Essays on Voting Power

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Für meine Eltern und meine Schwester

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Abstract

This thesis deals with the measurement of voting power in different decision environments. After a short introduction in Chapter 1, several established concepts for power analysis are reviewed and applied in Chapters 2 and 3. Chapters 4 and 5 step on new ground by introducing a power index for a decision environment that has not been formalized before.

More specifically, the second chapter contrasts the textbook claim that Luxembourg was a null player in the first period of the European Economic Community (EEC) with a more comprehensive picture of Luxembourg's role in EEC's voting system. It turns out that the assessment of Luxembourg's voting power is sensitive to the role played by the European Commission in the decision-making procedure and to the measurement concepts underlying power evaluations.

The third chapter analyzes the European Union's codecision procedure as a bargaining game between the Council of the European Union and the European Parliament. The relative influence of these institutions on legislative decision-making in the EU is assessed under a priori preference assumptions. In contrast to previous studies, the chapter does not consider the codecision procedure in isolation but includes several aspects of the EU's wider institutional framework. The finding that the Council is more influential than the Parliament is robust to adding "context" to the basic model but the imbalance is considerably smaller than was previously diagnosed.

The fourth chapter considers collective decisions between more than two alternatives by a given number of differently sized voter groups. *Weighted committee games* are introduced in order to describe corresponding decision-making in a similar fashion as *weighted voting games* model binary decision environments. The chapter compares different voting weight configurations for plurality, Borda, Copeland, and antiplurality rule. The respective geometries and distinct numbers of structurally non-equivalent committees have escaped notice so far.

Finally, the fifth chapter seeks to clarify if – and quantify the extent to which – adoption of a particular collective choice rule in a weighted committee game creates a (dis)advantage for specific groups a priori. It extends established methods for quantifying influence from weighted voting on binary options to several voting rules for three or more alternatives. Voting weights and decision rules interact in more complicated ways than traditional voting power indices can capture.

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Chapter 1

Introduction

1.1 Motivation and Scope

Voting and elections shape democratic participation at all levels of legislature and extend into many areas of economic activity and everyday life. Hiring committees vote on job market candidates; shareholders of private companies elect board members, who may in turn vote on a new CEO; high school students elect class representatives; families vote on the location of the next family vacation; etc.

The goal of these elections is to aggregate individual, often diverging opinions into a collective opinion. A wide range of methods and procedures exist for this purpose – from the complicated system used for electing the German Bundestag or weighted voting rules employed by the EU Council of Ministers to computationally hard Schulze method and simple plurality rule.

These procedures may differ significantly in what they construe as the collective opinion. That is, for the same voter preferences, different voting systems often select different winners. A real-world example concerned the decision to move the German Bundestag and federal government from Bonn to Berlin after the German Reunification. Making plausible assumptions about preferences, Leininger (1993) showed that a different voting system could very well have kept Bonn the seat of government and national parliament in the newly constituted Federal Republic of Germany. Similarly, Tabarrok and Spector (1999) analyzed the crucial US presidential election in 1860, which ultimately led to the US Civil War. Abraham Lincoln had only received a narrow plurality of the popular vote but won with a majority of votes in the Electoral College. Tabbarok and Spector argue that Lincoln was not the presidential candidate who best represented the preferences of the voters; his victory rather was a result of the choice of the voting system. It is likely that two of the three other candidates, Stephen Douglas and John Bell, could have won under reasonable alternative electoral systems – and averted the war.

Voting methods also differ in their structural properties. Among those are well-known characteristics like Condorcet efficiency or liability to monotonicity paradoxes. For example, the plurality runoff system that is used for the French presidential election may display a behavior that runs counter to the most obvious principles of democratic decision-making: *increased support* for a winning candidate can be *harmful*, i.e., make him or her non-winning. Other properties and staggering paradoxes are surveyed, e.g., in Nurmi (1987) and Felsenthal and Nurmi (2017, 2018).

When a voting system is given, it is often of interest to evaluate a voter's influence on collective decisions. Consider, for instance, a private company's shareholder committee that consists of three shareholders. Further assume that the first shareholder commands 45 % of the shares, the second 40 %, and the third the remaining 15 %. One might naively conclude that the large stockholder automatically has more say than his smaller peers. But closer inspection reveals that this intuition is misleading: if the committee has to decide between two alternatives (conceive, e.g., of a simple "accept" or "reject" decision), any two shareholders can jointly implement their preferred alternative and thus have the same *voting power*, i.e., ability to influence the voting outcome. Voting power is, in general, not proportional to voting weight. Related observations date back at least to Luther Martin, who was an anti-federalist delegate from Maryland to the US Constitutional Convention in Philadelphia 1787 and fought for fair representation of small states under the US Constitution. Martin (as cited in Riker 1986, p. 294) already realized that "though Delaware has one delegate, and Virginia but ten, yet Virginia has more than ten times as much power and influence in the government as Delaware."

As voting weights are an unreliable proxy for voting power, more sophisticated mathematical tools are needed to evaluate who has how much influence on the voting outcome. An *index of voting power* is exactly such an analytical tool. It tries to quantify the *a priori power* of voters under the voting rule at hand. Taking an a priori perspective makes the analysis focus on the power that a voter derives from the voting rule itself and abstracts from other factors that may be important to determine *actual* voting power. That is, it neither considers particular political interests, preferences, or diplomatic skills nor does it analyze any specific issues that are voted on. It rather takes the analysis behind a "veil of ignorance".

Starting with Penrose (1946), mathematicians, economists, political scientists and lawyers have defined a plethora of indices.[1] They all capture different aspects and meanings of power. Most of them only apply to the simple binary case ("yes" or "no"). More general frameworks, however, can capture richer decision environments with more than two alternatives and account for strategic interaction between the voters.

These richer decision environments and corresponding power investigations are the focus of this dissertation. Its contribution is twofold: first, it reviews and applies basic tools for the binary case (Chapter 2) as well as for spatial voting (Chapter 3) to EU decision-making. Second, it steps on new ground by introducing the framework of *weighted committee games* in order to describe weighted committee decisions on three or more alternatives (Chapter 4). It also introduces a power index for such games (Chapter 5). The latter is applied to analyze the voting power distribution in the Executive Board of the International Monetary Fund (IMF) and to derive general recommendations regarding the beneficiaries of particular voting methods.

1.2 Structure

All chapters are designed to be self-contained and can be read in any sequence. This comes at the cost of some overlap. Chapter 2 was recently published in *Games*. Chapter 3 is based on joint work with Nicola Maaser, published 2016 in *Social Choice and Welfare*. Chapters 4 and 5 are based on collaboration with Sascha Kurz and Stefan Napel.

1.2.1 Luxembourg in the Early Days of the EEC

Chapter 2 considers binary voting and reassesses an old and well-established stylized fact in the voting power literature: Luxembourg supposedly was powerless in the first period of the European Economic Community (EEC) from 1958 to 1972.

"Null players" are voters whose voting behavior never matters for the outcome of a vote. Luxembourg's role in the Council of Ministers during the first period of the EEC is often cited as a real-world case. Although it wielded one vote out of a total of 17 votes, there was not a single configuration of votes in which Luxembourg's

[1] According to Riker (1986), Luther Martin's informal reasoning was already close in spirit to ideas later formalized by Banzhaf (1965) and Deegan and Packel (1978). Felsenthal and Machover (1998), Laruelle and Valenciano (2008), and Napel (2018) provide surveys of the literature on the measurement of voting power.

decision could theoretically have made a difference. Consequently, standard binary power indices like the *Shapley-Shubik index* (Shapley and Shubik 1954) and the *Penrose-Banzhaf index* (Penrose 1946, Banzhaf 1965) assign zero power to Luxembourg.

The chapter contrasts this claim, which often serves as a textbook example in the analysis of voting power, with a more comprehensive picture of Luxembourg's role in the EEC's early voting system. It is shown that the situation was actually more nuanced.

First, the evaluation of Luxembourg's voting power depends on the role played by the European Commission in the decision-making procedure. That is, Luxembourg was only a null player if the proposal in question did not require an initial proposal of the European Commission. Unfortunately, this observation seems to be mainly unnoticed in the literature.

Moreover, the chapter identifies other sources of influence for Luxembourg – sources that cannot be captured by standard power indices. In particular, it employs a *composite game* with a Benelux Union and uses a popular power index for *games with communication structure* to account for the deep bond between Belgium, the Netherlands and Luxembourg and the historical importance of Luxembourg for European integration. Taking these factors behind the "veil of ignorance" changes the qualitative and quantitative assessment of Luxembourg's influence in early EEC decision-making.

1.2.2 Codecision in Context

Chapter 3 leaves the simple binary framework and considers a spatial voting model in which voters are assumed to have Euclidean preferences. It makes use of the "power as outcome sensitivity" approach (Napel and Widgrén 2004) and applies it to the EU's *codecision procedure*.

This procedure requires consensus to be reached between the European Parliament and the Council of Ministers through alternating amendments. If no agreement is achieved during the first two readings, a compromise is sought by means of a conciliation committee, the third and final phase of codecision.

The chapter portrays the codecision procedure as a bargaining game between the Parliament and the Council and assumes one-dimensional spatial preferences for members of the Parliament and delegates of the Council. The analysis provides an assessment of the relative influence of the two "co-legislators" on EU decision-making under a priori preference assumptions. In contrast to previous studies, it does not consider the codecision procedure in isolation but includes several aspects of the EU's wider institutional framework. It does so from a constitutional perspective which considers only biases stemming from the institutional structure rather than from today's preferences or individual personalities. That is, a more realistic picture of the codecision procedure is offered without giving up an a priori perspective.

The expected influence of the Parliament, the Council and of individual Council members on EU decisions can be quantified using the "power as outcome sensitivity approach". This approach conceives of a posteriori power as the sensitivity of an equilibrium outcome with respect to small changes in a player's preferences. The *strategic measure of power* (Napel and Widgrén 2004) then evaluates a priori power as expected a posteriori power, using a probability measure with a priori credentials.

The main result is that the quantitative assessment of the power relation between the EU's key institutions strongly depends on how much context is taken into account. The stark a priori bias in favor of the Council is greatly moderated when moving to more context-rich models. The qualitative assessment of the balance of power is, however, remarkably robust: the Parliament and the Council do not co-legislate on a par. Decision rules make the latter more influential.

1.2.3 Weighted Committee Games

Chapter 4 considers committee decisions on three or more alternatives that cannot be ordered in a natural fashion. It assumes that the committee uses a standard voting rule that treats votes anonymously but allows the committee members to wield asymmetric numbers of votes. Applications range from shareholders of a private company that have to fill a position in the company's supervisory board to supranational organizations like the IMF whose Executive Board has to select its next Managing Director.

The chapter formalizes such decision environments as *weighted committee games*, which are defined as the combination of a set of voters, a set of alternatives, and a particular weighted voting rule. The latter amounts to the combination of an anonymous voting rule with voting weights associated to the relevant players.

Rather than being concerned with power evaluations, the analysis then focuses on *structural investigations* of weighted committee games. It is motivated by the observation that different distributions of voting weights that are equivalent for binary majority decisions may be non-equivalent for more than two alternatives. For example, a committee consisting of three voters in which the first voter wields 4 votes, the second 3 votes, and the third 2 votes is equivalent to a committee in which all voters have the same voting weight if the committee has to decide on two alternatives: every coalition of at least two voters can then implement its preferred alternative. Things are different if the committee has to decide between three alternatives and uses plurality rule to do so. This can most easily be seen by assuming that the three voters have diverging interests, i.e., each has a different most preferred alternative. The voting outcome (assuming sincere voting) will then be the most preferred alternative of the first voter for weights of (4,3,2). If, however, votes are distributed equally, all three alternatives get the same number of votes and some tie-breaking rule will determine the outcome. Thus, the latter and the former voting weight distribution are non-equivalent: there is at least one preference profile for which they induce different winning alternatives.

Repeating the same exercise for all (infinitely many) distributions of voting weights between three players, the chapter identifies and compares all structurally distinct weight distributions for committees which use Borda, Copeland, plurality, or antiplurality rule. Their geometry and differing numbers of equivalence classes – e.g., 51 for Borda vs. 4 for Copeland rule if three voters decide on three alternatives – have so far escaped notice. The partition of the set of weight distributions into structurally distinct equivalence classes can be useful to identify the respective distribution of power in weighted committee games. This is done in Chapter 5.

1.2.4 Influence in Weighted Committee Games

Chapter 5 is based on the weighted committee framework developed in Chapter 4. It introduces an a priori measure of influence (or power) for weighted committee games similar to what Penrose, Banzhaf, Shapley, Shubik and others have done for binary decision environments. This allows to make new statements about the power implications of social choice rules. The chapter focuses on five rules: plurality, plurality runoff, Borda, Copeland, and Schulze rule. This complements previous comparisons of rule-specific properties like Condorcet efficiency or susceptibility to various voting paradoxes. The analysis illustrates how voting weights *and* procedures jointly determine the distribution of influence between differently sized homogeneous groups of voters for more than two alternatives.

The influence of a voter is quantified as the expected sensitivity of the outcome to changes in this voter's preferences. "Sensitivity" refers to the effect of arbitrary perturbations of the original preference ordering (perhaps caused by a well-endowed lobbyist who successfully bribes the voter or a spontaneous change of mind). Identifying the average effect of preference perturbations on the voting outcome for each preference profile and then – assuming that all preference profiles are equally likely – averaging over all conceivable profiles, gives a measure of a voter's a priori influence in the corresponding weighted committee game. It is a generalization of the Penrose-Banzhaf index.

The new index is applied to determine the power distribution in the IMF Executive Board. Voting weights as well as the selection process for the IMF's Managing Director have been reformed in 2016. It is investigated in the chapter whether – and to which extent – the aim of granting emerging market economies more say in the IMF's decision-making process was successfully achieved.

In the final part of the chapter, it is asked if winners and losers from adopting a particular voting rule can also be identified a priori if the precise distribution of votes is not yet known. Making use of diagrams similar to those used for illustrating the geometry of weighted committee games in Chapter 4, surprisingly general "rules of thumb" regarding the beneficiaries of particular voting rules can be identified.

Chapter 2

Luxembourg in the Early Days of the EEC: Null Player or Not?

On 25 March 1957, Germany, France, Italy and the Benelux countries signed the Treaty of Rome which established the European Economic Community (EEC). Sixty years later, it seems a well-established stylized fact that the founding fathers of what has now become the European Union unwittingly chose an odd voting rule: they rendered Luxembourg a "null player" in the Council of Ministers (CM) from 1958 to 1972.

The case of Luxembourg is often invoked as a textbook example in the analysis of voting power. It illustrates that a positive number of votes does not necessarily come with a positive share of power. Although it wielded one vote out of a total of 17 votes, there was not a single configuration of votes in which Luxembourg's decision could theoretically have made a difference. This matches the definition of a *null player*. Consequently, standard power indices like the *Shapley-Shubik index* (Shapley 1953; Shapley and Shubik 1954) and the *Penrose-Banzhaf index* (Penrose 1946; Banzhaf 1965) indicate zero voting power for Luxembourg.

The first thing which is forgotten in this context is that Luxembourg was formally a null player only under specific conditions. That is, when the EEC Treaty required the Council to (*i*) use weighted voting and (*ii*) act on a proposal of the European Commission. A closer look into the decision rules specified in the EEC Treaty reveals that in all other cases – in particular if Council decisions had to be taken by weighted voting *without* an initial proposal of the Commission – Luxembourg was *not* a null player, i.e., had positive voting power.

Secondly, if one recalls the deep bond between the Benelux countries and the important role that Luxembourg has always played (and still plays) in fostering European integration, there were probably other sources of influence for Luxembourg – sources that cannot so straightforwardly be captured by power indices which rest on the assumption of voters acting independently of each other. We provide historical arguments that support this thesis. We also use a *composite game* with a Benelux Union and invoke a popular power index for *games with communication structure* in order to get a more refined picture of Luxembourg's role in the first period of the EEC.

The remainder of this chapter is organized as follows: Section 2.1 will first introduce the different decision rules used by the then Council of Ministers and give a short account of the historical background which motivates later modeling choices. Then we introduce notation, weighted voting games and the two most prominent standard power indices in Section 2.2. Section 2.3 recalls the results of traditional power analysis for the early EEC. Section 2.4 takes account of the Benelux Union by means of a composite game; communication structures within the EEC are captured in Section 2.5. Section 2.6 concludes.

2.1 Historical Background

2.1.1 The Council's Internal Decision Rule

The manner in which the Council of Ministers took decisions between 1958 to 1972 was established in Art. 148 of the EEC Treaty. Even though Art. 148(1) allowed for simple majority voting, most of the Council decisions had to be taken by weighted voting and required a so-called *prescribed majority* as specified in Art. 148(2).[1] The corresponding weights were four votes each to Germany, France and Italy, two votes each to the Netherlands and Belgium, and one vote to Luxembourg.

For those decisions that had to be made by weighted voting, two different cases were distinguished. If the EEC Treaty required the Council to act on a proposal of the European Commission, the decision rule was a *pure weighted rule* that asked for at least 12 out of the 17 total votes in order to adopt a decision. For decisions in policy areas that did not require an initial proposal of the Commission, a *double majority rule* had to be used: (*i*) at least twelve votes and (*ii*) the approval of at least four of the six

[1]The original version of the EEC Treaty used the term "prescribed majority". It was changed to "qualified majority" later. *Constitutional measures* and *derogation from the Treaty* always required unanimity. For further information on weighted voting in the Council of Ministers during the first period of the EEC, see Felsenthal and Machover (1998, ch. 5.2).

member states were needed.[2]

2.1.2 The Benelux as a Key Driver of European Integration

Economic partnership between Belgium, the Netherlands and Luxembourg dates back at least to 1922 when Belgium and Luxembourg established an economic union, the *Union Economique Belgo-Luxembourgeoise*.[3] In the 1930s, the three Benelux and the Scandinavian states signed the *Oslo Convention* and *Ouchy Convention* with the aim to lower tariffs. At the end of the Second World War, the three Benelux countries agreed on an even deeper economic relationship among themselves. They signed the *Benelux Monetary Agreement* in 1943 and the *Netherlands-Belgium-Luxembourg Customs Convention* in 1944. The former fixed the exchange rates between the Dutch guilder and the Belgian-Luxembourg franc; the latter established a tariff union between the three Benelux countries. They introduced a common external tariff, unified their tax rates and established common institutions like a Council of the Economic Union, a Council for Trade Agreements and an Administrative Council for Customs. In 1945 the Benelux countries and France signed the *Economic Agreement on Mutual Consultation*, setting up a *Tripartite Council for Economic Cooperation*. This was not very successful and only lasted until 1948.

In the following years, the Benelux states continued to extend their partnership on different levels. Having signed a military treaty in 1948, they agreed on a common agricultural market in 1955 and unified their labour markets in 1956. Two years later, they signed a treaty that established a new economic union – the well-known *Benelux Union* – beginning in 1960 and intended to last for 50 years.[4] The so-called *Benelux Treaty* aimed at achieving free movement of capital, goods, people and services. Moreover, the signatories agreed on coordinating economic, financial and social policies. Important common institutions of the Benelux Union are the Committee of the Ministers, consisting of the three foreign ministers, the Benelux Parliament and the Council of the Union.

The Benelux countries were one of the main drivers of European integration. After the *European Coal and Steel Community* (ECSC) had been established in 1951

[2]There was moreover a transition period of eight years (until 31 December 1965) during which every country had a veto right. Unanimity was also required if the Council wanted to amend a Commission proposal (Art. 149).

[3]The describtion in this section is mainly based on Urwin (1995), Dedman (1996) and Gilbert (2012).

[4]Two years before expiration, an unlimited extension was agreed.

by the Treaty of Paris with France, Germany and Italy, the Benelux states insisted on a *Council of Ministers* to monitor the so-called *High Authority* of the ECSC and to defend the interests of the smaller member states. The inaugural session of the High Authority and the first session of the Council of Ministers both took place in Luxembourg in 1952.

The most important contribution of the Benelux countries to European integration was then made in 1955. They preferred much deeper integration than that achieved by the ECSC and therefore suggested a comprehensive economic community. The so-called *Benelux Memorandum* of 1955 resulted in the meeting of Messina, Italy – chaired by Joseph Beck, the Prime Minister of Luxembourg. This was the starting point for the 1957 Treaty of Rome that established the European Economic Community. The *Messina Declaration* was heavily based on common positions of the Benelux states regarding sectoral integration and a common market.

The deep integration between Belgium, Luxembourg and the Netherlands was explicitly acknowledged in the EEC Treaty. According to Art. 233 "[t]he provisions of this Treaty shall not be an obstacle to the existence or completion of regional unions between Belgium and Luxembourg, and between Belgium, Luxembourg and the Netherlands, in so far as the objectives of these regional unions are not achieved by application of this Treaty."

