Time Variation of the Matter Content of the Expanding Universe in the Framework of Brans-Dicke Theory

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Abstract

In the framework of Brans-Dicke theory, a cosmological model regarding the expanding universe has been formulated by considering an inter-conversion of matter and dark energy. A function of time has been incorporated into the expression of the density of matter to account for the non-conservation of the matter content of the universe. This function is proportional to the matter content of the universe. Its functional form is determined by using empirical expressions of the scale factor and the scalar field in field equations. This scale factor has been chosen to generate a signature flip of the deceleration parameter with time. The matter content is found to decrease with time monotonically, indicating a conversion of matter into dark energy. This study leads us to the expressions of the proportions of matter and dark energy of the universe. Dependence of various cosmological parameters upon the matter content has been explored.

Keywords: Matter to dark energy conversion, Brans-Dicke theory, Time variation of matter and dark energy, Gravitational constant, Density of matter, Cosmology.

1 Introduction

It has been established beyond doubt, by a number of recent astrophysical observations, that the universe is expanding with acceleration [1 - 3]. It has been shown by several studies on cosmology that nearly seventy percent of all constituents of the universe has a large negative pressure and is referred to as dark energy, which is found to have a major role in driving the expansion process with acceleration. It has not yet been possible to determine its true nature. The cosmological constant is a widely known parameter of the general theory of relativity, which is regarded as one of the most suitable candidates acting as the source for this repulsive gravitational effect and it conforms to observational data reasonably well, despite its own limitations [4]. So far the researchers have proposed a large number of models regarding dark energy and their characteristics have been studied extensively [4, 5]. It is important to note that this accelerated expansion is a very recent phenomenon and it follows a phase of expansion of universe with deceleration. For the successful nucleosynthesis and also for the structure formation of the universe, this is important. On the basis of observational findings, beyond a certain value of the redshift (z), the universe certainly had a decelerated phase of expansion [6]. Therefore, the evolution of the dark energy component has been such that its effect on the dynamics of the universe is appreciable only during the later stages of the matter dominated era. On the basis of an analysis of supernova data, a recent study has shown that there has certainly been a change of sign of the deceleration parameter (q) of the universe, from positive to negative, implying that a decelerated expansion preceded the present state of accelerated expansion [7].

Apart from the models regarding the accelerated expansion, involving the cosmological constant, many other models of dark energy have been proposed and applied for the sake of a proper explanation of findings [8, 9]. A negative value of the deceleration parameter, implying accelerated expansion, has been successfully generated by all these models. One of the most significant of these models is a scalar field with a positive potential which produces an effective negative pressure if the kinetic term is dominated by the potential term. One refers to this scalar field as the quintessence scalar field. In scientific literature there are a large number of models regarding quintessence potentials and they have been extensively used. One may go through a study by V Sahni on this field, to have detailed information in this regard [10]. A proper physical explanation or background is lacking about the origins of models of most of the quintessence potentials.

In order to ascertain the behaviors and roles played by these entities, one has to take into account the possibility of an interaction between the different components of the constituents of universe, such as the matter and dark energy, which is expected to give rise to a transfer of energy from one field to another. Researchers have proposed many models where a transfer of energy takes place from the component of dark matter to the component of dark energy, in such a manner that during the later period of evolution, the dark energy predominates over matter and an accelerated expansion of universe is caused [11, 12]. But the interactions between two components, upon which the models were constructed, were chosen to be arbitrary in most cases, without a strong logical foundation of a physical theory. For a proper interpretation of the role of dark energy in causing accelerated expansion, a prolonged search has been going on for the theory of an interaction between matter and scalar field, on the basis of which a cosmologically viable model can be formulated.

