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Irreversibility at zero temperature from the perspective of the environment

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We address the emergence of entropy production in the nonequilibrium process of an open quantum system from the viewpoint of the environment. By making use of a dilation-based approach akin to the Stinespring theorem, we derive an expression for the entropy production that comprises two fundamental contributions. The first is linked to the rate of the creation of correlations between the system and environment whereas the second highlights the possibility for the environment to modify its state in light of its coupling to the system. Both terms are shown to be associated with irreversible currents within the system and the environment, which pinpoint the emergence of irreversibility in the Markovian limit. Finally, we discuss how such a change of perspective in the study of entropy production has fecund implications for the study of non-Markovian open-system dynamics.

Introduction. Irreversibility is an emergent concept, stemming from the unavoidable loss of information that occurs during the evolution of systems containing a macroscopically large number of degrees of freedom. Despite considerable research for more than a century, several open questions remain as to the physical origins and implications of this concept. Recently, however, this field has experienced a boom of advances, motivated partially by improvements in the experimental control of mesoscopic systems [1–5] and quantum technology platforms [6,7]. This has allowed us to have direct control over the number of degrees of freedom [8] and the magnitude of classical and quantum fluctuations [9], opening the way to directly experiment with the emergence of irreversibility. Such advances have also been accompanied by fundamental progress in our theoretical understanding of out-of-equilibrium thermodynamics, such as the discovery of fluctuation theorems [10–17], the role of quantum correlations [18,19], and the interplay between thermodynamics and information [20–23].

Irreversibility is traditionally characterized by the concept of entropy production. But entropy production is not a physical observable and must therefore be related to observables by means of a theoretical framework [24–37]. For instance, entropy production can be associated with irreversible work in a work protocol [10,11] or with the heat exchanged between two systems [12]. A more sophisticated approach is based on the idea of stochastic trajectories [38,39], in which case the entropy production is associated with the ratio between forward and time-reversed path probabilities [10,16,31].

One aspect that has so far not been explored in detail concerns an understanding of entropy production from the perspective of the environment and the global unitary dynamics of the system plus environment. This problem was studied in

Ref. [40], where the authors identified a relation between entropy production and system-environment correlations. However, it is generally not fully known what is the importance of this contribution and what other possible contributions there may exist. The problem is also aggravated by the fact that such an answer, by construction, cannot be unique, for two main reasons. First, there is an infinite number of environments and interactions which leads to the same reduced dynamics of the system. And second, most of these reduced dynamics are obtained by means of a series of approximations (e.g., Born-Markov [35]), which make it impossible to keep track of the true physical contributions to the entropy production.

The goal of this Rapid Communication is to address the emergence of entropy production from the perspective of the environment and the system-environment interaction. To accomplish this, we consider the simple yet nontrivial model of a bosonic mode undergoing amplitude damping in contact with a zero-temperature bath, as described by a Lindblad master equation. The key of our approach is to consider all possible Gaussian dilations [41] of such dynamics, that is, all possible Gaussian unitary transformations, acting in the Hilbert space of both the system and the environment, which generate exactly the master equation that we aim at addressing. This eliminates the arbitrariness concerning the choice of the bath and allows us to avoid the use of any approximations that may hamper our ability to describe the entropy production.

The Gaussianity of the global dynamics allows us to construct a theory of entropy production using the idea of Wigner entropy, introduced recently in Ref. [42]. This theory has the advantage of operating in quantum phase space, where one may identify quasiprobability currents that represent the microscopic manifestations of the irreversible motion [14,31,32]. Looking then at the global unitary dynamics, we are able to identify two contributions to the entropy production, one related to the creation of mutual information between the

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system and its bath and the other related to the displacement of the bath from equilibrium. Moreover, we identify what are the irreversible currents acting within the system *and* the bath, and which are responsible for the emergence of irreversibility on the dynamics of the bath. Literature identifying the role played by these irreversible bath currents in the emergence of irreversibility is lacking. Finally, we also exploit the fact that our framework is readily applicable to non-Markovian systems, which allows us to identify such contributions to the entropy production as witnesses of non-Markovianity from the perspective of the bath. Potential applications to quantum heat engines are also discussed.