In the first crisis of the EEC in 1965, France boycotted every meeting of the Council. It was Pierre Werner, the Prime Minister of Luxembourg and later regarded as the "father" of the euro by many, who proposed a compromise during a number of meetings held in Luxembourg in 1966.[5] This compromise entailed that whenever a member state's vital national interests were affected adversely, negotiations had to continue as long as a mutually acceptable compromise was reached. That gave each member an informal veto right.

2.2 Preliminaries

2.2.1 Simple and Weighted Voting Games

A *simple game* is a special case of a cooperative game (N, v) in which $N = \{1, \dots, n\}$ denotes the non-empty and finite *set of players* and $v\colon 2^N \to \{0, 1\}$. A coalition S is referred to as *winning* if $v(S) = 1$ and as *losing* if $v(S) = 0$. A winning coalition is called

[5] Among the reasons for France's so-called "policy of the empty chair" were discrepancies regarding the admission of Britain, farm prices and increased budgetary power of the European Parliament.

minimal winning if every proper subcoalition $T \subset S$ is losing. It is generally required from (N, v) that (*i*) the empty coalition $\emptyset$ is losing (i.e., $v(\emptyset) = 0$), (*ii*) the grand coalition N is winning (i.e, $v(N) = 1$) and (*iii*) v is *monotone* (i.e., $S \subseteq T \Rightarrow v(S) \leq v(T)$).

A *weighted voting game* is a simple game that can be represented by a set of non-negative weights $\mathbf{w} = (w_1, \ldots, w_n)$ and a positive quota q such that $v(S) = 1$ if and only if $\sum_{i \in S} w_i \geq q$. We then write $[q; \mathbf{w}]$ interchangeably with (N, v), i.e., $(N, v) = [q; \mathbf{w}]$.[6]

A few definitions pertaining to players' roles in a simple game are worth recalling.

Definition 2.1. *A player $i \in N$ in game (N, v) is called a **null player** if for all $S \subseteq N$*

$$v(S) = v(S \cup \{i\}).$$

A null player contributes nothing to any coalition. If i is a null player, then $v(\{i\}) = 0$.

Definition 2.2. *A player $i \in N$ in game (N, v) is called a **dummy player** if for all $S \subseteq N \setminus \{i\}$*

$$v(S \cup \{i\}) = v(S) + v(\{i\}).$$

Intuitively, a dummy player i only contributes his standalone value to any coalition $S \subseteq N \setminus \{i\}$, i.e., his cooperation creates no complementarities and he has no meaningful strategic role in the game. Every null player is also a dummy player. Moreover, if (N, v) is a simple game, then a dummy player i can either be a null player or a dictator (i.e., $v(S) = 1 \Leftrightarrow i \in S$).

To our knowledge, the notion of a null player goes back to von Neumann and Morgenstern (1953, Ch. 10). They dedicated a full chapter's attention to simple and weighted games and pointed out that it may happen "that no minimal winning coalition contains a certain player *i*" (p. 436) – without directly referring to such a player as a null player. Presumably the first to explicitly make use of the term "dummy player" as defined above was Shapley (1953). The subtle difference between null and dummy players got somewhat lost in subsequent years. The literature often speaks of a "dummy player" but is actually referring to a "null player". The recent book by Maschler et al. (2013) is among the few that explicitly distinguishes both player types.

[6] Not every simple game has such a weighted representation. Examples include the qualified majority rule of the Council of the EU as specified in the Treaty of Lisbon, or the Canadian Constitution. See Taylor and Zwicker (1999) and Kurz and Napel (2016). Ongoing progress on the problem of verifying if a given simple game is weighted is, e.g., reported by Freixas et al. (2017).

2.2.2 Power Indices

A *power index* for simple games is a family of functions which map each simple game (N, v) to a vector of real numbers $f(N, v) = (f_1(N, v), \ldots, f_n(N, v))$, where $f_i(N, v)$ indicates the voting power of player i in game (N, v).

The two most prominent indices are the Shapley-Shubik index (SSI) and the Penrose-Banzhaf index (PBI) defined by

$$f_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} p_S^i \cdot [v(S \cup \{i\}) - v(S)], \quad i \in N,$$

where $\{p_S^i : S \subseteq N \setminus \{i\}\}$ is a probability distribution over the coalitions not including player i. For the PBI

$$p_S^i = \frac{1}{2^{n-1}}$$

and for the SSI

$$p_S^i = \frac{s!(n-s-1)!}{n!}.$$

Here s denotes the cardinality of coalition S and 0! is defined to equal 1. Regarding its probabilistic interpretation, the PBI assumes that all coalitions $S \subseteq N$ are equally likely while the SSI assumes all coalition sizes s to be equally likely and any coalition of a *given* size to be equally likely.[7]

The bracketed term $v(S \cup \{i\}) - v(S)$ is usually referred to as player i's *marginal contribution* to coalition S. In a simple game $v(S \cup \{i\}) - v(S) = 1$ if and only if $S \cup \{i\}$ is winning and S is losing. Player i is then also called *pivotal* or *decisive* for coalition S. If $v(S \cup \{i\}) - v(S) = 0$, i.e., $S \cup \{i\}$ and S are either both winning or both losing, then player i contributes nothing to coalition S.

2.3 Luxembourg in Traditional Voting Power Analysis

The two versions of the weighted decision rule used by the Council of Ministers from 1958–1972 amount to two different weighted games. The *pure weighted rule* simply is $[12; 4, 4, 4, 2, 2, 1]$. The *double majority rule*, by contrast, is described in the EEC Treaty

[7] See Napel (2018) for a recent overview on different approaches to the measurement of voting power. PBI and SSI often give similar values, but not always. They induce the same ordinal ranking of players if the players can be ordered by Isbell's desirability relation (see Isbell 1956).

as the intersection of two weighted games $(N, v_t) = [q^t; w_1^t, \ldots, w_6^t], t = 1, 2$. The first one is $(N, v_1) = [12; 4, 4, 4, 2, 2, 1]$ and captures the *weight dimension*. The second is $(N, v_2) = [4; 1, 1, 1, 1, 1, 1]$ and refers to the *majority of countries dimension*. A coalition $S \subseteq N$ is winning if and only if it is winning in both dimensions, i.e.,

$$(v_1 \wedge v_2)(S) = \begin{cases} 1 & \text{if} \quad \sum_{i \in S} w_i^t \geq q^t, \quad t = 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

It turns out that the double majority rule can also be translated into a single-dimensional weighted voting game that can be represented by $[10; 3, 3, 3, 2, 2, 1]$. That is, the intersection of (N, v_1) and (N, v_2) is equivalent to $[10; 3, 3, 3, 2, 2, 1]$.

Applying the SSI and PBI to the two decision rules we get the results reported in Table 2.1. As one can immediately see, Luxembourg was formally powerless under the pure weighted rule. There was not a single configuration in which the vote of Luxembourg did matter, i.e., it was never part of a minimal winning coalition. If, however, in addition to at least twelve votes a majority of the six member countries had to approve the motion, then Luxembourg suddenly played a relevant role. In particular, there existed one coalition that was losing without the support of Luxembourg but turned winning with Luxembourg: the coalition comprising Germany, France and Italy had a total of twelve votes but failed to satisfy the majority of countries requirement; if Luxembourg joins, the losing coalition turned winning. Thus, claiming that Luxembourg was a null player crucially depends on which of the two decision rules of Art. 148(2) had to be used.

Member state	SSI		PBI	
	pure weighted	double majority	pure weighted	double majority
G(ermany)	0.23333	0.21667	0.31250	0.28125
F(rance)	0.23333	0.21667	0.31250	0.28125
I(taly)	0.23333	0.21667	0.31250	0.28125
N(etherlands)	0.15000	0.16667	0.18750	0.21875
B(elgium)	0.15000	0.16667	0.18750	0.21875
L(uxembourg)	0.00000	0.01667	0.00000	0.03125

Table 2.1 SSI and PBI in 1958–72 CM under pure weighted and double majority rule

Luxembourg's role in EEC decision-making often serves as a prominent example in the literature on voting power in general and the European Union in particular. See, e.g., Straffin (1994, p. 1131), Felsenthal and Machover (1997b, p. 43), Leech (2003, p. 831), Pacelli and Taylor (2009, p. 81) and Le Breton et al. (2012, p. 159).

Unfortunately, it is in most cases simply claimed that Luxembourg was a "dummy player" (more specifically a null player). Few authors mention the two different rules in Art. 148(2) and the positive power of Luxembourg under the double majority rule as well as under the simple majority and unanimity provisions of the Treaty. It rather seems like received wisdom that Luxembourg was a null player from 1958–72.

To our knowledge, Affuso and Brams (1976, 1985) and Felsenthal and Machover (1998) are the only investigations that explicitly refer to the different decision rules specified in Art. 148. Felsenthal and Machover (1998) also claim that Luxembourg was a "dummy player", but they make clear that they are just concerned with the pure weighted rule. Affuso and Brams (1976, 1985) stress the difference between the pure weighted and the double majority rule and highlight that "[t]hese slightly more stringent decision rules afforded Luxembourg some nonzero voting power on the 1958 Council [...]" (Brams and Affuso 1976, p. 43).

2.4 Power in a Composite Game with a Benelux Union

The case distinction between the pure and the double majority weighted voting game notwithstanding, one is tempted to wonder why Luxembourg agreed to being a null player under the pure weighted rule.[8] Were they unaware of the fact that there was not a single conceivable configuration of votes in which they could make a difference? Or, and this leads us to the next argument, did they anticipate that their role within the Benelux Union gave them an important say nonetheless?

Given the historical background which we presented in Section 2.1, it seems natural to assume that the Benelux countries agreed to speak with one voice in the EEC. This section therefore treats Belgium, the Netherlands and Luxembourg as a bloc that acts like a single player. They are presumed to internally agree on the bloc's position before meeting the other three players.[9]

The standard framework we have used so far is not able to take such an arrangement directly into account. It is best to employ a *composite game* to model the situation.[10] We can think of the Benelux Union's decision-making as the first stage

[8]For the rest of this paper, we will only deal with the pure weighted rule. Under the double majority rule, Luxembourg has positive voting power anyway.

[9]Other contributions dealing with coalition formation within the European Union are Widgrén (1994), Hosli (1996) and Felsenthal and Machover (2002, 2008).

[10]For a thorough treatment of composite games, see Owen (1995, Ch. 11–12) and Felsenthal and Machover (1998, Ch. 2).

in a two-staged process in which all union members are bound to vote according to the within-union decision in the second stage.[11]

Formally, let $N_1, \ldots, N_k$ be k nonempty and disjoint sets of players such that $N = \bigcup_{j=1}^{k} N_j$. Also let $\underline{v}_1, \ldots, \underline{v}_k$ describe k simple games with respective player sets $N_1, \ldots, N_k$ and $\bar{v}$ a simple game with player set $K = \{1, \ldots, k\}$. The so-called *$\bar{v}$-composition* of $\underline{v}_1, \ldots, \underline{v}_k$ is then denoted by

$$u = \bar{v}[\underline{v}_1, \ldots, \underline{v}_k].$$

The characteristic function of this *composite game u* is given by

$$u(S) = \bar{v}\Big(\Big\{j\colon \underline{v}_j(S \cap N_j) = 1\Big\}\Big)$$

for $S \subseteq N$.

The composite game (N, u) represents a division of player set N into disjoint subsets N_j. The members of N_j have to come to an internal decision via game $(N_j, \underline{v}_j)$. Intuitively, one can think of the members of N_j electing a representative who is committed to the internal decision. In the second stage, the k representatives then come together and play game $(K, \bar{v})$ among themselves.

Applied to our context with a Benelux Union and the EEC's pure weighted rule, player set $N = \{\mathrm{G}, \mathrm{F}, \mathrm{I}, \mathrm{N}, \mathrm{B}, \mathrm{L}\}$ is partitioned into sets $N_1 = \{\mathrm{N,B,L}\}$, $N_2 = \{\mathrm{G}\}$, $N_3 = \{\mathrm{F}\}$ and $N_4 = \{\mathrm{I}\}$. They face

$$u = \bar{v}[\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4]$$

where $(N_1, \underline{v}_1)$ is the internal simple game of the Benelux Union, and $(N_2, \underline{v}_2)$, $(N_3, \underline{v}_3)$ and $(N_4, \underline{v}_4)$ are the trivial simple games whose sole voter is Germany, France or Italy, respectively.

Regarding decision-making within the Benelux Union, two options immediately emerge as natural candidates for $(N_1, \underline{v}_1)$. One could (*i*) use simple majority voting with the weights being those assigned by the EEC Treaty or (*ii*) simply require the support of at least two of the three countries. The former can be represented by $[3; 2, 2, 1]$ and the latter by $[2; 1, 1, 1]$. From an analytical perspective it doesn't matter which of the two is used: they are just different representations of the same game. Namely, every two-player coalition is minimal winning in $(N_1, \underline{v}_1)$.

Finally, $(K, \bar{v})$ with $K = \{1, 2, 3, 4\}$ is the four-player simple game played between

[11] At least for the PBI, this has already been pointed out by Felsenthal and Machover (2008).

the Benelux Union, Germany, France and Italy and assuming that Belgium, the Netherlands and Luxembourg are bound to their within union decision and cast a joint voting weight of five. This can be represented by $[12; 5, 4, 4, 4]$.

There are two types of minimal winning coalitions in $(K, \bar{v})$. One consists of the three large countries Germany, France and Italy. The other involves just two large countries plus the Benelux Union. Overall, a coalition of the three large countries and every coalition that involves two large countries and two Benelux countries is minimal winning in the composite game (N, u).

Let $\mathrm{SSI}^{\mathrm{C}}$ and $\mathrm{PBI}^{\mathrm{C}}$ denote the SSI and PBI calculated for above composite game with a Benelux Union.[12] We obtain

$$SSI^{C}(N, u) = (0.2333, 0.2333, 0.2333, 0.1000, 0.1000, 0.1000)$$

and

$$PBI^{C}(N, u) = (0.3750, 0.3750, 0.3750, 0.1875, 0.1875, 0.1875).$$

To see why Luxembourg no longer is a null player in (N, u), consider the minimal winning coalition $S = \{\mathrm{G}, \mathrm{F}, \mathrm{B}, \mathrm{L}\}$. Obviously, $u(S) = 1$ since $\bar{v}(\{1, 2, 3\}) = 1$ and $u(S \setminus \{\mathrm{L}\}) = 0$ since $\bar{v}(\{2, 3\}) = 0$. The crucial point is that coalition S loses the five votes of the Benelux Union in one go if Luxembourg decides to leave S.

2.5 Power in a Game with Communication Structure

Another modeling option which reflects the probable real importance of Luxembourg in early EEC decision-making is by means of *games with communication structure*. The basic assumption underlying such games is that coalitions can only form between players that are connected in a communication graph, i.e., that can "communicate" with each other. This connection can, e.g., be interpreted as reflecting ideological or spatial proximity.

Formally, a *simple game with communication structure* (N, v, g) is a simple game (N, v) augmented by an *undirected and unweighted graph* $g \subseteq g^N = \{\langle i, j\rangle : i, j \in N, i \neq j\}$ on N where $\langle i, j\rangle \in g$ means that players i and j can communicate, i.e., they are linked to each other. If the underlying simple game (N, v) allows for a weighted representation,

[12] The calculation of the PBI is particularly easy: for the large countries it is just the PBI in the second stage game $(K, \bar{v})$; for a member of the Benelux Union it is the product of its PBI in the first-stage game $(N_1, \underline{v}_1)$ and the Union's PBI in the second stage game $(K, \bar{v})$. Unfortunately, the SSI does not have such a product property; we calculate it directly from its definition.

we call (N, v, g) a *weighted voting game with communication structure*.

Players i and j in a coalition S are said to be *connected by* g if either $\langle i, j\rangle \in g$ or there is a path within S from i to j, i.e., we can find players $p_1, \ldots, p_k \in S$ with $p_1 = i$, $p_k = j$ and $\langle p_1, p_2\rangle, \ldots, \langle p_{k-1}, p_k\rangle \in g$. Coalition S is called *connected by* g if all players $i, j \in S$ are connected.

A *power index* for games with communication structure is a family of functions which assign a vector of real numbers $f(N, v, g) = (f_1(N, v, g), \ldots, f_n(N, v, g))$ to each game (N, v, g), where $f_i(N, v, g)$ is interpreted as the power of player i in game (N, v, g).

One of several prominent power indices for games with communication structure is the *position value* (PV) (Borm et al. 1992).[13] It evaluates the power associated with players' links. It can be obtained by first calculating the SSI of the so-called *link game* in which links in g are treated as the "players" and then assigning each player half of the SSI of each link he participates in.[14]

In what follows, we will only focus on the full graph, i.e., cases where $g = g^N$. In our specific political context it seems natural to assume that every player is connected to all the other players. That is, for all players $i, j \in N$ the link $\langle i, j\rangle$ is a member of g^N.[15] Then,

$$PV_i(N, v, g^N) = \sum_{\langle i,j\rangle \in g^N} \frac{1}{2} SSI_{\langle i,j\rangle}(g^N, v^N), \quad i = 1, \ldots, n,$$

where (g^N, v^N) denotes the simple game played by links in the full graph and v^N is the characteristic function such that a coalition L of links is winning if and only if it connects a winning coalition of the original simple game (N, v).

Intuitively, one can think of the position value as reflecting scenarios in which links are established randomly one after another until all links have been activated and the coalition in question is connected. The key feature of the PV is that the worth $v^N(L \cup \{\langle i, j\rangle\}) - v^N(L)$ of a new link is shared equally between the two players i and j that are connected by it.

To illustrate the idea behind the PV before looking at the EEC's pure weighted rule consider the game (N, v, g^N) with $N = \{A, B, C\}$ and $(N, v) = [6; 5, 3, 2]$. The

[13] The main other ones are the *Myerson value* (Myerson 1977), the *restricted Banzhaf index* (Owen 1986) and the *average tree solution* (Herings et al. 2008, 2010).

[14] Ghintran (2013) provides an extension of the position value that allows for an unequal division of a link's SSI.

[15] For a general definition of the PV, also applying to cases $g \subset g^N$, one needs to introduce the concept of a *restricted game*. This would, however, just complicate the exposition without adding useful insights for our setting.

corresponding full graph g^N is depicted in Figure 2.1.

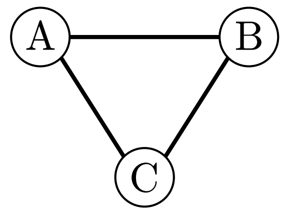

Figure 2.1 Full graph g^N with $N = \{A, B, C\}$

In a first step, one has to calculate the SSI of the three links $\langle A, B\rangle$, $\langle A, C\rangle$ and $\langle B, C\rangle$ in the link game. The minimal winning coalitions are $\{\langle A, B\rangle\}$ and $\{\langle A, C\rangle\}$. It is easy to see that link $\langle B, C\rangle$ is never pivotal and links $\langle A, B\rangle$ and $\langle A, C\rangle$ are each pivotal in half of the orderings. Thus, $SSI(g^N, v^N) = (0.5, 0.5, 0)$. The position value of a player is the sum of half of the SSI of all links in which this player is involved, i.e., $PV(N, v, g^N) = (0.50, 0.25, 0.25)$.

When applying the PV to the early EEC under the assumption that all six players are connected to each other and with $(N, v) = [12; 4, 4, 4, 2, 2, 1]$, we get

$$PV(N, v, g^N) = (0.1983, 0.1983, 0.1983, 0.1525, 0.1525, 0.1003).$$

The corresponding full graph g^N is depicted in Figure 2.2.

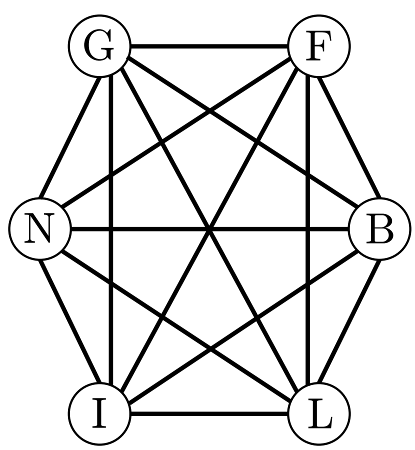

Figure 2.2 Full graph g^N with $N = \{\text{G}, \text{F}, \text{I}, \text{N}, \text{B}, \text{L}\}$

The fact that Luxembourg does have power in (N, v, g^N) is no surprise. A null player i in a full graph connects the grand coalition and is thus involved in some links that have a positive marginal contribution in the link game, which implies

$PV_i(N, v, g^N) > 0$ if (N, v) is non-dictatorial. For illustration, conceive of a sequence of bilateral talks before the actual voting takes place. Assume, e.g., that first France talks to Germany and they agree to support the proposal in question. Then France also manages to get the support of Belgium. This coalition of links $\langle F, G\rangle$ and $\langle B, F\rangle$ already captures ten votes. If we next assume that after some talking to each other, also the Netherlands and Luxembourg are in favor of the proposal, the coalition of links $\{\langle F, G\rangle, \langle B, F\rangle, \langle L, N\rangle\}$ is minimal winning. That is, the link between the Netherlands and Luxembourg is pivotal and Luxembourg gets half of its worth.

