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***Abstract*—In this paper, polynomial chaos expansions are applied to angle-of-arrival based localization. By using a polynomial chaos expansion on a least squares estimator, a new positioning method is designed. Simulation results show that the proposed method returns precise information about the statistical distribution of the position.**

***Index Terms*—Polynomial chaos, Localization, Angle-of-Arrival, Positioning**

I. Introduction

Accurate localization is one of the fundamental requirements of future Internet-of-Things (IoT) networks. In a cellular infrastructure, high localization accuracies allows network providers to offer additional services related to contextualized information delivery, targeted advertising or security applications. Traditional localization approaches based on the estimation of the signal ToA/TDOA rely on high communication bandwidths, and can thus not be applied to the localization of IoT nodes (which typically operate with low data rates and low bandwidths). By contrast, angle-of-arrival (AoA) based estimation is less dependent on the bandwidth of the communication system, making it a suitable candidate for IoT localization.

Recently, the deployment of fixed reference nodes, referred to as anchors, which can communicate with the user equipment (UE) to be localized, has been considered [1]. In this work we propose a localization system for a network of densely deployed anchors using AoA measurements at the different anchors. Localization algorithms using AoA measurements have been previously investigated in literature. In [2] the authors use a least squares (LS) estimator, and [3] investigates the use of a linearized LS estimator. While these methods show good efficiency, they do not take into account from the outset the uncertainty of each estimated AoA. We propose to apply polynomial chaos expansion theory to the localization of a RF transmitter by exploiting the AoA measurements at the different anchors, associated with their known uncertainties. This allows us to obtain the location of the transmitter, as well as its statistical distribution. Polynomial chaos expansions have already been used in electromagnetics for a variety of applications, such as ray-tracing [4], angle-of-arrival estimation [5] or dosimetry [6]. However, to the best of our knowledge, it has not been used in localization of RF transmitters. Polynomial chaos expansion (PCE) allows one to determine the statistical properties of the output of a process, based on the probability density function (PDF) of the input random variables of the process [7].

II. Method

A. AoA-based localization

We consider the situation where N anchors collect the angle-of-arrival measurements θ_i obtained from the signal emitted by one UE. The actual location of the UE is denoted as $\mathbf{x} = (\mathrm{x}, \mathrm{y})$. The locations of the anchors are $\mathbf{x}_i = (\mathrm{x}_i, \mathrm{y}_i)$, and the AoA measurements are given by θ_i, for $i = 1, ..., N$. If there are more than two anchors in this two-dimensional problem, the system becomes overdetermined. If the AoA measurements θ_i are the actual AoA's, it is easily shown that this deterministic problem can be expressed by the following matrix equation [2]:

$$\begin{bmatrix} -\mathrm{x}_1 \sin\theta_1 + \mathrm{y}_1 \cos\theta_1 \\ \vdots \\ -\mathrm{x}_N \sin\theta_N + \mathrm{y}_N \cos\theta_N \end{bmatrix} = \begin{bmatrix} -\sin\theta_1 & \cos\theta_1 \\ \vdots & \vdots \\ -\sin\theta_N & \cos\theta_N \end{bmatrix} \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \end{bmatrix} \tag{1}$$

that can be rewritten as

$$\mathbf{b} = \mathbf{H}\mathbf{x} \tag{2}$$

However, if the measurements θ_i are noisy, equation (2) will not have any solution, but an estimate of the solution can be obtained. The least squares estimate of the position $\hat{\mathbf{x}}$ is obtained by

$$\begin{aligned} \hat{\mathbf{x}} &= (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{b} && (3) \\ &\equiv \mathbf{H}^{\dagger}\mathbf{b} && (4) \end{aligned}$$

where † is the pseudo-inverse operator.

B. Polynomial chaos expansions

Consider that each measurement θ_i is associated with an uncertainty that is also estimated by the anchors and is assumed to have a Gaussian distribution with variance $\sigma^2_{\theta_i}$. If uncertainties are taken into account, the least squares estimate of the position of the UE is a function of the random variables θ_i, and is therefore a random variable. We denote this random variable by $\hat{\boldsymbol{x}}$. As shown in [7], $\hat{\boldsymbol{x}}$ can be expressed as:

$$\hat{\boldsymbol{x}} = \sum_{\alpha \in \mathbb{N}^N} \boldsymbol{c}_\alpha \Psi_\alpha(\{\theta_i\}_{i=1}^N) \tag{5}$$

where α is a multi-index and the polynomials $\{\Psi_\alpha(\{\theta_i\}_{i=1}^N)\}_{\alpha \in \mathbb{N}^N}$ form a polynomial chaos basis of the adequate Hilbert space containing $\hat{\boldsymbol{x}}$. These multivariate polynomials are products of univariate polynomials. For each input random variable θ_i, a series of univariate polynomials $\psi_k^{(i)}$, $k \in \mathbb{N}$, are constructed so that they are orthogonal with respect to the scalar product defined by the PDF of θ_i, φ_{θ_i}:

$$\langle \psi_j^{(i)}, \psi_k^{(i)} \rangle = \int \psi_j^{(i)}(u) \psi_k^{(i)}(u) \varphi_{\theta_i}(u) \mathrm{d}u = \gamma_j^{(i)} \delta_{jk} \tag{6}$$