In order to avoid the difficulties due to the arbitrariness of these models in the formulation of a particular Q-field, non-minimally coupled scalar field theories have been used to formulate models which can very effectively account for the transition from the decelerated phase to the accelerated phase of cosmic expansion. This has been made possible by the presence of the scalar field in the framework of the theory and it does not have to be incorporated separately. The most natural choice in this context is the Brans-Dicke theory which is regarded as the scalar-tensor generalization of general relativity, because of its simplicity and a possible reduction to the findings, predictions or results of general relativity in some limit. Thus the Brans–Dicke theory or its modified versions have been shown to account for the present acceleration of universe [13, 14]. An observation regarding the Brans-Dicke theory is that it can potentially generate sufficient acceleration in the matter dominated era even without taking into consideration an exotic quintessence field [15]. However, the researchers have been looking for a theory which can explain the change of cosmic expansion state from deceleration to acceleration. In most of the models the dark energy and dark matter components are considered to be non-interacting and are allowed to evolve independently. Due to the unknown nature of these two components, one is expected to get a relatively generalized framework for study by assuming an interaction between them. It has been shown by Zimdahl and Pavon that the interaction between dark energy and dark matter can be used effectively to solve the coincidence problem [16]. This idea may lead to the formulation of an interaction or inter-conversion of energy between the dark matter and the Brans-Dicke scalar field which is a geometrical field. Amendola had made a prediction earlier that there is a possibility of an inter-conversion of energy between the matter content of the universe and the non-minimally coupled scalar field [17].

It has been found from several applications of the Brans-Dicke theory that, in most of the models, the Brans-Dicke dimensionless parameter ω is required to have a very low value, typically of the order of unity, for an accelerated expansion of the universe [18]. It was once demonstrated in one of these studies that, considering the Brans-Dicke scalar field to be interacting with the dark matter, a generalized form of the Brans-Dicke theory can lead to an accelerated expansion even with a high value of ω [19]. For these studies, either one makes a modification of the Brans-Dicke theory to account for the findings properly or a quintessence scalar field is chosen to cause the required acceleration. Clifton and Barrow have shown in a recent study, which has also been shown by another group (Banerjee and Das), that no additional potential is necessary to generate the signature flip of the deceleration parameter from positive to negative [19, 20]. To explain the observational findings in this regard, they considered an interaction between the Brans-Dicke scalar field and the dark matter.

In the present model, a generalized form of Brans-Dicke theory has been used. In this form, the dimensionless parameter ω of Brans-Dicke theory is no longer treated as a constant. It is regarded as a function of the scalar field φ which is a time dependent quantity. This form of Brans-Dicke theory was first proposed by Bergman and it was expressed in a form of greater usefulness by Nordtvedt [21, 22]. The present study is not based upon any particular theoretical formulation regarding the mechanism of interaction between matter and the scalar field, causing an inter-conversion between matter and dark energy. A simple model has been proposed here by only taking into consideration a fact that the matter content of the universe is not conserved. This model has inherently kept open the possibilities of an inter-conversion between matter and some other form of energy, which might be regarded as dark energy, which is held responsible for generating the accelerated expansion of the universe, following a phase of deceleration. To express the density of matter (ρ) as a function of time, a function, denoted by $f(t)$, has been incorporated in its expression in a manner such that it accounts for the non-conservation of matter content of the universe. The modified expression of the density of matter (ρ) shows that if one chooses $f(t) = 1$ at all values of t, it would be conservation of the matter content of the universe represented by the expression of ρ. Empirical expressions of the scale factor (a) and the scalar field parameter (φ) have been used in Brans-Dicke field equations, in order to determine the functional form of $f(t)$. According to its definition, $f(t)$ is equal to the ratio of the matter content of the universe at any time (t) to the content of matter at the present epoch. This function $f(t)$ has been used here to formulate relevant expressions to determine the time variations of the proportions of matter and dark energy of the universe, assuming the mater content to be the only source of dark energy. The present study also explores the time dependence of the density of matter (ρ) and the gravitational constant (G). The purpose of formulating this model is to find out the effect of the change of matter content on the characteristics of the cosmic expansion. The effect of the dark energy content of the universe at any instant of time and the rate of its increase, upon the observational findings like gravitational constant, Hubble parameter and the deceleration parameter has been studied here. One finds that, the faster the generation of dark energy, more rapid would be the changes in these parameters.