Note that our finding of Luxembourg having a positive position value does not hinge on using the full graph g^N. It is sufficient to have one maximal losing coalition T that is connected and Luxembourg being linked to one country outside T.

2.6 Concluding Remarks

The case of Luxembourg is the textbook example for illustrating the concept of a null player (which is often confused with that of a dummy player). As shown above this is somewhat misleading: whether Luxembourg was a null player from 1958–1972 depends on the institutional context, i.e., the two different decision rules embodied in Art. 148(2) of the EEC Treaty. Moreover, even if one is aware of the exact rule being utilized, standard power indices like the SSI and PBI may not be able to pay proper account to the importance of Luxembourg for European decision-making. If we take factors like the Benelux Union or a potentially sequential bilateral structure of decision-making behind the "veil of ignorance" in a priori analysis of power, we see that unqualified claims that Luxembourg was a null player fall short of the truth.

Some may argue that this makes use of information that goes beyond a pure a priori perspective. To some extent, we agree. Power indices for composite games or games with communication structure hold a middle ground between fully a priori and a posteriori analysis. However, the respective analysis does neither presume any specific issues that are voted on. Nor does it take historical preferences or individual personalities into account. It analyzes the role of Luxembourg from a *constitutional* perspective that considers important legal and structural elements of the EEC's wider institutional framework, notably the Benelux Union which is explicitly referred to in the EEC Treaty.

To give a conclusive answer for the question raised in the title of this chapter, one needs to answer a more general question: which properties that go with different indices fit a specific application best? Aumann (2008, p. 535) remarked:

> "Which solution concept is 'right'? None of them; they are indicators, not predictions. Different solution concepts are like different indicators of an economy; different methods for calculating a price index; different maps [...]; different stock indices [...]; different batting statistics [...]; different kinds of information about rock climbs [...]; accounts of the same event by different people or different media; different projections of the same three-dimensional object [...]. They depict or illuminate the situation from different angles; each one stresses certain aspects at the expense of others."

For the specific application considered in this paper, both the concepts of games with a priori unions and games with communication structure in our view provide a better perspective than standard simple games. They give a more fitting description of the institutional environment during the first period of the EEC than the models underlying traditional power indices like the PBI and SSI. Be it Luxembourg's role within the Benelux Union or its importance for maintaining intra-European communication (recall the "empty chair" crisis), Luxembourg clearly influenced decision-making in the EEC. Taking all relevant aspects into the model, Luxembourg was no null player in the early days of the EEC.

Chapter 3

Codecision in Context: Implications for the Balance of Power in the EU

With the Lisbon Treaty's entry into force, codecision has become the *ordinary legislative procedure* for decision-making in the European Union (EU). As a step towards a better democratic functioning of the EU, it was introduced in the Treaty of Maastricht in 1993. Its primary objective was to strengthen the role of the directly elected European Parliament (EP). The Treaties of Amsterdam (1999), Nice (2003) and Lisbon (2009) subsequently amended the procedure and extended its scope.[1] The commonly drawn conclusion among EU observers is that the Parliament and the Council of the European Union (CEU) are now legislators on an equal footing. According to the EP's own description (European Parliament 2012, p. 5) the "ordinary legislative procedure is based on the principle of parity between the [...] European Parliament, representing the people of the Union, and the Council, representing the governments of Member States."

The codecision procedure has inspired a number of theoretical and empirical studies which aimed to answer the question of who has how much influence on EU legislation. Bargaining theory suggests that factors such as agents' evaluation of the status quo, or their patience and attitude towards risk determine the outcome of negotiations.[2] While the importance of these aspects seems uncontroversial in general, applied models on negotiations between the EP and the Council differ

[1] The codecision procedure applied to only 15 areas of community activity in its Maastricht version. This number increased in the Treaties of Amsterdam, Nice and Lisbon to now more than 80 areas of Community activity. The procedural rules in place today are essentially those laid down in the Treaty of Amsterdam, the only difference being that the Council now decides by qualified majority in all policy domains, including those which before required unanimity.

[2] Empirical studies, e.g., König et al. (2007), generally confirm these theoretical claims.

widely with respect to the game form used to describe the codecision procedure. As a result, theoretical findings vary from a genuine, balanced two-chamber system (Crombez 1997, 2000; Tsebelis and Garrett 2000; Moser 1996, 1997; Scully 1997) to a pronounced asymmetry in favor of the Council (Steunenberg and Dimitrova 2003; Napel and Widgrén 2006).[3]

In this study, we explore how robust predictions about the relative power of the two "co-legislators" are when several important but so far neglected elements of the EU's institutional framework are taken into account. We do so from a constitutional perspective which considers only biases stemming from the institutional structure rather than, say, from today's preferences or individual personalities. It turns out that an a priori bias in favor of the Council still persists when more institutional context is modeled. We take the widely-cited model of Napel and Widgrén (2006, henceforth N&W) as our reference point. In our view, their work succeeds well in providing a picture of the codecision procedure taken in isolation. Yet, looking at the wider institutional situation in which codecision is embedded, we suggest several modifications of their assumptions. Specifically, we consider (*i*) the fact that members of the Council are representatives of national governments which came off as winners in national general elections, (*ii*) the fact that citizens generally exhibit heterogeneity across member states rather than being all independent and identical in their preference distribution, and (*iii*) the observation that negotiations between the EP and the Council are characterized by mutual concessions. We then quantify how power is distributed both between the EP and the Council and inside the Council for a priori random, one-dimensional spatial preferences.

The remainder of the chapter is organized as follows. In Section 3.1 we describe existing theoretical models of the codecision procedure and discuss how conflicting predictions about the distribution of power come about. We present the N&W model of legislative politics in the EU in more detail in Section 3.2. Section 3.3 then proposes three modifications to that model. Section 3.4 presents the results from the quantitative analysis of these modifications. Section 3.5 concludes.

3.1 EU Codecision: Rules and Models

The "ordinary legislative procedure" as laid down in Article 294 of the Treaty on the Functioning of the European Union (TFEU) requires consensus to be reached between

[3]See Crombez and Vangerven (2014) for an extensive survey.

the EP and the Council through alternating amendments, based on a Commission proposal.[4] It consists of up to three readings with the possibility to conclude at any reading if the EP and the Council reach an overall agreement in the form of a joint text. If they cannot agree during the first two readings, a compromise is sought by means of a Conciliation Committee – the third and final phase of codecision. The Committee is made up of 28 delegates representing the members of the Council and an equal number of EP delegates.[5] The Commission has no formal say in the negotiations, but fulfills a mediating and facilitating role. In case of successful conciliation, the Committee's final joint text is voted upon under closed rule, i.e., neither institution can amend the proposal. A simple majority of the votes cast in the EP and a qualified majority in the Council are required for approval; otherwise (or if no joint text has been produced) the proposal fails and the legal status quo prevails.

The standard approach to modeling EU decision-making under codecision is to represent alternative policies as points in a policy space and to assume that political actors have Euclidean preferences over these points. The procedure is most naturally formalized by a finite extensive form game (see Figure 3.1). It follows from backward induction logic that codecision outcomes are determined by the anticipated outcomes of the last stage, i.e., the Conciliation Committee. The Commission is – at least formally – no substantial player because in the Conciliation Committee, the EP and the Council can jointly enact any policy on which they agree, without scope for a Commission veto. Which equilibrium policies are predicted then depends on assumptions about the location of the status quo, players' preferences and (im-)patience, and the theorist's conception of the bargaining process between Council and Parliament.

There already exist various theoretical, mostly qualitative models of the codecision procedure – amongst others by Crombez (1997, 2000), Steunenberg and Dimitrova (2003) and Tsebelis and Garrett (2000).[6] Crombez (1997, 2000) uses a spatial

[4]The Commission has no formal gate-keeping power since the Parliament and the Council may – under Art. 225 and Art. 241 TFEU, respectively – request the Commission to submit an appropriate proposal. Moreover, in specific cases proposals can also be submitted on the initiative of a group of member states, on a recommendation by the European Central Bank, or at the request of the Court of Justice (see Art. 294(15) TFEU).

[5]Despite being of equal size, delegations are potentially not symmetric because the Council is fully represented in the sense that each of its members is involved in the negotiation, whereas the Parliament's delegates are agents whose interests may or may not be completely aligned to those of their principal (see Franchino and Mariotto 2013). Empirically, Rasmussen (2008) finds that the Parliament's conciliation delegation is representative of the chamber as whole.

[6]Another strand of applied studies has focused on the intra-institutional distribution of power in the Council, using measures of voting power which originate in cooperative game theory. For

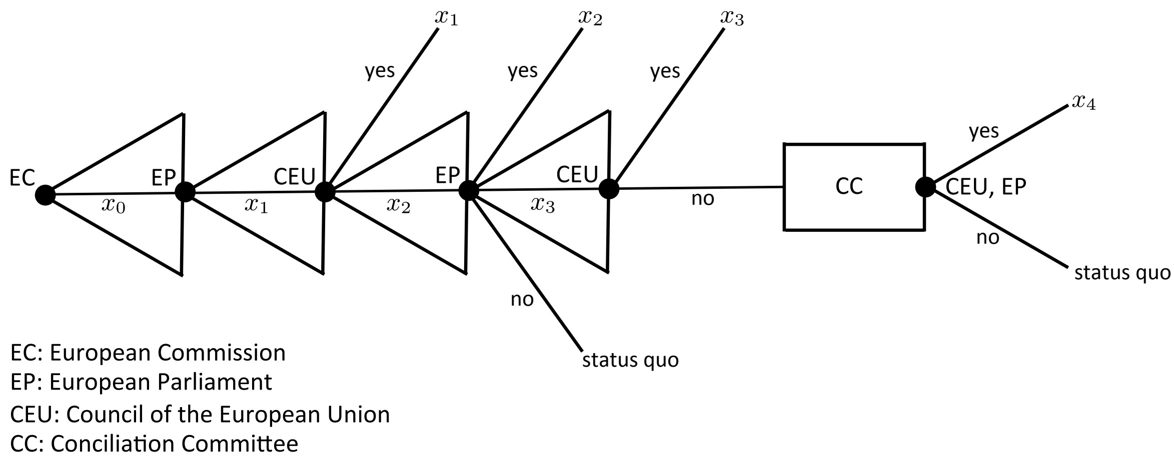

Figure 3.1 Stylized codecision game tree

model to analyze the effects of the Treaties of Maastricht and Amsterdam on the equilibrium policy and the corresponding powers of the EP, the Council and the Commission. He argues that under the Maastricht version of codecision, the EP and the Council are genuine co-legislators because both need to approve Commission proposals. The striking difference between the two versions of the procedure is that under the Treaty of Maastricht, the Council can revert to the original proposal of the Commission at the end of the procedure. This is no longer possible under the Treaty of Amsterdam, which has the Conciliation Committee as the final stage. The members of the Council thus compare the proposal of the EP to the status quo and no longer to the Commission's initial proposal. The author concludes, first, that agenda setting power now resides with both the EP and the Council, and second, that the Commission becomes powerless under the Amsterdam procedure "because its proposal no longer provides a reversion policy in case the Conciliation Committee fails to agree to a joint text" (Crombez 2000, p. 53). He suggests that the EP's preferences are similar to those of the Commission, which leads to the conclusion that the Amsterdam version may have decreased the EP's power relative to the Maastricht version.

Focusing on the Treaty of Amsterdam, Steunenberg and Dimitrova (2003) assume Euclidean preferences for all players and model the Conciliation Committee as an ultimatum bargaining game with the Council as the agenda setter. In their model, the

example, Le Breton et al. (2012) use the nucleolus to analyze past and current decision rules in the Council. Felsenthal and Machover (1998), Laruelle and Valenciano (2008), and Napel (2018) provide good overviews.

Council President drafts a bill in the first stage which is put to a vote in the Council in the second stage. Conditional on agreement, the common Council proposal is then submitted to the Parliament, which can only veto but not amend the proposal. Taking into account that a strategic Council President will only propose a bill that is preferred by a qualified majority of the Council members, the equilibrium policy derived via backward induction is the initial proposal of the Presidency. Subsequently, Steunenberg and Dimitrova (2003) estimate the power of the EP, the Council and the Commission by applying the concept of Steunenberg et al. (1999), which is based on expected distances. Their results suggest that both the Council members and the Parliament prefer the codecision procedure over the assent, consultation and cooperation procedures of the EU. Not surprisingly, they also ascribe greater power to the Council.

Tsebelis and Garrett (2000) use a one-dimensional spatial model of the legislative procedure to predict the codecision outcome for the Treaties of Maastricht and Amsterdam using a seven-member Council and treating the EP as a unitary actor. They highlight the dominance of agenda setting over veto power and argue that it is always advantageous to be the agenda setter if there are gains from trade. Similar to Crombez (2000) they point out that, under the Maastricht version and after the Conciliation Committee has broken down, the Council can make a "take-it-or-leave-it" proposal to the EP which can only be vetoed by an absolute majority of the EP. The Council is thus an unconstrained agenda setter "because it could essentially propose to the Parliament any variation of its common position that it wanted" (Tsebelis and Garrett 2000, p. 23). In contrast, under the Amsterdam version, the Conciliation Committee is the final stage and agenda setting now resides with both the EP and the Council. Tsebelis and Garrett (2000) see an institutional advantage for neither the EP nor the Council.

N&W assume spatial preferences for individual members of the EP, the Council and their respective representatives. Treating the codecision procedure as a non-cooperative game as in Figure 3.1 and assuming that all political actors reason strategically, the bargaining outcome of the codecision game can be determined by using backward induction: the outcome of the codecision game depends only on the outcome which the EP and the Council expect to result from engaging the Conciliation Committee. Assuming further that the time duration of the procedure does not significantly affect its outcomes, i.e., neither negotiator cares about reaching agreement a few weeks sooner or later, N&W use the Nash bargaining solution (Nash 1950) as a prediction for the codecision outcome. They find that the EP and the Council do not agree on some policy "in the middle", but that, with a unidimensional policy space and linear utility, the more conservative institution gets exactly its ideal point (see N&W, Prop. 1). Which institution is closer to the status quo and thus enjoys greater influence on codecision outcomes turns out to be determined by the respective intra-institutional decision quotas, i.e., qualified majority applied in the Council and simple majority in the EP. Applying the "power as outcome sensitivity" framework (see Napel and Widgrén 2004) to quantify the influence of the two institutions, the authors conclude that the Council is considerably more influential than the EP.

3.2 Basic Model

Following N&W and Napel et al. (2013), we present the basic theoretical model of negotiations in the Conciliation Committee as the last and strategically decisive stage of the codecision procedure.

We consider a convex *unidimensional* policy space $X \subseteq \mathbb{R}$, i.e., an interval of alternatives. Let $q \in X$ denote the *status quo* regarding the issue in question. All political actors, i.e., the currently 751 members of the EP and the 28 members of the Council, are assumed to have *single-peaked preferences*. For an individual i with ideal point $\lambda_i \in X$ preferences are represented by the utility function $u_i(x) = -|\lambda_i - x|$, i.e., utility falls linearly in distance between λ_i and policy $x \in X$. The ordered individual ideal points of the members of the EP (MEPs) will be denoted by $\pi_{(1)} \leq \cdots \leq \pi_{(751)}$; those of individual members of the Council by $\mu_{(1)} \leq \cdots \leq \mu_{(28)}$.

From the perspective of classical bargaining theory, finding a Conciliation compromise between the EP and the Council amounts to selecting a particular point (u^*_{EP}, u^*_{CEU}) in the utility possibility set $\mathcal{U}$. The latter is constructed by mapping each possible policy $x \in X$ to a utility pair $(u_{EP}(x), u_{CEU}(x))$. Suppose that the delegations of the EP and the Council enter negotiations with each other with respective bargaining positions π and μ. Whenever there are *gains from trade*, i.e., $\text{sign}(q - \pi) = \text{sign}(q - \mu)$, rational players can be expected to agree on a policy that is Pareto-efficient; the subset of such policies forms the *Pareto set* connecting π and μ. If the EP and the Council have opposite positions relative to the status quo, they fail to agree and the status quo will persist. In utility space, the status quo corresponds to the *disagreement point* $d = (u_{EP}(q), u_{CEU}(q))$.

In principle, various models from non-cooperative game theory and concepts from cooperative game theory could be applied to the bilateral bargaining situation

$(\mathcal{U}, d)$. Section 3.3.1 below will explore the implications of employing the Kalai-Smorodinsky solution.

Before conciliation begins, the respective bargaining positions π and μ have to be agreed on under the respective institution's internal decision-making rules. MEPs decide on any Conciliation compromise by simple majority rule, which – at least in theory – renders the median MEP pivotal. We will hence assume that the ideal point of the EP's representatives can be restricted to $\pi = \pi_{(376)}$.[7]

Agreement on the Council's ideal point μ is internally governed by the Lisbon voting rules today. These replaced the earlier Nice rules.[8] The latter were finally replaced in April 2017. For comparison reasons, we will consider both the Nice and Lisbon rules in the following. If the Council considers replacing the status quo q by a policy to its left, the countries which hold the left-most positions $\mu_{(1)}, \mu_{(2)}$, etc. will be the most enthusiastic about this. The critical Council member is then the country that first brings about the required qualified majority as less and less enthusiastic supporters of change are added to the coalition which endorses the new policy. For policies $x < q$, we refer to this critical member as the Council's *right pivot* R^{Nice} and R^{Lisbon}, respectively. Analogously, we identify the *left pivot* L^{Nice}, respectively L^{Lisbon}, for polices $x > q$.

In EU28, the Nice and Lisbon rules yield

$$R^{Nice} = \min\Big\{r \in \{15, \dots, 28\} \colon \sum_{i=1}^{r} w(\mu_{(i)}) \geq 260 \wedge \sum_{i=1}^{r} p(\mu_{(i)}) \geq 0.62 P^{EU28}\Big\}, \tag{3.1}$$

$$L^{Nice} = \max\Big\{l \in \{1, \dots, 14\} \colon \sum_{i=l}^{28} w(\mu_{(i)}) \geq 260 \wedge \sum_{i=l}^{28} p(\mu_{(i)}) \geq 0.62 P^{EU28}\Big\}, \tag{3.2}$$

$$R^{Lisbon} = \min\Big\{\min\Big\{r \in \{16, \dots, 28\} \colon \sum_{i=1}^{r} p(\mu_{(i)}) \geq 0.65 P^{EU28}\Big\}, 25\Big\}, \tag{3.3}$$

---

[7]This abstracts away from agency problems and other reasons for why the preferences of the EP delegation might not be congruent or at least sensitive to the EP's median voter.

[8]The Nice decision rule was a triple majority requirement. In addition to traditional weighted voting with a quota of roughly 73.9 % (i.e., 260 out of 352 votes), a qualified majority had to consist of at least a simple majority of member states (i.e., 15 out of 28) and had to represent at least 62 % of the total EU population. Under the Treaty of Lisbon, the old system of weighted voting has been replaced by a dual majority system. A qualified majority must now consist of at least 55 % of member states (i.e., 16 out of 28) and must represent at least 65 % of total EU population. Additionally, a blocking minority must include at least four Council members.

and

$$L^{Lisbon} = \max\Big\{\max\Big\{l \in \{1, ..., 13\}: \sum_{i=l}^{28} p(\mu_{(i)}) \geq 0.65P^{EU28}\Big\}, 4\Big\}, \tag{3.4}$$

where P^{EU28} refers to EU28's total population, $w(\mu_{(i)})$ denotes the voting weight of the Council member with ideal point $\mu_{(i)}$ and $p(\mu_{(i)})$ the population size he represents (see Table 3.1, columns (1) and (2)). We assume that the corresponding ideal points define the Council's aggregate position when contemplating a replacement of q by a policy to its left or right, respectively. They are denoted by μ_R^{Nice} respectively μ_R^{Lisbon} for the Council's *right pivot position*, and by μ_L^{Nice} respectively μ_L^{Lisbon} for the Council's *left pivot position*.

