases the number of same obstacles retrieved for computing the obstructed distances of the POIs.

To further improve the efficiency of our approach, we propose two algorithms: a single point retrieval method (SPRM) and a multiple point retrieval method (MPRM), to retrieve new POIs from the database when a moving user leaves the current safe region. The aim of SPRM and MPRM is to refine the POI search space, i.e., reduce the number of the retrieval of new POIs, for identifying $k$ obstructed nearest detour POIs with respect to the current location $l_{c}$ and destination $d$ of a moving user. A smaller number of retrieved POIs reduces the computational overhead and I/O cost for retrieving obstacles from the database.

The key difference between SPRM and MPRM is that SPRM incrementally retrieves obstructed nearest detour POIs with respect to the first location and the destination of the moving user (e.g., $l_{c}$ and $d$ in Figure 1), whereas for MPRM the obstructed nearest detour POIs are retrieved with respect to few of the current locations and destination of the moving user (e.g., $l_{c}$ and $d$, and $l_{c}{ }^{\prime}$ and $d$ in Figure 1). SPRM does not retrieve the same POI multiple times but may retrieve additional POIs, whereas MPRM reduces the retrieval of additional POIs in return of increasing the number of obstructed distance computations with respect to multiple locations (e.g., $l_{c}$ and $l_{c}{ }^{\prime}$ in Figure 1).

We summarize our key contributions below:

- We introduce and formulate $\mathrm{CO} k \mathrm{D}$ queries in spatial databases that allow pedestrians to monitor the nearest detour POIs in the presence of obstacles.
- We develop an efficient safe-region based solution for processing COkD queries. To the best of our knowledge, we are the first to address the problem of $\mathrm{CO} k \mathrm{D}$ queries.
- We develop two algorithms, SPRM and MPRM, to refine the POI search space and retrieve new POIs in the refined search space.
- We perform extensive experiments using a real data set to show the efficiency and effectiveness of our proposed solution.

Table 1 A List of Symbols.

| Notation | Description | Notation | Description |
| :--- | :--- | :--- | :--- |
| $k$ | The number of required nearest detour POIs | $x$ | The number of auxiliary POIs |
| $l_{c}$ | The current location | $d$ | The destination |
| $l_{f}$ | The location from where a user starts to move | $o_{i}$ | An obstacle |
| $l_{s}$ | The location used to compute safe regions | $O$ | The set of obstacles |
| $z$ | The $(k+x)^{t h}$ nearest POI of $l_{s}$ | $P$ | The set of all POIs |
| $p_{i}$ | A POI | $L$ | The list (set) of $(k+x)$ obstructed nearest detour POIs |
| $A$ | The set of $k$ obstructed nearest detour POIs for $l_{c}$ and $d$ | $T_{p}$ | POI R-tree |
| $d_{e}(p, q)$ | Euclidean distance between $p$ and $q$ | $d_{\Delta}(p, q)$ | Obstructed distance between $p$ and $q$ |
| $s_{e}(a, b, c)$ | Summation of $d_{e}(a, b)$ and $d_{e}(b, c)$ | $s_{e \Delta}(a, b, c)$ | Summation of $d_{e}(a, b)$ and $d_{\Delta}(b, c)$ |
| $s_{\Delta}(a, b, c)$ | Summation of $d_{\Delta}(a, b)$ and $d_{\Delta}(b, c)$ | $T_{o}$ | Obstacle R-tree |

## 2 Problem Formulation

In a $\mathrm{CO} k \mathrm{D}$ query, initially, a moving user provides her current location $l_{c}$, a destination $d$ and the number $k$ of desired nearest (detour) POIs. Later the moving user periodically updates her current location $l_{c}$. The obstructed space may include obstacles like buildings, parks, lakes, etc. An obstructed path is calculated as the shortest path between two points in the obstructed space, where a path does not intersect the interior of an obstacle. The obstructed distance between two points is the length of the obstructed path between those points. The obstructed detour distance $s_{\Delta}\left(l_{c}, p_{i}, d\right)$ of a POI $p_{i}$ is measured as the summation of the obstructed distances from $p_{i}$ to $l_{c}$ and $p_{i}$ to $d$. Similar to existing work in the obstructed space $[16,15]$, the POIs and obstacles are indexed using two separate $R$-trees [7], POI $R$-tree and obstacle $R$-tree in the database. Table 1 summarizes the symbols used in the paper.

A COkD query is formally defined as follows.

- Definition 1. A Continuous Obstructed $\boldsymbol{k}$ Detour Query: Given a set of POIs $P$ and a set of obstacles $O$, the current location $l_{c}$ of a moving user, a destination $d$, and the required number of the obstructed nearest detour POIs $k$, a $\operatorname{CO} k \mathrm{D}$ query returns $A$, a set of $k$ obstructed nearest detour POIs that have $k$ smallest obstructed detour distances with respect to every instance of $l_{c}$ and $d$, i.e., $s_{\Delta}\left(l_{c}, p_{i}, d\right) \leq s_{\Delta}\left(l_{c}, p_{j}, d\right)$ for $p_{i} \in A$ and $p_{j} \in P-A$.