2 Field equations and the theoretical model

The action in the generalized Brans-Dicke theory is given by [23],

$$S=\int[\frac{\varphi R}{16\pi G} + \frac{\omega(\varphi)}{\varphi}\varphi_{,\mu}\varphi^{,\mu} + L_m]\sqrt{-g}\, d^4x, \quad (1)$$

where R is the Ricci scalar, L_m is the matter Lagrangian, φ is the Brans-Dicke scalar field and ω is a dimensionless parameter which is considered to be a function of φ here instead of being regarded as a constant.
For a spatially flat Robertson-Walker space-time, the line element is given by,

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2\, d\theta^2 + r^2 sin^2\theta\, d\varphi^2] \tag{2}$$

Here a is the scale factor of the Universe.
For a spatially flat Friedmann-Robertson-Walker space-time, variation of action (1) with respect to the metric tensor components yields the field equations of the generalized Brans-Dicke theory as [24],

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{\varphi} + \frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}, \tag{3}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - 2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\ddot{\varphi}}{\varphi}. \tag{4}$$

Combining (3) and (4) one gets,

$$2\frac{\ddot{a}}{a} + 4\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{\varphi} - 5\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\ddot{\varphi}}{\varphi}. \tag{5}$$

From equation (3), the expression of $\omega(\varphi)$is obtained as,

$$\omega(\varphi) = 2\left[3\left(\frac{\dot{a}}{a}\right)^2 - \frac{\rho}{\varphi} + 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}\right]\left(\frac{\dot{\varphi}}{\varphi}\right)^{-2} \tag{6}$$

In many theoretical models the content of matter (dark + baryonic) of the universe has been assumed to remain conserved [24]. Following equation expresses the conservation of matter content of the universe.

$$\rho a^3 = \rho_0 a_0^3 = \rho_0 \quad \text{(taking } a_0 = 1\text{)} \tag{7}$$

There are some studies on Brans-Dicke theory of cosmology where one takes into account an interaction between matter and the scalar field. A possibility of an inter-conversion between dark energy and matter (both dark and baryonic matter) is taken into consideration in these studies. Keeping in mind this possibility, we propose the following relation for the density of matter (ρ).

$$\rho a^3 = f(t)\, \rho_0 a_0^3 = f(t)\, \rho_0 \quad \text{(taking } a_0 = 1\text{)} \tag{8}$$

In the present study we have not considered any theoretical model to explain or analyze the mechanism of interaction between matter and the scalar field. We have only considered a simple fact that the right hand side of equation (7) cannot be independent of time when one takes into account non-conservation of matter due to its generation from dark energy or its transformation into dark energy. We propose to introduce a function of time $f(t)$ in equation

(7) to get a new relation represented by equation (8). This function $f(t)=\frac{\rho a^3}{\rho_0 a_0^3}$, at any instant of time t is the ratio of matter content of the universe at the time t to the matter content at the present instant ($t=t_0$). Thus $f(t)$ can be regarded as proportional to the total content of matter (dark+baryonic) $M(t)$ of the universe at the instant of time t. We have denoted this ratio by R_1 where $R_1 \equiv f(t)=M(t)/M(t_0)$. We have defined a second ratio $R_2=\frac{1}{f}\frac{df}{dt}=\frac{1}{M}\frac{dM}{dt}$ which represents fractional change of matter per unit time. If, at any instant, R_2 is negative, it indicates a loss of matter or a change of matter into some other form due to its interaction with the scalar field. We have also defined a third ratio $R_3=f-1=\frac{M(t)-M(t_0)}{M(t_0)}$ indicating a fractional change of matter content from its value at the present time.

One may assume that the process of conversion of matter into dark energy started in the past at the time of $t=\gamma t_0$ where $\gamma<1$. Hence, $M(\gamma t_0)=M(t_0)R_1(\gamma t_0)$ is the total matter content of the universe, at $t=\gamma t_0$, when no dark energy existed. Thus $M(\gamma t_0)$ is the total content of matter and dark energy at all time. Assuming matter to be the only source of dark energy, the proportion of dark energy in the universe at any time t is given by the following ratio (R_4).

$$R_4=\frac{M(\gamma t_0)-M(t)}{M(\gamma t_0)}=\frac{f(\gamma t_0)-f(t)}{f(\gamma t_0)} \quad \gamma<1 \tag{9}$$

Thus $(R_4\times 100)$ is the percentage of dark energy present in the universe. Nearly 70% of the total matter-energy of the universe is dark energy at the present time [18]. For a proper choice of γ and k (to be defined later), we must have $R_4(t_0)\times 100 = 70$ approximately.