The choice of studying the zero-temperature amplitude damping channel is also motivated by a more fundamental reason. Despite being one of the simplest examples of an irreversible process, this problem cannot be described by the standard formalism of entropy production, which uses the von Neumann entropy [33–36]. The reason is ultimately related to the divergence of the quantum relative entropy when the reference state becomes pure [43–46] and leads to a divergence of the entropy production in the limit $T \to 0$. But whether or not this divergence has a physical significance has so far been an open question. The results presented here indicate that the divergence of the entropy production at zero temperature is nothing but a mathematical limitation of the quantum relative entropy.

The model. We consider a bosonic system (S) with Hamiltonian $H_S = \omega a^\dagger a$, where a ($a^\dagger$) is the system annihilation (creation) operator. We assume that S is subjected to a zero-temperature amplitude-damping channel described, in the interaction picture with respect to H_S, by the Lindblad master equation

$$\frac{d\rho_S}{dt} = 2\kappa \left[a\rho_S a^\dagger - \frac{1}{2}\{a^\dagger a, \rho_S\} \right], \tag{1}$$

where κ is the decay rate. We work in phase space by introducing the Wigner function $W_S(\alpha, \alpha^*)$ and transforming Eq. (1) into the quantum Fokker-Planck equation

$$\partial_t W_S = \partial_\alpha J_S + \partial_{\alpha^*} J_S^*, \tag{2}$$

where

$$J_S(W_S) = \kappa \left(\alpha + \frac{\partial_{\alpha^*}}{2} \right) W_S. \tag{3}$$

Equation (2) has the form of a continuity equation, thus allowing us to attribute to J_S the meaning of a current in phase space. This is further corroborated by the fact that J_S itself vanishes in the equilibrium state, which in this case is the vacuum $W_S^\infty = e^{-2|\alpha|^2}/\pi$.

The standard formalism of entropy production, which uses the von Neumann entropy, gives diverging results for this model. To circumvent this difficulty, we have shown in Ref. [42] that for Gaussian states one could use instead the Wigner entropy $S(W_S) = -\int d^2\alpha \, W_S \ln W_S$ (which also coincides with the Rényi-2 entropy [47]). The entropy production associated with Eq. (2) was then found to be [42]

$$\Pi = -\frac{d}{dt}S(W_S||W_S^\infty) = \frac{4}{\kappa}\int d^2\alpha \, \frac{|J_S(W_S)|^2}{W_S}, \tag{4}$$

where $S(W_1||W_2) = \int d^2\alpha \, W_1 \ln(W_1/W_2)$ is the Wigner relative entropy. The second equality in Eq. (4) establishes a direct relation between irreversibility and the existence of the current J_S. In fact, within the classical context, the quantity J_S/W_S is usually interpreted as a velocity in phase space [14,30,48].

Gaussian dilations. We now wish to describe the physics behind Eq. (4) from the perspective of the global dynamics of the system (S) plus environment (E). To do so, we ask what are the possible dilations which reproduce exactly the full dynamics of Eq. (1). We assume that the environment is bosonic, consisting of a set of modes b_k initially prepared in the global vacuum $|0\rangle_E$. Moreover, since Eq. (1) is Gaussian preserving, the same must also be true for the global unitary. Then, the most general Gaussian Hamiltonian must have the form [49–51]

$$H_T = \omega a^\dagger a + \sum_k \Omega_k b_k^\dagger b_k + \sum_k \gamma_k(a^\dagger b_k + b_k^\dagger a), \tag{5}$$

where Ω_k is the frequency of mode k and γ_k the corresponding coupling constant. Squeezing terms ($a^\dagger b_k^\dagger$) are not allowed due to the fact that the global vacuum $|0\rangle_S \otimes |0\rangle_E$ must be a fixed point of the unitary. Moreover, any other quadratic Hamiltonian (for instance, containing interactions between the bath modes) can be cast into the form (5) by a suitable normal mode transformation and renormalization of parameters.