## 3 Related Work

Efficient approaches have been proposed in the literature for variants of spatial queries in the obstructed space. Processing spatial queries in the presence of obstacles has been first addressed in [19]. In [6, 17, 19], the authors developed algorithms to find the nearest POIs with respect to a static location in the obstructed space. In [5], the authors developed an algorithm to process continuous obstructed nearest neighbor queries. In [4] and [1], the authors developed solutions for efficient processing of obstructed reverse nearest neighbor queries and obstructed optimal sequenced route queries, respectively. In [20], the authors addressed obstructed range nearest neighbor queries. Obstructed group nearest neighbor (OGNN) queries that return a POI with the minimum obstructed aggregate distance have been addressed in $[15,16]$. An OGNN query transforms to an obstructed detour query if the number of query location is two (i.e., a user's current location and destination). This paper focuses on the COkD query, which is different from all of the above mentioned queries.

In [19], the authors proposed the first algorithm to compute the obstructed distance between two locations. Instead of directly applying the obstructed distance computation algorithm between two locations, in [16], the authors developed an algorithm to efficiently compute multiple obstructed distances with respect to a single point without retrieving same obstacles multiple times. To compute the obstructed detour distance, we need to compute two obstructed distances from a common POI, and thus, we use the algorithm in [16].

Continuous nearest neighbor queries [3, 11] and continuous detour queries $[12,14,18]$ have been addressed in road networks that ignore the presence of the obstacles. In [12], the authors proposed an incremental approach using a shortest path tree to process continuous detour queries in the road network. In [14, 18], the authors developed a solution for detour queries with an assumption that a user travels in a predetermined path towards a destination. In $\mathrm{CO} k \mathrm{D}$ queries, a pedestrian's path towards a destination is not known before and can be obstructed by the obstacles.

Researchers have already shown that computing safe regions can significantly reduce the query processing overhead for processing moving nearest neighbor queries $[8,10,13]$. However, none of these approaches take the destination into account, and thus, the computed safe regions are not applicable for a $\mathrm{CO} k \mathrm{D}$ query, where a pedestrian moves towards a fixed destination.

## 4 Safe Regions

We develop a safe region based approach for processing COkD queries. The underlying idea is to identify the safe regions based on already retrieved POIs, obstructed integrated safe region (OISR) and the intersection of obstructed safe regions (OSRs), where the query answer does not change and any new POI does not need to be retrieved from the database for a moving user, respectively. These regions can help us to reduce the computational overhead and the retrieval of same POIs multiple times from the database. The larger the safe regions, the smaller is the number of times POIs need to be retrieved from the database. Considering this issue, we retrieve auxiliary POIs in addition to the required number $(k)$ of POIs with an intuition that additional POIs can reduce the processing overhead. The number of auxiliary POIs $x$ is decided in experiments.

Suppose that $L$ is a set of ordered $k+x$ obstructed nearest detour POIs that have been retrieved from the database with respect to a moving user's location $l_{s}$ and a fixed destination $d$ for $x \geq 0$. An OSR of a retrieved POI represents the area, where a user's movement cannot incur another POI that has not yet been retrieved from the database to have a smaller obstructed detour distance than the retrieved POI. Thus, additional POIs are not retrieved from the database if a user moves inside the intersection of the OSRs of the retrieved POIs. An OISR represents an area where the current COkD answer for a moving user does not change. To compute the OISR, in addition to OSRs we need to know the obstructed fixed rank region (OFRR) that represents the area where a user's movement does not change the relative ranking (based on the obstructed detour distance) of the retrieved POIs in $L$.

In Sections 4.1 and 4.2 , we show how the presence of a fixed destination $d$ makes the computation of OSRs and OFRR different from the existing OSR and OFRR computation techniques for obstructed nearest neighbor queries [10]. In Section 4.3, we combine OSRs and OFRR to compute an OISR.

### 4.1 Obstructed Safe Region (OSR)

Let $z$ represent the POI that has the $(k+x)^{t h}$ smallest obstructed detour distance with respect to $l_{s}$ and $d$. Based on the retrieved $k+x$ obstructed nearest POIs with respect to $l_{s}$ and $d$, we first define the obstructed known region: a set of points that have equal or smaller obstructed detour distances than that of $z$ with respect to $l_{s}$ and $d$. Figure 2(a) shows an obstructed known region for $k=2$ and $x=1$, where $p_{1}, p_{2}$, and $p_{3}$ have been retrieved as $k+x$ obstructed nearest detour POIs with respect to $l_{f}$ and $d$. Note that $l_{f}$ is the location from where a user starts to move, and $l_{s}$ is the location used to compute safe regions. Thus,

(a) Known Region.

(b) $O S R_{\Delta}\left(l_{s}, p_{1}\right)$.

(c) $O S R_{\Delta}\left(l_{s}, p_{2}\right)$.