The proportion of matter (dark + baryonic) in the universe is therefore given by,

$$R_5=1-R_4=1-\frac{M(\gamma t_0)-M(t)}{M(\gamma t_0)}=\frac{M(t)}{M(\gamma t_0)}=\frac{f(t)}{f(\gamma t_0)} \tag{10}$$

Thus $(R_5\times 100)$ is the percentage of matter (dark + baryonic) present in the universe.
The purpose of the present study is to determine a functional form of $f(t)$ to explore the time dependence of the ratios $R_1, R_2, R_3, R_4 and\ R_5$.
Using these parameters, the density of dark energy can be expressed as,

$$\rho_d=\frac{R_4}{R_5}\rho=\frac{f(\gamma t_0)-f(t)}{f(t)}\rho \tag{11A}$$

Thus, the density of total matter and energy is, $\rho_t=\rho_d+\rho=\frac{\rho}{R_5}=\frac{f(\gamma t_0)}{f(t)}\rho$ (11B)

To formulate the expression of $f(t)$ we have used the following relation which is based on equation (8).

$$f(t)=a^3\frac{\rho}{\rho_0} \tag{12}$$

Here, the density of matter (ρ) can be obtained from equation (5). For this purpose one needs to choose some suitable functional form of the Brans-Dicke scalar field φ. In the present study we have chosen an empirical forms of φ, following some recent studies in this regard [24, 25]. The proposed ansatz for φ is expressed as,

$$\varphi = \varphi_0 \left(\frac{a}{a_0}\right)^k = \varphi_0 a^k \tag{13}$$

Here k is a constant which determines the rapidity with which the parameter φ $\left(\equiv \frac{1}{G}\right)$ changes with time.

Combining equation (13) with equation (5), one gets the following expression of density of matter of the universe (ρ).

$$\rho = \varphi H^2[k^2 + (4-q)k + (4-2q)] \tag{14}$$

Using equation (14), ρ_0 can be written as,

$$\rho_0 = \varphi_0 {H_0}^2[k^2 + (4-q_0)k + (4-2q_0)] \tag{15}$$

Substituting from equations (14) and (15) into equation (12) we get,

$$f(t) = a^3 \frac{\varphi H^2[k^2+(4-q)k+(4-2q)]}{\varphi_0 {H_0}^2[k^2+(4-q_0)k+(4-2q_0)]} \tag{16}$$

In our derivation of the equations (14) we have used the standard expressions of Hubble parameter (H) and deceleration parameter (q), which are, $H = \dot{a}/a$ and $q = -\ddot{a}a/\dot{a}^2$ respectively. In the expression of $f(t)$, in the equation (16), the parameters φ, H and q are all functions of time . Their time evolution depends upon the time variation of the scale factor from which they are calculated. To calculate $f(t)$ using equation (16), we have used an empirical scale factor. This scale factor has been chosen in order to satisfy a recent observation regarding the deceleration parameter $q(\equiv -\ddot{a}a/\dot{a}^2)$. According to this observation the universe had a state of decelerated expansion before the present phase of acceleration began [18, 19, 24]. Thus, the deceleration parameter had a positive value before reaching the present stage of negative values. The functional form of our chosen scale factor is such that the deceleration parameter, calculated from it, shows a change of sign as a function of time. This scale factor, also used by Roy *et al.* and Pradhan *et al.*, is expressed as [25, 27],

$$a = a_0\ (t/t_0)^\alpha\ Exp[\beta(t-t_0)] \tag{17}$$

Here, the constants $\alpha, \beta > 0$ to ensure an increase of scale factor with time. The scalar field parameter (φ), Hubble parameter (H) and deceleration parameter (q), based on this scale factor are given by,

$$\varphi = \varphi_0 \left(\frac{a}{a_0}\right)^k = \varphi_0\ (t/t_0)^{k\alpha}\ Exp[\beta k(t-t_0)] \tag{18}$$

$$H = \dot{a}/a = \beta + \frac{\alpha}{t} \quad (19)$$

$$q = -\ddot{a}a/\dot{a}^2 = -1 + \frac{\alpha}{(\alpha+\beta t)^2} \quad (20)$$

Here, for $0 < \alpha < 1$, we get $q > 0$ at $t = 0$ and, for $t \to \infty$, we have $q \to -1$.
It clearly means that the chosen scale factor generates an expression of deceleration parameters which changes sign from positive to negative as time goes on. The values of constant parameters (α, β) have been determined from the following conditions.

