The relativity of simultaneity and presentism

Mario Bacelar Valente

Abstract

According to conventional wisdom, presentism is at odds with the theory of relativity. This is supposed to be shown quite simply just by considering the relativity of simultaneity. In this paper I will show that conventional wisdom is wrong. In fact by clarifying the physical meaning of the relativity of simultaneity one can inform the philosophical debate and endorse a presentist view.

1 Introduction

There is in the philosophical debate around the conception of time in the 1905 theory of relativity different levels. There is one important debate regarding to what point philosophy of time might or might not be informed by the theory of relativity (or other physical theories); one clear example of this being the presentism/eternalism debate and to what point physical theories can guide/help settle issues within the debate (see, e.g., Hawley 2006).

At another level of the debate it is usually accepted implicitly the relevance of scientific conceptions for an informed philosophy of time (see, e.g., Mozersky 2000, Arthur 2006). This is particularly clear in the case of the presentism/eternalism debate in which the relativity of simultaneity is taken by a majority of authors to show that presentism is not an option when one accepts a philosophically informed view taking into account the theory of relativity (see, e.g., Savitt 2000, Saunders 2002). It is at this level that this work is developed. The work is presented as follows.

In section 2 I will address what has been considered since its inception an important factor for changing from the previous notions of time: the so-called relativity of simultaneity. Also I will address one example of a philosophical view informed by the theory of relativity, consisting in a criticism of presentism (Petkov 2009).

In section 3 I will provide a clarification of the relativity of simultaneity by recovering Einstein early insights, and making clear their meaning. This will enable to see that the relativity of simultaneity is simply a relativity (i.e. dependence on the reference frame) of the synchronization of distant clocks.

In section 4 I will return to the criticism of presentism presented in section 2 and, using the results from section 3, show that it is based on a wrong interpretation of the relativity of simultaneity. In this way, I will make the case that the relativity of simultaneity is compatible with presentism.

2 The relativity of simultaneity and an example of a related informed philosophical view

Einstein identified the relativity of simultaneity as the most important consequence of the theory of relativity (Einstein 1914, 4), even referring to it as the relativity of time (Einstein 1915, 254).

Let us follow Einstein's account leading to the so-called relativity of simultaneity. First of all the time at a particular location (within a reference frame) is

measured/defined by a clock located in the immediate vicinity (Einstein 1907, 255). By a clock one understands a closed physical system that undergoes a recurrent process (Einstein 1912-1914, 29; Einstein 1915, 252-3). According to Einstein, "by means of the determination of this clock, every event that is spatially infinitely close to the [clock] can be assigned a temporal determination, the "time coordinate," or, in brief, the "time" of the event" (Einstein 1912-1914, 29). It is important to notice that "only the times of events occurring in the immediate vicinity of the clock can be ascertained directly by means of the clock" (Einstein 1915, 253). This means that at this moment one only has a notion of 'time' in the vicinity of the chosen clock. To go beyond this point one must consider a set of identical clocks that we can imagine to be located at different places of a rigid measuring framework, i.e. constituting with rods an inertial reference frame that enables to measured length and time intervals. In this way "to determine time at each point in space we can imagine it populated with a very great number of clocks of identical construction" (Einstein 1910, 125). At this point one can associate to each location a 'time coordinate', but these are unrelated. As Einstein called the attention to, "the totality of these clock readings does not yet give us the "time" as we need it for physical purpose" (Einstein 1907, 256). According to Einstein "to get a complete physical definition of time, we have to take an additional step. We have to say in what manner all of the clocks have been set at the start of the experiment" (Einstein 1910, 126). This can be achieved by synchronizing the clocks of the reference frame in a way that we can say that the different clocks are in phase; in simple terms, the hands of the clocks are made to be all pointing to the same time reading, and since they are identical clocks they run at the same pace, in this way maintaining the same phase. In this way we arrive at what Einstein calls the physical definition of time: "the totality of the readings of all clocks in phase with one another is what we will call the physical time" (Einstein 1910, 127).

This notion/definition of time is made in relation to a particular reference frame where identical clocks are synchronized. This means that "a statement on time has a meaning only with reference to a reference system" (Einstein 1907, 257).