We emphasize at this point that the choice of dealing with a Gaussian model is not actually a restriction, but more of a necessity. This is due to two main reasons. First, the problem of irreversibility at zero temperature cannot be handled with the standard measures of entropy production, leaving the Wigner framework, as of now, as the only available method. Second, as mentioned in the Introduction, in order to properly account for the entropy production from the perspective of the environment, one must deal with exactly soluble models of system-environment interactions. Again, in this case, Gaussian models are among the very few that are available. That being said, it is also worth mentioning the possible routes for generalizing these results to the non-Gaussian case. First, a phase-space framework may be developed in this case by means of the Husimi-Q function and its corresponding phase-space entropy [52]. Second, finite-temperature effects could be introduced for non-Gaussian systems using the concept of thermal operations [53], which are the family of dilations reproducing Davies master equations.

The dynamics generated by Eq. (5) depends only on two auxiliary functions, $g(t)$ and $f_k(t)$, which satisfy (see Supplemental Material [54])

$$\frac{dg}{dt} = -i\sum_k \gamma_k e^{i(\omega - \Omega_k)t} f_k(t), \tag{6}$$

$$\frac{df_k}{dt} = -i\gamma_k e^{-i(\omega - \Omega_k)t} g(t), \tag{7}$$

with initial conditions $g(0) = 1$ and $f_k(0) = 0$. We also assume that S starts in a Gaussian state, which is therefore characterized by the numbers (μ, N, M), where $\mu = \langle a \rangle_0$, $N = \langle \delta a^\dagger \delta a \rangle_0$, and $M = \langle \delta a \delta a \rangle_0$ (here, $\delta a = a - \langle a \rangle$). The first moments are then $\langle a \rangle_t = \mu g(t)$ and $\langle b_k \rangle_t = \mu f_k(t)$. The expression for the covariance matrix, including all SE correlations, is presented in Ref. [54].

Substituting the formal solution of Eq. (7) into (6) yields an integrodifferential equation for $g(t)$, which is in general non-Markovian. All results in this Rapid Communication will be given in terms of $g(t)$ and therefore hold also in the non-Markovian case. The Markovian limit of Eq. (1) corresponds to $g(t) = e^{-\kappa t}$ and is recovered asymptotically via a Wigner-Weisskopf approximation [55,56], after certain assumptions on the spectral density of the system [54]. We shall refer to this as the time-independent Markovian (TIM) limit.

Quantum Fokker-Planck equations. In the interaction picture, the global Wigner function $W_{SE}(\alpha, \alpha^*, \beta_1, \beta_1^*, \ldots)$ will satisfy the unitary equation

$$\partial_t W_{SE} = \partial_\alpha \mathcal{J}_S + \partial_{\alpha^*} \mathcal{J}_S^* + \sum_k \left(\partial_{\beta_k} \mathcal{J}_k + \partial_{\beta_k^*} \mathcal{J}_k^* \right), \quad (8)$$

where $\mathcal{J}_S$ and $\mathcal{J}_k$ are unitary currents. They can be expressed in terms of the auxiliary functions (6) and (7) as

$$\mathcal{J}_S(W_{SE}) = \frac{1}{g^*} \left(\sum_k \dot{f}_k^* \beta_k \right) W_{SE}, \quad (9)$$

$$\mathcal{J}_k(W_{SE}) = -\frac{\dot{f}_k}{\dot{g}} \mathcal{J}_E(W_{SE}), \quad (10)$$

where $\dot{f}_k = df_k/dt$. Moreover, we have defined the global bath currents $\mathcal{J}_E(W_{SE}) = \frac{\dot{g}}{g} \alpha W_{SE}$, which act collectively on all bath modes.

Integrating Eq. (8) over the bath degrees of freedom yields a quantum Fokker-Planck equation for S, which has the form of Eq. (2) with the marginal current $J_S(W_S) = \int d^2\boldsymbol{\beta} \, \mathcal{J}_S(W_{SE})$. As shown in Ref. [54], this current can be written for an arbitrary number of modes in a time-local form, in the spirit of Ref. [57], as

$$J_S(W_S) = \Gamma(t) \left(\alpha + \frac{\partial_{\alpha^*}}{2} \right) W_S, \quad (11)$$

where $\Gamma(t) = -\mathrm{Re}(\dot{g}/g)$ [58]. This current has the same form as Eq. (3) so that $\Gamma(t)$ may be associated with the loss rate in Eq. (1). Indeed, in the TIM limit we get precisely $\Gamma(t) = \kappa$. Note also that, since our master equation has only a single dissipator, it then follows that one may directly associate Markovianity with the positivity of $\Gamma(t)$ [57].