Figure 2 (a) Known Region, and (b)-(c) OSRs.
the first time when a safe region is computed, both $l_{f}$ and $l_{s}$ point to the same location. In all figures, we only show $l_{f}$ and we assume that the safe regions are computed for the first time and $l_{s}$ points to $l_{f}$.

Let $p_{o}$ be a POI located outside the obstructed known region that has not yet been retrieved from the database. For a POI $p_{i}$ in the obstructed known region, the obstructed safe region with respect to $p_{i}$, denoted by $O S R_{\Delta}\left(l_{s}, p_{i}\right)$, is defined as follows:

$$
\begin{align*}
& O S R_{\Delta}\left(l_{s}, p_{i}\right)=\left\{l \mid s_{\Delta}\left(l, p_{i}, d\right) \leq s_{\Delta}\left(l, p_{o}, d\right)\right\} \\
& =\left\{l \mid d_{\Delta}\left(l, p_{i}\right)+d_{\Delta}\left(p_{i}, d\right) \leq d_{\Delta}\left(l, p_{o}\right)+d_{\Delta}\left(p_{o}, d\right)\right\} \tag{1}
\end{align*}
$$

Here $l$ refers to a point location. Thus $O S R_{\Delta}\left(l_{s}, p_{i}\right)$ is a set of points, where each point $l$ satisfies $s_{\Delta}\left(l, p_{i}, d\right) \leq s_{\Delta}\left(l, p_{o}, d\right)$.

From the definition of the known region, $d_{\Delta}\left(l_{s}, p_{o}\right)+d_{\Delta}\left(p_{o}, d\right) \geq d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)$. Rearranging we have, $d_{\Delta}\left(l_{s}, p_{o}\right) \geq d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)-d_{\Delta}\left(p_{o}, d\right)$. On the other hand, according to the triangular inequality, $d_{\Delta}\left(l_{s}, l\right)+d_{\Delta}\left(l, p_{o}\right) \geq d_{\Delta}\left(l_{s}, p_{o}\right)$. By rearranging and replacing $d_{\Delta}\left(l_{s}, p_{o}\right)$ with its tighter bound, we have the tighter bound of $d_{\Delta}\left(l, p_{o}\right)$ as $d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)-d_{\Delta}\left(p_{o}, d\right)-d_{\Delta}\left(l_{s}, l\right)$.

In Equation 1, if we can guarantee that $\left(d_{\Delta}\left(l, p_{i}\right)+d_{\Delta}\left(p_{i}, d\right)\right)$ is less than or equal to a tighter bound of $\left(d_{\Delta}\left(l, p_{o}\right)+d_{\Delta}\left(p_{o}, d\right)\right)$, i.e., $d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)-d_{\Delta}\left(l_{s}, l\right)$, then $d_{\Delta}\left(l, p_{i}\right)+$ $d_{\Delta}\left(p_{i}, d\right) \leq d_{\Delta}\left(l, p_{o}\right)+d_{\Delta}\left(p_{o}, d\right)$ is satisfied. Thus, we can redefine $O S R_{\Delta}\left(l_{s}, p_{i}\right)$ as follows:

$$
\begin{align*}
& O S R_{\Delta}\left(l_{s}, p_{i}\right) \\
& =\left\{l \mid d_{\Delta}\left(l, p_{i}\right)+d_{\Delta}\left(p_{i}, d\right) \leq d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)-d_{\Delta}\left(l_{s}, l\right)\right\} \\
& =\left\{l \mid d_{\Delta}\left(l, p_{i}\right)+d_{\Delta}\left(l_{s}, l\right) \leq d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)-d_{\Delta}\left(p_{i}, d\right)\right\} \tag{2}
\end{align*}
$$

Figures 2(b) and 2(c) show OSRs for $p_{1}$ and $p_{2}$, respectively for the same example shown in Figure 2(a). According to Equation 2, if a moving user's current location $l_{c}$ satisfies $d_{\Delta}\left(l_{s}, l_{c}\right)+d_{\Delta}\left(l_{c}, p_{i}\right) \leq d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)-d_{\Delta}\left(p_{i}, d\right)$, then the user is inside the OSR of $p_{i}, O S R_{\Delta}\left(l_{s}, p_{i}\right)$, and any POI $p_{o}$ outside the obstructed known region cannot have a detour distance smaller than that of $p_{i}$ with respect to $l_{c}$ and $d$. If the user's current location $l_{c}$ is inside the intersection of the OSRs for all $(k+x)$ POIs in the obstructed known region, i.e., $\bigcap_{i=1}^{k+x} O S R_{\Delta}\left(l_{s}, p_{i}\right)$, then it is guaranteed that any POI $p_{o}$ outside the obstructed known region cannot have a detour distance smaller than those for $(k+x)$ POIs in the obstructed known region with respect to $l_{c}$ and $d$.

