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## LEANING TOWER OF PISA: RECENT STUDIES ON DYNAMIC RESPONSE AND SOIL-STRUCTURE INTERACTION

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### ABSTRACT

The Leaning Bell Tower of Pisa has been included in the list of the World Heritage Sites by UNESCO since 1987. Over the last 20 years, the Tower has successfully undergone a number of interventions to reduce its inclination. The Tower has also been equipped with a sensor network for seismic monitoring. In this study, preliminary results on the dynamic behavior of the monument are presented, including a review of historical seismicity in the region, identification of vibrational modes, definition of seismic input, site response analysis, and seismic response accounting for soil-structure interaction. This includes calibration of the dynamic impedances of the foundation to match the measured natural frequencies. The study highlights the importance of soil-structure interaction in the survival of the Tower during a number of strong seismic events since the middle ages.

*Keywords: Dynamic response, Leaning Tower, Soil-structure interaction, Seismic hazard assessment*

### 1. INTRODUCTION

The Bell Tower of *Piazza del Duomo* in Pisa (Italy) was built during a period of two centuries. Its construction began in 1173 and was completed in 1370 with the erection of the belfry. The periods of construction are summarized in Figure 1 (a). At the beginning of construction the Tower started leaning to the north, reaching a maximum tilt of about  $0.5^\circ$ . The leaning gradually switched to the South reaching a maximum tilt of about  $5.5^\circ$  in the early 1990s with brought the structure to the verge of collapse.. The total height of the Tower is equal to 58.4 m measured from the foundation. Its cross-section is ring-shaped, with an external diameter of 19.6 m at the base. The current tilt of the structure is about  $5^\circ$  towards the South, leading to an offset at the top of about 5 m (Squeglia and Bentivoglio, 2015). The estimated weight of the Tower is 14500 tons and the elevation of the center of mass is about 23 m from the base, as indicated in Figure 1 (b).

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The last abrupt increase in inclination of the Tower was due to the digging of the *Catino* (Italian word for basin) in 1838 by Alessandro della Gherardesca.

In the last thirty years the Tower became the subject of a series of successful interventions to reverse its tilting. The inclination was reduced by approximately  $0.5^\circ$  following stabilization works which began with the installation of 800 tons of lead ingots on the uplifted North side of the monument in 1993.

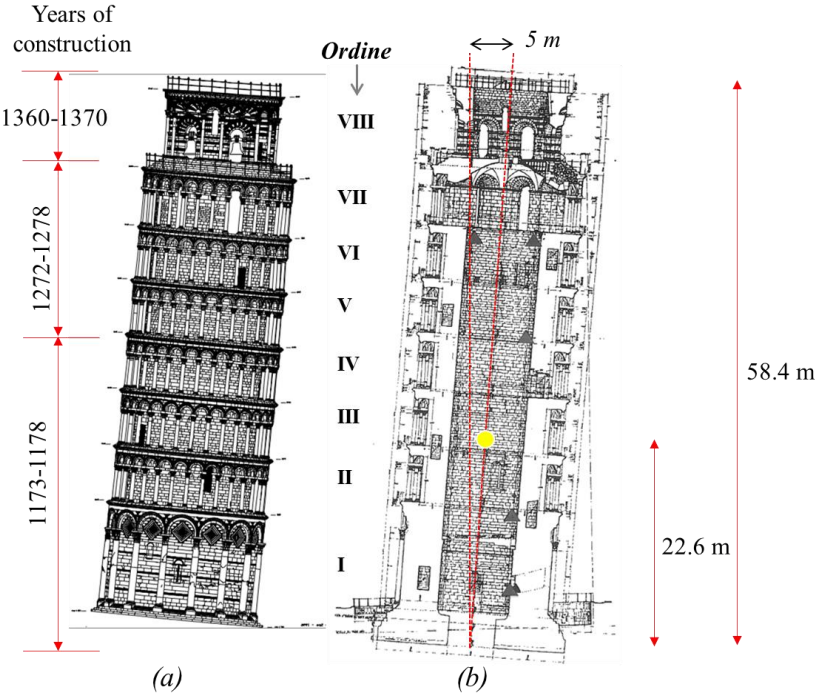


Figure 1. (a) Side view of the Tower of Pisa with indication of the years of construction and the definition of the levels (Ordine); (b) cross section of the Tower with indication of the location of the center of mass.

The reduction in inclination was achieved following works of under-excavation and extraction of approximately  $40\text{m}^2$  of soil from the uplifted North side of the Tower between 1999 and 2001. Figure 2 shows the inclined boreholes used to realize these works.



Figure 2 Boreholes for final works of under-excavation

The seismic response of the Tower under earthquake action was first studied by Grandori and Faccioli (1993), who presented results from dynamic analyses performed on a simplified finite-element (FE)

model of the structure in which the seismic input was defined in terms of a response spectrum. Experimental assessments of the monument began in 1994, with the identification of modal parameters by ISMES by means of forced vibrations using a vibrodyne. In light of the severe inclination and the lack of ductility of the Tower, there is a need to study its dynamic behavior under seismic action. The paper at hand presents an update on recent work carried out by the co-authors (Fiorentino et al., 2017, 2018).

