Time Evolution of Density Parameters for Matter and Dark Energy and their Interaction Term in Brans-Dicke Gravity

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Abstract

In the framework of Brans-Dicke (BD) theory, the present study determines the time dependence of BD parameter, energy density and equation of state (EoS) parameter of the cosmic fluid in a universe expanding with acceleration, preceded by a phase of deceleration. For this purpose, a scale factor has been so chosen for the present model that the deceleration parameter, obtained from it, shows a signature flip with time. Considering the dark energy to be responsible for the entire pressure, the time evolution of energy parameters for matter and dark energy and the EoS parameter for dark energy have been determined. A term, representing interaction between matter and dark energy, has been calculated. Its negative value at the present time indicates conversion of matter into dark energy. It is evident from the present study that the nature of dependence of the scalar field upon the scale factor plays a very important role in governing the time evolution of the cosmological quantities studied here. This model has an inherent simplicity in the sense that it allows one to determine the time evolution of dark energy for a homogeneous and isotropic universe, without involving any self-interaction potential or cosmological constant in the formulation.

Keywords: Brans-Dicke Theory, Dark Energy, EoS Parameter, Density Parameters, Interaction Term, Cosmology.

1. Introduction

The accelerated expansion of the universe is one of the most interesting and important phenomena in the field of cosmology that have been obtained through astrophysical observations in recent times [1-5]. An exotic form of energy, with a negative pressure, has been found to be responsible for the accelerated expansion of the universe. This energy is known as dark energy (DE). In the fields of physics and astronomy, an extensive research is now taking place, throughout the world, on the nature and dynamics of DE. A number of models have been proposed to explain this accelerated expansion of the universe, following a phase of deceleration. In most of the models, DE is represented by cosmological constant [6].

The present article is based on Brans-Dicke (BD) theory of gravitation. The BD theory is characterized by a scalar field (ϕ) and a dimensionless coupling parameter (ω) that govern the dynamics of space-time geometry. It can be regarded as a natural extension of the general theory of relativity which is obtained in the limit of an infinite ω and a constant value of ϕ [7]. The BD theory of gravity can be regarded as one of the most important theories, among all prevalent alternative theories of gravitation, which have very successfully explained the early and late time behaviours of the universe and solved the problems of inflation [8]. As an extension of the original BD theory, a generalized version was proposed, where ω is regarded as a function of the scalar field ϕ [9-11]. Several models regarding the expanding universe have been formulated on the basis of this theory [7, 12-15].

In the present study, BD field equations (for a spatially flat, homogeneous and isotropic space-time) have been used to determine time dependence of the coupling parameter (ω), equation of state (EoS) parameter (γ) and energy density (ρ). Time dependence of ω was previously studied by many groups, using scale factors that have power law dependence upon time [7, 14, 15]. These scale factors lead to constant deceleration parameters that are not likely to describe the cosmic expansion properly. According to astrophysical observations, the universe has made a transition from a phase of decelerated expansion to a phase of accelerated expansion, implying that the deceleration parameter has a dependence upon time [12, 16]. Taking this fact into consideration, we have used a scale factor (a) that leads to a time dependent deceleration parameter which changes sign from positive to its present negative value. For simplicity, we have assumed power law dependence of ω upon ϕ and also ϕ upon a. Time dependence of ρ and the EoS parameter (γ), in the present study, are consistent with the results obtained from some recent studies based on general relativity with anisotropic space-time [17, 18]. A theoretical model has been proposed to determine the time variation of density parameters (Ω) for matter and dark energy. The results from this model are in agreement with those obtained from studies based on completely different premises [19, 20]. An effective interaction term (Q_{eff}) between matter and dark energy has been calculated here and its time dependence has been studied. Its time variation is found to be consistent with studies based on some other models [21, 22].

2. Metric and Field Equations

The action, in the generalized Brans-Dicke theory, is expressed as,

$$\mathcal{A} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R + \frac{\omega(\phi)}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \mathcal{L}_m\right) \qquad (01)$$

In equation (1), g is the determinant of the metric tensor $g^{\mu\nu}$, L_m is the Lagrangian for matter, R denotes the Ricci scalar, φ is the Brans-Dicke scalar field and ω is a dimensionless coupling parameter. This parameter is regarded as a function of φ in generalized BD theory.
The variation of action in equation (1) leads to the following field equations.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{\phi}T_{\mu\nu} - \frac{\omega}{\phi^2}\left[\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\beta}\phi^{,\beta}\right] - \frac{1}{\phi}\left[\phi_{\mu;\nu} - g_{\mu\nu}\left\{\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi\right\}\right] \qquad (02)$$

$$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi = \frac{1}{2\omega+3}T \qquad (03)$$

$R_{\mu\nu}$ is the Ricci tensor and $T_{\mu\nu}$ is the energy-momentum tensor. In the above equations, a semicolon stands for a covariant derivative and a comma indicates an ordinary derivative with respect to x^β.
The energy-momentum tensor ($T_{\mu\nu}$) for the cosmic constituents is given by,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + g_{\mu\nu}p \qquad (04)$$

Here, ρ denotes the energy density, p is the isotropic pressure, u_ν is the four-velocity vector and T is the trace of T_j^i. This form of the energy-momentum tensor is based on an assumption that the total matter-energy content of the expanding universe is a perfect fluid.

In a co-moving coordinate system, $u^\nu = (0,0,0,1)$, having the characteristic of $g_{\mu\nu}u^\mu u^\nu = 1$. The line element for a homogeneous and isotropic universe, in Friedmann-Robertson-Walker (FRW) cosmology, can be expressed as,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\xi^2\right] \tag{05}$$

Where $a(t)$ denotes the scale factor, t is the cosmic time, k is the spatial curvature parameter. Three spherical polar coordinates are denoted by r, θ and ξ respectively.