We may also take the opposite route and trace Eq. (8) over the system to obtain a non-Markovian equation for the environment, which has the form

$$\partial_t W_E = \sum_k \partial_{\beta_k} J_k + \partial_{\beta_k^*} J_k^*, \quad (12)$$

where $J_k = -(\dot{f}_k/\dot{g}) J_E$ and $J_E(W_E) = \int d^2\alpha \, \mathcal{J}_E(W_{SE})$ is the marginal current, which can be written as [54]

$$J_E(W_E) = \dot{g} \left\{ \mu - \sum_q \left(N f_q^* \partial_{\beta_q^*} + M f_q \partial_{\beta_q} \right) \right\} W_E. \quad (13)$$

The system therefore acts as a non-Markovian environment for E, which introduces displacements, thermal fluctuations, and squeezing, depending on the initial conditions (μ, N, M).

Entropy production. Having the full solution for W_{SE}, we may now proceed to analyze the entropy production from the perspective of the bath. Equation (5) conserves the total number

of quanta in the system. This allows one to derive the following entropic conservation law,

$$\frac{dS(W_{SE} || W_S^\infty W_E(0))}{dt} = 0, \quad (14)$$

which means that the entropic distance to the global vacuum remains the same at all times during the evolution. Using this result, one may then express the entropy production rate Π in Eq. (4) as

$$\Pi = \frac{d\mathcal{I}_{SE}}{dt} + \frac{dS(W_E || W_E(0))}{dt}, \quad (15)$$

where $\mathcal{I}_{SE} = S(W_{SE} || W_S W_E)$ is the Wigner mutual information between S and E [54]. This shows that the entropy production rate, which is usually expressed as a local quantity of S, has two clear contributions: One is a local quantity representing the production of entropy within E and the other is a nonlocal term related to the rate at which S-E correlations build up. We note that these two mechanisms were also studied in Ref. [59], where they were related with the possibility of observing non-Markovianity. Equation (15) thus holds the potential for pinpointing the effects of non-Markovianity in irreversible nonequilibrium processes, a topic of large interest both fundamentally and technologically.

As a further remark, a similar argument was found in Ref. [40], where the entropy production resulting from a nonequilibrium process was ascribed to the difference between the (in general, quantum-correlated) system-environment state and the tensor product between the reduced state of the system and the equilibrium state of the environment. Equation (15) clearly identifies both the above contributions to the entropy production, but expresses them from the perspective of the environment, thus providing an original (and indeed fruitful) take to the effects of system-environment interaction.

We may now express the quantities in Eq. (15) in terms of the irreversible currents J_S and J_E generated within the system and the environment. First, Eq. (4) is simply replaced by

$$\Pi = -\frac{d}{dt} S(W_S || W_S^\infty) = \frac{4}{\Gamma} \int d^2\alpha \frac{|J_S(W_S)|^2}{W_S}, \quad (16)$$

which holds for arbitrary time-dependent Γ. Next, we do the same for the last term in Eq. (15), which surprisingly can be cast almost in exactly the same form, as

$$\frac{dS(W_E || W_E(0))}{dt} = \frac{4}{\Gamma} \int d^2\boldsymbol{\beta} \, \frac{|J_E(W_E)|^2}{W_E}. \quad (17)$$

Despite having almost the same structure as Eq. (4), this result refers to the contribution of entropy production generated within the bath. We therefore see that part of the entropy production is due to the creation of irreversible currents J_E within the environment. The remaining part, related to the mutual information, is nonlocal but may be written using Eqs. (4), (15), and (17) as

$$\frac{d\mathcal{I}_{SE}}{dt} = \frac{4}{\Gamma} \int d^2\alpha \, d^2\boldsymbol{\beta} \, W_{SE} \left\{ \frac{|J_S|^2}{W_S^2} - \frac{|J_E|^2}{W_E^2} \right\}. \quad (18)$$