Condition 1: $H = H_0$ at $t = t_0$ (21 A)

Condition 2: $q = q_0$ at $t = t_0$ (21 B)

Combining the equations (21A) and (21B) with the equations (19) and (20) respectively, one obtains,

$$\alpha = (1 + q_0)(H_0 t_0)^2 = 4.76 \times 10^{-01} \quad (22\text{ A})$$

$$\beta = \frac{H_0 t_0 - \alpha}{t_0} = \frac{H_0 t_0 - (1+q_0)(H_0 t_0)^2}{t_0} = 1.25 \times 10^{-18} \quad (22\text{ B})$$

The values of different cosmological parameters used in the present study are,

$$H_0 = 72\left(\frac{Km}{Sec}\right) per\ Mega\ Parsec\ = 2.33 \times 10^{-18} sec^{-1}$$
$$t_0 = 14\ billion\ years = 4.415 \times 10^{17} sec$$
$$\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} m^{-3} Kgs^2$$
$\rho_0 = 2.831 \times 10^{-27} Kg/m^3$ [present density of matter (dark+baryonic)]
$$q_0 = -0.55$$

To determine the value of $f(t)$ from equation (16), one must apply the expressions of (18), (19), (20), (22A, B) and use the above mentioned values of cosmological parameters.

The function $f(t)$ is defined by the relation $\rho a^3 = f(t)\ \rho_0 a_0^3$. According to this relation, the value of $f(t)$is always positive and, $f(t) = 1$ at $t = t_0$(taking $a_0 = 1$). The functional form $f(t)$ in equation (16) ensures that $f(t) = 1$ at $t = t_0$. The values of k for which $f(t)$ is positive over the entire range of study (say, from $t = 0.5t_0$ to $t = 1.5t_0$) is given below.

$k < (k_-)_{min}$ or $k > (k_+)_{max}$ over the entire range of study. Here,

$(k_-)_{min} = (q-2)_{min}$ over the range from $t = 0.5t_0$ to $t = 1.5t_0$ (23)

$(k_+)_{max} = (q-2)_{max}$ over the range from $t = 0.5t_0$ to $t = 1.5t_0$ (24)

For our range of study, i.e. from $t = 0.5t_0$ to $t = 1.5t_0$, we find,

$(k_-)_{min} - 2.73$ and $(k_+)_{max} = -2.14$

Thus we have a lower and an upper range of permissible values for k which are $k < (k_-)_{min}$ or $k > (k_+)_{max}$ respectively. The upper range, $k > (k_+)_{max}$, includes both positive and negative values of k and the lower range, $k < (k_-)_{min}$, has only negative values. According to equation (13), the parameter φ is a decreasing function of time for negative values of k, causing the gravitational constant $(G = \frac{1}{\varphi})$ to increase with time. Therefore we find that the upper range of k allows G to be both increasing and decreasing function of time, although the lower range of k causes G to be an increasing function of time. To choose between these two ranges of k, we have to determine the values of ω_0 at different values of k and compare them with those obtained from other studies. Using equation (6) we can write the following expression (eqn. 25) regarding ω for this model.

$$\omega = \frac{2}{k^2}\left[3(1+k) - \frac{\rho}{\phi H^2}\right] \tag{25}$$

Using equations (25) we get the following expression of ω_0 (the value of ω at the present epoch).

$$\omega_0 = \frac{2}{k^2}\left[3(1+k) - \frac{\rho_0}{\phi_0 {H_0}^2}\right] \tag{26}$$

As per several studies on Brans-Dicke theory, ω_0 has a negative value close to (-1) [18].