The following three equations are the field equations of BD theory for a universe filled with a perfect fluid, in the geometry of FRW space-time expressed by equation (5). These equations have been obtained from the field equations (2) and (3) in which $T_{\mu\nu}$ from equation (4) has been incorporated and the components of the metric tensor $g_{\mu\nu}$ have been taken from the line element expressed by equation (5).

$$3\frac{\dot{a}^2+k}{a^2} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} - \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} = \frac{\rho}{\phi} \tag{06}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2+k}{a^2} + \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{\gamma\rho}{\phi} \tag{07}$$

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{a}\dot{\phi}}{a\phi} = \frac{\rho}{\phi}\frac{(1-3\gamma)}{2\omega+3} - \frac{\dot{\phi}}{\phi}\frac{\dot{\omega}}{2\omega+3} \tag{08}$$

In these field equations, $\gamma(\equiv P/\rho)$ is the equation of state (EoS) parameter for the cosmic fluid, which has been treated as a function of time in the present study.

3. Theoretical Model

Combining equations (6), (7) and (8), for zero spatial curvature ($k = 0$), one obtains,

$$\dot{\omega} + \left(2\frac{\ddot{\phi}}{\dot{\phi}} + 6\frac{\dot{a}}{a} - \frac{\dot{\phi}}{\phi}\right)\omega - 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right)\frac{\phi}{\dot{\phi}} = 0 \tag{09}$$

In the present study we have assumed the following ansatzes for ϕ and ω. These parameters have been assumed to have power-law relations with a and φ respectively.

$$\phi = \phi_0\left(\frac{a}{a_0}\right)^n \tag{10}$$

$$\omega = \omega_0\left(\frac{\phi}{\phi_0}\right)^m \tag{11}$$

The expression of ϕ in equation (10) has been taken from some earlier studies in this regard [12, 16].The reason for choosing the above empirical relation for ω is the fact that this is regarded as a function of the scalar field in the generalized Brans-Dicke theory [12].

Using the expressions (10) and (11) in equation (9), one obtains,

$$m\omega n + (n + 4 - 2q)\omega - \frac{6}{n}(1 - q) = 0 \tag{12}$$

Here, q $(\equiv -\ddot{a}a/\dot{a}^2)$ is the deceleration parameter.
Writing $\omega = \omega_0$ and $q = q_0$ (i.e., their values at $t = t_0$) in equation (12), we get,

$$m = \frac{6(1-q_0)}{n^2\omega_0} - \frac{n+4-2q_0}{n} \tag{13}$$

Combining equation (6) with (10) and taking $k = 0$ (for flat space), one gets,

$$\rho = \phi H^2\left(3 + 3n - \frac{\omega}{2}n^2\right) \tag{14}$$

Here, H $(\equiv \dot{a}/a)$ is the Hubble parameter.
Replacing all parameters in equation (14) by their values at $t = t_0$, one obtains,

$$\omega_0 = \frac{2}{n^2{H_0}^2}\left(3{H_0}^2 + 3n{H_0}^2 - \rho_0/\phi_0\right) \tag{15}$$

Using equation (15) in equation (13) one gets,

$$m = \frac{3(1-q_0){H_0}^2}{3{H_0}^2+3n{H_0}^2-\rho_0/\phi_0} - \frac{n+4-2q_0}{n} \tag{16}$$

Equations (15) and (16) show that both ω_0 and m depends upon the parameter n, which determines the time dependence of ϕ, as per equation (10).
Using equations (10) and (14) in (7), for $k = 0$ (zero spatial curvature) and writing $\frac{\ddot{a}}{a} = -qH^2$ one obtains the following expression of the equation of state parameter,

$$\gamma = \frac{2q-1-0.5\omega n^2-n-n^2+nq}{3+3n-0.5\omega n^2} \tag{17}$$

The value of the equation of state parameter at the present time is therefore given by,

$$\gamma_0 = \frac{2q_0-1-0.5\omega_0 n^2-n-n^2+nq_0}{3+3n-0.5\omega_0 n^2} \tag{18}$$

To use the expression of ω in equation (11), the values of ω_0 and m should be taken from equations (15) and (16) respectively. The value of ω, required for the expressions of ρ and γ, in equations (14) and (17) respectively, should then be taken from equation (11). The value of ϕ, required for equation (14), should be taken from equation (10).
To determine the time dependence of several cosmological parameters $(\phi, \omega, \rho, \gamma)$, following empirical expression of the scale factor has been used in the present model.

$$a = a_0 Exp\left[\alpha\left\{(t/t_0)^\beta - 1\right\}\right] \tag{19}$$

This scale factor has been so chosen that it generates a deceleration parameter $(q = -\ddot{a}a/\dot{a}^2)$ which changes sign from positive to negative, as a function of time. This signature flip indicates a transition of the universe from a phase of decelerated expansion to a phase of accelerated expansion, in accordance with several recent studies based on astrophysical observations [12].We have taken $a_0 = 1$ for all calculations.
Here α, β should have the same sign to ensure an increase of the scale factor with time. Using equation (19), the Hubble parameter (H) and the deceleration parameter (q) are obtained as,

$$H = \frac{\dot{a}}{a} = \frac{\alpha\beta}{t_0}\left(\frac{t}{t_0}\right)^{\beta-1} \tag{20}$$

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{1-\beta}{\alpha\beta}\left(\frac{t}{t_0}\right)^{-\beta} \tag{21}$$

For $0 < \beta < 1$ and $\alpha > 0$ one finds that, $q \to +\infty$ as $t \to 0$ and $q \to -1$ as $t \to \infty$, showing clearly a signature flip of q with time.
Taking $H = H_0$ and $q = q_0$, at $t = t_0$, the values of the constants α and β are obtained as,

$$\alpha = \frac{H_0 t_0}{1 - H_0 t_0 (1 + q_0)} \tag{22}$$

$$\beta = 1 - H_0 t_0 (1 + q_0) \tag{23}$$

It is often necessary to find the evolution of a cosmological quantity as a function of the redshift parameter (z) where $z = \frac{a_0}{a} - 1$. Using equation (19) one gets the following expression as a relation between redshift parameter and time.