3.3 Adding Context: Three Modifications

3.3.1 Kalai-Smorodinsky Solution

Without any empirical or theoretical reasons to consider either the EP or the Council a more patient or skilled bargainer, it is natural to use a symmetric bargaining solution in order to model the outcome of negotiations in the Conciliation Committee. Economic as well as political applications of formal bargaining theory focus almost exclusively on the Nash bargaining solution. A frequently cited reason is that the Nash solution enjoys non-cooperative support via Rubinstein's (1982) alternating offers bargaining game (see Binmore 1987). But there are other negotiation procedures whose equilibrium outcomes correspond to different bargaining solutions. Moreover, these non-cooperative "foundations" are often only valid in the limit, where players' incentives to reach an agreement in finite time vanish. Thus, in the absence of detailed information about how the negotiations unfold, non-cooperative implementation does not provide a sound basis to discriminate between different bargaining solutions.

Especially with regard to free-form bargaining situations like the Conciliation Committee, a good reason to favor a particular bargaining solution is the appeal and the descriptive plausibility of its axiomatic characterization. The Nash solution is determined by the rather controversial property of independence of irrelevant alternatives (along with efficiency, symmetry and invariance to equivalent payoff representations). While this axiom may be plausible if bargaining is about rational

Member state	Population (1)	Nice weight (2)	EP seats (3)	SMP Nice ($\times 10^{-2}$) (4)	SMP Lisbon ($\times 10^{-2}$) (5)
Germany	80 780 000	29	96	4.42	8.16
France	65 856 609	29	74	4.41	6.34
United Kingdom	64 308 261	29	73	4.40	6.17
Italy	60 782 668	29	73	4.40	5.80
Spain	46 507 760	27	54	4.09	4.29
Poland	38 495 659	27	51	4.08	3.62
Romania	19 942 642	14	32	2.09	2.18
Netherlands	16 829 289	13	26	1.94	1.89
Belgium	11 203 992	12	21	1.79	1.40
Greece	10 992 589	12	21	1.79	1.38
Czech Republic	10 512 419	12	21	1.79	1.34
Portugal	10 427 301	12	21	1.79	1.33
Hungary	9 879 000	12	21	1.79	1.29
Sweden	9 644 864	10	20	1.48	1.27
Austria	8 507 786	10	18	1.48	1.17
Bulgaria	7 245 677	10	17	1.48	1.06
Denmark	5 627 235	7	13	1.04	0.93
Finland	5 451 270	7	13	1.04	0.91
Slovakia	5 415 949	7	13	1.04	0.91
Ireland	4 604 029	7	11	1.04	0.84
Croatia	4 246 700	7	11	1.04	0.81
Lithuania	2 943 472	7	11	1.04	0.70
Slovenia	2 061 085	4	8	0.60	0.63
Latvia	2 001 468	4	8	0.60	0.63
Estonia	1 315 819	4	6	0.60	0.57
Cyprus	858 000	4	6	0.59	0.53
Luxembourg	549 680	4	6	0.59	0.51
Malta	425 384	3	6	0.44	0.49
CEU aggregate				52.87	57.15
EP				1.97	11.45

Table 3.1 2014 population, Nice weights, EP seats, and power in basic scenario under the Nice and Lisbon Treaty rules for EU28 members

arbitration, it is less acceptable as a description of how agents actually bargain.[9] From that perspective, solution concepts which satisfy certain monotonicity properties appear to be more desirable. The predominant concept here is the Kalai-Smorodinsky (1975) solution. It is also deemed an attractive option in the specific context of decision-making in the EU (see, e.g., Achen 2006, p. 100; Schneider et al. 2010).

We call the maximum feasible utility that player i can achieve in the bargaining problem i's *aspiration level* $a_i(\cdot)$; it corresponds to an agreement where i extracts all the surplus, given that the other player receives at least his payoff from disagreement.[10] Typically, the so-called *utopian point* u^* whose coordinates correspond to the aspirations of both players will not be feasible. The Kalai-Smorodinsky solution suggests that both players cut back from u^* proportionally in a way that preserves the ratio of their aspirations. More precisely, the Kalai-Smorodinsky solution is defined by

$$\xi^{KS}(\mathcal{U}, d) = d + \bar{\lambda}(u^* - d),$$

where $\bar{\lambda} = \max\{\lambda \in \mathbb{R} : d + \lambda(u^* - d) \in \mathcal{U}\}$.

The feature of mutual concessions by both parties with respect to their utopian point seems to reflect actual codecision negotiations well. For example, Tsebelis et al. (2001) analyze the process of "give and take" between the EP and the Council under the Maastricht version of codecision (see Section 3.1), tracking legislative proposals through the different stages of the amendment process. Elgström and Smith (2000, p. 676) note that EU negotiations are "influenced by an informal principle of *juste retour*, i.e., that all members are supposed to gain something from an ongoing round of negotiation." Arguably, decision-makers have learnt to see codecision "as an interlinked, continuous procedure where it is essential and normal that there be intensive contacts throughout the procedure from before first reading onwards" (Shackleton and Raunio 2003, p. 173), resulting in a more cooperative mode of negotiation. Given these observations, our first modification to the baseline model suggests to consider

[9] For illustration, consider the bargaining problem defined by $\mathcal{U} = \{u_{\text{EP}}(x), u_{\text{CEU}}(x) \colon 0 \leq x \leq 1\}$, $\pi = 0.4$, $\mu = 0.6$ and $q = 0$. Now suppose that, e.g., due to a judicial decision, the bargaining set is restricted to $\mathcal{U}' = \{u_{\text{EP}}(x), u_{\text{CEU}}(x) \colon 0 \leq x \leq 0.5\}$. Independence of irrelevant alternatives implies that the Nash solution is $u^N = (u^N_{\text{EP}}(x), u^N_{\text{CEU}}(x)) = (0, -0.2)$ in both problems since $u^N \in \mathcal{U}' \subset \mathcal{U}$. So despite the fact that the Council sees its most preferred alternative disappear, and the EP does not, the Nash solution is unchanged. Also see Dubra (2001).

[10] The Kalai-Smorodinsky solution is defined by the following individual monotonicity axiom in lieu of Nash's independence of irrelevant alternatives: if player j's aspiration levels $a_j(\mathcal{U})$ and $a_j(\mathcal{U}')$ coincide in two bargaining problems $(\mathcal{U}, d)$ and $(\mathcal{U}', d')$ where the set of feasible payoffs $\mathcal{U}'$ is a subset of $\mathcal{U}$, then player i will receive at least as much utility in $(\mathcal{U}, d)$ as in $(\mathcal{U}', d')$.

the Kalai-Smorodinsky solution instead of Nash's bargaining solution.

Without loss of generality and for illustrational purposes we assume in the following that $\text{sign}(q-\pi) = \text{sign}(q-\mu)$, i.e., gains from trade, and $|\pi - q| \leq |\mu - q|$, i.e., the EP's ideal point π is closer to q than the Council's ideal point μ. It immediately follows that $u^*_{\text{EP}} = 0$ and

$$u^*_{\text{CEU}} = \begin{cases} 0 & \text{if } |\pi - q| \geq |\pi - \mu| \\ -(|\pi - \mu| - |\pi - q|) & \text{otherwise.} \end{cases}$$

This is illustrated in Figure 3.2. As soon as π is located between q and μ, $u^*_{\text{EP}} = 0$ because in this case the Council *always* prefers an implemented policy that is equal to π to a policy that is equal to q. Regarding u^*_{CEU}, things are slightly more complicated. If, as in the upper panel of Figure 3.2, π is closer to μ than to q, $u^*_{\text{CEU}} = 0$ because the EP prefers an implemented policy that is equal to μ to a policy that is equal to q. If, however, π is closer to q than to μ, as in the lower panel of Figure 3.2, the best the Council can get *given* that the EP receives at least its utility from disagreement is $u^*_{\text{CEU}} = -(|\pi - \mu| - |\pi - q|)$. Moving from this point, which is equal to $\pi + (\pi - q)$, even closer to μ would give the EP *less* utility than in case of disagreement.

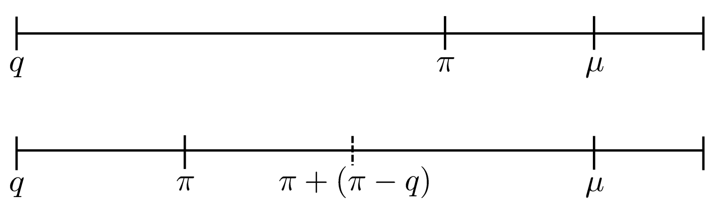

Figure 3.2 Ideal point configurations with $u^* = (0, 0)$ in the upper panel and $u^* = (0, -|\pi - \mu| - |\pi - q|)$ in the lower panel

Geometrically, the Kalai-Smorodinsky bargaining outcome $\xi^{KS}(\mathcal{U}, d)$ is just the intersection of $\mathcal{U}$'s Pareto frontier and the straight line connecting the disagreement point d, say $(-\pi, -\mu)$ for $q = 0$, and the utopian point u^* (see Figure 3.3). We obtain the following prediction for the implemented policy $x^{\text{KS}}(\pi, \mu, q)$:

Proposition 3.1. *Assume that preferences of the EP and the Council are represented by utility functions $u_i(x) = -|\lambda_i - x|$ for $\lambda_i, x \in X \subseteq \mathbb{R}$ where X is a non-empty interval. Whenever there are gains from trade, the Kalai-Smorodinsky solution to the bargaining problem $(\mathcal{U}, d)$ corresponds to agreement on a policy x^{KS} which is located on the Pareto frontier but nearer to the ideal point which is closer to the status quo. More specifically,*

$$\operatorname{sign}(q-\pi) = \operatorname{sign}(q-\mu) \Rightarrow x^{\mathrm{KS}}(\pi,\mu,q) = \begin{cases} \pi + \frac{\mu-\pi}{1+(\mu-q)/(\pi-q)} & \textit{if } |\pi-q| \leq |\mu-q| \textit{ and } |\pi-q| \geq |\pi-\mu|, \\ \mu + \frac{\pi-\mu}{1+(\pi-q)/(\mu-q)} & \textit{if } |\pi-q| > |\mu-q| \textit{ and } |\mu-q| > |\pi-\mu|, \\ \pi + \frac{\pi-q}{3} & \textit{if } |\pi-q| \leq |\mu-q| \textit{ and } |\pi-q| < |\pi-\mu|, \\ \mu + \frac{\mu-q}{3} & \textit{if } |\pi-q| > |\mu-q| \textit{ and } |\mu-q| \leq |\pi-\mu|. \end{cases}$$

The proof is presented in the appendix to this chapter. The left and right panels of Figure 3.3 illustrate the result for utopian points $u^* = (0,0)$ and $u^* = (0, -|\pi-\mu|-|\pi-q|)$, respectively. In contrast to the Nash prediction of N&W, the Kalai-Smorodinsky solution gives an interior solution. Nevertheless, the agreed policy is still nearer to the ideal point of the more conservative institution.[11] As can be seen in the right panel of Figure 3.3, the status quo bias is more extreme for $u^* = (0, -|\pi-\mu| - |\pi-q|)$. The bias is also more pronounced the closer d is located to the utopian point.

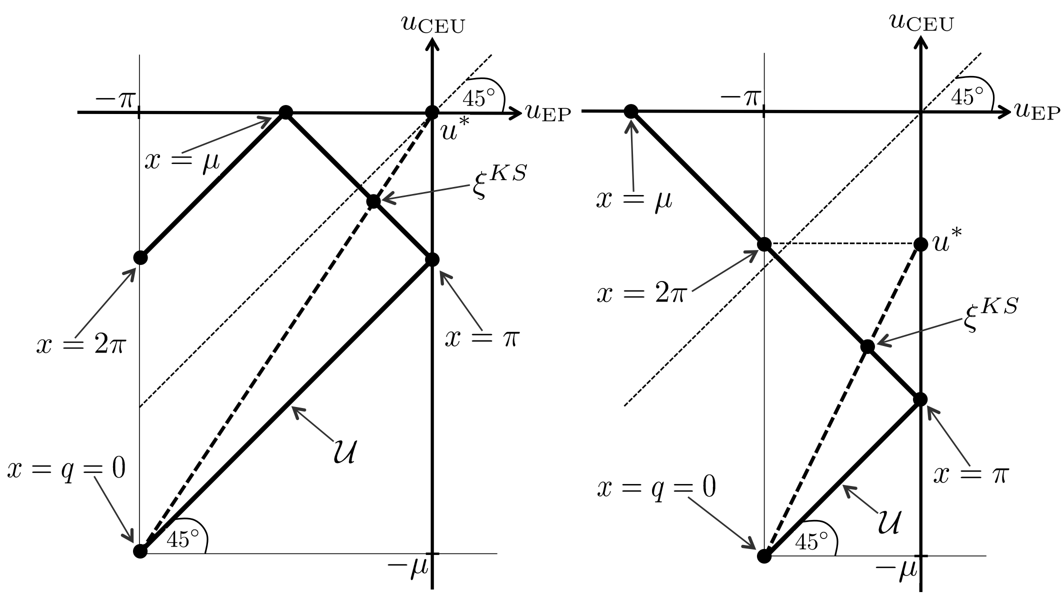

Figure 3.3 Kalai-Smorodinsky bargaining solution with $u^* = (0,0)$ in the left panel and $u^* = (0, -|\pi-\mu| - |\pi-q|)$ in the right panel

[11]The result that the Kalai-Smorodinsky agreement is closer to the institution with smaller status quo distance remains valid for multidimensional policy spaces. A proof is available from the authors upon request. While the bilateral bargaining situation between the EP and the Council can still be readily analyzed, multidimensional spaces make it much harder to predict which collective positions MEPs and members of the Council will adopt in the first place. A possible approach could be to use a point solution like the Copeland winner, or to assume an exogenous ordering of dimensions on which individuals vote sequentially.

We substantiate the suggestion to use the Kalai-Smorodinsky solution by a tentative empirical evaluation of how well that model predicts decision outcomes compared to the basic setting. The analysis relies on the DEUII dataset (Thomson et al. 2006, 2012) which is based on expert judgements of member states' positions in a one dimensional policy space. The dataset reports countries' preferences for 158 policy issues in EU27 as well the EP's preferences, the status quo and the policy outcome. Unfortunately, a number of issues had to be excluded when the data contained no information on either the EP's preferences, the status quo or the outcome. We calculated the Council's common ideal point and then excluded issues for which the EP and the Council had diverging interests of whether to move to the left or to the right of the status quo. For the remaining 33 issues, we identified the Nash and the Kalai-Smorodinsky predictions and computed their respective distance to the actual outcome. The Kalai-Smorodinsky model performed better for 19 issues, whereas the Nash solution had the edge in seven issues, and seven other issues were ties. The Wilcoxon signed-rank test on the equality of distances indicates that the Kalai-Smorodinsky model is indeed more accurate than the Nash solution (Z-statistic 1.70, p-value 0.0895).

In order to obtain quantitative statements regarding the expected influence of the EP or individual Council members on EU decisions, we apply the "power as outcome sensitivity" approach (Napel and Widgrén 2004) to the analysis of power in collective decision-making. This framework merges traditional power index analysis with a non-cooperative game-theoretic approach. It conceives of *a posteriori* power as the sensitivity of the equilibrium outcome with respect to marginal changes in a player's behavior or preferences. The *strategic measure of power (SMP)* then evaluates *a priori* power as *expected* a posteriori power, using a probability measure with a priori credentials.

Rewriting Prop. 3.1, the Kalai-Smorodinsky solution predicts

$$x^{\mathrm{KS}}(\pi,\mu,q)=\begin{cases}\frac{(\pi+\mu)q-2\pi\mu}{2q-\pi-\mu} & \text{if } (q<\pi\le\mu \text{ or } \mu<\pi<q) \text{ and } |\pi-q|\ge|\pi-\mu|,\\ \frac{(\pi+\mu)q-2\pi\mu}{2q-\pi-\mu} & \text{if } (q<\mu<\pi \text{ or } \pi\le\mu<q) \text{ and } |\mu-q|>|\pi-\mu|,\\ \frac{4\pi-q}{3} & \text{if } (q<\pi\le\mu \text{ or } \mu<\pi<q) \text{ and } |\pi-q|<|\pi-\mu|,\\ \frac{4\mu-q}{3} & \text{if } (q<\mu<\pi \text{ or } \pi\le\mu<q) \text{ and } |\mu-q|\le|\pi-\mu|,\\ q & \text{otherwise}\end{cases}$$

as the equilibrium codecision outcome.

Taking the partial derivatives of the predicted outcome, the a posteriori power of EP for a *given* realization of status quo q and ideal points $\pi_1, ..., \pi_{751}$ and $\mu_1, ..., \mu_{28}$ then is

$$\frac{\partial x^{\mathrm{KS}}(\pi, \mu, q)}{\partial \pi} = \begin{cases} \frac{(q-2\mu)(2q-\pi-\mu)+(\pi+\mu)q-2\pi\mu}{(2q-\pi-\mu)^2} & \text{if } (q < \pi < \mu \text{ or } \mu < \pi < q) \text{ and } |\pi - q| > |\pi - \mu|, \\ \frac{(q-2\mu)(2q-\pi-\mu)+(\pi+\mu)q-2\pi\mu}{(2q-\pi-\mu)^2} & \text{if } (q < \mu < \pi \text{ or } \pi < \mu < q) \text{ and } |\mu - q| > |\pi - \mu|, \\ \frac{4}{3} & \text{if } (q < \pi < \mu \text{ or } \mu < \pi < q) \text{ and } |\pi - q| < |\pi - \mu|, \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, for an individual member k of the Council, we obtain

$$\frac{\partial x^{\mathrm{KS}}\big(\pi, \mu(\mu_1, ..\mu_{28}), q\big)}{\partial \mu_k} = \begin{cases} \frac{(q-2\pi)(2q-\pi-\mu)+(\pi+\mu)q-2\pi\mu}{(2q-\pi-\mu)^2} & \text{if } (q < \pi < \mu \text{ or } \mu < \pi < q) \text{ and } |\pi - q| > |\pi - \mu|, \\ \frac{(q-2\pi)(2q-\pi-\mu)+(\pi+\mu)q-2\pi\mu}{(2q-\pi-\mu)^2} & \text{if } (q < \mu < \pi \text{ or } \pi < \mu < q) \text{ and } |\mu - q| > |\pi - \mu|, \\ \frac{4}{3} & \text{if } (q < \mu < \pi \text{ or } \pi < \mu < q) \text{ and } |\mu - q| < |\pi - \mu|, \\ 0 & \text{otherwise,} \end{cases}$$

where $\mu = \mu_k$, i.e., member k is the Council's pivotal member.

Note, however, that we are *not* interested in a player's influence on a single issue but rather in *expected* influence. We measure this *a priori power* by computing the average of a posteriori power over a large number of uniformly distributed issues (cf. Section 3.4).

3.3.2 Representatives in the EP and in the Council

Pointing to the normative character of their analysis, N&W assume that individual decision-makers' ideal points in both the Council and the EP come from an a priori identical, uniform distribution. The Council's internal qualified majority voting with its demanding supermajority requirements then implies that the distribution of the ideal point of the Council's pivotal member (see formulas (3.1) – (3.4)) is skewed with a peak rather close to the status quo.

This neglects, however, a basic design feature of the EU: the Council represents the states and the Parliament represents the citizens. Decision-makers in the Council are representatives of national governments, who usually have the support of a majority of voters in their member state. By contrast, the EP is composed of MEPs who are organized into various transnational party groups, each consisting of multiple (and sometimes rather distantly related) national member parties from the 28 EU countries. Political competition is governed by some variant of proportional rule of which the precise forms are determined by the member states.[12] Election thresholds also vary from country to country; 14 member states now require no minimum percentage of votes for a party to obtain seats in the EP (see European Parliament 2014, p. 16 and p. 92f). Efforts to design a uniform electoral procedure, while mandated by Art. 223(1) TFEU, have as yet failed to reach consensus. National (or regional) parties control nominations to European elections and run election campaigns. As a consequence, MEPs answer to both national and EP-level principals, giving rise to a dual agency problem that is probably one reason why policy cohesion of the EP's political groups is relatively low. Difficulty of maintaining discipline and cohesion also stems from the fact that, unlike parliamentary democracies at the national level, the EP does not need to form and sustain a EU government by means of stable majorities.[13]

The fundamentally different modes of election and mandates of MEPs and members of the Council are institutional rules, too, which should be taken into account in theoretical analysis.[14] Thus, our second modification of the N&W model is a very stylized representation of these differences but yet richer than the model where all decision-makers' preferences are identically distributed.

As a first step, we include individual citizens' preferences into the model from Section 3.2: consider the partition $\mathfrak{C} = \{C_1, \ldots, C_{28}\}$ of the EU voter population into 28 *constituencies* with $n_j = |C_j| > 0$ members each. We assume:

(SPA) All individual voters have spatial preferences, characterized by ideal point ν^i in policy space X.