**2. EXPERIMENTAL SEISMIC RESPONSE**

The Leaning Tower of Pisa is equipped with a network of accelerometers for seismic monitoring. The location of sensors is depicted in Figure 3.

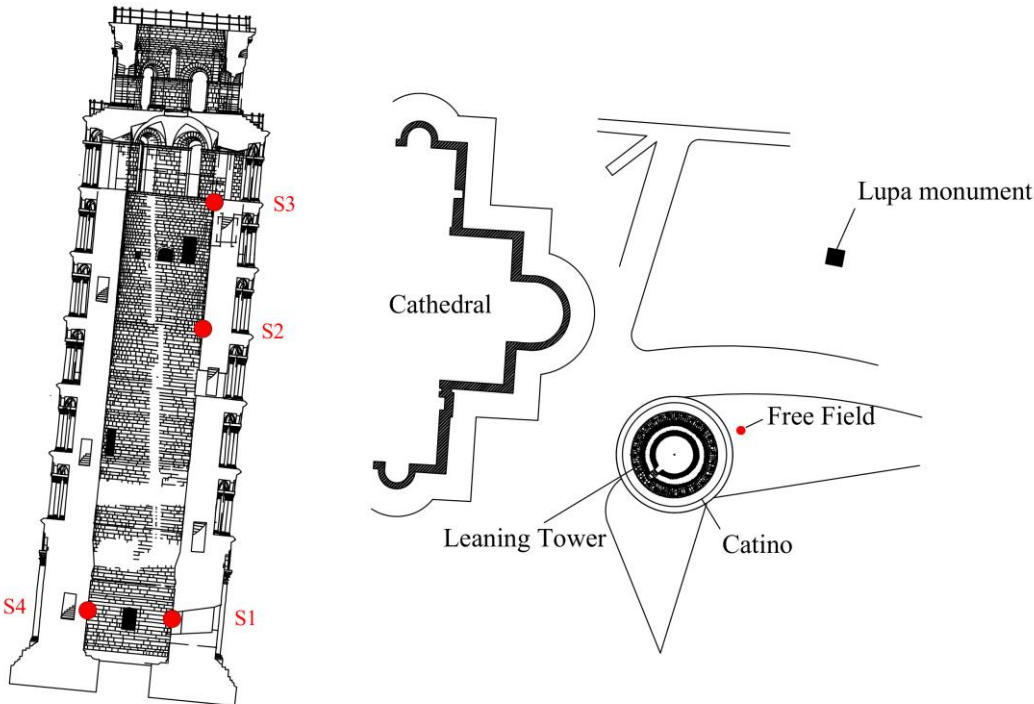


Figure 3 Location of accelerometers on the Tower (S1, S2, S3, S4) and the Free Field.

The recorded strong ground motions have been analyzed using different methodologies, including Fast Fourier Transforms (FFT), Continuous Wavelet Transforms (CWS) and Wavelet Cross Spectra (WCS). These analyses (see Figure 4) allowed identification of the frequencies of four natural modes, as reported in Table 1. The first two modes are associated with bending in N-S and E-W direction, respectively, and have a natural frequency of about 1 Hz. The third is a vertical mode with a frequency of about 3 Hz. It is worth noting that the only information available in the literature about the vertical mode comes from Nakamura (1999). Moreover, a frequency of 6.3 Hz was also identified, possibly relating to a torsional mode.

Table 1 Identified vibration modes and Experimental frequencies

Vibration mode	Experimental frequency [Hz]
1° bending mode (NS)	0.95
2° bending mode (EW)	1
3° vertical mode	3
4° torsional (?) mode	6.3