### 4.2 Obstructed Fixed Rank Region (OFRR)

The OFRR represents an area where the ranking of $k$ obstructed nearest detour POIs in $L$ does not change. We compute an OFRR using the concept of a dominant region. In [10],


Figure 3 (a)-(b) Dominant Regions, (b) Approximate OFRR (c), and (d) OFRR (Shaded Areas).
for a moving obstructed nearest POI query, an obstructed dominant region of POI $p_{i}$ over POI $p_{j}$ is defined as $\beta_{\Delta}\left(p_{i}, p_{j}\right)=\left\{l \mid d_{\Delta}\left(l, p_{i}\right)<=d_{\Delta}\left(l, p_{j}\right)\right\}$. We modify the definition of a dominant region for a $\mathrm{CO} k \mathrm{D}$ query as follows:

$$
\begin{equation*}
\beta_{\Delta}\left(p_{i}, p_{j}\right)=\left\{l \mid s_{\Delta}\left(l, p_{i}, d\right)<=s_{\Delta}\left(l, p_{j}, d\right)\right\} \tag{3}
\end{equation*}
$$

For a COkD query, an OFRR for an ordered POI set $L$ can be computed as follows:

$$
\begin{equation*}
O F R R(L)=\bigcap_{i=1}^{|L|-1} \beta_{\Delta}\left(p_{i}, p_{i+1}\right) \tag{4}
\end{equation*}
$$

To reduce the complexity of the computation of OFRRs, we first approximate a dominant region of POI $p_{i}$ over POI $p_{j}$ as $\beta_{e \Delta}{ }^{\prime}\left(p_{i}, p_{j}\right)=\left\{l \mid s_{e \Delta}\left(l, p_{i}, d\right)<=s_{e \Delta}\left(l, p_{j}, d\right)\right\}$.

Using the approximate dominant regions, we compute the approximate OFRR (AOFRR) for $L$ as follows:

$$
\begin{equation*}
\operatorname{AOFRR}(L)=\bigcap_{i=1}^{|L|-1} \beta_{e \Delta}{ }^{\prime}\left(p_{i}, p_{i+1}\right) \tag{5}
\end{equation*}
$$

We continue with the same example shown in Figure 2(a). Figures 3(a) and 3(b) show the approximate dominant region of $p_{1}$ over $p_{2}, \beta_{e \Delta}{ }^{\prime}\left(p_{1}, p_{2}\right)$ and the approximate dominant region of $p_{2}$ over $p_{3}, \beta_{e \Delta}{ }^{\prime}\left(p_{2}, p_{3}\right)$, respectively.

After computing the $\operatorname{AOFRR}(L)$ using Equation 5, we identify the non visible region inside $\operatorname{AOFRR}(L)$ for every POI in $L$. Let $N V R_{i}$ be a non visible region for a POI $p_{i}$ and $N V R$ be the union of non visible regions with respect to all POIs in $L$. Thus, an OFRR for an ordered POI set $L$ can be computed from the approximated OFRR as follows:

$$
\begin{equation*}
O F R R(L)=\operatorname{AOFRR}(L)-N V R \tag{6}
\end{equation*}
$$

Figures 3(c) and 3(d) show $\operatorname{AOFRR}(L)$ and $\operatorname{OFRR}(L)$, respectively, where $L=\left\{p_{1}, p_{2}, p_{3}\right\}$. $\operatorname{AOFRR}(L)$ (shaded area) in Figure 3(c) is computed as the intersection areas between the shaded areas, $\beta_{e \Delta}{ }^{\prime}\left(p_{1}, p_{2}\right)$ and $\beta_{e \Delta}{ }^{\prime}\left(p_{2}, p_{3}\right)$, in Figures 3(a) and 3(b), respectively. $O F R R(L)$ in Figure 3(d) is computed by removing the nonvisible regions of $p_{1}, p_{2}$, and $p_{3}$ from $\operatorname{AOFRR}(L)$, i.e., $\operatorname{OFR} R(L) \subseteq \operatorname{AOFRR(L)}$.

### 4.3 Obstructed Integrated Safe Region (OISR)

The obstructed integrated safe region, denoted by OISR, is the area, where a user's movement does not change the COkD query answer. It is the intersection of OSR and OFRR. Formally the OISR can be defined as follows:

$$
\begin{equation*}
\operatorname{OISR}\left(l_{s}, L\right)=\operatorname{OFRR}(L) \cap \bigcap_{i=1}^{k} O S R_{\Delta}\left(l_{s}, p_{i}\right) \tag{7}
\end{equation*}
$$



Figure 4 OISR.


Figure 5 Non Visible Region for $p_{1}$.

The shaded area in Figure 4 shows the OISR for the same example shown in Figure 2(a). However, computing intersections of safe regions for every POI is expensive. The following theorem shows that the intersection of $\operatorname{OFRR(L)}$ and $O S R_{\Delta}\left(l_{s}, p_{k}\right)$ is enough to generate $\operatorname{OISR}\left(l_{s}, L\right)$.