$$z = \frac{a_0}{a} - 1 = Exp\left[-\alpha\left\{(t/t_0)^\beta - 1\right\}\right] - 1 \tag{24}$$

The values of different cosmological parameters used in this article are:

$H_0 = \frac{72\frac{Km}{s}}{Mpc} = 2.33 \times 10^{-18} sec^{-1}$, $q_0 = -0.55$, $\rho_0 = 9.9 \times 10^{-27} Kgm^{-3}$, Ω_{D0}=0.7
$\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} Kgs^2m^{-3}$, $t_0 = 1.4 \times 10^{10} Years = 4.42 \times 10^{17} s$

4. Determination of Density Parameters

The total pressure (P) of the entire matter-energy content of the universe is contributed by dark energy because, the whole matter content (dark matter + baryonic matter) is regarded as pressureless dust [12, 23]. Thus we can write,

$$P = \gamma\rho = \gamma_D \rho_D \tag{25}$$

Here, γ_D is the EoS parameter for dark energy and ρ_D is the density of dark energy.
We propose the following empirical relationship between γ_D and γ.

$$\gamma_D = A\,\gamma t^\mu \tag{26}$$

Here A and μ are constants.

Using equations (25) and (26) we can write,

$$\Omega_D = \frac{\rho_D}{\rho} = \frac{\gamma}{\gamma_D} = A^{-1} t^{-\mu} \tag{27}$$

Here, Ω_D denotes the density parameter for dark energy.
Taking $\Omega_D = \Omega_{D0}$ at $t = t_0$, in equation (27), we get,

$$A = \frac{1}{\Omega_{D0} {t_0}^{\mu}} \tag{28}$$

Using equation (28) in (27) we get,

$$\Omega_D = \Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu} \tag{29}$$

Using equation (29), the density parameter for matter can be expressed as,

$$\Omega_m = \frac{\rho_m}{\rho} = \frac{\rho - \rho_D}{\rho} = 1 - \Omega_D = 1 - \Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu} \tag{30}$$

In deriving equation (30), we have used the fact that the universe is composed mainly of matter and dark energy, considering all other forms of energy to be negligibly small [24, 25]. Using equations (29) and (30), the expressions for densities of dark energy and matter can be respectively written as,

$$\rho_D = \rho \Omega_D = \rho \Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu} \tag{31}$$

$$\rho_m = \rho \Omega_m = \rho \left[1 - \Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu}\right] \tag{32}$$

Using equation (28) in (26), the equation-of-state parameter for dark energy (γ_D) becomes,

$$\gamma_D = \frac{\gamma}{\Omega_D} = \frac{\gamma}{\Omega_{D0}} \left(\frac{t}{t_0}\right)^{\mu} \tag{33}$$

Using equation (24), the expressions of Ω_D and Ω_m can be written in terms of redshift (z) as,

$$\Omega_D = \Omega_{D0} \left(1 + \frac{1}{\alpha} ln \frac{1}{z+1}\right)^{-\mu/\beta} \tag{34}$$

$$\Omega_m = 1 - \Omega_{D0} \left(1 + \frac{1}{\alpha} ln \frac{1}{z+1}\right)^{-\mu/\beta} \tag{35}$$

The values of Ω_{D0} is close to 0.7 according to several astrophysical observations [24, 25]. As per the history of evolution of density parameters of the universe, there was a time in the recent past when $\Omega_D = \Omega_m = 0.5$ and, the corresponding z value was lying somewhere in the range of $0 < z < 1$, as obtained from some recent studies [19, 20]. That phase of the universe might be same as (or close to) the one when there was a transition from decelerated expansion to an accelerated expansion of the universe because it is the dark energy that is known to be responsible for the accelerated expansion. According to a recent study, the transition of the universe from a phase of decelerated expansion to its present state of accelerated expansion took place in the past at $z = 0.6818$ which was around 7.2371×10^9 years ago [25]. Taking z_c to denote the value of the redshift parameter at which the universe had $\Omega_D = \Omega_m$, we get the following expression of μ from equations (34) and (35).

$$\mu = \frac{\beta \, ln(2\Omega_{D0})}{ln\left[1 + \frac{1}{\alpha} ln\left(\frac{1}{z_c+1}\right)\right]} \tag{36}$$

Using equations (10), (19), (29) and (30), Ω_D and Ω_m can be expressed as functions of the scalar field (ϕ) by the following two equations.

$$\Omega_D = \Omega_{D0}\left(1 + \frac{1}{n\alpha} ln \frac{\phi}{\phi_0}\right)^{-\mu/\beta} \quad (37)$$

$$\Omega_m = 1 - \Omega_{D0}\left(1 + \frac{1}{n\alpha} ln \frac{\phi}{\phi_0}\right)^{-\mu/\beta} \quad (38)$$

5. Calculation of an Effective Interaction Term

The energy conservation equation is expressed as,

$$\dot{\rho} + 3H(\rho + P) = 0 \quad (39)$$

Taking the pressure $P = \gamma\rho = \gamma_D\rho_D$ (eqn. 25) and $\rho = \rho_m + \rho_D$ in equation (39) one gets,

$$\dot{\rho}_m + \dot{\rho}_D + 3H[\rho_m + \rho_D(1 + \gamma_D)] = 0 \quad (40)$$

If it is assumed that the two entities, matter and dark energy the universe, have been interacting with each other, causing the generation of one of them at the cost of the other, one may define a parameter representing their interaction, on the basis of equation (40). The interaction term (Q) can be represented by the following equation [19, 21, 23].

$$\dot{\rho}_m + 3H\rho_m = Q \quad (41)$$

$$\dot{\rho}_D + 3H\rho_D(1 + \gamma_D) = -Q \quad (42)$$

A negative value of Q represents a transfer of energy from the matter field to the field of dark energy and a positive value of Q represents conversion of dark energy into matter. It is evident from equations (41) and (42) that Q has a dependence upon time and, its values, obtained from these two equations, would not be the same. Let us denote the values of Q, obtained from these two equations, by Q_1 and Q_2 respectively.
Using the relation $\rho = \rho_m + \rho_D$ and equation (31) in (41) we get,

$$Q_1 = (1 - \Omega_D)\dot{\rho} - \rho\dot{\Omega}_D + 3H\rho(1 - \Omega_D) \quad (43)$$