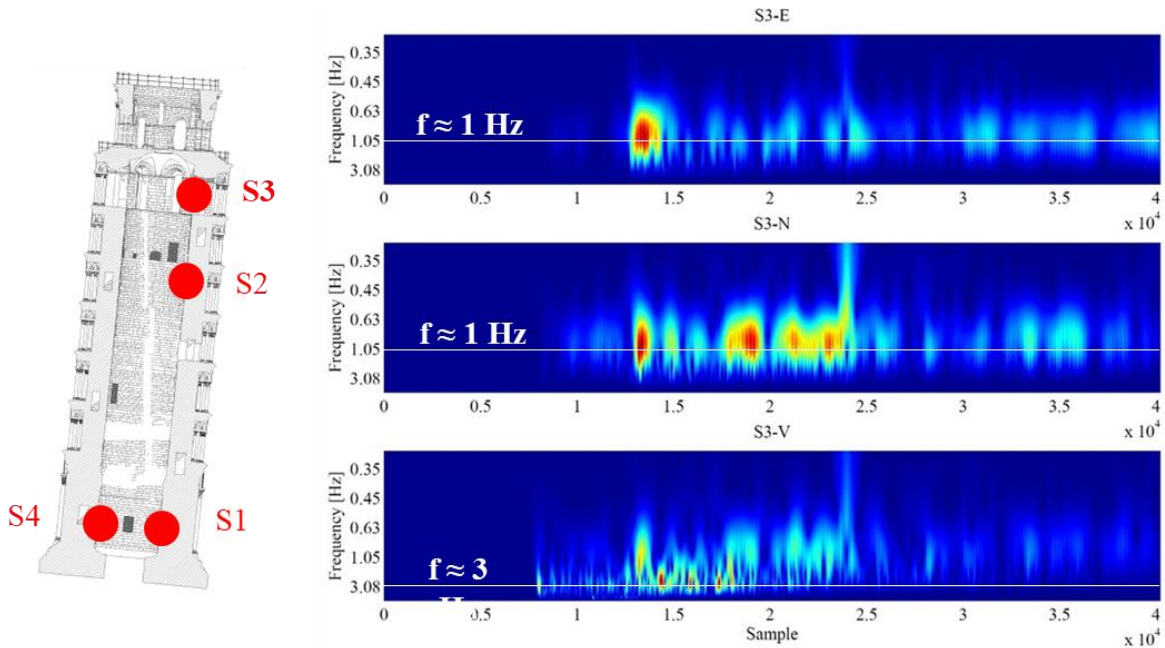


Figure 4 Location of sensors on the Tower (left) and CWT of the response recorded at S3 (right). S3-E = East West component; S3-N = North-South component;

### 3. SEISMIC INPUT AND SOIL CHARACTERIZATION

#### 3.1 Historical seismicity

The area around Pisa is characterized by moderate seismicity. The main seismic sources in the region are located in the area of Pisa hills which are responsible of the 1846 M 6 Orciano Pisano earthquake) and the Garfagnana area, which released the 1920 M 6.5 Garfagnana earthquake. According to the Italian Database of historic earthquakes (Rovida et al., 2016), from year 1117 to 2018 eight earthquakes with intensity levels ( $I_{MCS}$ ) equal or greater to 6 struck Pisa. This is considered a lower bound for inflicting structural damage according to the EMS98 damage scale (Grunthal, 1998).

Seven of these earthquakes took place after the completion of the Tower in 1370, therefore the Tower has withstood a considerable number of earthquakes with  $I_{MCS}$  ranging from 6 to 7. Figure 5 and Table 2 summarize estimated  $I_{MCS}$  values ( $\geq 5$ ) from historical earthquakes in the area.

Regarding the damage to the local building stock, an extensive study was carried out by Moroni and Stucchi (1993) and published as an appendix in the work by Grandori and Faccioli (1993). The historical sources cited in that study reported moderate to heavy damage to buildings.

The earliest information dates back to 1322 ( $I_{MCS} = 5-6$ ), when one source reported the fall of a marble plate depicting a *Madonna* from the *Duomo* façade.

A greater amount of information is available for more recent events. With reference to the 1767 earthquake of Versilia, the historical documents mention heavy damage including damage to chimneys, balcony collapses, many cracks in buildings and some collapses in perimeter walls.

Cracks in the majority of masonry residential buildings and cracks on the church of S. Giovanni in Pisa were reported during the 1814 Livorno earthquake ( $I_{MCS} = 6$ ).

The most damaging earthquake which took place during the life of the Tower was the August 14 1846 M 6 earthquake, known as Colline Pisane or Orciano Pisano earthquake. The maximum intensity was assigned to the village of Orciano Pisano, where the earthquake resulted in the partial or total collapse of the majority of the buildings ( $I_{MCS} = 9$ ). An Intensity  $I_{MCS} = 7$  was assigned to Pisa, where extensive damage was observed on masonry buildings and on some public buildings and churches. No damage was reported on the Tower of Pisa.

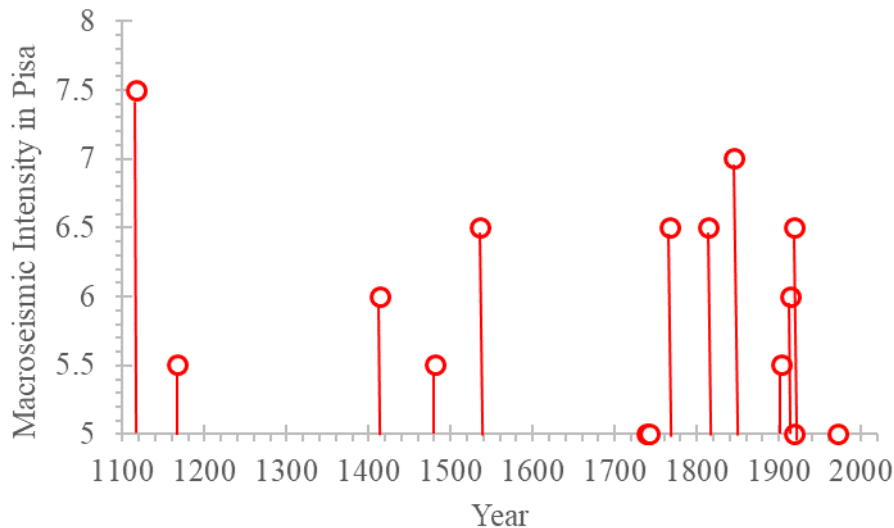


Figure 5 Macroseismic intensities in Pisa from year 1100 A.D. to year 2018 as reported in the CPTI15 catalog of Italian Earthquakes (after Locati et al. 2016, Rovida et al. 2016).