To have $\omega_0 < 0$, the condition to be satisfied by k is given by,

$$k < \frac{\rho_0}{3\,\phi_0 {H_0}^2} - 1 \quad \text{or,} \quad k < -0.9884 \tag{27}$$

For the entire lower range of k values and for a part of its upper range, the above condition is satisfied.
The gravitational constant (G), which is reciprocal of the Brans-Dicke scalar field parameter (φ) is given by,

$$G = \frac{1}{\varphi} = \frac{(a/a_0)^{-k}}{\varphi_0} = \frac{1}{\varphi_0} Exp\left[k\alpha t_0^\beta\right] Exp\left[-k\alpha t^\beta\right]$$

Thus, $$\frac{G}{G_0} = Exp\left[k\alpha t_0^\beta\right] Exp\left[-k\alpha t^\beta\right] \tag{28}$$

An experimentally measurable parameter $\frac{\dot{G}}{G}$ is given by,

$$\frac{\dot{G}}{G} = \frac{1}{G}\frac{dG}{dt} = -k\frac{\dot{a}}{a} = -kH \tag{29}$$

Using equation (29) we get,

$$\left(\frac{\dot{G}}{G}\right)_{t=t_0} = -kH_0 \tag{30}$$

The value of k should be so chosen that $\left(\frac{\dot{G}}{G}\right)_{t=t_0} < 4 \times 10^{-10}\ Yr^{-1}$ [26].

3 Graphical interpretation of theoretical findings

We have plotted ω_0 as a function of k in Figure 1. For the lower range of k values, the values of ω_0 are negative and also close to the values obtained from other studies [18]. For the upper range of k, we have both positive and negative values of ω_0, the positive values being totally contrary to the findings of other studies in this regard [18].

According to equation (30), the gravitational constant increases with time for negative values of k. Thus, for the entire lower range of k values, G increases with time. Only for the negative values of the upper range, which constitute a small part of this range, G increases with time. There are experimental observations and theoretical models where G has been shown to be increasing with time [27].

In Figure 2 we have plotted the Brans-Dicke parameter ω as a function of time for a value of k in the lower range and for a negative value in its upper range. This is found to be an increasing function of time, with all negative values, for the lower range values of k and this behavior is quite consistent with other studies [18]. But its behavior is different for the upper range of k values. For the positive values of the upper range of k, the values of ω are all positive, as obtained from equation (25), which is inconsistent with the findings of other studies

On the basis of observations in the last three paragraphs, we have found it logical to choose values from the lower range of k to determine the time dependence of $f(t)$ and other relevant parameters connected to it in the present study.

Figure 3 shows the variation of $R_1(\equiv f)$ as a function of time for three different values of k in its lower range. It shows that the matter content of the universe $[M(t) = f(t)M_0]$ decreases with time and the rate of its change becomes faster for more negative values of k.

The variation of R_2 and R_3 as functions of $R_1(\equiv f)$ has been shown in Figure 4. From Figure 3 we know that R_1 decreases with time monotonically. By definition, R_2 is simply proportional to the rate of change of R_1. As R_1 approaches its present value of unity, R_2 becomes less and less negative, implying slower rate of production of dark energy from matter. R_3 is found to be positive in the past ($R_1 > 1$) and negative in future ($R_1 < 1$), as expected from its definition and the behavior of R_1.

Plots of R_4 and R_5 as functions of time are shown in Figure 5. The proportion of dark energy (R_4) in the universe is found to increase with time. Since it is assumed to be generated from matter (dark + baryonic), the proportion of matter in the universe (R_5) decreases with time. Thus, the sum of R_4 and R_5 is unity since the beginning of the conversion process at $t = \gamma t_0$. With $k = -4$ and $\gamma = 0.6$ these curves give nearly correct values at the present time $t = t_0$.

Figure 6 shows the plots of density of matter (ρ_m), dark energy (ρ_d) and total density (ρ_t) as functions of time. Equations (6), (9A, B) have been used for these plots. With $k = -4$ and $\gamma = 0.6$, one obtains approximately correct values from these curves for $t = t_0$.

Figure 7 shows plots of H and q as functions of R_1. As R_1 approaches its present value of unity, the dark energy content of universe becomes larger although its rate of generation from matter becomes slower. It can be inferred from these curves that larger proportion of dark energy causes faster changes in Hubble parameter and deceleration constant.

Figure 8 shows the variations of G/G_0 and $\dot{G}/G$ functions of R_1. As R_1 approaches its present value of unity, the dark energy content of universe becomes larger although its rate of generation from matter becomes slower. It appears from these curves that, as dark energy increases, the gravitational constant increases. One may also infer that, as $|R_2| \equiv |\dot{f}/f|$ becomes smaller with time, the values of $\dot{G}/G$ decreases. As the parameter k is made more and more negative, the fractional rate of depletion of matter content ($|R_2|$) increases and the rate of change of the gravitational constant, Hubble parameter and deceleration parameter increases. Therefore the dark energy content and its rate of production is found to have a role in governing the behavior of various cosmological parameters connected to the expansion of universe.