The point now is what to make of a particular statement on time made in reference to a particular reference frame S from the perspective of another reference frame S' in relative motion. It is here that we start to address questions related to the notion of simultaneity. Let us consider the statement on time $\{stat\} = \{two \text{ spatially distant point events are simultaneous with respect to a reference frame S}. We know that in Newtonian mechanics this statement is valid for all inertial reference frames (see, e.g., Torretti 1996, 28); but is this the case in the theory of relativity? Let us follow Einstein on this issue:$

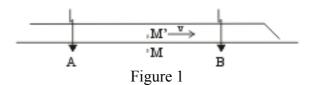
Consider two nonaccelerated coordinate systems S and S' in uniform translational motion with respect to one another. Suppose that each of these systems is provided with a group of clocks invariably attached to it, and that all the clocks belonging to the same system are in phase. Under these conditions the readings of the group attached to S will define the physical time with respect to S; analogously, the readings of the group attached to S' define the physical time with respect to S'. Each elementary event will have a time coordinate t with respect to S, and a time coordinate t' with respect to S'. But, we have no right to assume a priori that the clocks of the two groups can be set in such a manner that the two time coordinates of the elementary event would be the same, or in other words, in such a way that t would be equal to t'. (Einstein 1910, 128)

In fact, since the physical definition of time is particular to each reference frame it will turn out that the previous statement on time $\{stat\}$ is not valid in all the inertial reference frames in relative motion. We can see this easily by considering the Lorentz transformation that relates space and time determinations made in two reference frames in relative motion (Einstein 1915, 256; Stephenson and Kilmister 1958, 37). Let us consider two events with coordinates (t_1, x_1) and (t_2, x_2) in the reference frame S, and coordinates (t_1', x_1') and (t_2', x_2') in the reference frame S'. According to the Lorentz transformation $t_1' - t_2' = \beta[t_1 - t_2 - v(x_1 - x_2)/c^2]$, where $\beta = 1/$ sqrt $(1 - v^2/c^2)$, c is the velocity of light in vacuum, and v is the relative velocity between S and S'. In the particular case of two events that in S are simultaneous but occurring at different locations we have $t_1 = t_2$ and $x_1 \neq x_2$, and so $t_1' - t_2' = \beta v(x_1 - x_2)/c^2$, i.e. in S' the two events are not simultaneous. In this reference frame it is considered to exist a time interval between the two events.

It is possible to give a less mathematical presentation of the relativity of simultaneity (see, e.g., Einstein 1916, 274-6; Bergmann 1942, 30-2). Let us imagine a train in relative motion in relation to an embankment and that two thunderbolts strike simultaneously with reference to the embankment, leaving permanent marks on a train and on the embankment (see figure 1). The light from the thunderbolts is reflected and directed towards two observers (one on the train, the other on the embankment) located midway between the marks, at M' and M. The light beams arrive at the same time at M. however M' is in relative motion. When the thunderbolts strike, M' coincides with M but it is moving with the train with a velocity v. In this way, an observer locate at M' is

hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A. Hence the observer will see the beam of light emitted from B earlier than he will see that emitted from A. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightning flash B took place earlier than the lightning flash A. (Einstein 1916, 275)

We arrive then at the so-called relativity of simultaneity: two events (in this case the two thunderbolts striking) are taken to be simultaneous in a reference frame (the embankment), while in another reference frame (the train) are taken not to be simultaneous.

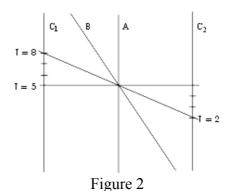


This result is regarded as quite important on physically informed philosophical views on time. In fact, some consider the relativity of simultaneity a decisive element against presentist views and in agreement with eternalist views (see, e.g. Petkov 2006; Peterson and Silberstein 2010). I will focus just on one exemplifying argument made by Petkov (2009).¹

¹ In this work I will only consider Petkov views related to the relativity of simultaneity presented in Petkov (2009).

By considering the relativity of simultaneity Petkov makes the case that the only way in which presentism can accommodate the relativity of simultaneity is by relativizing what he calls the three-dimensional (3D) world. For an eternalist (four-dimensionalist) a 3D object is just a slice of a four-dimensional (4D) worldline of a timelessly existing 4D world (or block universe) in which all the slices (i.e. the 3D objects) are actually all given at once. For a presentist, the 3D world consists of all 3D objects and fields existing simultaneously at the moment 'now' or 'present'. In this view there is a clear differentiation between past, present and future. The past consists in the previous states of the 3D world. What we call the future refers to the expected forthcoming states. In this way "the past and the future do not exist on the presentist view - they are merely states of the 3D world which exists solely at the present moment" (Petkov 2009, 126).