Damage observed in Pisa during the 1846 earthquake is reported in a historical document put together by Professor Leopoldo Pilla (1846), a famous geologist of the University of Pisa (and a martyr in the first war of Italian Independence in 1848). The vault of the church of S. Michele collapsed, there was damage to a vault in the church of S. Francesco. Cracks were reported on the Clock Tower of *Palazzo Pretorio*, in the columns of the peristyle.

In the *Piazza del Duomo*, one cross of the roof and a marble square stone of the outer wall of the *Duomo* fell down. Some light cracks were observed in the *Camposanto* (Cemetery) and the *Battistero* (Baptistry). No damage was observed on the Tower of Pisa (*Campanile*).

Table 2 Historical earthquakes around Pisa with macroseismic Intensity  $I_{MCS} \geq 5$ .

Meizoseismal area	Date (DD/MM/YYYY)	Intensity in Pisa	Maximum intensity
Monti Pisani	03/01/1117	7-8	-
Pisa	10/01/1168	5-6	5-6
Pisa	? / ?/1322	5-6	5-6
Colline Metallifere	07/08/1414	6	7-8
Monti Pisani	06/02/1481	5-6	5-6
Appennino settentrionale	17/08/1536	6-7	6-7
Garfagnana	06/03/1740	5	8
Livorno	27/01/1742	5	6
Lunigiana	21/01/1767	6-7	7
Costa pisano-livornese	03/04/1814	6-7	6-7
Colline Pisane	14/08/1846	7	9
Lucca	27/10/1914	6	7
Mugello	29/06/1919	5	10
Garfagnana	07/09/1920	6-7	10
Appennino settentrionale	25/10/1972	5	5

The 1920 earthquake of Garfagnana was very damaging in the meizoseismal region, but only slight damage was observed in Pisa.

### 3.2 Seismic input

Seismic hazard assessment was performed by the authors by combining a probabilistic (PSHA) and a deterministic approach (DSHA). To this end, SP96 (Sabetta and Pugliese, 1996) and AB10 (Akkar and Bommer, 2010) Ground Motion Predictive Equations (GMPE) were employed. Uniform Hazard Spectra (UHS) on rock were computed for Return Periods (RPs) of 130 and 500 years. These values are based on the correlations between MCS intensity and RP, already used by Grandori and Faccioli (1993). Disaggregation results were employed to identify controlling earthquakes. Based on the Italian seismic catalogue CPTI15 (Rovida et al., 2016), it was possible to identify two key earthquake scenarios: a M 5.2 event with distance from source of about 20 km associated with a return period (RP) of 130 years (e.g. Livorno 1742), and a M 5.7 earthquake with a distance from source of about 20 km (e.g. Orciano Pisano 1846) associated with a return period of 500 years. These are related to MCS intensities VI and VII, respectively. The target response spectrum for EC8 class B site was evaluated by means of the Akkar and Bommer GMPE, including the subsoil term associated with  $V_{s,30}$ . Eight accelerograms were selected for each RP from the European Strong Motion Database (Luzi et al. 2016), considering  $5 < M < 5.5$  for 130 years RP, and  $5.3 < M < 6.2$  for 500 years RP. The selected components of the horizontal accelerograms were scaled in such a way so that the average spectrum of each set of accelerograms approximates well the target spectrum for Soil B. This task has been accomplished using In-Spector software (Acunzo et al., 2014). The scaling was carried out in the range of the fundamental periods 0.3-1.1 s in order to take into account the periods of the first two bending modes (about 1 s) and that of the third (vertical) mode (about 0.3 s) of the structure, thus obtaining the proper scaling factor SF for each record. To obtain the vertical time histories on Soil B, each original vertical record taken from the European Strong Motion Database was scaled with the corresponding SF (see Figure 6).

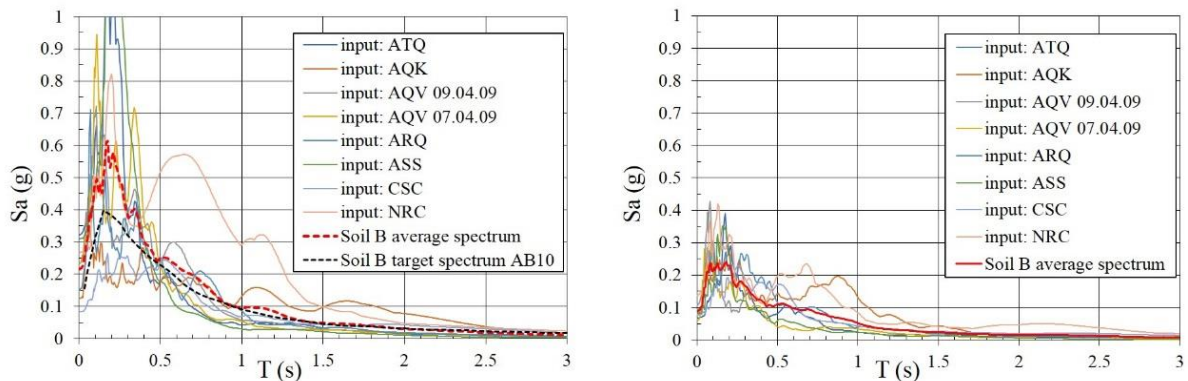


Figure 6. Spectrum-compatible horizontal acceleration time histories for EC8 Soil B with 500 years Return Period: horizontal (left) and vertical (right) components