Using equations (31) and (33) in equation (42) we get,

$$Q_2 = -\Omega_D\dot{\rho} - \rho\dot{\Omega}_D - 3H\rho(\Omega_D + \gamma) \quad (44)$$

To estimate the difference (ΔQ) between $Q_1$and Q_2, we write the following expression.

$$\Delta Q = Q_1 - Q_2 = \dot{\rho} + 3H\rho(1 + \gamma) \quad (45)$$

To have an estimate of an effective interaction between matter and dark energy, we define a parameter Q_{eff} as the average of $Q_1$and Q_2. Thus we have,

$$Q_{eff} = \frac{Q_1 + Q_2}{2} = \left(\frac{1}{2} - \Omega_D\right)\dot{\rho} - \rho\dot{\Omega}_D + \frac{3}{2}H\rho(1 - 2\Omega_D - \gamma) \quad (46)$$

To judge the plausibility of using Q_{eff} as a measure of interaction between matter and dark energy, one may calculate $\Delta Q/Q_{eff}$, from the following expression (using eqns. 45, 46).

$$\frac{\Delta Q}{Q_{eff}} = \frac{\dot{\rho}+3H\rho(1+\gamma)}{\left(\frac{1}{2}-\Omega_D\right)\dot{\rho}-\rho\dot{\Omega}_D+\frac{3}{2}H\rho(1-2\Omega_D-\gamma)} \qquad (47)$$

To determine the time dependence of the interaction term (Q), we have calculated $\dot{\rho}$ and $\dot{\Omega}_D$, based on the expressions of ρ and Ω_D in equations (14) and (29) respectively.

$$\dot{\rho} = \rho H\left[n-2(1+q)-\frac{\omega m n^3}{6+6n-\omega n^2}\right] \qquad (48)$$

$$\dot{\Omega}_D = \frac{-\mu\Omega_D}{t} = \frac{-\mu\Omega_D}{t_0}\left(\frac{Ht_0}{\alpha\beta}\right)^{\frac{1}{1-\beta}} \qquad (49)$$

Equations (10) and (11) have been used, along with the definitions of H and q, to calculate $\dot{\rho}$ (eqn. 48). Equation (20) has been used to calculate $\dot{\Omega}_D$ (eqn. 49).
Using equations (48) and (49), in (43), (44), (46) and (47) one obtains,

$$Q_1 = (1-\Omega_D)\rho H\left[n-2(1+q)-\frac{\omega m n^3}{6+6n-\omega n^2}\right]+\frac{\mu\rho\Omega_D}{t_0}\left(\frac{Ht_0}{\alpha\beta}\right)^{\frac{1}{1-\beta}}+3H\rho(1-\Omega_D) \qquad (50)$$

$$Q_2 = -\Omega_D\rho H\left[n-2(1+q)-\frac{\omega m n^3}{6+6n-\omega n^2}\right]+\frac{\mu\rho\Omega_D}{t_0}\left(\frac{Ht_0}{\alpha\beta}\right)^{\frac{1}{1-\beta}}-3H\rho(\Omega_D+\gamma) \qquad (51)$$

$$Q_{eff} = \left(\frac{1}{2}-\Omega_D\right)\rho H\left[n-2(1+q)-\frac{\omega m n^3}{6+6n-\omega n^2}\right]+\frac{\mu\rho\Omega_D}{t_0}\left(\frac{Ht_0}{\alpha\beta}\right)^{\frac{1}{1-\beta}}+\frac{3}{2}H\rho(1-2\Omega_D-\gamma) \qquad (52)$$

$$\frac{\Delta Q}{Q_{eff}} = \frac{\rho H\left[n-2(1+q)-\frac{\omega m n^3}{6+6n-\omega n^2}\right]+3H\rho(1+\gamma)}{\left(\frac{1}{2}-\Omega_D\right)\rho H\left[n-2(1+q)-\frac{\omega m n^3}{6+6n-\omega n^2}\right]-\rho\frac{-\mu\Omega_D}{t_0}\left(\frac{Ht_0}{\alpha\beta}\right)^{\frac{1}{1-\beta}}+\frac{3}{2}H\rho(1-2\Omega_D-\gamma)} \qquad (53)$$

If the value of $\frac{\Delta Q}{Q_{eff}}$ is found to be sufficiently small, it would be quite reasonable to believe that Q_{eff}, which is actually an average of $Q_1$and Q_2, will truly represent the interaction between matter and dark energy.
In the table below, i.e. Table-1, we have listed the values of $\left(Q_{eff}\right)_{t=t_0}$ and $\left(\frac{\Delta Q}{Q_{eff}}\right)_{t=t_0}$ for different values of the parameter *n*. These are the values of these two quantities at the present time. Table-1 shows that the present value of the interaction term between matter and dark energy is of the order of 10^{-44} and it is negative. The present value of $\Delta Q/Q_{eff}$ is so small ($\sim 10^{-15}$) that Q_{eff}, as defined by eqn. 46, is likely to describe the interaction with sufficient accuracy. Its negative value indicates decay of matter into dark energy at the present time.
These values show that as *n* becomes more negative, $\left(Q_{eff}\right)_{t=t_0}$ becomes more negative. It clearly implies that a greater rate of change of the scalar field (ϕ) or gravitational constant ($G = 1/\phi$) with time has certainly some connection to a larger interaction between mater and dark energy of the universe, causing a larger rate of generation of dark energy at the expense of matter.