The creation of mutual information is thus related to the global average mismatch between the phase-space velocities in the system (J_S/W_S) and the bath (J_E/W_E).
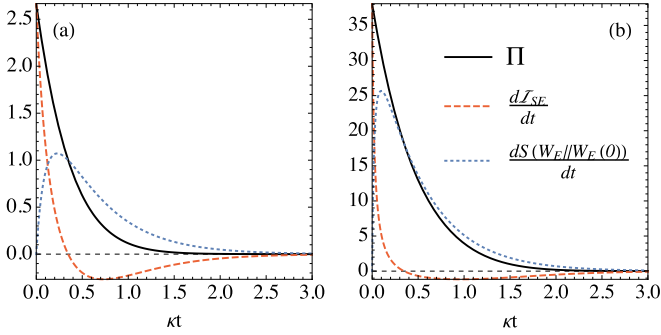
FIG. 1. Example of the three terms in Eq. (15) for a thermal state ($\mu = M = 0$) with (a) $N = 1$ and (b) $N = 10$. The inset in (a) shows the corresponding integrated quantities, from 0 to t.

Examples. The expressions of all entropic quantities appearing in this Rapid Communication can be written in terms of μ, N, M, and g, and are presented in the Supplemental Material [54]. Here, let us analyze some specific examples. A very special case, which is of particular interest, is when the system starts in a coherent state $\rho_S(0) = |\mu\rangle\langle\mu|$ [49] (that is, $N = M = 0$). This situation is atypical because the solution of Eq. (1) turns out to also be a coherent state with $\mu_t = \mu g(t)$, which means that the global system remains in a product state throughout. Consequently, $S(W_S)$ and $S(W_E)$ are both constant in time and hence $\mathcal{I}_{SE} = 0$ throughout the motion. Despite being a very particular case, this example serves to show that irreversibility may emerge even in the complete absence of correlations. Moreover, it serves as a counterexample to show that the divergence of the standard von Neumann entropy production [33–36], discussed previously, is not due to any sophisticated feature of the system-bath interaction, but is merely a mathematical limitation of the standard formalism.

Next, we consider a thermal initial state where $\mu = M = 0$ and $N = (e^{\beta\omega} - 1)^{-1}$ is the mean excitation number for S. In this case, the state continues to be thermal but with an occupation number $N|g|^2$. The three quantities appearing in Eq. (15) are shown in Fig. 1 for $N = 1$ and $N = 10$ for the TIM limit. As can be seen, for larger values of N, the contribution to $d\mathcal{I}_{SE}/dt$ becomes smaller, except at very short times. Thus, in the high-temperature limit, the correlation between the system and bath contributes negligibly to the irreversible behavior.

Non-Markovianity. Our main focus so far has been on the dilations of the Markovian dynamics described in Eq. (1). However, all results presented here also hold in the non-Markovian case and therefore provide us with an ideal platform to understand the interplay between the emergence of irreversibility and non-Markovianity. The evolution of the entropic quantities considered here provide us with several witnesses of non-Markovianity. Surprisingly, the mutual information $\mathcal{I}_{SE}$ is not one of them since $d\mathcal{I}_{SE}/dt$ does not have a well-defined sign, even in the Markovian case (see Fig. 1). Instead, one can witness Markovianity by monitoring the distance of S or E from their respective vacua, that is, $S(W_S||W_S^\infty)$ and $S(W_E||W_E(0))$. As an overall Markovian dynamics occurs when $\Gamma(t) > 0$, we see from Eqs. (4) and (17) that, in the Markovian case, the system will relax monotonically towards the vacuum, whereas the bath will distance itself from it

monotonically. Thus, any reversal in the velocity at which these processes occur can be taken as witnesses of non-Markovianity.