- Theorem 2. Given a set of retrieved ordered POIs L with respect to a moving user's locations $l_{s}$ and $d$, the obstructed safe region $O S R_{\Delta}\left(l_{s}, p_{i}\right)$ for every $i^{\text {th }}$ nearest POI $p_{i}$ of $l_{s}$ in $L$, the obstructed fixed rank region $\operatorname{OFRR}(L)$, then $\operatorname{OFRR}(L) \cap \bigcap_{i=1}^{k} O S R_{\Delta}\left(l_{s}, p_{i}\right)=$ $O F R R(L) \cap O S R_{\Delta}\left(l_{s}, p_{k}\right)$.
Proof. Suppose $l_{c}$ is a location in $\operatorname{OFRR}(L) \cap \bigcap_{i=1}^{k} O S R_{\Delta}\left(l_{s}, p_{i}\right)$. Since $l_{c}$ is a location inside $\operatorname{OFRR}(L)$, for $i \in[1 . . k-1]$, the following equation also holds:

$$
\begin{equation*}
d_{\Delta}\left(l_{c}, p_{i}\right)+d_{\Delta}\left(p_{i}, d\right) \leq d_{\Delta}\left(l_{c}, p_{k}\right)+d_{\Delta}\left(p_{k}, d\right) \tag{8}
\end{equation*}
$$

Since $l_{c} \in O S R_{\Delta}\left(l_{s}, p_{i}\right)$, from Equation 2, we have

$$
\begin{equation*}
d_{\Delta}\left(l_{c}, p_{i}\right)+d_{\Delta}\left(p_{i}, d\right) \leq d_{\Delta}\left(l_{s}, z\right)+d_{\Delta}(z, d)-d_{\Delta}\left(l_{s}, l_{c}\right) \tag{9}
\end{equation*}
$$

Now if Equation 9 holds for location $l_{c}$ and $i=k$, then according to Equation 8, Equation 9 also holds for $l_{c}$ and $i \in[1 . . k-1]$. Thus, $\operatorname{OFRR}(L) \cap \bigcap_{i=1}^{k} O S R_{\Delta}\left(l_{s}, p_{i}\right)=$ $O F R R(L) \cap O S R_{\Delta}\left(l_{s}, p_{k}\right)$.

## 5 Algorithms

In this section, we present our COkD query processing algorithm (Algorithm 1) using safe regions computed in Section 4. The input to the algorithms are a current location $l_{c}$, a destination $d$, the number of required nearest detour POIs $k$, and the number of auxiliary POIs $x$. The output of the algorithm is the set of $k$ obstructed nearest detour POIs $A$. Both $l_{c}$ of a moving user and $A$ are updated periodically. The algorithm uses a priority queue $Q_{p}$ and lists $L$ and $L^{\prime}$ to process a COkD query. $Q_{p}$ is used to store already accessed $R$-tree nodes and POIs. $L$ is a set of ordered $k+x$ obstructed detour POIs with respect to $l_{c}$ and $d$. The list $L^{\prime}$ includes POIs that are not in $L$ but have been retrieved from the database for finding $k+x$ obstructed nearest detour POIs.

Algorithm 1 starts with initializing $l_{f}$ and $l_{s}$ as $l_{c}$, where $l_{f}$ is a moving user's start location and $l_{s}$ is a location used to compute the last safe regions. Then the algorithm retrieves $k+x$ obstructed nearest detour POIs with respect to $l_{f}$ and $d$ using the function RetrievePOIs in $L$ (Line 2), adds first $k$ obstructed nearest detour POIs in $L$ to $A$ (Line 3), and sends $A$ to the

```
Algorithm 1 COkD_Process.
Input: \(l_{c}, d, k, x\)
Output: \(A\)
    \(l_{f}, l_{s} \leftarrow l_{c}\)
    \(L \leftarrow\) RetrievePOIs \(\left(l_{f}, d, k, x\right)\)
    \(A \leftarrow\) FindAnswer \(\left(l_{c}, d, k\right)\)
    \(\operatorname{Send}(A)\)
    for every update of \(l_{c}\) do
    flagOISR,\(L \leftarrow\) CheckOISR \(\left(l_{s}, l_{c}, d, k, x, L\right)\)
    if \(\mathrm{flagOISR}=1\) then
        Send(A)
    else
        flagOSR \(\leftarrow\) CheckOSRs \(\left(l_{s}, l_{c}, d, k, x, L\right)\)
        if \(\mathrm{flag} O S R=0\) then
            \(L \leftarrow\) RetrieveNextPOIs \(\left(l_{f}, l_{c}, d, k, x, L\right)\)
            \(l_{s} \leftarrow l_{c}\)
        end if
        \(A \leftarrow F\) FindAnswer \(\left(l_{c}, d, k\right)\)
        \(\operatorname{Send}(A)\)
    end if
    end for
```

user (Line 4). The function RetrievePOIs incrementally retrieves Euclidean nearest detour POIs with respect to $l_{c}$ and $d$ from the database until $k+x$ obstructed nearest detour POIs for $l_{c}$ and $d$ have been identified. After every update of the current location $l_{c}$, the algorithm checks whether the current location $l_{c}$ is in $\operatorname{OISR}\left(l_{s}, L\right)$ using the function CheckOISR. The function returns 1 if $l_{c} \in O I S R, 0$ otherwise. The steps of the function CheckOISR are discussed in Section 5.1.