TABLE - 1		
Values of Q_{eff} and $\Delta Q/Q_{eff}$ at the present time for several values of the parameter *n* which determines the time variation of the scalar field (ϕ).		
n	$\left(Q_{eff}\right)_{t=t_0}$	$\left(\frac{\Delta Q}{Q_{eff}}\right)_{t=t_0}$
-1.60	-1.153×10^{-44}	1.296×10^{-14}
-1.62	-1.606×10^{-44}	-3.719×10^{-15}
-1.64	-2.046×10^{-44}	2.433×10^{-15}
-1.66	-2.473×10^{-44}	-3.020×10^{-15}
-1.68	-2.886×10^{-44}	-1.021×10^{-15}
-1.70	-3.285×10^{-44}	-9.549×10^{-15}
-1.72	-3.670×10^{-44}	-4.883×10^{-15}
-1.74	-4.042×10^{-44}	-4.188×10^{-15}
-1.76	-4.400×10^{-44}	-2.602×10^{-15}
-1.78	-4.744×10^{-44}	-2.001×10^{-15}
-1.80	-5.075×10^{-44}	-1.014×10^{-15}
-1.82	-5.392×10^{-44}	-6.463×10^{-15}
-1.84	-5.696×10^{-44}	2.797×10^{-15}
-1.86	-5.986×10^{-44}	-1.663×10^{-15}

6. Choice of Values for the Parameter *n*

The parameter *n* controls the time evolution of the scalar field (ϕ). Time variations of ω, ρ and γ also depend on *n*, as we know that both m and ω_0 are functions of *n*. To estimate its value, one may use the fact that the scalar field (ϕ) is the reciprocal of the gravitational constant (G) [12, 26]. Hence, using equations (10) and (19), we can write,

$$G=\frac{1}{\phi}=\frac{1}{\phi_0}\left(\frac{a}{a_0}\right)^{-n}=\frac{1}{\phi_0}Exp\left[-n\alpha\left\{(t/t_0)^{\beta}-1\right\}\right] \quad (54)$$

According to some studies, the gravitational constant increases with time [27-29]. In the expression of G in equation (54), the scale factor (a) is an increasing function of time. For negative values of the parameter n, G would be an increasing function of time. Therefore, in the present study we have used only negative values of the parameter *n*.

Using equations (20) and (54), we get,

$$\frac{\dot{G}}{G}=-n\frac{\dot{a}}{a}=-nH=-n\frac{\alpha\beta}{t_0}\left(\frac{t}{t_0}\right)^{\beta-1} \quad (55)$$

Equation (55) suggests that the value of the parameter *n* can be estimated from the experimental observations of the fractional rate of change of the gravitational constant ($\dot{G}/G$), at the present time, as reported in articles on time varying G [30].

According to a study by S. Weinberg, the largest possible value of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ is 4×10^{-10} per year [32].

TABLE – 2 Values of ω_0, m, γ_0, γ_{D0}, $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ for several values of the parameter n which determines the time variation of the scalar field (ϕ).					
n	ω_0	m	γ_0	γ_{D0} ($z_c = 0.7$)	$\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ (Yr^{-1})
-1.63	-1.514	-0.183	-1.796	-2.566	1.198×10^{-10}
-1.64	-1.518	-0.168	-1.691	-2.416	1.205×10^{-10}
-1.65	-1.522	-0.154	-1.588	-2.268	1.212×10^{-10}
-1.66	-1.525	-0.140	-1.486	-2.123	1.220×10^{-10}
-1.67	-1.529	-0.127	-1.386	-1.979	1.227×10^{-10}
-1.68	-1.532	-0.115	-1.287	-1.839	1.234×10^{-10}
-1.69	-1.535	-0.104	-1.190	-1.700	1.242×10^{-10}
-1.70	-1.538	-0.093	-1.095	-1.564	1.249×10^{-10}
-1.71	-1.540	-0.083	-1.001	-1.430	1.256×10^{-10}
-1.72	-1.543	-0.073	-0.909	-1.299	1.264×10^{-10}
-1.73	-1.545	-0.064	-0.819	-1.170	1.271×10^{-10}
-1.74	-1.547	-0.055	-0.730	-1.043	1.279×10^{-10}
-1.75	-1.549	-0.046	-0.643	-0.918	1.286×10^{-10}
-1.76	-1.551	-0.038	-0.558	-0.796	1.293×10^{-10}
-1.77	-1.553	-0.031	-0.474	-0.677	1.301×10^{-10}
-1.78	-1.554	-0.024	-0.392	-0.559	1.308×10^{-10}
-1.79	-1.555	-0.017	-0.311	-0.444	1.315×10^{-10}
-1.80	-1.557	-0.011	-0.232	-0.332	1.323×10^{-10}
-1.81	-1.558	-0.005	-0.155	-0.221	1.330×10^{-10}
-1.82	-1.559	0.001	-0.079	-0.113	1.337×10^{-10}
-1.83	-1.560	0.006	-0.005	-0.008	1.345×10^{-10}
-1.84	-1.561	0.012	0.067	0.095	1.352×10^{-10}
-1.85	-1.561	0.016	0.137	0.196	1.359×10^{-10}

Table-2 shows the values of ω_0, m, γ_0, γ_{D0}, $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ for different values of the parameter n which governs the time variation of the scalar field (ϕ), as per equation (10). It is found that, for the values of n over the range of $-1.76 < n < -1.67$, the values of the present EoS parameter (γ_0) in this table are consistent with the range of values (approximately $-1.7 < \gamma_0 < -0.6$) obtained from astrophysical observations, as reported in some recent articles on the time variation of EoS parameter, based on an anisotropic space-time [17, 18].Over this entire range, the values of the parameter m are negative, indicating that ω would be less negative as ϕ increases, as evident from equation (11), where ω_0 has a negative value. For $n < -1.81$ in this table we have $m > 0$, implying that ω would be more and more negative as ϕ increases. Both of these natures of variations of $\omega(\phi)$ are reflected in the plots of some other studies on Brans-Dicke theory [13, 31]. But the second type of variation is not physically valid, since it corresponds to those values of EoS parameter which are not in conformity with the astrophysical observations in this regard. The values of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ in this table are absolutely within the range of permissible values envisaged by S. Weinberg [32].