To make his case Petkov considers two observers A and B in relative motion. We can regard the observers to be located at the origin of two inertial reference frames. Let us consider two clocks C_1 and C_2 from A's reference frame (at the locations -d and +d). Figure 2 gives a schematic representation of the clocks' worldlines and the planes of simultaneity for A and B.



As it is usually done, we consider that A and B meet at an event M (i.e. that there is a moment in which the origins of the reference frames coincide) and that they set the clocks to $t_A = t_B = 5$. To Petkov, for a presentist it only makes sense to consider that the clocks exist only at the moment 'now' of their proper time. Accordingly, "to observer A, both clocks exist at the 5th second of the coordinate time measured in A's reference frame" (Petkov 2009, 129); in this way "A comes to the conclusion that C_1 and C_2 both exist at the 5th second of their proper times" (Petkov 2009, 129). For Petkov this result is to be expected since, for presentists, all objects share the same 'now'.

The problem facing presentism is that, "what is simultaneous for A, however, is not simultaneous for B" (Petkov 2009, 129). When considering B's plane of simultaneity we see that

what is simultaneous for B at the 5th second of B's time (when B meets A at M) is clock C_1 existing at the 8th second of its proper time and clock C_2 existing at the 2th second of its proper time. Therefore, for B the moment 'now' of the proper time of C_1 is the 8th second, whereas the present moment of C_2 is the 2th second of its proper time. (Petkov 2009, 129)

According to Petkov interpretation of the relativity of simultaneity "when A and B meet at M, they will disagree on which is the present moment of each of the clocks and on what exists for them at the moment of meeting" (Petkov 2009, 129). Petkov concludes that the "relativity of simultaneity is possible in the framework of the presentist view if different pairs of clocks exist for A and B at M" (Petkov 2009, 129), i.e. the presentist must regard the existence of the 3D clocks as relative (observer- or frame-dependent). If this was the case, one should endorse Petkov view that

the concept of existence employed by a relativized version of presentism is so twisted that Nature is unlikely to be impressed by this pushing of the human imagination to such an extreme that allows observer A to claim that C_1 at its 8th second does not exist for him but exists for B ... This shows why the price presentism should pay to avoid a contradiction with relativity is an ontological relativization of existence. (Petkov 2009, 130-1)

I will postpone a critical evaluation of Petkov views until I make a clarification of the so-called relativity of simultaneity. However, this example shows that, if one accepts that our philosophical views on time should be in agreement with what one considers to be the content/consequences of our physical theories, physically informed arguments can be quite elaborated and consistent with the adopted interpretations.²

3 The relativity of simultaneity as (just) a relativity of synchronization of distant clocks

Petkov interpretation of the relativity of simultaneity goes well beyond Einstein's presentation. Returning to Einstein's example of a train in motion relative to an embankment, Einstein mentions that

Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train, and vice versa (relativity of simultaneity). Every reference-body (co-ordinate system) has its own particular time; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event. (Einstein 1916, 275-6)

It is important to have in mind that Einstein arrives at these views not from a vague and intuitive notion of time, but by following a very defined narrative in which he gives a definition of physical time; and the so-called relativity of simultaneity is revealed by statements on time related to this narrative. In a nutshell we have: (1) identical clocks; (2) the 'distribution' of the clocks to make a space-time reference frame; (3) the synchronization of the clocks in the particular reference frame; (4) the adoption of the same procedure for all other inertial reference frames that are in relative motion or rest between all of them.

By following steps 1 to 3 one defines physical time in a particular reference frame; by applying 4 one defines physical time in other reference frames. Einstein's views on time are views on the physical time defined in this way.

By following Einstein's own account I will now show that by giving a closer look at the step 3, i.e. by analysing the synchronization procedure, one arrives at an

² This does not mean that it is not possible to present philosophical arguments that 'resist' scientifically informed views (see, e.g., Dorato 2006)

interpretation of the so-called relativity of simultaneity quite different from the one being made by Petkov.

By taking the velocity of light to be isotropic, i.e. not dependent on any direction, Einstein considers that two clocks located at A and B are synchronous if the time intervals (measured using the identical clocks A and B) that light takes to go from A to B and from B to A are equal: "suppose a ray of light leaves from A toward B at "A-time" t_A , is reflected from B toward A at "B-time" t_B , and arrives back at "A-time" t_A . The two clocks are synchronous by definition if $t_B - t_A = t_A' - t_B$ " (Einstein 1905, 142).