We can also link the backflow of information, a key figure of merit of non-Markovianity, with the entropy flux, defined as the mismatch between the entropy production and the total change in the system entropy, $\Pi - dS(W_S)/dt$. Using the definition of $\mathcal{I}_{SE}$ in Eq. (15), one may write Φ as

$$\Phi = \frac{dS(W_E)}{dt} + \frac{dS(W_E||W_E(0))}{dt} = -4\Gamma\langle a^\dagger a\rangle_t. \quad (19)$$

For finite-temperature environments, the entropy flux can be either positive or negative, depending on whether the system was initially warmer or colder than the bath. But for a zero-temperature bath, in the Markovian limit the entropy flux can only be from S to E. A backflow of entropy from E to S can therefore be directly related to a backflow of information.

Finally, we may also witness non-Markovianity by monitoring the entanglement of the system S with an ancilla A [60]. To do that, we consider again the specific example of a thermal state with occupation N, but suppose instead that this thermal state actually stems from the two-mode squeezing between the system mode a and an ancilla mode c. That is, $\rho_{AS}(0) = V|0\rangle_{AS}\langle 0|V^\dagger$, where $V = e^{z(a^\dagger c^\dagger - ac)}$ and $N = \sinh^2(z)$. Then, as shown in Ref. [54], the system-ancilla mutual information may be related to the other entropic quantities appearing in Eq. (15) as

$$\begin{aligned}
\frac{d\mathcal{I}_{AS}}{dt} &= -\frac{2N}{N+1}\left\{|g|^2\left[N(1 - |g|^2) + \frac{1}{2}\right]\right\}\Pi \\
&= -\frac{2N}{N+1}\left\{(1 - |g|^2)\left[N|g|^2 + \frac{1}{2}\right]\right\}\frac{dS(W_E||W_E(0))}{dt} \\
&= \frac{N}{N+1}(1 - 2|g|^2)\frac{d\mathcal{I}_{SE}}{dt}.
\end{aligned}$$

Thus, we see that $d\mathcal{I}_{AS}/dt$ is related to both Π and $dS[W_E||W_E(0)]/dt$ by nonpositive prefactors, so that in the Markovian case $\mathcal{I}_{AS}$ will decay monotonically. Conversely, $d\mathcal{I}_{AS}/dt$ is linked to $d\mathcal{I}_{SE}/dt$ by a factor which does not have a definite sign, therefore showing that the system environment correlations cannot be used as a witness of non-Markovianity.

Applications. Although we have focused on the conceptual implications of the emergence of irreversibility, our results also have potential applications, for instance, in designing strategies that minimize losses in quantum heat engines. To that end, suppose that the relaxation process described here actually corresponds to one of the strokes of a heat engine. The total entropy produced, which is the time integral of Eq. (4), will depend only on the initial and final states of the system. However, from Eq. (15), we see that this entropy production will partially go to the production of entropy within the environment and partially to the buildup of SE correlations. From the perspective of a real machine, therefore, it is desired to minimize the entropy production in the bath, at the cost of increasing the system-bath correlations. This may be particularly important for finite-sized environments operating continuously, whose degradation will be closely related to non-Markovian effects and could inspire different types of quantum control methods applicable to the physics of quantum heat engines [61,62].

Conclusions. We have made use of a dilationlike approach to study the open dynamics of a quantum system and characterize the occurrence of entropy production resulting from a nonequilibrium process. This approach has allowed us to identify two fundamental mechanisms that are responsible for the production of entropy: On one hand, the dynamical bath introduced through the dilation mechanism may get correlated to the system. On the other hand, its state might differ from the equilibrium one in light of its interaction with the system. Both features are independently responsible for the emergence of irreversible entropy and can be associated with the existence of probability currents within the system and the environment, thus playing a key role in the phenomenology of irreversibility. It is also very thought-provoking that both mechanisms can be linked to the occurrence of non-Markovian dynamics [59], which remarks the relevance of our approach to the characterization of the features of open quantum systems, and paves the way to the investigation on the emergence of objective reality through the concept of quantum Darwinism [63]. The formalization of such a link will be the focus of forthcoming investigations.

[1] J. Liphardt, S. Dumont, S. B. Smith, I. Tinoco, and C. Bustamante, Science (N.Y.) **296**, 1832 (2002).

[2] T. Speck, V. Blickle, C. Bechinger, and U. Seifert, Europhys. Lett. **79**, 30002 (2007).