4 Conclusion

The purpose of the present study is to find out the time evolution of the matter content of the universe by using standard empirical expressions of scale factor and the Brans-Dicke scalar field parameter. The density of matter has been assumed to depend upon a function of time $f(t)$, which is proportional to the matter content of the universe. No specific form of this function has been assumed in the present model. Its form has been determined from the field equations. One may choose a particular form of this function $f(t)$ to get a better model. This function decreases with time, indicating a conversion of matter into dark energy. It is found through graphical analysis that, as the proportion of dark energy content of the universe increases with time, the deceleration parameter becomes more and more negative and the gravitational constant is found to increase with time. The implications of all these observations are that there is a possibility of a dependence of the gravitational constant and the deceleration parameter on the content of dark energy and the rate of its generation. It has been found in the present study that if the matter of the universe is regarded as the only source of dark energy, the present proportions of both of them must depend on how long this process of conversion has been continuing. The experimentally determined values of the present proportions of matter and dark energy can be obtained from this model by a proper tuning of parameters. As a future plan to continue this study, one may improve this model by changing the functional form of the empirically chosen Brans-Dicke scalar field parameter (φ) and also by choosing a specific form of $f(t)$ in terms of φ and the scale factor. The choice of the functional form of $f(t)$, which accounts for the non-conservation of matter content of the universe, should be such that its value must be unity at the present time, as per the requirement of its defining relation expressed by equation (10). From different functional forms of $f(t)$, one may determine the nature of dependence of the cosmological parameters like gravitational constant, Hubble parameter, deceleration parameter etc. upon the dark energy content of the universe and also the rate of conversion of matter into dark energy.

FIGURES

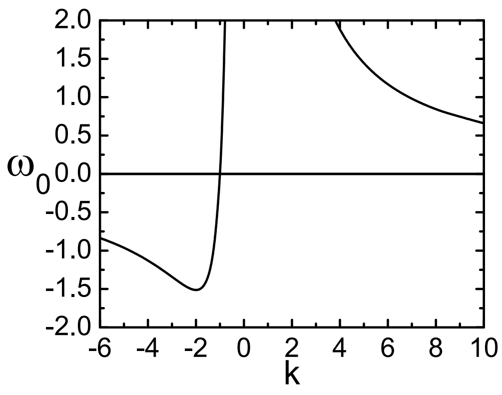

Figure 1: Plot of the Brans-Dicke dimensionless parameter ω at the present epoch $(t = t_0)$ as a function of k.

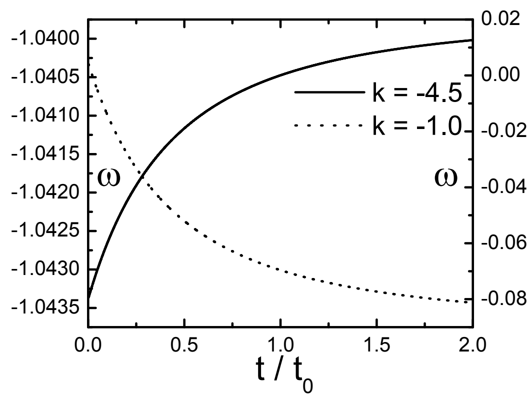

Figure 2: Plot of ω as a function of time, for a value of k in the upper range (*dotted*) and one in the lower range (*solid*), shown along the right and left vertical axes respectively.

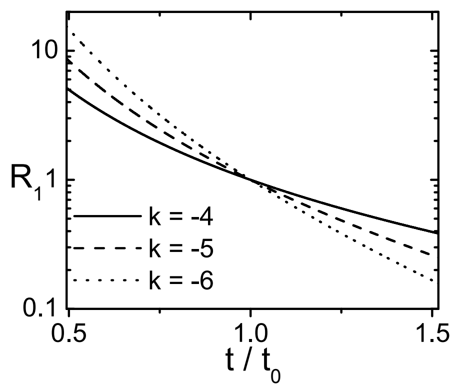

Figure 3: Plot of $R_1[\equiv f(t)]$ as a function of time for three different values of k in its lower range.