### 3.2 Geophysical tests

A 2D geophysical array (SESAME 2005) was deployed to provide a shear-wave velocity profile of the soil underlying *Piazza del Duomo*. The layout of the instruments is displayed in Figure 7 (a). This kind of test can reach depths of approximately 100 m, which is significantly larger compared to the depths reached by available Down-Hole and Cross-Hole tests. The measurements were performed using nine REFTEK130 stations equipped with Lennartz 3D 5s velocimeters and deployed in a triple equilateral triangle configuration. The central station was located near the Baptistery. The set-up is depicted in Figure 7 (b) and 7 (c).



Figure 7. (a) Array2D test in Piazza del Duomo in Pisa: a) layout of the instruments in the square; (b) Installation of one the seismic stations; (c) Measurement station formed by REFTEK130 seismic stations and Lennartz 3D 5s velocimeters

The software GEOPSY was employed in the analyses. The inversions revealed the presence of a rigid layer ( $V_s \approx 500 \text{ m/s}$ ) at a depth of about 100m (Figure 6). A single station analysis was performed within the same test to evaluate H/V spectral ratios, from which a resonance peak at 1.3Hz, associated with an interface at 40m depth was identified. Another peak was identified at 0.3Hz, which is possibly associated with an interface at 500m depth or so. The H/V ratio and the associated resonance peaks are shown in Figure 8.

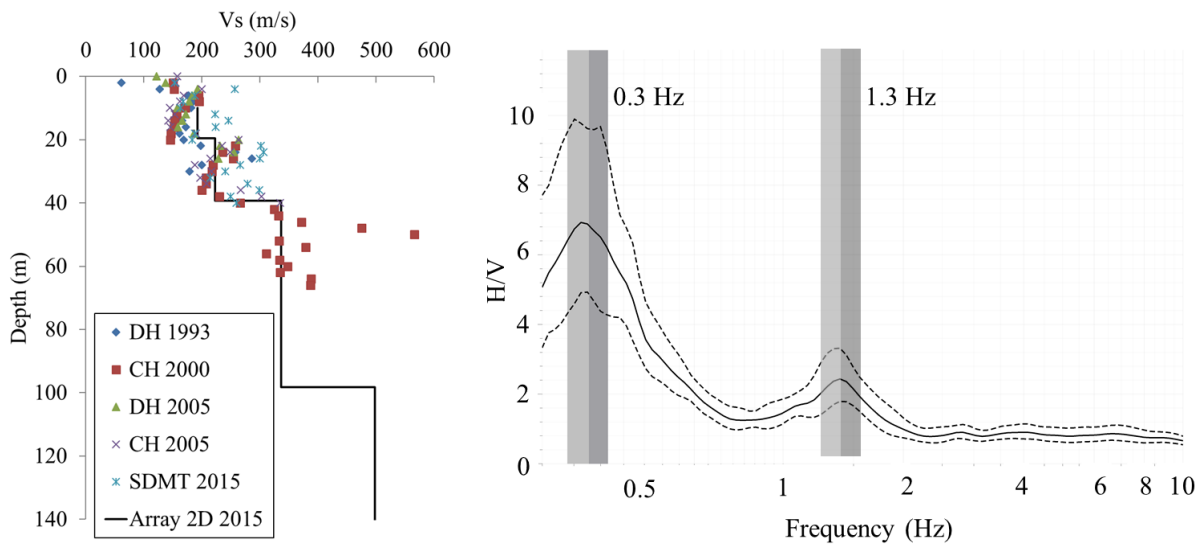


Figure 8. Comparison between inversions from Array2D test and old DH/CH tests (left graph) and H/V spectral curve for a single station (right graph)

### 3.3 Subsoil model and site response analysis

The soil model adopted for the site response analyses is reported in Table 2. It is based on the proposal by Viggiani and Pepe (2005) and takes into account geotechnical investigations carried from 1970 to 1993. In this model three distinct horizons are identified: A (sandy and clayey silt), B (marine clays) and C (dense sand), which can be further subdivided into the strata described in Table 3. Thickness and unit weight for each stratum were estimated according to the data reported in the above study. The



assumed shear wave velocity profile is based on the aforementioned seismic array inversions. However, it should be pointed out that this profile is in good agreement with the  $V_S$  values measured by other geophysical tests in the upper strata (e.g. SDMT tests carried out in 2015 up to 40 m and a 65 m deep cross-hole test in 1999). It should be noticed that a nominal seismic bedrock ( $V_S > 800$  m/s) is not encountered over the explored upper 95 m.