FIGURES

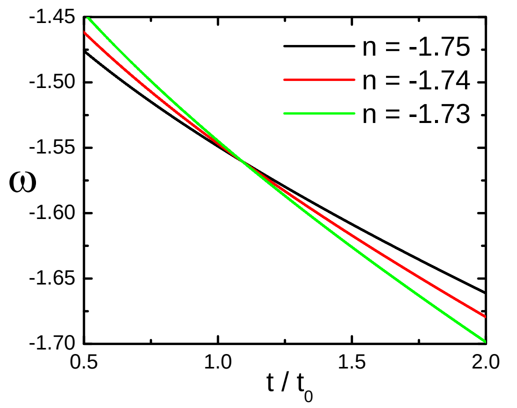

Figure 1: Plots of Brans-Dicke parameter versus time for three different values of the parameter *n*.

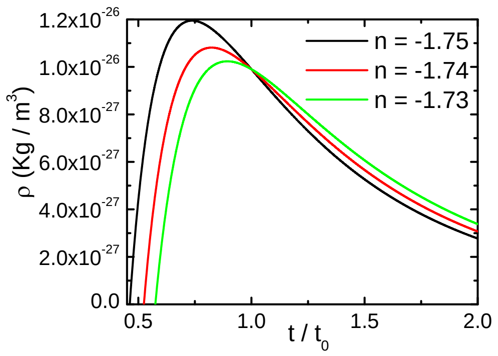

Figure 2: Plots of energy density versus time for three different values of the parameter *n*.

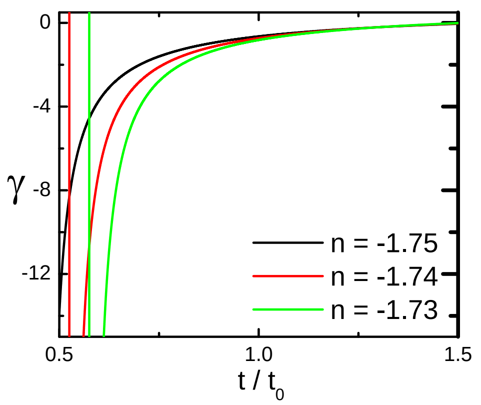

Figure 3: Plots of the equation of state parameter versus time for three different values of the parameter *n*.

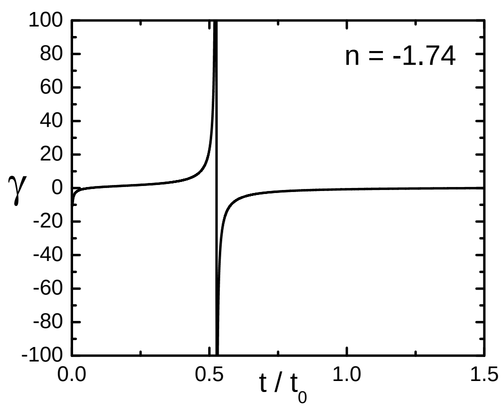

Figure 4: Plot of the equation of state parameter versus time, for $n = -1.74$, on a larger time scale.

FIGURES

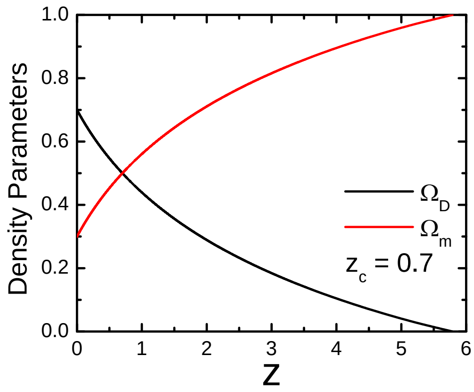

Figure 5: Plots of density parameters, for matter and dark energy, for $z_c = 0.7$, against the redshift parameter.

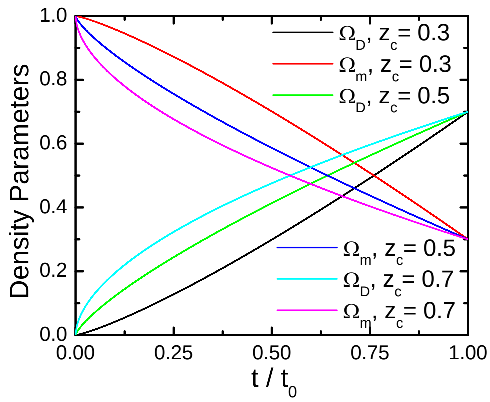

Figure 6: Plots of density parameters, for matter and dark energy, as functions of time, for three different values of z_c.

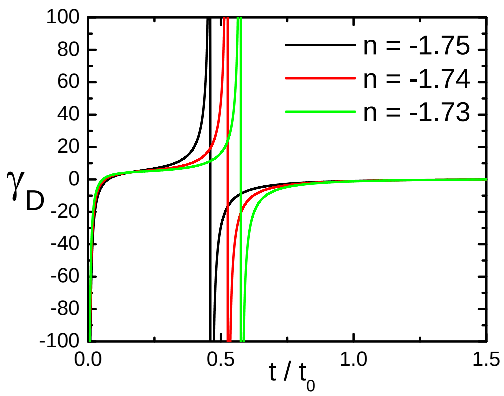

Figure 7: Plots of the equation of state (EoS) parameter for dark energy versus time for three different values of the parameter n.

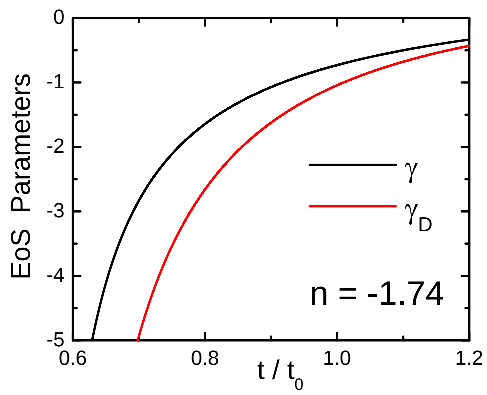

Figure 8: Plots of EoS parameters versus time, for total energy (γ) and dark energy (γ_D). For $n = -1.74$, $\gamma_0 = -0.73$ and $\gamma_{D0} = -1.04$.