If the function CheckOISR returns 1 (i.e., $l_{c} \in O I S R$ ), then Algorithm 1 sends $A$ to the user without any further computation (Lines 7-8). On the other hand, if the function CheckOISR returns 0 (i.e., $l_{c} \notin O I S R$ ), then Algorithm 1 checks whether $l_{c}$ is in the intersection of OSRs of POIs $\left\{p_{1}, p_{2}, \ldots, p_{k}\right\}$ in $L$ using the function CheckOSRs (Line 10). The function checks the condition stated in the last line of Equation in 2 to determine whether $l_{c} \in O S R\left(p_{i}\right)$ of a POI $p_{i}$. If the condition is false for the OSR of any POI in $\left\{p_{1}, p_{2}, \ldots, p_{k}\right\}$, the function returns 0 . If the condition is true for all POIs, then the function returns 1, i.e., $l$ is in the intersection of OSRs of POIs $\left\{p_{1}, p_{2}, \ldots, p_{k}\right\}$ in $L$.

If $\operatorname{flag} O S R=1$, then the algorithm does not need to retrieve any new POI. If $\operatorname{flag} O S R=$ 0 , then the algorithm retrieves $k+x$ nearest detour POIs in $L$ with respect to $l_{c}$ and $d$ using the function RetrieveNextPOIs (Line 12). For the function RetrieveNextPOIs, we develop two efficient methods: SPRM (Section 5.2) and MPRM (Section 5.3) with the aim to minimize the number of the retrieval of POIs for finding $k+x$ nearest detour POIs with respect to $l_{c}$ and $d$. After retrieving new POIs using the function RetrieveNextPOIs, $l_{s}$ is updated as $l_{c}$ (Line 13). Finally, Algorithm 1 adds first $k$ obstructed nearest detour POIs in $L$ to $A$ from $L$ and sends $A$ to the user (Lines 15-16).

```
Algorithm 2 CheckOISR.
Input: \(l_{s}, l_{c}, d, k, x, L\)
Output: flagOISR and \(L\)
    \(N V R \leftarrow\) ComputeNVR \(\left(L, l_{s}, d\right)\)
    if \(l_{c} \in N V R\) then
        return \(0, L\)
    end if
    flag,\(L \leftarrow\) CheckPOIOrder \(\left(L, l_{c}, d\right)\)
    if flag \(=1\) then
        return \(0, L\)
    else
        return \(l_{c} \in O S R\left(p_{k}\right), L\)
    end if
```


### 5.1 Function CheckOISR

The steps of this function is shown in Algorithm 2. The inputs to the algorithm are $l_{s}, l_{c}, d$, $k, x$, and $L$. The outputs are a flag flagOISR and the $L$. From Equation 7, we know that $\operatorname{OISR}\left(l_{s}, L\right)$ is the intersection of $\operatorname{OFRR}(L)$ and $\operatorname{OSR}\left(p_{k}\right)$. The function first computes non visible region $N V R$ as the union of non visible regions with respect to all POIs in $L$ (Line 1). if the direct path between a location and a POI is obstructed then the location is non-visible from the POI. Thus any location of a non visible region for a POI does not have a direct path to that POI. Figure 5 shows an example of non visible region (represented with two lines ab and ce) for POI $p_{1}$ with respect to obstacle $O_{1}$ by ignoring the presence of other obstacles. Non visible regions can be computed using a visibility graph [2, 9]. The vertices of a visibility graph represent POIs and corner points of the obstacles, and there is an edge between two vertices if the direct path between those vertices is not obstructed. To reduce the computational overhead, after computing a non visible region $N V R_{i}$ for a POI $p_{i}$, it is stored and reused in the query evaluation process unless any new obstacle is retrieved.

Since $\operatorname{OFRR}(L)$ can be computed as $\operatorname{AOFRR}(L)-N V R$ (Equation 6), if $l_{c}$ in $N V R$ then $l_{c}$ is not in $\operatorname{OFRR}(L)$. Again from Equation 7, $\operatorname{OISR}\left(l_{s}, L\right)=\operatorname{OFRR}(L) \cap O S R_{\Delta}\left(l_{s}, p_{k}\right)$. Thus, if $l_{c}$ in $N V R$ then $l_{c}$ is also not in $\operatorname{OISR}\left(l_{s}, L\right)$. In such a scenario, Algorithm 2 returns flagOISR as 0 and $L$ without any modification (Lines 2-4).

Otherwise, using the function CheckPOIOrder, Algorithm 2 computes obstructed detour distances of POIs in $L$ with respect to $l_{c}$ and $d$, and sorts the POIs in $L$, if the order of POIs based on computed obstructed detour distances changes (Line 5). If the order is changed, flag is set to 1 and Algorithm 2 returns flagOISR as 0 and updated $L$ (Lines 6-7). Otherwise, Algorithm 2 checks whether $l_{c} \in \operatorname{OSR}\left(p_{k}\right)$ using the condition stated in the last line of Equation in 2 and returns flagOISR as 1 or 0 and $L$ without any modification, if the condition stated in the last line of Equation in 2 is satisfied or not, respectively.