In particular, a Gaussian random variable generates the well known Hermite polynomials [7]. The coefficients $\boldsymbol{c}_\alpha$ can be obtained by different methods. In this work we use the projection method. The orthogonality of the polynomials $\psi_k^{(i)}$ allows one to express the coefficients $\boldsymbol{c}_\alpha$ as:

$$\boldsymbol{c}_\alpha = \frac{\mathbb{E}\left[\hat{\boldsymbol{x}} \Psi_\alpha\right]}{\mathbb{E}\left[\Psi_\alpha^2\right]} \tag{7}$$

where $\mathbb{E}$ is the mathematical expectation operator. A Gauss quadrature is used to evaluate the integrals appearing in the mathematical expectations, as described by Sudret [7]

In practice, the series appearing in (5) is truncated at a certain order such that the number of polynomials taken into account is typically low. We denote by $\mathcal{A}$ the subset of multi-indices corresponding to these polynomials. The interest of this method is that knowing the PDF of the input variables allows one to deduce statistical information of the output values with a limited number of runs of the LS estimator. Indeed, the mean and variance of the output are given by:

$$\mathbb{E}[\hat{\boldsymbol{x}}] = \boldsymbol{c}_0 \tag{8}$$

$$\sigma_{\hat{\boldsymbol{x}}}^2 = \mathrm{Var}\left[\sum_{\alpha \in \mathcal{A} \backslash \boldsymbol{0}} \boldsymbol{c}_\alpha \Psi_\alpha\right] = \sum_{\alpha \in \mathcal{A} \backslash \boldsymbol{0}} \boldsymbol{c}_\alpha^2 \|\Psi_\alpha\|^2 \tag{9}$$

in which the squares of the coefficients are calculated component-wise.

III. Results

In the simulations, $N = 3$ anchors have been considered, each of them making an independent AoA estimation with Gaussian distributed estimation errors. The position of the anchors and the AoA estimation error standard deviations are given in Table I.

TABLE I
Parameters of the Calculation

Anchor	1	2	3
(x_i, y_i)	$(-6, 4)$	$(11, -4.5)$	$(7, 16.6)$
σ_{θ_i}	$12°$	$10°$	$7°$

Our algorithm based on polynomial chaos was used with a fifth order quadrature to perform localization. The results of the calculation are given in Fig. 1. The representation of the polynomial chaos expansion of the position estimate (5) is called the response surface. The position estimate shown in Fig. 1 is the mean of the response surface (8). Moreover, the PCE of the position estimate allows one to deduce the statistical distributions of the position coordinates. From these distributions, the 90% confidence region (CR) of the position was calculated. The validity of the obtained region has been assessed with a Monte-Carlo calculation. In Fig. 1, the 90% CR obtained with the PCE is compared to the 90% confidence region obtained assuming that the distributions of the estimated position coordinates are Gaussian, with the variance $\sigma_{\hat{\boldsymbol{x}}}^2$ obtained from (9). We observe that the confidence region obtained from the response surface differs significantly from the ellipse, assessing the fact that the position distribution cannot be assumed to be Gaussian.

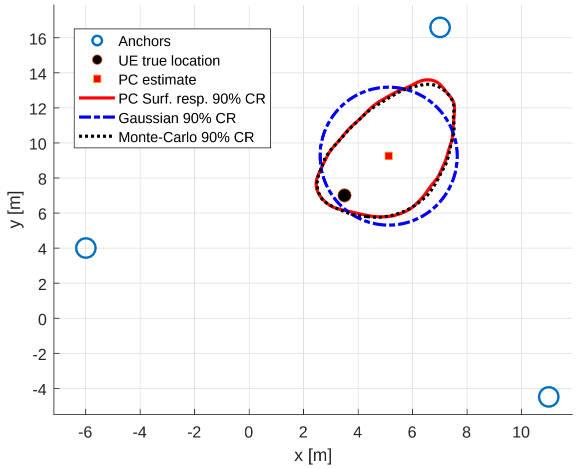

Fig. 1. Estimation of the position with the polynomial chaos (PC) method, as well as 90% confidence intervals, calculated with the polynomial chaos based response surface, with the variances assuming that the response surface is Gaussian-shaped and by Monte-Carlo

IV. Conclusion

A new two dimension position estimation method based on polynomial chaos, least squares estimator, and AoA measurements has been proposed. Compared to traditional positioning methods, it presents the advantage of taking into account the uncertainties on the AoA estimations from the outset. This allows one to compute the statistical distribution of the position, and consequently, confidence regions.

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