FIGURES

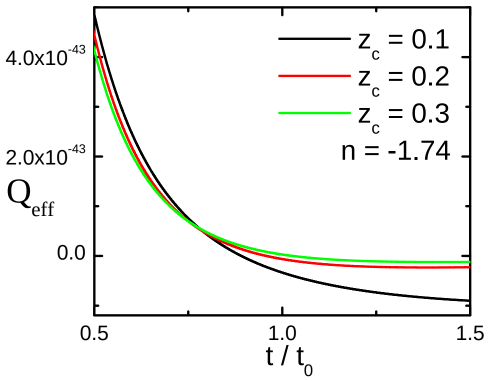

Figure 9: Plots of effective interaction term Q_{eff} versus time for three different values of z_c.

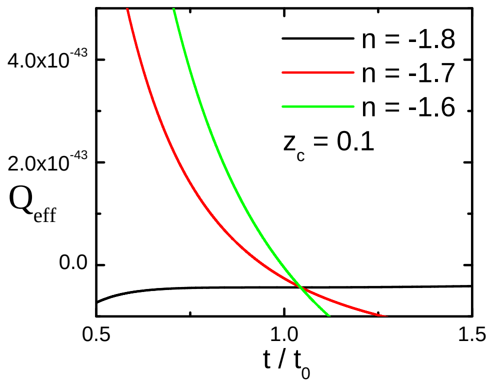

Figure 10: Plots of effective interaction term Q_{eff} versus time for three different values of the parameter n.

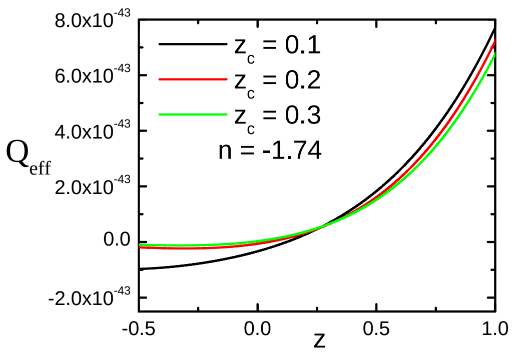

Figure 11: Plots of effective interaction term Q_{eff} versus redshift (z) for three different values of z_c.

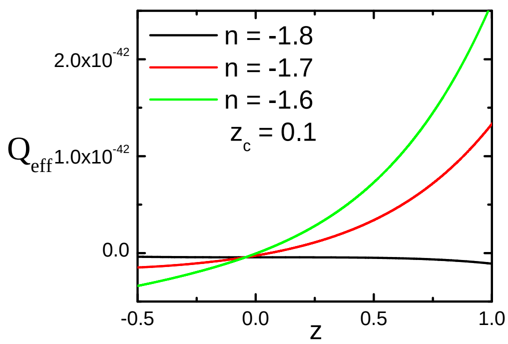

Figure 12: Plots of effective interaction term Q_{eff} versus redshift (z) for three different values of the parameter n.

7. Results and Discussion

Figure 1 shows the variation of the Brans-Dicke parameter (ω) as a function of time, for three different values of the parameter n. Here, ω has a negative value and it becomes more negative with time. For more negative values of n, it decreases less rapidly. This nature of ω, which is gradually becoming more negative with time, is also obtained from other studies on Brans-Dicke theory [13, 31]. The time dependence of ω, obtained from the present study, is based on a scale factor which leads to a time dependent deceleration parameter (that shows a signature flip with time from positive to negative), unlike some recent studies [7, 14, 15] that are based on time independent deceleration parameters. The value of ω_0 is found to be close to -1.55 which is consistent with the range provided in the conclusion of a recently published article based on Kantowski-Sachs space-time [15].

Figure 2 shows the variation of energy density (ρ) with time, for three different values of the parameter n. Here, ρ is found to rise steeply to a peak and then decreases at a slower rate. More negative values of n cause quicker attainment of the peak and also a larger peak value of energy density. It falls more steeply for more negative values of n.

Figure 3 shows the time dependence of the equation of state (EoS) parameter (γ) for three different values of the parameter n. These curves show a steep rise initially and then they gradually become asymptotic to a small negative value. Saturation is reached faster for less negative values of n. The value of γ at the present time ($t = t_0$) is -0.73 which is in agreement with the ranges of values obtained from astrophysical observations, as reported in earlier studies [17, 18]. These curves are similar to those obtained from dark energy models based on LRS Bianchi type-V metric in the framework of Einstein's general theory of relativity [18].

Figure 4 shows a single plot EoS parameter, for $n = -1.74$, on a longer time scale. It shows a pattern that is very much similar to the one depicted in one of the earlier studies based on dark energy models in anisotropic Bianchi type-I (B-I) space-time, in the framework of Einstein's general theory of relativity [17].

The plots in the figures 2-4 show very clearly that the time evolution of energy density and EoS parameter, obtained here for FRW space-time in the framework of Brans-Dicke theory, are quite in agreement with the results obtained from anisotropic space-time in the framework of general relativity.

Figure 5 shows the variations of density parameters, for dark energy and matter, as functions of the redshift parameter. Here, the value of μ has been taken to be -0.557, causing $z_c = 0.7$, implying that the dark energy took over at around $z = 0.7$. These plots are almost identical in nature to the plots obtained from some recent studies [19, 20], which were carried out on premises completely different from the present study.

Figure 6 shows the plots of density parameters, for dark energy and matter, as functions of time. Here we have three pairs (Ω_m and Ω_D) of density parameters, corresponding to three different values of the parameter μ, connected to three different values of z_c.

Figure 7 shows the time variation of the EoS parameter for dark energy (γ_D) for three different values of the parameter *n*. These plots are very much similar to the ones obtained from studies based on anisotropic space-time in the framework of general relativity [17, 18]. Plots with less negative values of *n* take longer time to attain saturation.