From a normative constitutional-design point of view it is appealing to presume:

(IID) All individual ideal points are independent and identically distributed (i. i. d.).

If electoral arrangements have any role to play for how citizens' preferences are represented in the Council and the EP, then it is clear that the ideal points of Council members and of MEPs cannot be identically distributed. It is less clear

[12] The most common system is list proportionality with the member state as a single constituency and d'Hondt's rule for seat allocation. But manifold deviations exist, see European Parliament (2014).

[13] Yet, voting cohesion has increased across parliamentary sessions, especially for the three largest political groups (see Hix et al. 2007); there also exists an agreement between the latter to support the current European Commission.

[14] Other facts such as, for example, the number of parliamentary groups into which MEPs are organized, or the degree of cohesion within these groups, are of a more transitory nature and should in our view not be held fixed behind the "veil of ignorance".

though *how* the different genesis of the two legislators' preferences should be formally modeled. While certainly not ideal, our two assumptions below provide, in our view, a reasonable first approximation and thus help to assess the potential biases induced by institutional rules more properly:

(MED) The preferences of country j's representative in the Council are congruent with the country's median voter. More formally, representative j has ideal point

$$\mu_j = \text{median}\{\nu^i : i \in C_j\}.$$

This assumption reflects that the median voter's position is crucial to the formation of a popular majority so that, in a competitive democracy, this voter's preferences can be expected to shape electoral campaigns, legislative decision-making and, eventually, the government's policy. This policy position corresponds to the electorate's *core* which can be understood as the result of sophisticated strategic interaction in the electorate and in the national legislature (see Banks and Duggan 2000). We consider (MED) to be a fair approximation, even in the light of the big variation among the actual national electoral systems in the EU member states (see McDonald et al. 2004). Note that a country's population size generally affects the distribution of its median. Specifically, if the ideal points ν^i of voters $i \in C_j$ are pairwise independent and come from an arbitrary identical distribution F with positive density f on X, then its median position μ_j is asymptotically normally distributed with mean $\bar{m} = F^{-1}(0.5)$ and standard deviation

$$\sigma_j = \frac{1}{2\, f(\bar{m})\, \sqrt{n_j}}$$

(see, e.g., Arnold et al. 1992, p. 223). The variance of the position of C_j's representative is smaller, the greater the population size n_j.

It is even more challenging to formulate an appropriate assumption about how MEPs' positions are connected to citizens' preferences. It is beyond the scope of this contribution to develop a model of endogenous entry and platform formation in the EP and to our best knowledge, no model exists so far that encompasses the observations made above.[15] In the absence of such a model, we propose to represent the institutional realities described above by:

[15] A natural starting point seems to be some variant of a citizen-candidate model in which citizens can freely form parties and seats are distributed proportionally (see, e.g., Hamlin and Hjortlund 2000). An important difficulty in modeling proportional systems in general is that elections may fail to produce a clear winner, so that the policy output depends on the legislative bargaining game occurring after the election. The link between composition of the EP and policy formation is even less

(CRD) MEPs who are elected in country j are a clustered random draw from that country's electorate. More formally, let s_j denote the number of seats allocated to country j. If μ_j is the median voter position in C_j, then the ideal points $\pi_1^j, \ldots, \pi_{s_j}^j$ of j's MEPs are distributed according to the symmetric triangular distribution $F(a_j, \mu_j, b_j)$ on the interval $[a_j, b_j]$ with peak location μ_j, where a_j and b_j are the lower and upper bound, respectively, of country j's policy space.

This assumption reflects, first, that as a result of proportional representation and the fragmentation of European elections into separate national contests, MEPs are ideologically very diverse and, in fact, occupy the entire range of the political spectrum (see, e.g., McElroy and Benoit 2012). Second, generally low election thresholds make it easier for new parties or even individual candidates to enter successfully, and raise the odds that radical positions get represented in the EP. Finally, the positions of MEPs from a certain country and the position of the national government as reflected by μ_j both derive from the same voters' preferences; this interdependency should not be ignored. The triangular distribution achieves this by clustering ideal points $\pi_1^j, \ldots, \pi_{s_j}^j$ around the country median. Moreover, it is widely used in "limited knowledge" applications (see, e.g., Law and Kelton 2000, Sect. 6.11).

3.3.3 Heterogeneity among Member States

Even for the kind of constitutional-design exercise that we are carrying out, assumption (IID) in the previous section may be unduly restrictive on the joint distribution of voter ideal points in $C_1, \ldots, C_{28}$. Namely, treating all individuals as homogeneous ignores that the partition $\mathfrak{C}$ may have reasons. These reasons (e.g., geographic barriers, ethnics, language, religion) are likely to involve or give rise to closer political connections between voters within constituencies than across them. With a view to the EU, this is rather obvious. We consider, for example, the fact that elections to the EP are conducted organizationally independently in each country, with different rules and different parties to choose from, as revealing of the preference heterogeneity across member states.

On the other hand, it is true that a constitutional analysis should ignore knowledge about specific preferences for normative reasons. This implies that all citizens should

clear than for national parliaments. We conjecture that, in such a model, the absence of rents from government participation – as in the EP – will give rise to a very large number of dispersed parties in equilibrium.

be considered *identical* a priori, i.e., every ideal point ν^i should be drawn from the same marginal probability distribution F. However, the "veil of ignorance" perspective does not necessarily entail that citizens' preferences must also be considered as *independent* of each other.

As an alternative to the benchmark assumption (IID) that all ideal points ν^i with $i \in \bigcup_j C_j$ are drawn independently from the same marginal distribution F, we therefore also explore the idea that preferences within a country are positively correlated with each other. This gives rise to a special type of heterogeneity among countries. In particular, we determine individual ideal points ν^i by a two-step random experiment: first, we draw a constituency-specific shock θ_j independently for each $j = 1, \ldots, 28$ from a distribution G with standard deviation σ_{ext}. This parameter captures the degree of *external heterogeneity* between $C_1, \ldots, C_{28}$ for the policy issue at hand. Parameter θ_j is taken to reflect the expected ideal point of citizens from C_j. Each citizen $i \in C_j$ is then assigned an individual ideal point ν^i from a distribution F_j which has mean θ_j and is otherwise just a shifted version of the same distribution F for each constituency $j = 1, \ldots, 28$.[16] F's standard deviation σ_{int} is a measure of the *internal heterogeneity* in any constituency. It intuitively reflects the opinion differences within any given C_j. In summary, we account for heterogeneity among countries by assuming:

(HET) The ideal points of all citizens are identically distributed with convoluted a priori distribution $G * F$ but not independent: citizens in constituency C_j experience shock θ_j, which is independent of θ_k for any $k \neq j$.

The introduction of heterogeneity makes it worthwhile to include an additional institutional fact, namely the *degressive proportionality* in the EP's national composition. There is a fixed number of MEPs to be elected in each country and smaller states elect more MEPs than would be proportional to their populations. For example, Spain's population is more than one hundred times that of Malta, whereas its number of seats in the EP is only nine times that of Malta (see Table 3.1, column (3)). Under assumption (IID), the fact that MEPs come from different constituencies is obviously inessential; under (HET), by contrast, links are established between citizens' preferences in a given constituency, the position of the constituency's member in the Council (assumption (MED)) and the preferences of its EP delegation (assumption (CRD)).

[16] Specifically, we draw θ_j from a uniform distribution $\mathbf{U}(-a, a)$ with variance σ^2_{ext}, and then obtain $\nu^i = \theta_j + \varepsilon$ with $\varepsilon \sim \mathbf{U}(0, 1)$.

Basic scenario	Nash solution Citizens' preferences i. i. d.
Scenario I	Kalai-Smorodinsky solution Citizens' preferences i. i. d.
Scenario II	Kalai-Smorodinsky solution Citizens' preferences correlated within countries
Scenario III	Kalai-Smorodinsky solution Citizens' preferences correlated within countries EP-Council distinction

Table 3.2 Overview of different scenarios

3.4 Simulation Results

In this section, we quantify the effects of our modifications on both the inter- and intra-institutional distribution of power in EU codecision. Our results are based on Monte-Carlo simulations.[17] In a first step, we draw 751 random numbers as the ideal points of MEPs and 28 random numbers as the ideal points of the Council members from distributions $F_1, \ldots, F_{28}$. In a second step, we sort the realized ideal points and determine the EP's and the Council's pivot positions according to their respective internal decision rules presented in Section 3.2. We are thus able to identify the policy outcome and, by repeating above procedure up to 10^9 times, to obtain numerical estimates of the SMP values of the EP, the whole Council and its individual members.

In our basic setting, which has already been considered by N&W and which we use as our reference point, we assume the distributions F_j as well as the distribution of the status quo to be a $[0, 1]$-uniform distribution. As a predictor for the bargaining outcome, we use the Nash solution. Building on this benchmark, we distinguish three scenarios which combine the modifications described in Section 3.3. We provide an overview in Table 3.2; the corresponding results are reported in Table 3.3. The effects of moving from our benchmark model to the different scenarios under the Nice rules and the Lisbon rules are also illustrated in Figure 3.4.

We report simulation results of the basic scenario in Table 3.1, columns (4) and (5).

[17]We obtain an estimate of the intra-Council distribution of power in EU28 according to the *Shapley-Shubik-index* (see Shapley and Shubik 1954) as an intermediate result to the inter-institutional simulation and can also calculate it exactly by standard methods. This permits use of the former as a control variate for our SMP estimator. The variance reduction obtained in this way is up to 45 %. Remaining inaccuracies are due to the simulative nature of our results.

The SMP values confirm the finding of N&W that the Council is much more influential than the EP. For example, under the Nice rules, a shift in the Council's ideal point by a small amount $\triangle\mu$ in expectation shifts the outcome by $0.529 \cdot \triangle\mu$, whereas a shift in the EP's position is only passed through at a rate of 0.020. As N&W (p. 143) have pointed out, this remarkable asymmetry is due to the extreme status quo bias of bargaining which "translates the event '[the Council] is more conservative' into '[the Council] defines x^*' and '[the Council] has power'."

3.4.1 Scenario I

Scenario I continues to take the distributions F_j as well as that of the status quo to be $[0,1]$-uniform, but applies the Kalai-Smorodinsky rather than the Nash solution to describe bargaining in the Conciliation Committee (see Section 3.3.1). Under the Nice decision rules and compared to our basic scenario, the Council's ex ante power increases from 0.529 to 0.597 and the EP's ex ante power from 0.020 to 0.075. Considering the Lisbon rules, the Council's ex ante power increases from 0.572 to 0.598 and the EP's ex ante power from 0.115 to 0.190.

The most important observation is that the EP now is ascribed considerable influence already under the Nice Treaty. However, the Council is still the more powerful institution. Although the ideal points of the MEPs and the Council members come from the same distribution, the Council's internal qualified majority requirement results in a more conservative distribution of its collective ideal point compared to the EP whose ideal point is determined by simple majority. The reason for why both the Council's and the EP's ex ante power increase is due to two effects. To illustrate these effects, assume gains of trade and $|\mu - q| < |\pi - q|$. First note that, in contrast to the basic scenario, even the institution with greater distance to the status quo may exert influence, namely if $|\mu - q| > |\pi - \mu|$. Regarding the second effect, we have to distinguish between (a) $|\mu - q| < |\pi - \mu|$ and (b) $|\mu - q| > |\pi - \mu|$. In case (a), only a (marginal) shift in the common position of the more conservative institution can affect the location of the agreed policy x^{KS}. However, $\partial x^{KS}/\partial\mu = 4/3$ (see Section 3.3.1) implies that the effect on x^{KS} is always *larger* than the initial shift of the more conservative institution. Similarly, we find that in case (b), we always have $\partial x^{KS}/\partial\pi + \partial x^{KS}/\partial\mu > 1$. Since the only difference between Scenarios I and II is the choice of the bargaining model, the probabilities of gains of trade and of one player being more conservative than the other remain unchanged. Thus, whenever we have gains of trade, the Nash solution implies either $\partial x^{NB}/\partial\pi = 1$ and $\partial x^{NB}/\partial\mu = 0$

Member state \ Scenario	SMP Nice ($\times 10^{-2}$) I	II	III	SMP Lisbon ($\times 10^{-2}$) I	II	III
Germany	5.03	5.77	5.72	8.68	9.27	9.19
France	5.02	5.78	5.78	6.73	7.21	7.20
United Kingdom	5.01	5.77	5.78	6.54	7.01	7.00
Italy	5.01	5.77	5.77	6.14	6.57	6.57
Spain	4.65	5.37	5.40	4.54	4.88	4.92
Poland	4.63	5.34	5.38	3.84	4.14	4.17
Romania	2.35	2.68	2.71	2.27	2.42	2.48
Netherlands	2.17	2.47	2.51	1.97	2.10	2.16
Belgium	2.00	2.28	2.32	1.45	1.55	1.60
Greece	2.00	2.28	2.32	1.43	1.53	1.58
Czech Republic	2.00	2.28	2.32	1.38	1.48	1.54
Portugal	2.00	2.28	2.32	1.38	1.47	1.53
Hungary	2.00	2.28	2.32	1.33	1.42	1.48
Sweden	1.66	1.88	1.91	1.31	1.40	1.46
Austria	1.66	1.88	1.92	1.20	1.29	1.34
Bulgaria	1.66	1.88	1.92	1.09	1.17	1.23
Denmark	1.16	1.31	1.34	0.95	1.02	1.08
Finland	1.16	1.31	1.33	0.93	1.01	1.06
Slovakia	1.16	1.31	1.33	0.93	1.00	1.05
Ireland	1.16	1.31	1.33	0.86	0.93	0.98
Croatia	1.16	1.31	1.33	0.83	0.89	0.95
Lithuania	1.16	1.31	1.33	0.71	0.77	0.82
Slovenia	0.67	0.75	0.77	0.64	0.69	0.74
Latvia	0.67	0.75	0.77	0.63	0.69	0.74
Estonia	0.66	0.75	0.77	0.57	0.62	0.67
Cyprus	0.66	0.75	0.77	0.53	0.58	0.63
Luxembourg	0.66	0.75	0.76	0.50	0.55	0.60
Malta	0.49	0.55	0.56	0.49	0.54	0.59
CEU aggregate	59.65	68.14	68.79	59.84	64.21	65.36
EP	7.45	15.55	18.37	19.00	25.39	26.31

Table 3.3 Strategic power in EU28 under Scenarios I, II and III, Nice and Lisbon Treaty rules

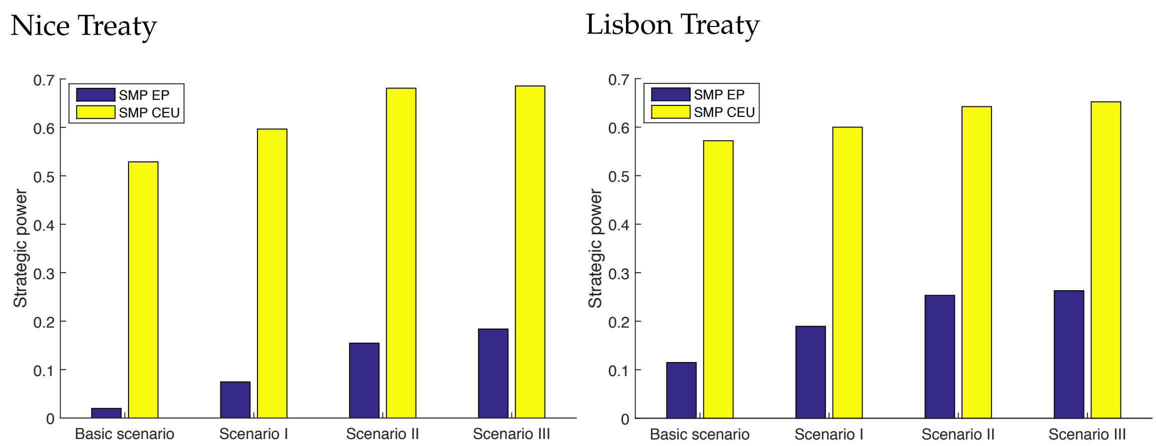

Figure 3.4 Inter-institutional distribution of power in EU28 under different scenarios

or $\partial x^{\mathrm{NB}}/\partial \pi = 0$ and $\partial x^{\mathrm{NB}}/\partial \mu = 1$, while the Kalai-Smorodinsky solution always implies $\partial x^{\mathrm{KS}}/\partial \pi + \partial x^{\mathrm{KS}}/\partial \mu > 1$. Intuitively, the compromise culture reflected by the Kalai-Smorodinsky solution makes the outcome more sensitive to slight preference changes of either institution than the all-or-nothing Nash bargaining outcomes.

Looking at the intra-institutional distribution of power in the Council, we find that all 28 countries gain absolute power under the Nice rules compared to the basic scenario (see Table 3.1). The same holds under the Lisbon rules, except for the smaller countries beginning with Lithuania, whose SMP numbers remain essentially unchanged. Moreover, it is worth mentioning that for every country the absolute gains under the Lisbon rules are smaller than those under the Nice rules.

3.4.2 Scenario II

Our Scenario II replaces the assumption that all citizens' preferences are i. i. d. with heterogeneity among member states (cf. Section 3.3.3). To incorporate assumption (HET) in our analysis, we draw a country specific shock θ_j which reflects the expected ideal point of citizens from constituency C_j. So voters' ideal points from different countries come from different distributions F_j with mean θ_j that are shifted versions of some distribution F. We draw MEPs' ideal points from shifted uniform distributions $F_1, \ldots, F_{28}$ and also take degressive proportionality in the EP's national composition into account, i.e., we draw 96 ideal points from Germany's uniform distribution, 74 from France's uniform distribution, etc. (cf. Table 3.1, column (3)). The ideal point of each Council member similarly comes from the respective shifted uniform distributions $F_1, \ldots, F_{28}$. The status quo in Scenarios II and III is still drawn from a uniform distribution, but now over a larger interval that captures all possible preferences in a heterogenous EU.

Taking heterogeneity among member states along with degressive proportionality into account has a further positive effect on the SMP values of the Council and the EP under both the Nice and the Lisbon rules. Under the Nice Treaty the EP's power increases from 0.075 to 0.156 and that of the Council from 0.597 to 0.681. The effects under the Lisbon Treaty are slightly smaller. The reason for why both institutions gain by a similar magnitude lies in the probability of gains of trade which changed as a result of our modeling choices. We draw the status quo from a uniform distribution on the interval of *all* possible policy preferences but the ideal points of the Council members and of MEPs come from a uniform distribution on the respective country's shifted unit interval. Thus, both the ideal points of the Council members and the MEPs display a lower variance than the status quo, which leads to an *increased* incidence of gains of trade. In particular, the probability of gains of trade under the Nice Treaty increases from 55 % under Scenario I to 73 % under Scenario II. Similarly, we identify an increase from 69 % to 82 % under the Lisbon Treaty.

3.4.3 Scenario III

We now turn to Scenario III. We would claim it to be the most realistic. It incorporates all our modifications; first, the Kalai-Smorodinsky solution, second, heterogeneity among member states (along with degressive proportionality in the EP's national composition) and third, the fact that Council members represent countries' governments (MED) while MEPs represent citizens and are clustered around the respective country's median (CRD). Specifically, we draw MEPs' ideal points from (shifted) triangular distributions with the respective country's median as the peak. The ideal points of the 28 Council members are drawn from (shifted) beta distributions $F_1, \dots, F_{28}$ with parameters $((n_j + 1)/2, (n_j + 1)/2)$. This follows from our assumption (IID) for the case of uniformly distributed voter ideal points (see Arnold et al. 1992, p. 13f).

Compared to Scenario II, both the EP and the Council gain absolute power under both Treaties but to a much smaller degree than before. While under the Nice Treaty there is basically no effect on the influence of the Council, the EP's influence increases from 0.156 to 0.184. The effects under the Lisbon Treaty are much more balanced and increase the influence of the EP from 0.254 to 0.263 and the influence of the Council from 0.642 to 0.654.