[3] D. Collin, F. Ritort, C. Jarzynski, S. B. Smith, I. Tinoco, and C. Bustamante, Nature (London) **437**, 231 (2005).

[4] S. Ciliberto, A. Imparato, A. Naert, and M. Tanase, Phys. Rev. Lett. **110**, 180601 (2013).

[5] J. Roßnagel, S. T. Dawkins, K. N. Tolazzi, O. Abah, E. Lutz, F. Schmidt-Kaler, and K. Singer, Science **352**, 325 (2016).

[6] C. Myatt, B. King, and Q. Turchette, Nature (London) **403**, 269 (2000).

[7] S. Groeblacher, A. Trubarov, N. Prigge, M. Aspelmeyer, and J. Eisert, Nat. Commun. **6**, 7606 (2015).

[8] I. A. Martínez, É. Roldán, L. Dinis, D. Petrov, and R. A. Rica, Phys. Rev. Lett. **114**, 120601 (2015).

[9] T. B. Batalhão, A. M. Souza, L. Mazzola, R. Auccaise, R. S. Sarthour, I. S. Oliveira, J. Goold, G. De Chiara, M. Paternostro, and R. M. Serra, Phys. Rev. Lett. **113**, 140601 (2014). T. B. Batalhão, A. M. Souza, R. S. Sarthour, I. S. Oliveira, M. Paternostro, E. Lutz, and R. M. Serra, *ibid.* **115**, 190601 (2015).

[10] G. E. Crooks, J. Stat. Phys. **90**, 1481 (1998); Phys. Rev. E **61**, 2361 (2000).

[11] C. Jarzynski, Phys. Rev. Lett. **78**, 2690 (1997); Phys. Rev. E **56**, 5018 (1997).

[12] C. Jarzynski and D. K. Wójcik, Phys. Rev. Lett. **92**, 230602 (2004).

[13] H. Tasaki, arXiv:cond-mat/0009244.

[14] U. Seifert, Rep. Prog. Phys. **75**, 126001 (2012).

[15] I. J. Ford and R. E. Spinney, Phys. Rev. E **86**, 021127 (2012).

[16] G. Manzano, J. M. Horowitz, and J. M. R. Parrondo, arXiv:1710.00054.

[17] R. Dorner, J. Goold, C. Cormick, M. Paternostro, and V. Vedral, Phys. Rev. Lett. **109**, 160601 (2012).

[18] J. F. Leandro and F. L. Semião, Opt. Commun. **282**, 4736 (2009).

[19] S. Popescu, A. J. Short, and A. Winter, Nat. Phys. **2**, 754 (2006).

[20] J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, Nat. Phys. **11**, 131 (2015).

[21] N. Cottet, S. Jezouin, L. Bretheau, P. Campagne-Ibarcq, Q. Ficheux, J. Anders, A. Auffèves, R. Azouit, P. Rouchon, and B. Huard, Proc. Natl. Acad. Sci. U.S.A. **114**, 7561 (2017).

[22] P. A. Camati, J. P. S. Peterson, T. B. Batalhão, K. Micadei, A. M. Souza, R. S. Sarthour, I. S. Oliveira, and R. M. Serra, Phys. Rev. Lett. **117**, 240502 (2016).

[23] J. Goold, M. Paternostro, and K. Modi, Phys. Rev. Lett. **114**, 060602 (2015).

[24] L. Onsager, Phys. Rev. **37**, 405 (1931); **38**, 2265 (1931).

[25] S. Machlup and L. Onsager, Phys. Rev. **91**, 1512 (1953).

[26] S. R. de Groot and P. Mazur, *Non-Equilibrium Thermodynamics*, 1st ed. (North-Holland, Amsterdam, 1961), p. 536.

[27] L. Tisza and I. Manning, Phys. Rev. **105**, 1695 (1957).

[28] J. Schnakenberg, Rev. Mod. Phys. **48**, 571 (1976).

[29] T. Tomé and M. J. de Oliveira, Phys. Rev. Lett. **108**, 020601 (2012).