### 5.2 SPRM

POIs are indexed using an $R$-tree in the database. To identify $(k+x)$ obstructed nearest detour POIs for $l_{c}$ and $d$, SPRM incrementally retrieves Euclidean detour POIs with respect to $l_{f}$ and $d$, where from $l_{f}$ the user starts to move towards a destination $d$. A priority queue $Q_{p}$ stores already accessed $R$-tree nodes and POIs in order of the minimum Euclidean detour

Table 2 Experiment Settings.

| Parameter | Range | Default value | Parameter | Range | Default value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $k$ | $1-20$ | 10 | $x$ | $1-20$ | 12 |
| Query Range $R$ | $500-3000$ units | 1500 units | $\|P\| /\|O\|$ Ratio | $50-350$ | 200 |

distances with respect to $l_{f}$ and $d$. To avoid the retrieval of the same POIs multiple times and reduce I/O access, SPRM does not start the search for the POIs from the root node of the $R$-tree while incrementally retrieving the POIs with respect to $l_{f}$ and $d$.

The POI search space that has been already traversed is an ellipse with foci at $l_{f}$ and $d$ and the major axis having the length equal to the Euclidean detour distance of the last retrieved POI with respect to $l_{f}$ and $d$ from $Q_{p}$. SPRM determines the current $(k+x)^{\text {th }}$ smallest obstructed detour distance of $l_{c}$ and $d$ based on the already retrieved POIs. The ellipse expands with the retrieval of new POIs from $Q_{p}$. With the retrieval of a new POI, SPRM updates the current $(k+x)^{t h}$ smallest obstructed detour distance of $l_{c}$ and $d$ if it becomes smaller. The search ends when the minimum Euclidean detour distance of $l_{c}$ and $d$ from the boundary of the ellipse becomes greater than or equal to the current $(k+x)^{t h}$ smallest obstructed detour distance of $l_{c}$ and $d$.

### 5.3 MPRM

Since SPRM expands the POI search space (i.e., ellipse) with respect to fixed locations $l_{f}$ and $d$, some retrieved POIs may never become part of the $\mathrm{CO} k \mathrm{D}$ answer with respect to the updated location $l_{c}$ and $d$. To avoid the retrieval of those additional POIs, MPRM retrieves new POIs with respect to $l_{c}$ and $d$ instead of $l_{f}$ and $d$. Similar to SPRM, MPRM does not start the search from the root of the POI $R$-tree node and reuses the already traversed nodes of the POI $R$-tree. However, MPRM incurs additional processing overhead for computing the minimum Euclidean detour distances with respect to $l_{c}$ and $d$ for the nodes/POIs stored in $Q_{p}$.

MPRM sorts the already retrieved POIs according to the obstructed detour distance with respect to $l_{c}$ and $d$. Then MPRM resorts the elements in $Q p$ based on their Euclidean detour distances with respect to $l_{c}$ and $d$. The algorithm continues to retrieve the next Euclidean nearest detour POI $p$ with respect to $l_{c}$ and $d$ from $Q_{p}$ as long as the Euclidean detour distance of $p$ with respect to $l_{c}$ and $d$ is smaller than the current $(k+x)^{t h}$ smallest obstructed detour distance of $l_{c}$ and $d$ based on already retrieved POIs.

## 6 Experiments

We present the performance of our safe region based approach using both SPRM and MPRM and compare them with a naive approach (NA) that independently finds $k$ obstructed nearest detour POIs for every location update of a moving user using the $\mathrm{O} k \mathrm{D}$ algorithm proposed in [16] (please see Section 3) for details. We use both real and synthetic data sets. The total space is normalized into $10,000 \times 10,000$ square units. The real dataset of Germany consists of 36334 Minimum Bounding Rectangles (MBRs) of railway lines (rrlines) and 76999 MBRs of hypsography data (hypos). In this dataset, end points of hypos represent POIs, and rrlines are the obstacles. Though we use MBRs to represent obstacles, our approach is applicable for obstacles of any shape. We also use the real datasets of rivers and lakes in Greece as obstacles, and generate synthetic POIs using uniform random distribution. We denote the synthetic dataset (Greece dataset) by ' $S$ ' and Germany dataset by ' $G$ '.


Figure 6 Effect of the number of required POIs $k$.

We use a 2.4 GHz Intel i5 CPU and 16 GB main memory. Table 2 shows the range and default values of our experiment parameters. To observe the effect of a parameter in an experiment, we set other parameters to their default values.