Table 3. Subsoil model adopted for site response analyses over the upper 95 m. LT = Lithotype,  $\Delta H$  = layer thickness,  $\gamma$  = unit weight,  $V_S$  = shear wave velocity, NL = Nonlinear Characterization ([R] = Rollins et al., 1998; [D] = Darendeli, 2001), SB=Seismic Bedrock

LT	$\Delta H$ (m)	$\gamma$ (kN/m <sup>3</sup> )	$V_S$ (m/s)	NL	LT	$\Delta H$ (m)	$\gamma$ (kN/m <sup>3</sup> )	$V_S$ (m/s)	NL
MG	3.0	18.5	180	average [R]	B7	4.6	18.5	230	RC tests
A1	5.4	19		DSDSS test S4-C2 $\sigma'_v=65$ kPa	B8	1.4	18.5		RC tests
A2	2.0	18		PI=30 $\sigma'_v=55$ kPa	B9	4.0	19		RC tests
B1	3.5	17		[D]	B10	2.6	19.5	RC tests	
B2	2.0	17.5		RC tests	C1	27.5	20.5	340	PI=0 $\sigma'_v=350$ kPa
B3	4.9	16.5		RC tests	C2	11	20.5		PI=15 $\sigma'_v=500$ kPa
B4	1.2	19.5		RC tests	C3	16	20.5		PI=0 $\sigma'_v=600$ kPa
B5	3.0	20	RC tests	SB	-	21	500		-
B6	2.4	19	230	PI=8 $\sigma'_v=200$ kPa	(C3)	-	-	-	
				[D]					

For this reason, considering the uncertainties in the  $V_S$  values at higher depths, the input motion for site response analysis was defined according to EC8 class B classification (i.e. instead of rock - class A). Regarding the nonlinear properties, most of the strata are characterized based on resonant column (RC) tests (Impavido et al., 1993).

Stratum A1, for which no cyclic data are available, was characterized through DSDSS (Double Specimen Direct Simple Shear) tests conducted on a soil sample extracted from a depth of 6.3 m (S4-C2) (D'Elia et al. 2003). The cyclic tests were conducted for different vertical consolidation stresses  $\sigma'_{vc}$  (65-130-260 kPa); corresponding results are reported in Figure 9 (left graph) in terms of normalized secant shear modulus and damping ratio as a function of shear strain amplitude.

Given the lack of experimental data, literature curves obtained for similar soils were employed for the rest of soil strata (Darendeli, 2001; Rollins et al., 1998) (see Table 3). Site response analyses were carried out using the 1D frequency-domain equivalent linear code STRATA (Kottke and Rathje, 2008). The results for a return period of 500 years are reported in Figure 9 (right graph) in terms of horizontal acceleration response spectra computed at ground surface (averaged over all input motions employed). The average input spectrum at seismic bedrock level is also shown for comparison. Moderate amplification phenomena take place in the medium-to-long periods with a maximum amplification ratio slightly higher than 2 at around 1.2 s, where the corresponding average spectral accelerations are about 0.2g. Spectral accelerations as high as 0.5-0.6g (average values) develop at ground surface in the 0.2-0.4 s period range.

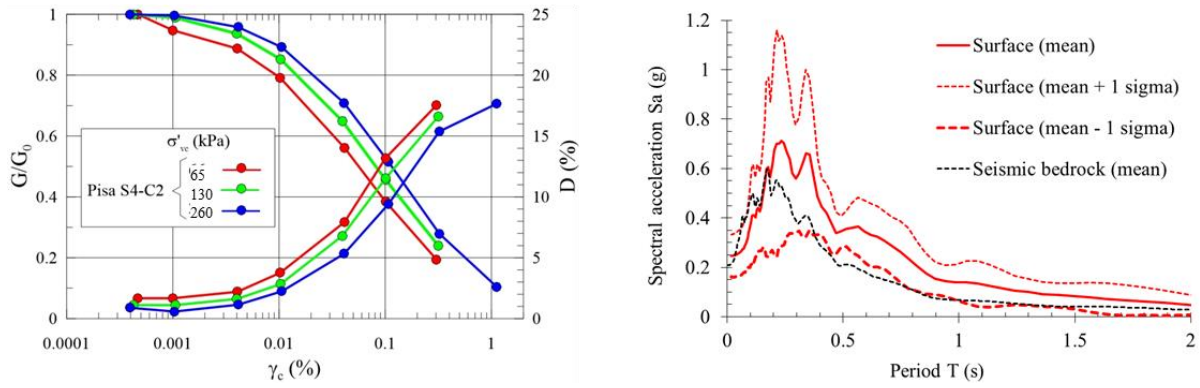


Figure 9.  $G/G_0$  and  $D$  curves obtained through DSDSS test on S4-C2 sample for A1 stratum (left graph); input and output average response spectra obtained from equivalent-linear site response analyses carried out for 500 years return period (right graph).

#### 4. SOIL-STRUCTURE INTERACTION AND DYNAMIC RESPONSE

A simplified FE stick model, depicted in Figure 10 was built considering the inclination of the Tower in the N-S direction. For each storey of the Tower, the coordinates of the centroid were defined, based on the study by Macchi and Ghelfi (2005). For each centroid, 3 translational masses were defined.