[30] T. Tomé and M. J. de Oliveira, Phys. Rev. E **82**, 021120 (2010).

[31] R. E. Spinney and I. J. Ford, Phys. Rev. E **85**, 051113 (2012).

[32] G. T. Landi, T. Tomé, and M. J. de Oliveira, J. Phys. A **46**, 395001 (2013).

[33] H. Spohn, J. Math. Phys. **19**, 1227 (1978).

[34] H.-P. Breuer, Phys. Rev. A **68**, 032105 (2003).

[35] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2007), p. 636.

[36] S. Deffner and E. Lutz, Phys. Rev. Lett. **107**, 140404 (2011).

[37] M. J. de Oliveira, Phys. Rev. E **94**, 012128 (2016).

[38] D. J. Evans, E. G. D. Cohen, and G. P. Morriss, Phys. Rev. Lett. **71**, 2401 (1993); D. J. Evans and D. J. Searles, Phys. Rev. E **50**, 1645 (1994).

[39] G. Gallavotti and E. G. D. Cohen, Phys. Rev. Lett. **74**, 2694 (1995); J. Stat. Phys. **80**, 931 (1995).

[40] M. Esposito, K. Lindenberg, and C. Van Den Broeck, New J. Phys. **12**, 013013 (2010).

[41] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, U.K., 2000).

[42] J. P. Santos, G. T. Landi, and M. Paternostro, Phys. Rev. Lett. **118**, 220601 (2017).

[43] R. L. Frank and E. H. Lieb, J. Math. Phys. **54**, 122201 (2013).

[44] M. Müller-Lennert, F. Dupuis, O. Szehr, S. Fehr, and M. Tomamichel, J. Math. Phys. **54**, 122203 (2013).

[45] K. M. R. Audenaert, J. Math. Phys. **55**, 112202 (2014).

[46] S. Abe, Phys. Rev. A **68**, 032302 (2003).

[47] G. Adesso, D. Girolami, and A. Serafini, Phys. Rev. Lett. **109**, 190502 (2012).

[48] C. Van den Broeck and M. Esposito, Phys. Rev. E **82**, 011144 (2010).

[49] A. L. de Paula, J. G. G. de Oliveira, J. G. P. de Faria, D. S. Freitas, and M. C. Nemes, Phys. Rev. A **89**, 022303 (2014).

[50] Á. Rivas, K. P. A. Douglas, S. F. Huelga, and M. B. Plenio, New J. Phys. **12**, 113032 (2010).

[51] F. Caruso, J. Eisert, V. Giovannetti, and A. S. Holevo, New J. Phys. **10**, 083030 (2008).

[52] A. Wehrl, Rev. Mod. Phys. **50**, 221 (1978).

[53] J. Oppenheim, M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **89**, 180402 (2002); M. Horodecki and J. Oppenheim, Nat. Commun. **4**, 2059 (2013); M. Lostaglio, D. Jennings, and T. Rudolph, *ibid.* **6**, 6383 (2015).

[54] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.97.050101 for details on the calculations of the system-enviroment dynamics.

[55] V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1930).

[56] M. O. Scully and M. Suhail Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, U.K., 1997).

[57] M. J. W. Hall, J. D. Cresser, L. Li, and E. Andersson, Phys. Rev. A **89**, 042120 (2014).

[58] In general, g will be complex and the imaginary part of $\dot{g}/g$ will give rise to a Lamb shift in the oscillator frequency. However, this imaginary term enters into the unitary evolution and therefore does not contribute to any entropic quantities. We shall therefore omit this contribution for the purpose of clarity. However, note that all results in this Rapid Communication are also valid in the presence of the Lamb shift.

[59] L. Mazzola, C. A. Rodríguez-Rosario, K. Modi, and M. Paternostro, Phys. Rev. A **86**, 010102(R) (2012).

[60] Á. Rivas, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **105**, 050403 (2010).

[61] A. del Campo, J. Goold, and M. Paternostro, Sci. Rep. **4**, 6208 (2014).

[62] O. Abah and E. Lutz, arXiv:1611.09045; arXiv:1707.09963.

[63] W. H. Zurek, Nat. Phys. **5**, 181 (2009).