Einstein asks us to imagine a rod in motion in relation to a reference frame where the clocks were previously synchronized, and with clocks located at the two ends. Importantly one supposes that these clocks are also synchronous with the clocks of the reference frame without taking into account that they are in relative motion: "the two ends (A and B) of the rod are equipped with clocks that are synchronous with the clocks of the system at rest, i.e., whose readings always correspond to the "time of the system at rest" at the locations they happen to occupy" (Einstein 1905, 144-5). Now we suppose that next to these clocks (that give the time reading of the reference frame in relation to which the rod is in motion), are two observers that "apply to the two clocks the [previously mentioned] criterion for synchronism" (Einstein 1905, 145). From the perspective of the observers, the time it takes light to go from A to B is given by $t_B - t_A = r_{ab}/(v - c)$, where r_{ab} is the length of the 'moving' rod as measured in the reference frame (Einstein 1905, 144-5). This time interval is different from the time it takes the light to go back from B to A: t_A ' $- t_B = r_{ab}/(v + c)$. From this Einstein concludes that

the observers co-moving with the moving rod would thus find that the two clocks do not run synchronously while the observers in the system at rest would declare them synchronous.

Thus we see that we must not ascribe absolute meaning to the concept of simultaneity; instead, two events that are simultaneous when observed from some particular coordinate system can no longer be considered simultaneous when observed from a system that is moving relative to that system. (Einstein 1905, 145)

This is an immensely important statement. Einstein presents for the first time the idea of relativity of simultaneity by concluding that the synchronization procedure is frame-

³ I think Eddington's related presentation is clearer than Einstein's (Eddington 1923, 28). Let us consider two reference frames S and S' in relative motion (with velocity v; i.e., in which S' is moving with a relative velocity v in the positive x-direction of the reference frame S) that have their clocks synchronized according to Einstein's prescription. Let A and B be two clocks from reference frame S'. A signal is emitted from A at time t₁', arriving at B at time t_B', being reflected back an arriving at A at time t₂'. Since the clocks where synchronized, the instant t_B at B is simultaneous with the instant $\frac{1}{2}(t_1' + t_2')$ at A. However things look different from the perspective of S. For S the two clocks are moving; in this way, for S, the time the signal takes in going from A to B is x/(c-v), where x is the distance between A and B as measured in S, while the time it takes the signal to return to A from B is x/(c-v). Since x/(c-v) $\beta^2 x/c^2$ (c + v) and $x/(c-v) = \beta^2 x/c^2$ (c - v), to S the instant of arrival t_B at B is $\beta^2 x/c^2$ later than the halfway instant $\frac{1}{2}$ (t₁ + t₂), which is the time of arrival at B according to S' as measured by S (i.e. βt_B '). We see that to S the clocks A and B are not synchronized. Taking into account the so-called length contraction we see that the desynchronization factor of B when seen from S is given by $\beta x'v/c^2$, where x' is the distance between A and B as measured in S'. If we consider that the two reference frames S and S' had set their time to zero at the moment the origin of the reference frames coincided and that A is located at the origin of S', then $t_B = \beta t_{B'} - \beta v x_{B'}/c^2$, where $x_{B'}$ is the location of the clock B in the reference frame S'. This means that a clock C located further away from the origin of S'at $x_C' > x_B'$ will from the perspective of S be even more desynchronised than B in relation to the clock located at the origin of S' (see also López-Ramos 2008).

dependent, i.e. that the synchronization procedure is relative to the reference frame where it is applied. Clocks taken to be synchronous in one reference frame are taken to be desynchronised from the perspective of another reference frame.⁴ The synchronization procedure is not absolute; it is relative to the reference frame where it is made. This is what the relativity of simultaneity is all about. All of Einstein's comments regarding the 'relativity of time' or the 'relativity of simultaneity' are made in the context of defining physical time in each reference frame by a frame-dependent synchronization procedure.

In the next section by analysing again Petkov example we will see what a different does it make this clarification.

4 Why the relativity of simultaneity can be in agreement with presentist views.

Let us look again at figure 2, now thinking in terms of a relativity of synchronization. All the clocks are spatially located, one in relation to another, and this independently of being in relative motion or relative rest. The clocks C_1 and C_2 at rest in relation to A are synchronized with A according to Einstein's prescription. The clock B passing by A is not in the past of A or in the future of A; it is co-existent with A. As Einstein showed, the procedure that synchronizes C_1 and C_2 to A, enabling to define the physical time in this reference frame, is not for B a synchronization; this is due to the fact that for B the clocks have a phase lag given by a desynchronization factor $-\beta vx/c^2$, where x is the clock's position in A's reference frame.