To explain these effects consider first the Nice Treaty. If we use the Nash solution instead of the Kalai-Smorodinsky solution here, an SMP of 0.775 for the Council and of 7.62×10^{-6} for the EP results. Intuitively, this indicates that the Council is now the more conservative institution in almost all cases where we have gains of trade. In fact, the number of cases with gains of trade in which the Council is more conservative increased by 0.054 compared to Scenario II, while the number of such cases in which the EP is more conservative dropped to nearly zero. In order to see why the EP benefits more than the Council in Scenario III, next recall that with the Kalai-Smorodinsky solution, it plays a crucial role how far the institutions' positions are apart from each other. Our assumptions (CRD) and (MED) affect the distribution of the pivot positions π and μ, and in particular reduce their expected distance. This is most obvious when we assume that the pivotal member of the Council and the median MEP come from the same constituency; here the clustering of each country's MEP delegation around the country median clearly reduces the expected distance compared to the "pure" heterogeneity model in Scenario II.[18]

Assuming gains of trade and taking into account that under Scenario III the Council is in almost all cases the more conservative institution, we either have (a) $|\mu - q| < |\pi - \mu|$ or (b) $|\mu - q| > |\pi - \mu|$. In case (a), *only* a marginal shift in the Council's common position has an effect on the policy outcome x^{KS}. In case (b), the CM *and* the EP can shift x^{KS}, but a marginal change in the common position of the Council has a larger effect on x^{KS} than a marginal change in the EP's common position. However, the smaller the distance between π and μ, the smaller the Council's advantage, whereas the influence of the EP on x^{KS} increases. In summary, our assumptions (CRD) and (MED) have two effects. They render the Council the more conservative institution in almost all cases and they reduce the expected distance between μ and π. This leads to a higher number of (b) cases, which over-compensate the EP's loss of influence in (a) cases.

Similar reasoning applies to the Lisbon Treaty. Again, the number of cases in which the Council (the EP) is more conservative increases (decreases) when moving from Scenario II to Scenario III. This has a positive effect on both institutions, but again a larger effect on the Council. Note that the Council now benefits more strongly from the greater incidence of gains of trade because the number of cases in which it is more conservative is now nearly twice as large as under the Nice Treaty. Regarding

[18]Specifically, in our simulations, the average distance between π and μ in case of gains of trade and the Nice Treaty (Lisbon Treaty) decreases from 0.254 (0.166) under Scenario II to 0.225 (0.144) under Scenario III.

the average distance between π and μ in case of gains of trade we find a reduction by nearly the same magnitude as before (see fn. 18). Together, this explains why the effects of Scenario III on the EP's and the Council's SMP values are much more balanced under the Lisbon Treaty than under the Nice Treaty.

Comparing Scenario III to the benchmark under the Nice rules, the EP is now attributed more than nine times as much a priori power. By contrast, the Council's power is larger by only about 30 %. Regarding the Lisbon rules, the relative effects are smaller but still favor the EP: its influence more than doubles; the Council's power increases only by about 14 %. Of course, the Council still remains the more powerful institution in Scenario III, but the asymmetry between the EU's main decision-makers is much smaller than in our basic setting (see Figure 3.4).

Turning to the Council's intra-institutional distribution of power, we find that the absolute power of all 28 countries has increased by roughly the same percentage, both under the Nice and Lisbon rules. In other words, values of a normalized SMP, which indicate relative influence would remain essentially unchanged for all countries.

3.5 Concluding Remarks

Existing models of the codecision procedure which take a constitutional perspective, i.e., which base claims on institutional rules rather than current preferences, yield competing conjectures about the inter-institutional balance of power in the EU. We followed this literature and introduced several new aspects into the bargaining game between the EP and the Council. As a first modification, we suggested that there is reason to consider the Kalai-Smorodinsky solution rather than the Nash solution as a predictor for the bargaining outcome since actual negotiations in the Conciliation Committee seem marked by mutual concessions. We then incorporated heterogeneity between different constituencies. Finally, we added the observation that MEPs represent citizens while Council members are representatives of national governments.

Our first main result is that the quantitative assessment of the players' power relation strongly depends on how much context is taken into account. The stark power divide predicted by the basic setting of N&W seems somewhat exaggerated. It is greatly moderated when moving to more context-rich models. In Scenario III, which gives in our view the so far most realistic description of the EU's wider institutional framework, the inter-institutional gap (i.e., the EP's SMP relative to the

Council's SMP) has only about one seventh the original size from the basic scenario when considering the Nice rules; it is still reduced by half under the Lisbon rules. However, and this may be deemed more important, the qualitative assessment of the balance of power is remarkably robust: the EP and the Council do not co-legislate on a par. The latter remains more influential due to its more conservative internal voting rule.

Plenty of other modifications of the considered model are conceivable. For example, the motive to reach agreement is provided by the risk of breakdown of negotiations. While we treated players in the EP and the Council to be risk-neutral, one could argue that the Council is more risk-averse due to the higher visibility of national representatives compared to the EP. This would suggest applying a concave transformation to the Council's utility function, which would change bargaining outcomes in favor of the EP (see, e.g., Kihlstrom et al. 1981). Another potential source of built-in asymmetry between the Council and the EP in codecision could be the requirement that an absolute majority of MEPs is needed to amend a Council proposal in the second reading (see Hagemann and Høyland 2010). Other possible modifications for future research include the fact that – besides the EP and the Council – the Commission, national parliaments, lobbyists, rapporteurs and political parties could also be regarded as relevant players in the codecision game.

Our contribution offers a robustness check on a key theoretical result in the literature on the power distribution in codecision. But it should also be seen as a cautionary note on a more general level. While we fully agree with N&W (p. 138) that "any systematic bias in influence must result from institutional rules rather than differences between the politicians involved", the practical problem of how to adequately reflect complex institutional realities in applied social choice analysis has no ready solution.

3.6 Appendix
Proof of Proposition 3.1

For $|\pi - q| = |\mu - q|$ the result is trivial. So consider gains from trade and $|\pi - q| < |\mu - q|$. This implies $u^*_{EP} = 0$. The proof is split in two parts. First consider $|\pi - q| \geq |\pi - \mu|$ such that $u^*_{\mathrm{CEU}} = 0$. The Pareto frontier on $[-|\pi - \mu|, 0]$ can be described by

$$u_{\mathrm{CEU}} = -|\pi - \mu| - u_{\mathrm{EP}}$$

and the straight line connecting d and u^* by

$$u_{\mathrm{CEU}} = \frac{|\mu - q|}{|\pi - q|} u_{\mathrm{EP}}.$$

The Kalai-Smorodinsky bargaining solution is located where the two lines cross, i.e.,

$$\begin{aligned} -|\pi - \mu| - u_{\mathrm{EP}} &= \frac{|\mu - q|}{|\pi - q|} u_{\mathrm{EP}} \\ \Leftrightarrow u_{\mathrm{EP}} &= \frac{-|\pi - \mu|}{1 + \frac{|\mu - q|}{|\pi - q|}} > -\frac{|\pi - \mu|}{2} \\ \Rightarrow u_{\mathrm{CEU}} &= -|\pi - \mu| + \frac{|\pi - \mu|}{1 + \frac{|\mu - q|}{|\pi - q|}} < -\frac{|\pi - \mu|}{2}. \end{aligned}$$

Above inequalities can be easily obtained by recalling $|\mu - q|/|\pi - q| > 1$ from above. The result is equivalent to $x^{\mathrm{KS}} = \pi + \frac{\mu - \pi}{1 + (\mu - q)/(\pi - q)} \in (\pi, \pi + \frac{1}{2}(\mu - \pi))$. This completes the first part of the proof.

Now consider $|\pi - q| < |\pi - \mu|$ such that $u^*_{\mathrm{CEU}} = -(|\pi - \mu| - |\pi - q|)$. While this has no effect on the Pareto frontier, the straight line connecting d and u^* is now given by

$$u_{\mathrm{CEU}} = -(|\pi - \mu| - |\pi - q|) + 2u_{\mathrm{EP}}.$$

The intersection point is then given by

$$\begin{aligned} -|\pi - \mu| - u_{\mathrm{EP}} &= -(|\pi - \mu| - |\pi - q|) + 2u_{\mathrm{EP}} \\ \Leftrightarrow u_{\mathrm{EP}} &= -\frac{|\pi - q|}{3} > -\frac{|\pi - \mu|}{3} \\ \Rightarrow u_{\mathrm{CEU}} &= -|\pi - \mu| + \frac{|\pi - q|}{3} < -\frac{2|\pi - \mu|}{3}. \end{aligned}$$

This is equivalent to $x^{\mathrm{KS}} = \pi + \frac{\pi - q}{3} \in (\pi, \pi + \frac{1}{3}(\mu - \pi))$, which completes the proof.

Chapter 4

Weighted Committee Games

Consider a committee, council, corporate board, etc. that involves three players (parties, groups, shareholders, delegations). Suppose the first wields 6 votes, the second 5 votes, and the third 2 votes. Will their collective choices differ, ceteris paribus, from those resulting if each player wielded 5 votes? Or, say, from outcomes for a $(48\%, 24\%, 28\%)$ distribution of votes?

The way in which voting weights translate into collective decisions and how they affect the influence of the respective decision-makers are old concerns for institutional design. See Riker (1986), for instance, on reactions by delegate Luther Martin from Maryland to the Constitutional Convention in Philadelphia in 1787. Notwithstanding residual disagreement on which measures of power or success are the most meaningful in a given context, the structural properties of voting are today well understood for majority decisions on two alternatives. It is easy to see, e.g., that for all of the above weight distributions any pair of players can jointly implement their preferred alternative. The two form a winning coalition irrespective of whether they have an $11:2$, $7:6$, $10:5$, or $52\% : 48\%$ majority. As long as each player wields positive but less than half the total weight, *all* distributions of votes among three players are equivalent under simple majority rule. They amount to different *weighted representations* of the same mathematical structure, known as a *simple voting game*. The literature has formalized numerous related results (see, e.g., Taylor and Zwicker 1999).

But what if the committee is to choose from three or more candidates? Very little is known then. Consider the most basic case: the committee uses plurality rule and always selects the candidate who received the most votes. Now player 1 has greater say for weights of $(6, 5, 2)$ than for equal ones. Namely, whenever players 2 and 3 fail to agree, player 1 is decisive and his or her favorite candidate wins with a tally of $6:5:2$, $11:2$, or $8:5$. The same plurality winners would result for $(48\%, 24\%, 28\%)$, i.e., committees with voting weights of $(6, 5, 2)$ and $(48\%, 24\%, 28\%)$ are structurally equivalent – but one with $(5, 5, 5)$ is not. One can conceive of the former as different weighted representations of the same *committee game*, referring to the combination of a set of n players, a set of m alternatives, and a particular mapping from n-tuples of preferences to a winning alternative.

The goal of this chapter is to extend the knowledge on equivalent weighted voting environments from two to more alternatives. We study four standard aggregation methods: plurality, Borda, Copeland, and antiplurality rule. These can produce four different winners for the same profile of preferences. We show that the methods also differ widely in the degree to which voting weights matter. For instance, there exist only 4 structurally different Copeland committees but 51 Borda committees when $n = m = 3$. Our findings do not depend on whether sincere preference statements or strategic votes are considered.

Committees that decide between two alternatives have received wide attention. Von Neumann and Morgenstern (1953) started their formal analysis by introducing simple voting games. Shapley and Shubik (1954) and Banzhaf (1965) constructed corresponding indices of voting power. Their applications range from the US Electoral College, UN Security Council, and EU Council of Ministers to governing bodies of the IMF and private corporations. See Mann and Shapley (1962), Riker and Shapley (1968), Owen (1975), or Brams (1978) for seminal contributions. They and more recently Barberà and Jackson (2006), Felsenthal and Machover (2013), Koriyama et al. (2013), Kurz et al. (2017), and many others have sought to quantify the links between voting weights and collective choices to evaluate democratic playing fields from a fairness or welfare perspective.

The extent to which different voting weights make real rather than only cosmetic differences has practical relevance. Weighted committee games offer the potential to extend the respective analysis to decision bodies that face non-binary options. For example, voting rights among the 24 Directors of the International Monetary Fund's (IMF's) Executive Board were reformed in 2016. Is there a possibility that this will affect any decisions, such as its choice of the next IMF Managing Director? The Executive Board declared (IMF Press Release 2016/19) that in the future a winner from a shortlist of at most three candidates shall be adopted "by a majority of the votes cast". Suppose this means (*i*) receiving the most votes (plurality rule). Have changes of the distribution of IMF drawing rights, hence votes, then made a difference? And would it make a difference to interpret the declaration as calling instead for (*ii*) a two-candidate runoff if nobody gets an outright majority (plurality runoff rule) or (*iii*) securing the highest number of pairwise majority wins against competitors (Copeland rule)? Both types of questions – comparing distinct vote distributions for a given rule or different rules for a given distribution – are about equivalences between committees that we formalize in this chapter.

Here we are not so much concerned with any particular voting body but a parsimonious framework for classifying non-binary voting structures. We take different compositions of committees – monotonically related to an underlying scale such as population, shareholdings, etc. or not – and a voting rule as primitives and investigate their equivalence relations. We seek to identify all structurally distinct weight distributions to help assessing, for instance, if nominal differences in political representation translate into real ones. We provide minimal representations for all pertinent committee games for small n and m. Comprehensive lists of games only existed for $m = 2$ so far. The extensions could be applied, e.g., to establish sharp bounds on the numbers of voters and alternatives that permit certain monotonicity violations or paradoxes (cf. Nurmi 1987 and Felsenthal and Nurmi 2017, 2018); to generalize rule-specific findings on manipulability from one to infinitely many equivalent committees (see Aleskerov and Kurbanov 1999 and Smith 1999); or to check robustness of voting equilibria to small reallocations of voting weights (cf. Myerson and Weber 1993, Bouton 2013, or Buenrostro et al. 2013). We also give a glimpse of the beautiful geometry of weighted committee games.

4.1 Related Literature

Our analysis concerns arbitrary mappings from n-tuples of preferences over m alternatives to a winning one. We seek to connect a given mapping to an anonymous baseline decision rule in the same way as weighted representations of a simple voting game described by player set N and coalitional function v connect it to simple majority or supermajority rule.

Simple voting games and the subclass of weighted voting games (i.e., those that have weighted representations) received a complete chapter's attention by von Neumann and Morgenstern (1953, Ch. 10). Taylor and Zwicker (1999) devoted a full-length monograph to them and investigations continue. See, e.g., Kurz and Tautenhahn (2013) on open challenges in classifying and enumerating simple voting games in the tradition of Isbell (1956, 1958) and Shapley (1962). Machover and Terrington (2014) studied simple voting games as "mathematical objects in their own right" and have connected their algebraic structure to seemingly unrelated areas of mathematics. Beimel et al. (2008), Gvozdeva and Slinko (2011), Houy and Zwicker (2014) or Freixas et al. (2017) document ongoing progress on the problem of verifying if a given game (N, v) is weighted.

However, the literature has increasingly acknowledged that the presumption of dichotomous decision-making is a severe limitation. Many committee decisions allow more than two outcomes. And even for binary motions, voters usually can abstain, stay away from the ballot, express different intensities of support, etc.

This has led to generalizations of simple voting games to multiple levels of approval. For instance, Felsenthal and Machover (1997a), Tchantcho et al. (2008) and Parker (2012) have considered *ternary voting games* with the option to support a proposal, to abstain, or to reject it. *Quaternary voting games* introduced by Laruelle and Valenciano (2012) add the possibility not to participate in a ballot. The case of an arbitrary finite number of individual actions translating into one of finitely many collective outcomes has been addressed by Hsiao and Raghavan (1993) and Freixas and Zwicker (2003, 2009). In their *(j, k)-games* each player expresses one of j linearly ordered levels of approval and every resulting j-partition of player set N is mapped to one of k ordered output levels.

Committees that determine quantities like grades, interest rates, budget sizes, etc. can be modeled as (j, k)-games. But the assumption of ordered actions and outcomes is problematic for alternatives with multidimensional attributes – for instance, if the committee is to select from several policy options, locations of a facility, job candidates, etc. Pertinent extensions of simple voting games have been introduced as *multicandidate voting games* by Bolger (1986) and taken up as *simple r-games* by Amer et al. (1998). These are the most closely related concepts in the literature to weighted committee games as far as we are aware. In particular, weighted plurality committees (as defined below) have featured in the framework of Bolger and Amer et al. as "simple plurality games" and "relative majority r-games". However, the respective analysis concerned values and power indices rather than structural investigation of the underlying games. We seem the first to find, e.g., that there are no more than 36 distinct "simple plurality games" with four players[1] and so only 36 different distributions of power can arise.

[1]Chua et al. (2002) identified the eight games that cannot generate ties for $m = 3$.

4.2 Notation and Definitions

4.2.1 Committees and Simple Voting Games

We consider finite sets N of $n \geq 1$ players or voters such that each voter $i \in N$ has strict preferences P_i over the set $A = \{a_1, \ldots, a_m\}$ of $m \geq 2$ alternatives. $\mathcal{P}(A)$ denotes the set of all $m!$ strict preference orderings on A. A *(resolute) social choice rule* $\rho \colon \mathcal{P}(A)^n \to A$ maps each profile $\mathbf{P} = (P_1, \ldots, P_n)$ to a single winning alternative $a^* = \rho(\mathbf{P})$. The combination (N, A, ρ) of a set of voters, a set of alternatives and a particular social choice rule will be referred to as a *committee game* or just as a *committee*.

For given N and A, there are $m^{(m!^n)}$ distinct rules ρ. Those that treat all voters $i \in N$ symmetrically will play a special role in our analysis: suppose preference profile $\mathbf{P}'$ results from applying a permutation $\pi \colon N \to N$ to profile $\mathbf{P}$, so $\mathbf{P}' = (P_{\pi(1)}, \ldots, P_{\pi(n)})$. Then ρ is *anonymous* if for all such $\mathbf{P}$, $\mathbf{P}'$ the winning alternative $a^* = \rho(\mathbf{P}) = \rho(\mathbf{P}')$ is the same. We will write r instead of ρ if we want to highlight that a considered rule is anonymous, i.e., we impose no restrictions on general social choice rules denoted by ρ but require anonymity for rules denoted by $r \colon \mathcal{P}(A)^n \to A$.

For $m = 2$ and binary alternatives $a_1 = 1$ and $a_2 = 0$, it is common to describe ρ by a *coalitional function* $v \colon 2^N \to \{0, 1\}$ with $v(S) = 1$ when $1\, P_i\, 0$ for all $i \in S$ implies $\rho(\mathbf{P}) = 1$. Sets $S \subseteq N$ with $v(S) = 1$ are called *winning coalitions*. The pair (N, v) is referred to as a *simple voting game*: it can be viewed as a cooperative game in which the worths $v(S)$ of coalitions are restricted to $\{0, 1\}$.

A simple voting game (N, v) is *weighted* and also called *weighted voting game* if there exists a non-negative vector $\mathbf{w} = (w_1, \ldots, w_n)$ of weights and a positive quota q such that $v(S) = 1$ if and only if $\sum_{i \in S} w_i \geq q$. One then refers to pair $(q; \mathbf{w})$ as a *(weighted) representation* of (N, v) and denotes the respective game by $[q; \mathbf{w}]$, i.e., $(N, v) = [q; \mathbf{w}]$. It is without loss of generality to focus on integer numbers: given $q \in \mathbb{R}_{++}$ and $\mathbf{w} \in \mathbb{R}^n_+$ one can always find $q' \in \mathbb{N}$ and $\mathbf{w}' \in \mathbb{N}^n_0$ such that $[q; \mathbf{w}] = [q'; \mathbf{w}']$. Certificates for the non-weightedness of a given simple game (N, v) can be rather complex and characterization of weightedness remains an active field for $m = 2$ (see Section 4.1).

Somewhat involved analogues of winning coalitions and coalitional functions exist for $m > 2$. For instance, Moulin (1981) introduced *veto functions* to succinctly describe the outcomes that given coalitions of players could prevent if they coordinated. Different types of *effectivity functions* clarify the power structure associated with a rule ρ by enumerating the sets of alternatives that given coalitions of voters can force $\rho(\mathbf{P})$ to lie in. See Peleg (1984). We provide a different perspective by

Rule	Winning alternative at preference profile $\mathbf{P}$
Antiplurality	$r^A(\mathbf{P}) \in \arg\min_{a\in A} \left\|\{i \in N \mid \forall a' \neq a \in A\colon a'P_i a\}\right\|$
Borda	$r^B(\mathbf{P}) \in \arg\max_{a\in A} \sum_{i\in N} b_i(a,\mathbf{P})$
Copeland	$r^C(\mathbf{P}) \in \arg\max_{a\in A} \left\|\{a' \in A \mid a >^{\mathbf{P}}_M a'\}\right\|$
Plurality	$r^P(\mathbf{P}) \in \arg\max_{a\in A} \left\|\{i \in N \mid \forall a' \neq a \in A\colon aP_i a'\}\right\|$

Table 4.1 Considered anonymous social choice rules

investigating analogues to weightedness of a simple voting game on the domain of general committee games.

4.2.2 Four Anonymous Social Choice Rules

We will define weightedness of general rules ρ relative to some fixed anonymous rule r. For the latter we here focus on four standard social choice rules with lexicographic tie breaking. Their definitions are summarized in Table 4.1.