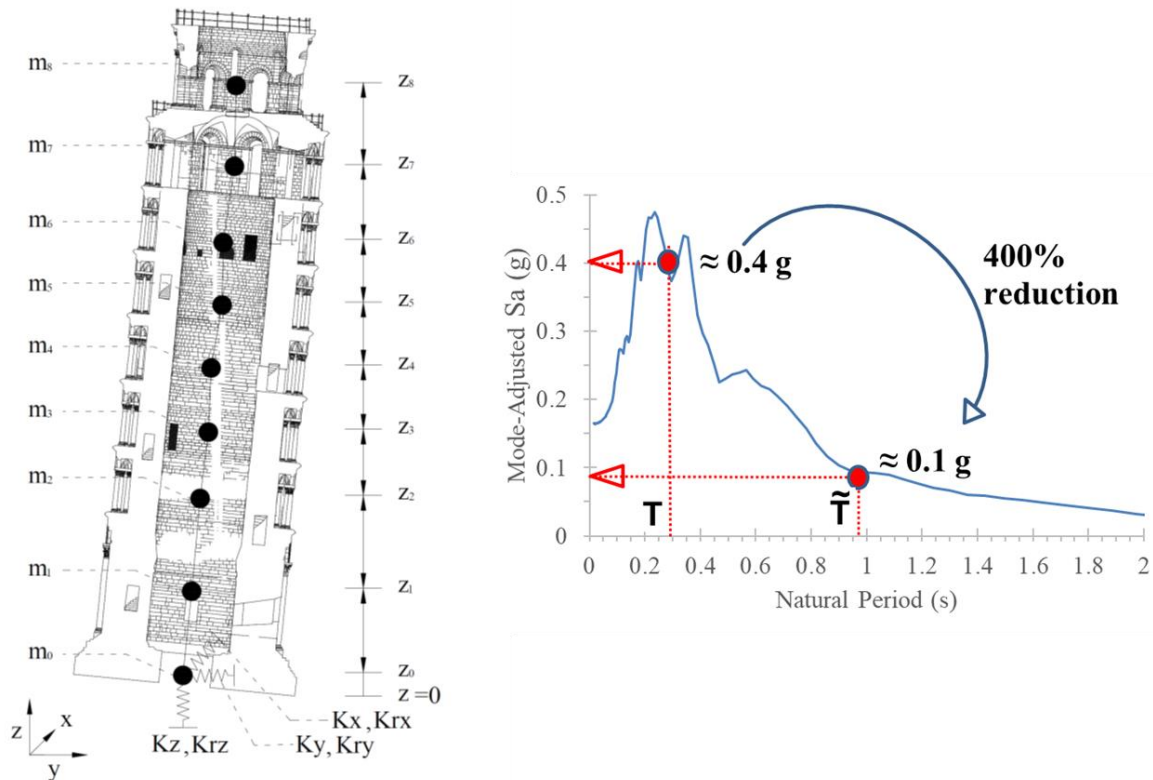


Figure 10. Simplified model of the Leaning Tower of Pisa.  $m_i$ : mass of the  $i^{\text{th}}$  storey;  $z_i$ : elevation of the  $i^{\text{th}}$  storey;  $K_x, K_y, K_z$  = translational base impedances;  $K_{rx}, K_{ry}, K_{rz}$  = rotational base impedances (left graph); reduction in seismic response due to soil-structure interaction for a mode-adjusted spectrum for 500 years return period (right graph).

Three translational springs and rotational springs were assigned at the base of the model using the tables by Mylonakis et al. (2006). Table 3 shows the comparison between the results of the modal analysis and the frequencies obtained experimentally. For a nominal soil shear modulus of  $G = 77$  GPa, the frequencies obtained by considering the foundation alone and the foundation with the "Catino" are 0.87

Hz and 0.88 Hz, respectively, which are about 10% lower than the measured values of 1Hz (Florentino et al 2018).

More satisfactory results were obtained for  $G = 95$  GPa, which lead to a natural frequency of 0.96 Hz in bending and 3.1 Hz in vertical mode. A further refinement was performed by varying the values of the foundation springs by  $\pm 20\%$ , to obtain improved agreements between the natural frequencies estimated experimentally and analytically (denoted as ‘‘Calibrated’’ in Table 3).

Table 3. Comparison between numerical and experimental frequencies before and after model updating

Exp. mode	Exp. freq. (Hz)	Ring Foundation only		Foundation with Catino		
		G=77GPa	G=95GPa	G=77GPa	G=95GPa	Calibrated
Bending N-S	0.96	0.87	0.96	0.88	0.97	0.95
Bending E-W	1	0.87	0.96	0.89	0.97	1
Vertical	3	2.8	3.1	2.8	3.1	3
Torsional	6.3	4.31	4.7	5.9	6.4	6.3

A response spectrum analysis was performed using the calibrated model, which provided some preliminary results on the force demand as reported in Table 4. The results are compared to those obtained for gravitational loading. Evidently, the base moment produced by seismic load with a return period of 130 years equal to 230MNm is about 20% lower than the gravitational one. On the other hand, the moment obtained for a return period of 500 years is considerably larger than the gravitational one.

Table 4. Force demand at the base of the Tower in terms of overturning moment.