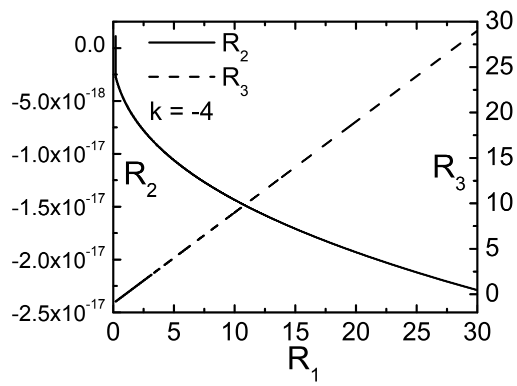

Figure 4: Plot of R_2 and R_3 as a function of R_1 time for three different values of k in its lower range.

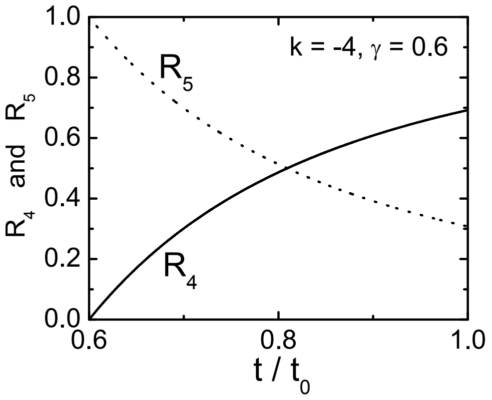

Figure 5: Plot of R_4 (*solid*) and R_5 (*dotted*), the proportions of dark energy and matter respectively, as functions of time. Matter to energy conversion began at $t = \gamma t_0$.

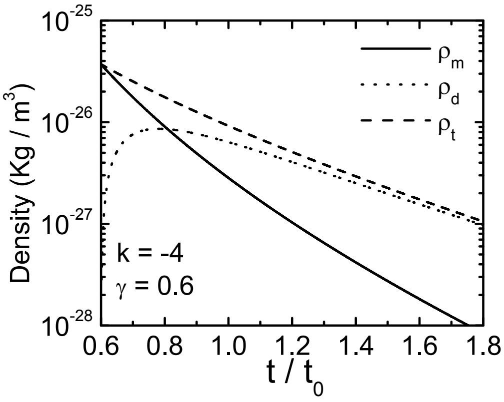

Figure 6: Plot of density of matter ($\rho_m \equiv \rho$), dark energy (ρ_d) and total density (ρ_t) as functions of time.

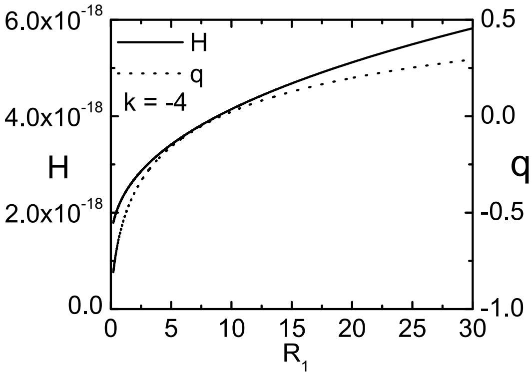

Figure 7: Plot of H and q as functions of R_1 along the left and right vertical axes respectively.

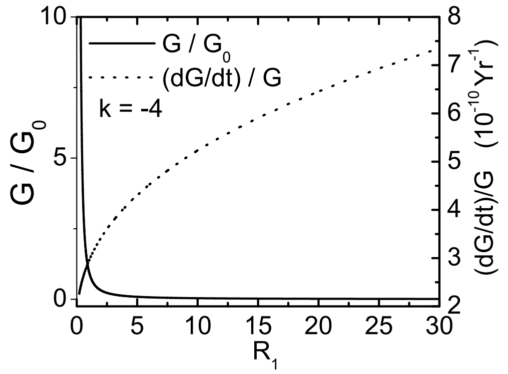

Figure 8: Plot of G/G_0 and $\dot{G}/G$ as functions of R_1 along the left and right vertical axes respectively.

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