Figure 8 shows the time variations of EoS parameters for total energy and dark energy. Here γ_D has a more negative value compared with γ. Their difference decreases with time, as the proportion of dark energy increases in the universe. For $n = -1.74$ the values of γ_0 and γ_{D0} are respectively -0.73 and -1.04 respectively. These values are quite consistent with the ranges of values obtained from several astrophysical observations and described in some studies regarding dark energy and time varying EoS parameter [17-20].

The parameter *n*, in this model, plays an important role in governing the time evolution of various cosmological quantities. Both ω_0 and *m*, which controls the value of ω, are dependent upon *n* (eqns. 15, 16). Larger negative value of *n* means faster change of ϕ $(\equiv 1/G)$ with time. Taking $n = -1.74$ we get, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} = 1.28 \times 10^{-10}\ Yr^{-1}$ from equation (55). This is well within its upper limit, i.e. $4 \times 10^{-10} Yr^{-1}$, predicted by S. Weinberg [32].

Figure 9 shows the variation of the effective interaction term Q_{eff} as a function of time for three different values of z_c. It is found to decrease with time from an initial positive value, gradually becoming negative at a later time. This behaviour of signature flip is consistent with the results of a recent study on holographic dark energy [21]. For smaller values of z_c (which correspond to more negative values of μ), Q_{eff} attains its negative phase faster. Its positive phase indicates that dark energy initially decayed into matter and, in the later stage, where $Q_{eff} < 0$, matter decays into dark energy.

Figure 10 shows the variation of Q_{eff} as a function of time for three different values of the parameter *n*. Two of these curves decrease with time and change their signs from positive to negative. These two curves show that the transfer of energy, between matter and dark energy, reverses its direction, producing dark energy at the expense of matter at the later stage. The third curve, with $n = -1.8$, is entirely in the negative domain, becoming gradually closer to zero with time. It clearly shows that, there exists a value of the parameter *n*, below which we have an unidirectional energy transfer, from the matter field to the field of dark energy, at a gradually decreasing rate.

Figure 11 depicts the change of Q_{eff} as a function of z, for three different values of z_c. It is found that, for smaller values of z_c, Q_{eff} attains its negative phase faster.

Figure 12 shows the variation of Q_{eff} as a function of the redshift parameter (z) for three different values of the parameter *n*. Like figure 10, one of these three plots also indicate the existence of a threshold value for n, below which the direction of energy transfer is always from the matter field to the field of dark energy with Q_{eff} becoming less negative with time. Above this threshold of n, the interaction term shows a signature flip from positive to negative as time progresses. The curve for $n = -1.8$ in this figure is similar to the behaviour obtained from a study on the interaction term between dark matter and dark energy [22]. Since around 80% of the matter content is dark matter, the interactions of dark energy with matter consist mainly of the interactions of dark energy with dark matter.

8. Concluding Remarks

The present study is based on a spatially flat, homogeneous and isotropic space-time, in the framework of Brans-Dicke theory. Here, the time evolution of Brans-Dicke parameter (ω), energy density (ρ) and EoS parameter (γ) have been determined from the field equations. Unlike many other studies, no cosmological constant and self interaction potential has been used in the formulation of the theory. The choice of a scale factor, that ensures a signature flip of the deceleration parameter with time, is likely to give more credibility to the results of this model. Without involving any self interaction potential (V_ϕ) and cosmological constant (Λ), the time evolutions of density parameters for matter and dark energy have been determined and these results are sufficiently consistent with those obtained through much more rigorous calculations [19, 20]. The parameter μ, in this model, governs the time dependence of Ω_D, Ω_m and γ_D. It has been shown that μ can be determined if one gets information from astrophysical observations regarding the time or redshift (z) at which we had $\Omega_D = \Omega_m$ in the recent past of the expanding universe. Assuming an interaction to be taking place between matter and dark energy, an effective interaction term has been calculated on the basis of the energy conservation equation. Its negative value at the present time implies a decay of matter into dark energy which is responsible for the accelerated expansion of the universe. The behaviour of this interaction term is consistent with the results of some recent studies [21, 22] carried out in a completely different way. The parameter n, which actually controls the rate of change of gravitational constant with time (as per eqn. 54), determines the nature of time variation of some cosmological quantities (ω, ρ and γ) mentioned above. It implies that the dependence of the scalar field ($\phi \equiv 1/G$) upon the scale factor (a) (eqn. 10) plays a very important role in cosmic expansion. The dependence of ω upon φ is controlled by ω_0 and m which are functions of n. Positive values of n leads to large negative values of γ_{D0}, contrary to its range of values close to -1, obtained from recent astrophysical observations [19]. This result is in favour of using negative values of n, implying an increase of gravitational constant with time, as per equation (54). This behaviour of gravitational constant is also obtained from other studies [25-27]. Keeping in view all these facts, one may think of improving this model by choosing a relation, between ϕ and a, which is different from the ansatz of equation (10), in order to have more than one tunable parameter like n controlling the time variation of the scalar field (ϕ). As a future project, one may think of determining the time dependence of ρ_m and ρ_D from equations (41) and (42) by assuming Q to be a function of the scalar field (ϕ) where ϕ may be assumed to have a power-law dependence upon time, as in some earlier studies [7]. For this purpose H can be taken from equation (20) and γ_D can be assumed to have a form like $\gamma_D = A\gamma t^{\mu}$, as in equation (26). Here γ can be replaced by γ_0, considering very slow time variation of γ, as evident from figure 4, over a span of time close to $t = t_0$. Using the solutions of equations (41) and (42), one can calculate Ω_D and Ω_m, taking the total energy density (ρ) from equation (14). Thus it would be possible to establish a relationship between the density parameters (Ω_D and Ω_m) and the scalar field (ϕ).

Acknowledgements

The authors of this article express their thanks and gratitude, very heartily, to the researchers and academicians whose works have enriched them immensely and inspired them to undertake the present study. S. Roy would like to thank all his colleagues in the Department of Physics, St. Xavier's College, Kolkata, India for creating and maintaining an ambience of erudition and an atmosphere of cooperation and support.

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