Load	Overturning moment [MNm]
Gravitational	280
Seismic input- mean 130 years	230
Seismic input - mean 500 years	420

The importance of soil-structure interaction on the vibrational characteristics of the tower can be explored based on the so-called wave parameter ( $1/\sigma$ ) introduced by Veletsos and co-workers (Veletsos & Meek 1974; NIST 2012; Maravas et al 2014),

$$\frac{1}{\sigma} = \frac{H^* f}{V_s} \quad (1)$$

$H^*$  being the height of an equivalent Single-Degree-Of-Freedom (SDOF) structure (about 23 m based on the elevation of the centre of mass of the Tower),  $f$  its fixed base natural frequency (about 3 Hz) and  $V_s$  the soil shear wave propagation velocity (about 225 m/s). Considering these figures, the wave parameter is estimated at around 0.3, a remarkably high value that exceeds all available data for building structures (Stewart et al 1999). The SSI period can be estimated in an approximate manner from the familiar expression (Veletsos & Meek 1974; NIST 2012; Maravas et al 2014)

$$\tilde{f} = f \left[ 1 + \frac{k}{K_x} \left( 1 + \frac{K_x H^{*2}}{K_{ry}} \right) \right]^{-1/2} \quad (2)$$

where  $k$  is the equivalent stiffness of the superstructure modelled as a SDOF oscillator (which can be evaluated as  $k = 4 \pi^2 m f^2 = (4 \pi^2 14500 \times 3^2 = 5.2 \times 10^6 \text{ kN/m})$  and  $K_x, K_{ry}$  are the foundation stiffnesses in horizontal translation and rocking (about  $5 \times 10^6 \text{ kN/m}$  and  $5 \times 10^8 \text{ kNm/rad}$ ), respectively. Substituting these values into the above equation, one obtains  $\tilde{f} \approx 1.1 \text{ Hz}$  which is reasonably close to the measured value of 1 Hz (the difference being mainly due to the SDOF idealization of the actual

infinite-degree-of-freedom system). The period shift due to SSI, ( $\tilde{T}/T \approx 1/0.3 \approx 3$ ) is the highest known for a structure of this height (Stewart et al 1999).

An equivalent analysis can be carried out considering that the natural frequencies in rocking oscillations of a perfectly rigid superstructure is  $f_{ry} = 1/2\pi (5 \times 10^8 \text{ kNm} / 1.1 \times 10^7 \text{ Mg m}^2)^{0.5} \approx 1.1\text{Hz}$ , the corresponding frequency in swaying of a rigid superstructure is  $f_x = 1/2\pi (5 \times 10^6 \text{ kN/m} / 14500 \text{ Mg})^{0.5} \approx 3\text{Hz}$ , and the natural frequency of the fixed-base structure is  $f = 1/2\pi (5.2 \times 10^6 \text{ kN/m} / 14500 \text{ Mg})^{0.5} \approx 3\text{Hz}$ . Combining the above results using Dunkerley's rule

$$\tilde{f} = (f^{-2} + f_x^{-2} + f_{ry}^{-2})^{1/2} \quad (3)$$

yields  $\tilde{f} \approx 1 \text{ Hz}$ , which is in meaningful agreement with the first estimate.

The role of SSI in the seismic response of the Tower can be assessed with the help of the mode-adjusted spectrum of Figure 10 (right graph), obtained by considering a modal participation coefficient of (2/3). Evidently, under fixed-base conditions the spectral response is on the order of 0.4g, whereas under flexible-base conditions it drops below 0.1g – a 400% reduction. Note that this reduction is probably a lower bound, as it does not account for period elongation due to non-linear soil response, increase in damping etc. The beneficial effect of SSI on the seismic response of the Tower of Pisa is massive.

## 5. CONCLUSIONS

The study at hand reports on a numerical and experimental characterization of the seismic behavior of the leaning Tower of Pisa, and a brief review of historical seismicity. The definition of the seismic input at bedrock level is based on the results of a new set of geophysical and geotechnical tests performed in Piazza del Duomo. On the basis of these measurements, a subsoil model was compiled to perform site response analyses, which provided the free-field input ground motion. A structural Finite-Element (FE) model was put together based on experimental measurements to explore the earthquake response of the Tower, including the effect of soil-structure interaction. The FE model was compiled considering the Tower inclination which can capture overturning moments due to vertical seismic motion. The foundation impedance matrix was evaluated based on solutions from the literature and was refined to match the experimental data. The shift in natural period due to SSI, from about 0.3s under fixed-base conditions to over 1s considering soil compliance ( $\tilde{T}/T \approx 3$ ) and a corresponding wave parameter ( $1/\sigma$ ) of about 0.3 are the largest known for a structure of this height. The reduction in spectral acceleration demand due to SSI is on the order of 400%, from 0.4g under fixed-base conditions, to less than 0.1g considering soil flexibility. This reduction is probably a lower bound, as it does not account for period elongation due to non-linear soil response and associated increase in damping. Evidently, the beneficial effect of SSI on the seismic response of the Tower is massive and may explain the lack of earthquake damage on the structure, despite its severe inclination, low strength and limited ductility. Apart from SSI, the modest seismicity in the area fundamentally contributed to the survival of the monument.

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