Under *plurality rule* r^P each voter names his or her top-ranked alternative and the alternative that is ranked first by the most voters will be chosen.[2] Analogously, *antiplurality rule* r^A selects the alternative that is ranked last by the fewest voters. *Borda rule* r^B requires each voter i to give $m-1, m-2, \ldots, 0$ points to the alternative that he or she ranks first, second, etc. These points $b_i(a,\mathbf{P}) := \left|\{a' \in A \mid aP_i a'\}\right|$ equal the number of alternatives that i ranks below a. The alternative with the highest total number of points, known as its *Borda score*, is selected. *Copeland rule* r^C considers pairwise majority votes between the alternatives. They define the *majority relation* $a >^{\mathbf{P}}_M a' \;:\Leftrightarrow\; \left|\{i \in N \mid aP_i a'\}\right| > \left|\{i \in N \mid a'P_i a\}\right|$ and the alternative that beats the most others according to $>^{\mathbf{P}}_M$ is selected. In particular, if some alternative a is a Condorcet winner, i.e., beats all others, then $r^C(\mathbf{P}) = a$.

We assume that whenever there is a non-singleton set $A^* = \{a^*_{i_1}, \ldots, a^*_{i_k}\}$ of optimizers in Table 4.1, the alternative $a^*_{i^*} \in A^*$ with lowest index $i^* = \min\{i_1, \ldots, i_k\}$ is selected. This amounts to *lexicographic tie breaking* for $A \subset \{a, \ldots, z, aa, ab, \ldots\}$ and has computational advantages over working with set-valued choices. In particular,

[2]The formal structure of a committee game is unaffected by whether voting is sincere or strategic. The difference only lies in the interpretation of $\mathcal{P}(A)^n$: it refers to profiles of true preferences in the former and stated ones in the latter case. So it is without loss of generality if we adopt the simpler vocabulary of sincere voting and say "ranked first" instead of "named as top-ranked".

only $m^{(m!^n)}$ distinct mappings from preference profiles to alternatives a^* need to be considered, compared to $(2^m - 1)^{(m!^n)}$ if each profile were mapped to a non-empty set $A^* \subseteq A$. The former entails no loss of information as we consider all $\mathbf{P} \in \mathcal{P}(A)^n$: the set of alternatives tied at $\mathbf{P}$ is fully determined by $a^* = r(\mathbf{P})$ and the respective winners $a^{**}, a^{***}, \ldots$ at profiles $\mathbf{P}', \mathbf{P}'', \ldots$ that swap a^* with alternatives $a', a'', \ldots$ that might be tied with a^* at $\mathbf{P}$.[3] The considered rules r^A, r^B, r^C, r^P and their set-valued versions are hence in one-to-one correspondence and exhibit the same structural equivalences.[4]

4.2.3 Weighted Committee Games

Committee games (N, A, ρ) that model real committees, councils, parliaments etc. are more likely than not to involve a non-anonymous social choice rule ρ. Designated members might have procedural privileges and veto rights. Or an anonymous decision rule r applies not at the level of voters but their respective shareholdings, IMF drawing rights, etc. Moreover, we may take the relevant players $i \in N$ in a committee game to be well-disciplined parties, factions, or interest groups with different numbers of seats. Anonymity of the underlying rule at the level of individual voters then is destroyed at the level of voter blocs.

The latter two cases – individual voters with different numbers of votes and groups of voters who act as monolithic blocs – amount to the same: the corresponding rule ρ can be viewed as the *combination of an anonymous social choice rule r with integer voting weights $w_1, \ldots, w_n$* attached to the relevant players.

In the following, we let r denote the entire family of mappings from n-tuples of linear orders over $A = \{a_1, \ldots, a_m\}$ to winners $a^* \in A$ determined by the considered rule (for all n and m). Then the indicated combination operation amounts to a simple replication. It defines the social choice rule $r|\mathbf{w}\colon \mathcal{P}(A)^{w_\Sigma} \to A$ by

$$r|\mathbf{w}(\mathbf{P}) := r(\underbrace{P_1, \ldots, P_1}_{w_1 \text{ times}}, \underbrace{P_2, \ldots, P_2}_{w_2 \text{ times}}, \ldots, \underbrace{P_n, \ldots, P_n}_{w_n \text{ times}})$$

for a given anonymous rule r and a non-negative, non-degenerate weight vector $\mathbf{w} = (w_1, \ldots, w_n) \in \mathbb{N}_0^n$ with $w_\Sigma := \sum_{i=1}^n w_i > 0$. In the degenerate case $\mathbf{w} = (0, \ldots, 0)$,

[3] Given $r(\mathbf{P}) = b$, for example, a tie with a can directly be ruled out; one sees if b was tied with c by checking whether $r(\mathbf{P}') = c$ or b where $\mathbf{P}'$ only swaps b's and c's position in every player's ranking P_i.

[4] Analogous reasoning would apply if ties were broken in a uniform random way, i.e., for the most basic type of probabilistic social choice. See Brandl et al. (2016) on differences between deterministic and probabilistic frameworks.

P_1	P_2	P_3	P_4
d	b	c	c
e	c	e	b
b	e	a	a
a	a	d	d
c	d	b	e

$\Rightarrow$

$r^A|\mathbf{w}(\mathbf{P}) = a$ (a has min. bottom ranks 0)
$r^B|\mathbf{w}(\mathbf{P}) = b$ (b has max. Borda score 28)
$r^C|\mathbf{w}(\mathbf{P}) = c$ (c has max. pairwise wins 3)
$r^P|\mathbf{w}(\mathbf{P}) = d$ (d has max. plurality tally 5)

Table 4.2 Choices for preference profile $\mathbf{P}$ when $\mathbf{w} = (5, 3, 2, 2)$

let $r|\mathbf{0}(\mathbf{P}) \equiv a_1$.

We say a committee game (N, A, ρ) is *r-weighted* for a given anonymous social choice rule r if there exists a weight vector $\mathbf{w} = (w_1, \ldots, w_n) \in \mathbb{N}_0^n$ such that

$$\rho(\mathbf{P}) = r|\mathbf{w}(\mathbf{P}) \text{ for all } \mathbf{P} = (P_1, \ldots, P_n) \in \mathcal{P}(A)^n.$$

Then – so when $(N, A, \rho) = (N, A, r|\mathbf{w})$ – we refer to $(N, A, r, \mathbf{w})$ as a *(weighted) representation* of (N, A, ρ). The corresponding game will also be denoted by $[N, A, r, \mathbf{w}]$.

If the anonymous rule in question is plurality rule r^P, we call $(N, A, r^P|\mathbf{w})$ a *(weighted) plurality committee*. Similarly, $(N, A, r^A|\mathbf{w})$, $(N, A, r^B|\mathbf{w})$, and $(N, A, r^C|\mathbf{w})$ are respectively referred to as an *antiplurality committee*, *Borda committee*, and *Copeland committee*. That such committees can crucially differ for a fixed distribution $\mathbf{w}$ is illustrated in Table 4.2: the winning alternative all depends on the voting rule r in use.[5]

4.3 Equivalence Classes of Weighted Committee Games

4.3.1 Equivalence of Committee Games

Weighted representations of given committee games are far from unique. Consider, e.g., the j-dictatorship game (N, A, ρ_j) where $\rho_j(\mathbf{P})$ equals the alternative that is top-ranked by P_j for every $\mathbf{P} \in \mathcal{P}(A)^n$. This coincides with $[N, A, r, \mathbf{w}]$ for $r \in \{r^C, r^P\}$ and any $\mathbf{w} \in \mathbb{N}_0^n$ with $w_j > \sum_{i \neq j} w_i$.

Committees $(N, A, r|\mathbf{w})$ and $(N', A', r'|\mathbf{w}')$ evidently are *equivalent* if $N = N'$, $A = A'$, and $r \neq r'$ or $\mathbf{w} \neq \mathbf{w}'$ but the respective mappings from preference profiles to outcomes

[5] Moreover, e wins under *approval voting* for suitable ballots (Brams and Fishburn 1978). See Felsenthal et al. (1993), Leininger (1993), or Tabarrok and Spector (1999) for related case studies.

a^* are the same; that is, when $r|\mathbf{w}(\mathbf{P}) = r'|\mathbf{w}'(\mathbf{P})$ for all $\mathbf{P} \in \mathcal{P}(A)^n$. We will focus on situations where $r = r'$ and try to capture structural equivalence in the sense that $(N, A, r|\mathbf{w})$ and $(N', A', r|\mathbf{w}')$ *reflect the same decision environment* even though weights and labels of players or alternatives might differ. The latter means there are bijective mappings $\pi\colon N \to N'$ and $\tilde{\pi}\colon A \to A'$ such that each player $i \in N$ and alternative $a \in A$ has the same role in $(N, A, r|\mathbf{w})$ as do player $\pi(i)$ and alternative $\tilde{\pi}(a)$ in $(N', A', r|\mathbf{w}')$. Accordingly, r-weighted committee games $(N, A, r|\mathbf{w})$ and $(N', A', r|\mathbf{w}')$ will be called *structurally equivalent* (or *equivalent up to isomorphism*) if

$$\Big\{a_j\, P_i\, a_k \Leftrightarrow \tilde{\pi}(a_j)\, P'_{\pi(i)}\, \tilde{\pi}(a_k)\Big\} \;\Rightarrow\; \tilde{\pi}\Big(r|\mathbf{w}(\mathbf{P})\Big) = r|\mathbf{w}'(\mathbf{P}')$$

for suitable bijections $\pi\colon N \to N'$ and $\tilde{\pi}\colon A \to A'$ that map every profile $\mathbf{P}$ of preferences P_i over A to a relabeled profile $\mathbf{P}'$ of preferences $P'_{\pi(i)}$ over A'.

This includes situations where $N = N'$ but weights $\mathbf{w}'$ are a permutation of $\mathbf{w}$. For instance, the Copeland committee $(N, A, r^C|\mathbf{w})$ has different attractiveness to a given player for $\mathbf{w} = (3, 1, 1)$, $(1, 3, 1)$, or $(1, 1, 3)$. However, the decision environment is the same: it involves a dictator player whose most-preferred alternative always wins and two null players whose preferences do not influence the outcome.

A given weight distribution $\mathbf{w} \in \mathbb{N}_0^n$ fixes the number of players. So as labels of players and alternatives do not matter, we write $(r, \mathbf{w}) \sim_m (r, \mathbf{w}')$ to denote that r-committee games with m alternatives are structurally equivalent for weight distributions $\mathbf{w}$ and $\mathbf{w}'$. Relation $\sim_m$ and a suitable reference distribution $\bar{\mathbf{w}} \in \mathbb{N}_0^n$ with $\bar{w}_1 \geq \bar{w}_2 \geq \ldots \geq \bar{w}_n$ serving as index jointly define an *equivalence class*

$$\mathcal{E}^r_{\bar{\mathbf{w}},m} := \Big\{\mathbf{w} \in \mathbb{N}_0^n \mid (r, \mathbf{w}) \sim_m (r, \bar{\mathbf{w}})\Big\}.$$

$\mathcal{E}^r_{\bar{\mathbf{w}},m}$ is the set of all weight distributions that give rise to weighted committee games equivalent to $[N, A, r, \bar{\mathbf{w}}]$ up to isomorphism. If voters use rule r for deciding between m alternatives, then all weight distributions $\mathbf{w}, \mathbf{w}' \in \mathcal{E}^r_{\bar{\mathbf{w}},m}$ come with identical monotonicity properties, voting paradoxes, manipulation incentives, strategic equilibria, implementation possibilities, etc.

4.3.2 Illustration

As an example, consider Borda rule r^B for $m = 3$ and reference weights $\bar{\mathbf{w}} = (5, 2, 1)$. To simplify the exposition, let us focus on the subset $\mathcal{E}^B_{(5,2,1),3} \subset \mathcal{E}^{r^B}_{(5,2,1),3}$ of vectors $\mathbf{w}$ with $w_1 \geq w_2 \geq w_3$.

Identity of $\rho = r^B|(5,2,1)$ and $r^B|\mathbf{w}$ implies two inequalities for each profile $\mathbf{P} \in \mathcal{P}(A)^3$: the Borda winner must beat each of the other alternatives. Writing abc in abbreviation of aP_ibP_ic, profile $\mathbf{P} = (cab, bac, abc)$, for instance, implies $r^B|\bar{\mathbf{w}}(\mathbf{P}) = c$ and hence the Borda score of (lexicograpically maximal) c under any equivalent weight vector $\mathbf{w}$ must strictly exceed that of a and b:

$$2w_1 > w_1 + w_2 + 2w_3 \tag{I}$$

$$2w_1 > 2w_2 + w_3. \tag{II}$$

Similarly, profile $\mathbf{P}' = (cab, abc, bac)$ makes a the winner. Being lexicographically minimal, this implies a's Borda score must not be smaller than b's and c's:

$$w_1 + 2w_2 + w_3 \geq w_2 + 2w_3 \tag{III}$$

$$w_1 + 2w_2 + w_3 \geq 2w_1. \tag{IV}$$

Profiles $\mathbf{P}'' = (abc, bca, bac)$ and $\mathbf{P}''' = (abc, bca, bca)$ similarly induce $\rho(\mathbf{P}'') = a$ and $\rho(\mathbf{P}''') = b$, which implies

$$2w_1 + w_3 \geq w_1 + 2w_2 + 2w_3 \tag{V}$$

$$2w_1 + w_3 \geq w_2 \tag{VI}$$

$$w_1 + 2w_2 + 2w_3 > 2w_1 \tag{VII}$$

$$w_1 + 2w_2 + 2w_3 \geq w_2 + w_3. \tag{VIII}$$

Condition (VIII) is trivially satisfied for any $\mathbf{w} \in \mathbb{N}_0^n$. (IV) and (V) imply $w_1 = 2w_2 + w_3$. This makes (I) equivalent to $w_2 > w_3$ and (VII) to $w_3 > 0$. Combining $w_1 = 2w_2 + w_3$ and $w_2 > w_3 > 0$ also verifies (II), (III) and (VI). The 212 remaining profiles $\mathbf{P} \in \mathcal{P}(A)^3$ turn out not to impose additional constraints. Hence

$$\mathbf{w} \in \mathcal{E}^B_{(5,2,1),3} = \Big\{(2w_2 + w_3, w_2, w_3) \in \mathbb{N}_0^3 \,:\, w_2 > w_3 > 0\Big\}.$$

The full class $\mathcal{E}^{r^B}_{(5,2,1),3}$ follows by permuting the weight distributions in $\mathcal{E}^B_{(5,2,1),3}$. Other equivalence classes, such as $\mathcal{E}^{r^B}_{(1,1,1),3}$, $\mathcal{E}^{r^B}_{(2,1,1),3}$, etc., can be characterized analogously. However, determining *all classes* is quite involved even for $n = m = 3$.

4.3.3 Relation between Equivalence Classes

As the number of distinct mappings from preference profiles to outcomes is finite for given n and m, there are only finitely many disjoint $\mathcal{E}^r_{\bar{\mathbf{w}},m}$ with $\bar{\mathbf{w}} \in \mathbb{N}_0^n$ for any given rule r. They partition the infinite space $\mathbb{N}_0^n$ of weight distributions into a collection $\left\{\mathcal{E}^r_{\bar{\mathbf{w}}_1,m}, \mathcal{E}^r_{\bar{\mathbf{w}}_2,m}, \ldots, \mathcal{E}^r_{\bar{\mathbf{w}}_\xi,m}\right\}$ of all r-weighted committees with n voters deciding on m alternatives. We will see below that the numbers ξ of elements of such a partition – hence the numbers of structurally distinct weighted committee games for given r, n, and m – vary widely across rules.

Let us first gather some results on the relation between equivalence classes for different rules r or parameters n and m; proofs are provided in Appendix A. Non-surprisingly, the degenerate weight vector $\mathbf{w}^0 = \mathbf{0}$ always forms its own class:

Lemma 4.1. *Let $m \geq 2$, $r \in \{r^A, r^B, r^C, r^P\}$ and $\mathbf{w} \neq \mathbf{0} \in \mathbb{N}_0^n$. Then $(r, \mathbf{0}) \nsim_m (r, \mathbf{w})$.*

We focus on non-degenerate weight vectors $\mathbf{w} \neq \mathbf{0}$ from now on. Another straightforward observation is that the considered rules do not differ for $m = 2$:

Proposition 4.1. *The partitions $\left\{\mathcal{E}^r_{\bar{\mathbf{w}}_1,2}, \ldots, \mathcal{E}^r_{\bar{\mathbf{w}}_\xi,2}\right\}$ of $\mathbb{N}_0^n$ coincide for $r \in \{r^A, r^B, r^C, r^P\}$.*

Furthermore, weighted committee games with $m = 2$ are in one-to-one relation to standard weighted voting games $[q; w_1, \ldots, w_n]$ with a 50%-majority quota: [6]

Proposition 4.2. *Let $N = \{1, \ldots, n\}$ and $A = \{a_1, a_2\}$. For any $\mathbf{w} \neq \mathbf{0} \in \mathbb{N}_0^n$ and $r \in \{r^A, r^B, r^C, r^P\}$*

$$r|\mathbf{w}(\mathbf{P}) = a_1 \Leftrightarrow v(S) = 1$$

where v is the coalitional function of weighted voting game $(N, v) = [q; \mathbf{w}]$ with $q = \frac{1}{2}\sum_{i \in N} w_i$ and coalition $S = \{i \in N \mid a_1 \, P_i \, a_2\} \subseteq N$ collects all players who prefer a_1 at profile $\mathbf{P} \in \mathcal{P}(A)^n$.

It follows that the respective partitions $\left\{\mathcal{E}^r_{\bar{\mathbf{w}}_1,2}, \ldots, \mathcal{E}^r_{\bar{\mathbf{w}}_\xi,2}\right\}$ of $\mathbb{N}_0^n$ coincide with those for weighted voting games with a simple majority quota. Their study and enumeration for $n \leq 5$ dates back to von Neumann and Morgenstern (1953, Ch. 10).

The remaining propositions consider equivalence classes for a fixed rule r as the number m of alternatives is varied.

Proposition 4.3. *For Copeland rule r^C, the partitions $\left\{\mathcal{E}^{r^C}_{\bar{\mathbf{w}}_1,m}, \ldots, \mathcal{E}^{r^C}_{\bar{\mathbf{w}}_\xi,m}\right\}$ of $\mathbb{N}_0^n$ coincide for all $m \geq 2$.*

[6] Weighted voting games $[q; \mathbf{w}]$ and $[q'; \mathbf{w}]$ with quota $q = \sum w_i/2$ and $q' = q + \epsilon$ for small $\epsilon > 0$ are in a well-defined sense *duals* of each other, i.e., are also in bijection.

Proposition 4.4. *For plurality rule r^P, the partitions $\left\{\mathcal{E}^{r^P}_{\bar{\mathbf{w}}_1,m}, \ldots, \mathcal{E}^{r^P}_{\bar{\mathbf{w}}_\xi,m}\right\}$ of $\mathbb{N}_0^n$ coincide for all $m \geq n$.*

Proposition 4.5. *For Borda rule r^B and given $m \geq 3$, every weight vector $\tilde{\mathbf{w}}_\mathbf{j} = (j, 1, 0, \ldots, 0)$ with $j \in \{1, \ldots, m-1\}$ identifies a different class $\mathcal{E}^{r^B}_{\tilde{\mathbf{w}}_\mathbf{j},m}$.*

It follows from Proposition 4.5 that, for any fixed number of players, the number ξ of structurally distinct Borda committee games grows without bound as m goes to infinity. Borda rule differs in this respect from Copeland, plurality, and also antiplurality plurality rule:

Proposition 4.6. *For antiplurality rule r^A, the partitions $\left\{\mathcal{E}^{r^A}_{\bar{\mathbf{w}}_1,m}, \mathcal{E}^{r^A}_{\bar{\mathbf{w}}_2,m}, \ldots, \mathcal{E}^{r^A}_{\bar{\mathbf{w}}_\xi,m}\right\}$ of $\mathbb{N}_0^n \setminus \{\mathbf{0}\}$ consist of $\xi = n$ equivalence classes identified by weight vectors $\bar{\mathbf{w}}_1 = (1, 0, \ldots, 0), \bar{\mathbf{w}}_2 = (1, 1, \ldots, 0), \ldots, \bar{\mathbf{w}}_\mathbf{n} = (1, 1, \ldots, 1)$ for all $m \geq n + 1$.*

The reference vectors in Propositions 4.5 and 4.6 have the lowest possible weight sum for the respective class of games. Before we elaborate on this be reminded that equivalence classes would be the same if we considered uniform tie breaking or set-valued choices (cf. end of Section 4.2.2).[7]

4.4 Identifying Weighted Committee Games

4.4.1 Minimal Representations and Test for Weightedness

Above rules have the property that $[N, A, r, \mathbf{w}] = [N, A, r, \mathbf{w}']$ when $\mathbf{w}$ is a multiple of $\mathbf{w}'$. Even if $\mathbf{w}$ represents the actual distribution of seats or vote shares in a given institution, it can be analytically more convenient to work with $\mathbf{w}'$. More generally, given $(N, A, \rho) = (N, A, r|\mathbf{w})$, we say that $(N, A, r, \mathbf{w})$ has *minimum integer sum* or is a *minimal representation* of (N, A, ρ) if $\sum_{i \in N} w'_i \geq \sum_{i \in N} w_i$ for all representations $(N, A, r, \mathbf{w}')$ of (N, A, ρ) that involve rule r. The games in a given equivalence class $\mathcal{E}^r_{\bar{\mathbf{w}},m}$ usually have a unique minimal representation.[8] The corresponding minimal weights are the focal choice for $\bar{\mathbf{w}}$. For instance, $(5, 2, 1)$ has minimal sum among all $\mathbf{w} \in \mathcal{E}^{r^B}_{(5,2,1),3}$ characterized in Section 4.3.2.