In this way, when clock C₁ is synchronized to A's clock, there is for B a desynchronization factor that must be taken into account (in this case of +3 seconds), and the same occurs in relation to clock C_2 (that has a phase lag, for B, of – 3 seconds). What follows is that A considers that C₁ and C₂ are in synchrony with A's clock, and because of this takes them to mark simultaneous events (i.e. the ticking of the different clocks are considered to be in phase). However, B (that takes the ticking of the clocks of B's reference frame to be in phase, i.e. simultaneous) considers that the ticking of all of the clocks in A's reference frame go at the same rate but that there is an increasing constant phase lag from a clock to another as one is further away from A' clock (i.e. that the clocks run at the same pace but where set at different 'initial' times by a 'wrong' synchronization procedure; e.g. A's clock was set at 12h, A₂'s clock set at 11h 45min, A₃'s clock set at 11h 30 min, and so on). Thus, for B, events that are simultaneous with the ticking of the clocks in A's reference frame are seen to correspond to different instants of time due to the lack of synchronization of A's clocks from B's perspective. It is clear from this that B is not 'seeing' clock C₁ 3 seconds in the future or 'seeing' clock C₂ 3 seconds in the past. The synchronization between clocks does not send the clocks into the past or into the future; B is simply measuring, due to the fact that the synchronization procedure adopted by A is relative to A's reference frame, a phase lag in the clocks synchronized in A's reference frame.

In this way, Petkov is wrong when considering that "for B clock C_1 exists at the 8th second of its proper time (at its 'now') and clock C_2 exists at the 2th second of its proper time (at its 'now')" (Petkov 2009, 129). This means that all of his argumentation relying in an eternalist interpretation of the relativity of simultaneity cannot be made to stand.

⁴ Unfortunately this result is not usually recognized, leading to a lot of confusion in the interpretation of the so-called relativity of simultaneity. One exception in a standard textbook is Bergmann (1942) that writes: "observed from the unstarred frame of reference, different S*-clocks go at the same rate, *but with a phase constant depending on their position* … two events that occur simultaneously with respect to S are not in general simultaneous with respect to S*, and vive versa" (Bergmann 1942, 38 [my emphasis]).

In fact by clarifying the physical meaning of the relativity of simultaneity as a relativity (frame-dependence) of the synchronization of distant clocks it turns out that, contrary to conventional wisdom, presentism seems not to be at odds with the relativity of simultaneity.

If we return to Einstein's definition of physical time for different reference frames, all that we have are clocks spatially located one in relation to another (at relative rest or motion), i.e. clocks that co-exist spatially. We only deal with what Petkov called the 3D world.

When applying the synchronization procedure in different reference frames, we find out that the procedure (or any equivalent procedure; see Eddington 2009, 27-9) is frame-dependent (i.e. relative). This relativity of the synchronization of distant clocks was 'baptized' by Einstein with the name 'relativity of simultaneity', but it does not mean that suddenly, just because B's clock is in relative motion in relation to A's clock (and C_1 and C_2), A's clock is co-existent with B's clock while clocks C_1 and C_2 slip into the future and the past.

As we have seen the only thing that happens is that B assigns different phases to C_1 and C_2 , and this can be done because C_1 and C_2 co-exist with B (i.e. they share the same now). In fact B measures the time light takes to go from A to each of the clocks, i.e. C_1 and C_2 are in the 'midst' of B's reference frame so that B can observe them and attribute to them a particular phase. Nowhere the notion of past and future enter the picture. We only have spatially co-existent clocks. In fact this spatially co-existence of all the clocks of all the reference frames can be called 'now' or 'present'. Presentism seems to be very much at home with the so-called relativity of simultaneity (or better, relativity of synchronization).

5 Conclusion

If one endorsers a scientifically informed philosophy of time, it turns out, contrary to conventional wisdom, that presentism does not have to be seen at odds with the relativity of simultaneity. This is made possible by clarifying what relativity of simultaneity is all about. The relativity of simultaneity arises from the frame-dependence of the synchronization of distant clocks. This means that clocks that are synchronized in one reference frame S are taken to be desynchronised in a reference frame S' in relative motion in relation to S. All this is going on between spatially co-existent clocks; if we take this spatial co-existence of distant clocks to be the definition of 'now' or 'present', there is no need to rely on the notions of 'past' or 'future' to give an account of the relativity of synchronization of distant clocks. In this way presentism can be regarded as a well-founded philosophical position informed by the relativity of simultaneity.

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