For every experiment, we consider 200 sample $C O k D$ queries and takes the average performance in terms of the computational time and I/O costs for retrieving POIs and obstacles from the database. For every COkD query sample, we randomly generate $l_{f}$ and $d$ according to the specified range in the experiment. Then we randomly generate $l_{c} \mathrm{~S}$ in the following way: a user moves towards the direction of $d$ but the followed path may not be the shortest one for arriving at $d$. Though the distance between two $l_{c} \mathrm{~s}$ is kept fixed, the number of $l_{c} \mathrm{~s}$ may vary for two paths having $l_{f}$ and $d$ in the same query range (e.g., 3000 units). Therefore, we show the average computational time and I/Os required per $l_{c}$ for a path as the cost of a COkD query sample.

### 6.1 Effect of the Number of Required POIs $k$

Figure 6 shows that the required computational time and I/Os are higher for the naive approach than those for our safe region based approach for varying $k$. From Figures 6(a) and $6(\mathrm{~d})$, we observe that the computational time increases rapidly for the naive approach than our safe region based approach for higher values of $k$. This is because with the increase of $k$, for both SPRM and MPRM, the safe regions become larger and the probability for $l_{c}$ to remain inside OISR increases, which avoids the re-computation of $\mathrm{CO} k \mathrm{D}$ answer. On the other hand, the naive approach requires to evaluate the obstructed nearest detour POIs for every update of $l_{c}$ and the time required for the evaluation increases for the higher values of $k$.

Figures 6(b), 6(e), 6(c) and 6(f) show that the I/O cost for both POIs and obstacles increases with the increase of $k$, which is expected because the number of POIs and obstacles retrieved from the database increase with the increase of $k$.


Figure 7 Effect of the number of auxiliary POIs $x$.

### 6.2 Effect of the Number of Auxiliary POIs $x$

Figure 7 shows that the computational time and I/O cost for the naive approach is higher than our approach but remain same irrespective of values of $x$ because the naive approach does not retrieve auxiliary POIs. On the other hand, we observe that for MPRM the performance improves with the increase of $x$ upto a certain threshold then again degrades. The reason is as follows. With the increase of $x$, the area of safe region becomes larger and the query processing overhead decreases, but after certain threshold with the increase of $x$, the cost for computing the non visible regions diminishes the gain achieved from the large safe regions. For SPRM, we observe that the performance degrades with the increase of $x$. This is because for SPRM, POIs are always retrieved with respect to $l_{f}$, and the retrieval of POIs that are not required increases with the increase of $x$.

### 6.3 Effect of the Query Range $\boldsymbol{R}$

In this experiment, we vary $R$ from 500 meter to 3000 meter by considering the typical travelling distance of a pedestrian. Figure 8 shows that SPRM performs better than MPRM, which can be explained from the underlying structure of SPRM and MPRM. It is expected that set of nearest detour POIs remain same for several timestamps, and the number of POIs and obstacles retrieved with respect to $l_{c}$ and $d$ is small. On the other hand, MPRM needs to compute obstructed detour distances for every element in $Q_{p}$ with respect to $l_{c}$. Therefore, SPRM performs better than MPRM.

The performance of both naive and safe region based approaches degrades with the increase of $R$. Since the distance between consecutive $l_{c} \mathrm{~S}$ increases with the increase of $R$, the probability that $l_{c}$ falls outside the safe region also increases and more POIs and obstacles need to be retrieved from the database.

### 6.4 Effect of POI-Obstacle Ratio $|\boldsymbol{P}| /|O|$

Figure 9 shows the comparative performance between the naive approach and the safe region based approach for varying the ratio of the number of POIs and the number of obstacles $|P| /|O|$. Increase of $|P| /|O|$ ratio means that the sample space contains more POIs than


Figure 8 Effect of the query range $R$.


Figure 9 Effect of POI-obstacle ratio $[P] /[O]$.
obstacles. With the increase of $|P| /|O|$, the I/O cost for POIs increases for both SPRM and MPRM, which is expected. For SPRM, the I/O cost of obstacles decreases because less number of obstacles are retrieved with respect to fixed locations $l_{f}$ and $d$. However, for MPRM, the I/O cost of obstacles increases because detour obstructed distances of POIs are computed with respect to different locations.

## 7 Conclusion

We have introduced and formulated $\mathrm{CO} k \mathrm{D}$ queries. We have proposed the first approach based on safe regions for efficient processing of $\mathrm{CO} k \mathrm{D}$ queries. We have further improved the efficiency of our approach by developing two POI retrieval algorithms: SPRM and MPRM.

We have performed experiments using both real and synthetic datasets. The results show that our approach for $\mathrm{CO} k \mathrm{D}$ queries with SPRM requires on average $67.3 \%$ less processing time, $62 \%$ less I/Os for obstacles and $72.6 \%$ less I/Os for POIs than the naive approach that applies the existing $\mathrm{O} k \mathrm{D}$ query processing algorithm to evaluate for $\mathrm{CO} k \mathrm{D}$ queries. On the other hand, our approach with MPRM requires on average $69.2 \%$ less processing time, $67 \%$ less I/Os for obstacles and $72 \%$ less I/Os for POIs than the naive approach.

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