[7]Lexicographic tie breaking can yield $r|\mathbf{w}(\mathbf{P}) = r|\mathbf{w}'(\mathbf{P}) = a^*$ even though the sets of alternatives tied at $\mathbf{P}$, say A^* and A'^*, differ between $r|\mathbf{w}$ and $r|\mathbf{w}'$. Then construct $\mathbf{P}'$ as follows: if $A^* \not\subset A'^*$, fix an alternative $a' \in A^* \setminus A'^*$ and swap positions of a^* and a' in $\mathbf{P}$. Now $r|\mathbf{w}(\mathbf{P}') = a^*$ is unchanged but $r|\mathbf{w}'(\mathbf{P}') \neq a^*$. If $A^* \subset A'^*$, consider $a' \in A'^* \setminus A^*$ analogously.

[8]If $m = 2$, minimal representations are unique for up to $n = 7$ players (Kurz 2012). Multiplicities for games with larger values of m or n arise but are rare.

Finding minimal representations of arbitrary Copeland committees simplifies to finding them for $m = 2$ by Proposition 4.3. And by Proposition 4.2 this amounts to finding minimal representations of specific weighted voting games. Linear programming techniques have proven helpful for this task and can be adapted to committees that apply rules r^A, r^B, or r^P. These all belong to the family of *positional* or *scoring rules*: winners can be characterized as maximizers of scores derived from alternatives' positions in $\mathbf{P}$ and a suitable *scoring vector* $\mathbf{s} \in \mathbb{Z}^m$ with $s_1 \geq s_2 \geq \ldots \geq s_m$. Specifically, let the fact that alternative a is ranked at the j-th highest position in ordering P_i contribute s_j points for a, and refer to the sum of all points received as a's *score*. Then score maximization for $\mathbf{s}^B = (m-1, m-2, \ldots, 1, 0)$ yields the Borda winner, $\mathbf{s}^P = (1, 0, \ldots, 0, 0)$ the plurality winner, and $\mathbf{s}^A = (0, 0, \ldots, 0, -1)$ or $(1, 1, \ldots, 1, 0)$ the antiplurality winner.

For an arbitrary scoring rule r that induces social choice rule ρ for appropriate weights, let us denote the index of the winning alternative at profile $\mathbf{P}$ by $\omega_\rho(\mathbf{P}) \in \{1, \ldots, m\}$, i.e., $\rho(\mathbf{P}) = a_{\omega_\rho(\mathbf{P})} \in A$; write $S_k(P_i) \in \mathbb{Z}$ for the unweighted $\mathbf{s}$-score of alternative a_k derived from its position in ordering P_i (e.g., for $m = 3$ and $a_3 = c$, we have $S_3(P_i) = s_2$ if either aP_icP_ib or bP_icP_ia). Then any solution to the following *integer linear program* yields a minimal representation $(N, A, r, \mathbf{w})$ of (N, A, ρ):

$$\min_{\mathbf{w} \in \mathbb{N}_0^n} \sum_{i=1}^{n} w_i \tag{ILP}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{i=1}^{n} S_k(P_i) \cdot w_i \leq \sum_{i=1}^{n} S_{\omega_\rho(\mathbf{P})}(P_i) \cdot w_i \, - \, 1 \quad \forall \mathbf{P} \in \mathcal{P}(A)^n \, \forall 1 \leq k \leq \omega_\rho(\mathbf{P}) - 1, \\ & \sum_{i=1}^{n} S_k(P_i) \cdot w_i \leq \sum_{i=1}^{n} S_{\omega_\rho(\mathbf{P})}(P_i) \cdot w_i \quad \forall \mathbf{P} \in \mathcal{P}(A)^n \, \forall \omega_\rho(\mathbf{P}) + 1 \leq k \leq m. \end{aligned}$$

The case distinction between scores of non-winning alternatives a_k with index $k < \omega_\rho(\mathbf{P})$ vs. $k > \omega_\rho(\mathbf{P})$ reflects the tie breaking assumption. If some (non-minimal) representation $(N, A, r, \mathbf{w}')$ of (N, A, ρ) is known and satisfies $w'_1 \geq w'_2 \geq \ldots \geq w'_n$ then adding constraints $w_i \geq w_{i+1}, \forall 1 \leq i \leq n-1$, to (ILP) helps to speed up computations.

If it is not yet known whether ρ is r-weighted, (ILP) provides a decisive *test for r-weightedness* for any scoring rule r.[9] Namely, the constraints in (ILP) characterize a non-empty compact set if and only if ρ is r-weighted. Checking non-emptiness of the constraint set for a given ρ answers the question of its r-weightedness. This can

[9]This extends to Copeland rule r^C by Propositions 4.1 and 4.3.

be done with optimization software (e.g., Gurobi or CPLEX) that also identifies the weight sum minimizer at little extra cost.

4.4.2 Algorithmic Strategy

We would like to characterize all r-committee games for fixed n and m. In principle, one could do this as follows: loop over the $m^{(m!^n)}$ different social choice rules $\rho\colon \mathcal{P}(A)^n \to A$; conduct above test for r-weightedness; if successful, determine a representation $(N, A, r, \bar{\mathbf{w}})$ and characterize $\mathcal{E}^r_{\bar{\mathbf{w}},m}$ as in Section 4.3.2; continue until all rules ρ have been covered.

The extreme growth of $m^{(m!^n)}$ prevents a direct implementation of this idea: $n = m = 3$ already gives rise to an intractable $3^{216} > 10^{103}$ different mappings. However, many mappings can be dropped from consideration in large batches. If $\rho(\mathbf{P}) = a_1$ for one of the $(m-1)!^n$ profiles $\mathbf{P}$ where a_1 is unanimously ranked last, for instance, then ρ cannot be r-weighted for $r \in \{r^A, r^B, r^C, r^P\}$. This rules out $m^{(m!^n-1)}$ candidate mappings in one go. Similarly, if weights $\mathbf{w}$ such that $r|\mathbf{w}(\mathbf{P}) = a_1$ turn out to be incompatible with $r|\mathbf{w}(\mathbf{P}') = a_2$ for two suitable profiles $\mathbf{P}, \mathbf{P}'$, then all $m^{(m!^n-2)}$ mappings ρ with $\rho(\mathbf{P}) = a_1$ and $\rho(\mathbf{P}') = a_2$ can be disregarded at once.

Branch-and-Cut Algorithm
Given n, m and r, identify every class $\mathcal{E}^r_{\bar{\mathbf{w}}_\mathbf{k},m}$ by a minimal representation.

Step 1	Generate all $J := (m!)^n$ profiles $\mathbf{P}^1, \ldots, \mathbf{P}^J \in \mathcal{P}(A)^n$ for $A := \{a_1, \ldots, a_m\}$. Set $\mathcal{F} := \varnothing$.
Step 2	For every $\mathbf{P}^j \in \mathcal{P}(A)^n$ and every $a_i \in A$, check if there is any weight vector $\mathbf{w} \in \mathbb{N}_0^n$ s.t. $r|\mathbf{w}(\mathbf{P}^j) = a_i$ by testing feasibility of the implied constraints (cf. Section 4.3.2). If yes, then append (i, j) to $\mathcal{F}$.
Step 3	Loop over j from 1 to J.
Step 3a	If $j = 1$, then set $C_1 := \{1 \le i \le m \mid (i, j) \in \mathcal{F}\}$.
Step 3b	If $j \ge 2$, then set $C_j := \varnothing$ and loop over all $(p_1, \ldots, p_{j-1}) \in C_{j-1}$ and all $p_j \in \{1, \ldots, m\}$ with $(p_j, j) \in \mathcal{F}$. If (ILP) has a solution for the restriction to the profiles $\mathbf{P^1}, \ldots, \mathbf{P^j}$ with prescribed winners $\rho(\mathbf{P^i}) = a_{p_i}$ for $1 \le i \le j$, then append $(p_1, \ldots, p_j)$ to C_p.
Step 4	Loop over the elements $(p_1, \ldots, p_j, \ldots, p_J) \in C_J$ and output minimal weights $\bar{\mathbf{w}}$ such that $r|\bar{\mathbf{w}} \equiv \rho$ with $\rho(\mathbf{P^j}) = p_j$ by solving (ILP).

Table 4.3 Determining the classes of r-weighted committees for given n and m

The branch-and-cut algorithm described in Table 4.3 operationalizes these considerations. It can be tuned – and performance significantly improved – if the rule r in question uses only partial information, such as top ranks. For plurality rule r^P it actually suffices to consider individual preferences for which all alternatives below the top are in lexicographic order. Then only m^n profiles instead of $(m!)^n$ need to be looped over. Analogous reasoning applies to antiplurality rule r^A.

Alas, the algorithm can still require too much memory and running time. The main alternative then is to loop over different weight distributions and check if they are structurally distinct from those already known. Namely, start with $w_\Sigma := 0$ and an empty list $\hat{\mathcal{W}}$ of weight vectors; increase the sum of weights w_Σ in steps of 1; generate the set $\mathcal{W}_{w_\Sigma} := \left\{\mathbf{w} \in \mathbb{N}_0^n \,\middle|\, w_1 \geq \cdots \geq w_n \text{ and } w_1 + \cdots + w_n = w_\Sigma\right\}$ and loop over all $\mathbf{w} \in \mathcal{W}_{w_\Sigma}$. The respective weight vector $\mathbf{w}$ is appended to $\hat{\mathcal{W}}$ if for every $\mathbf{w}' \in \hat{\mathcal{W}}$ we have $r|\mathbf{w}(\mathbf{P}) \neq r|\mathbf{w}'(\mathbf{P})$ for at least one $\mathbf{P} \in \mathcal{P}(A)^n$. The set $\hat{\mathcal{W}}$ then contains a growing list of minimal weight vectors that correspond to structurally distinct committee games $[N, A, r, \mathbf{w}]$. This method has the advantage of not requiring any weightedness test, such as (ILP). However, search needs to be stopped manually and just produces a lower bound on the actual number of classes.[10]

4.5 Number and Geometry of Weighted Committee Games

4.5.1 Number of Antiplurality, Borda, Copeland, and Plurality games

A combination of our analytical findings and indicated computational methods permits to identify all structurally distinct r-weighted committee games with $r \in \{r^A, r^B, r^C, r^P\}$ for small n and m. This can be useful in several ways: demonstrating, for instance, that a certain voting paradox does not occur for any of the 34 distinct plurality committees with $n = 4$, $m = 3$, which we list in Appendix B, would suffice to establish that at least five voter groups or four alternatives are needed for r^P to exhibit the paradox. Similarly, a characterization of voting equilibria for, say, the 7 weight vectors listed for antiplurality rule when $n = m = 4$ would automatically extend to all distributions of votes.

Table 4.4 summarizes our findings. Figures do not include the degenerate class $\mathcal{E}_{\mathbf{0},m}$. When less than 150 equivalence classes of games exist, we report a minimum

[10]One can compute upper bounds on the weight sum that guarantees coverage of *all* equivalence classes, analogously to bounds for minimal representation of weighted voting games (see Muroga 1971, Thm. 9.3.2.1). In our context such bounds are way too large to be practical, however.

sum integer representation for each in Appendix B. The list for $m = 2$ nests the games reported by Krohn and Sudhölter (1995) and Brams and Fishburn (1996); plurality committees with $m = 3$ nest the tie-free games listed by Chua et al. (2002) for $n = 3, 4$. The branch-and-cut approach required excessive memory for Borda committees when $m > 4$ or $n = m \geq 4$. We write "$\geq \ldots$" if the search algorithm appended no new games to set $\hat{\mathcal{W}}$ for long enough to support the conjecture that the indicated bound equals the exact number of games; we write "$\gg \ldots$" if we expect more computing power to yield yet more games.[11]

n, m \ r	Antiplurality	Borda	Copeland	Plurality
3,2	4			
4,2	9			
5,2	27			
6,2	138			
7,2	1 663			
8,2	63 764			
9,2	9 425 479			
3, 3	5	51	4	6
3, 4	3	505	4	6
3, 5	3	$\geq$ 2 251	4	6
4, 3	19	5 255	9	34
4, 4	7	$\gg$ 635 622	9	36
4, 5	4	$\gg$ 635 622	9	36
5, 3	263	$\gg$ 1 153 448	27	852
6, 3	$\geq$ 33 583	$\gg$ 1 153 448	138	$\gg$ 160 312

Table 4.4 Numbers of distinct weighted committee games

4.5.2 Geometry of Committee Games with n=3

In principle, one could characterize the full equivalence class of committee games for each reference distribution that we list in Appendix B. We have indicated how in Section 4.3.2. But computation of the respective partition of $\mathbb{N}_0^n$ is very arduous –

[11] We used 128 GB RAM and eight 3.0 GHz cores. Some instances ran for more than two months.

much more than determining into which classes given games $[N, A, r, \mathbf{w}]$ fall.

We have done the latter to obtain a first overview of the geometry of committee games. Our illustrations differ in content but echo the geometric approach to voting espoused by Saari (1995, 2001). His eponymous triangles concern $m = 3$ alternatives and consider an arbitrary number n of voters. They illuminate how collective rankings vary with the applicable voting procedure for given preferences.

We, by contrast, assume $n = 3$ voter groups or players and consider arbitrary fixed numbers of alternatives. The points in our triangles correspond to voting weight distributions; colors group them into equivalence classes. We use the standard projection of the 3-dimensional unit simplex of relative weights to the plane, illustrated in Figure 4.1: extreme points of the resulting equilateral triangle match committees in which only one of the groups has voting rights; the midpoint reflects equal numbers of votes for each group. The relative weight axes are suppressed in subsequent figures. Points of identical color correspond to structurally equivalent weight distributions, i.e., they induce isomorphic committee games for the voting rule r under investigation. We can thus depict the partition of all non-degenerate weight distributions $\mathbf{w} \in \mathbb{N}_0^3$ into equivalence classes $\mathcal{E}^r_{\mathbf{w},m}$ in terms of relative vote shares. When classes correspond to line segments or single points in the simplex, we have manually enlarged these in Figures 4.2–4.4 in order to improve visibility.

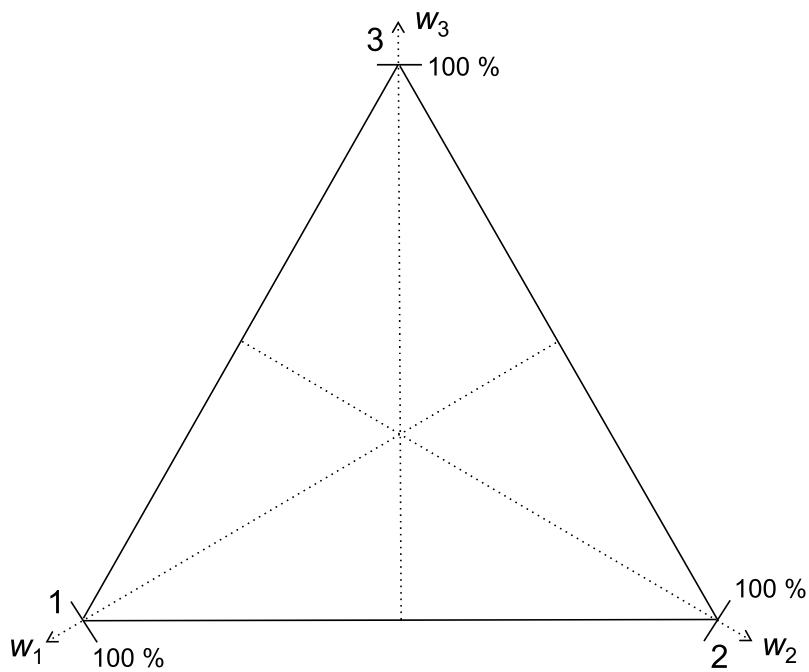

Figure 4.1 Simplex of all distributions of relative voting weights for $n = 3$

4.5.2.1 Copeland Committees

Figure 4.2 shows all Copeland committees with three players. The four equivalence classes $\mathcal{E}^{r^C}_{\bar{\mathbf{w}},m}$ with $\bar{\mathbf{w}} \in \{(1,0,0),(1,1,0),(1,1,1),(2,1,1)\}$, $m \geq 2$, can be identified as follows. The dark blue triangles in the corners collect all weight distributions in $\mathcal{E}^{r^C}_{(1,0,0),m}$: one group with more than 50% of the votes can impose its preferred alternative as a dictator. The green lines cover all weight distributions in $\mathcal{E}^{r^C}_{(2,1,1),m}$: one player holds 50% of the votes, the others share the rest in an arbitrary positive proportion. The three black points depict situations in which two players have equal positive numbers of votes while the third has no votes, i.e., $\mathcal{E}^{r^C}_{(1,1,0),m}$. The yellow triangle in the middle reflects the many equivalent weight configurations in $\mathcal{E}^{r^C}_{(1,1,1),m}$: each player wields a positive number of votes below half the total. As known from the analysis of binary weighted voting games, weights do not matter inside the central triangle: quite dissimilar distributions like (33, 33, 33) and (49, 49, 1) induce the same pairwise majorities; hence possibilities for players to influence outcomes and to achieve their goals are identical.

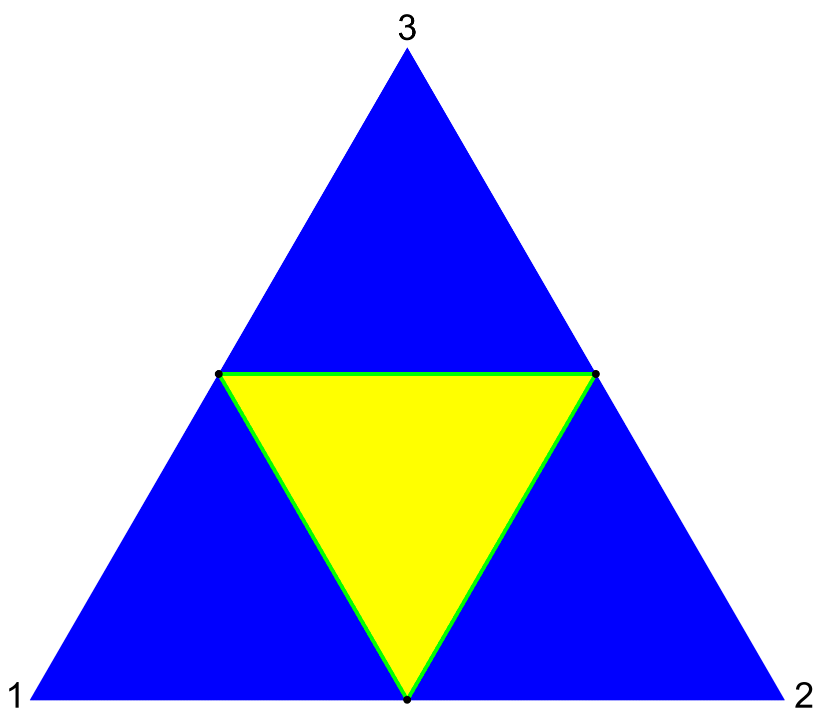

Figure 4.2 The four Copeland equivalence classes for $m \geq 2$

4.5.2.2 Plurality Committees

Figure 4.3 illustrates the situation for $m \geq 3$ when plurality rule r^P is used. Weight vectors **w** that belong to Copeland class $\mathcal{E}^{r^C}_{(1,1,1),m}$ split into the plurality classes $\mathcal{E}^{r^P}_{(1,1,1),m}$ with identical weights for all three players, $\mathcal{E}^{r^P}_{(2,2,1),m}$ and $\mathcal{E}^{r^P}_{(3,2,2),m}$. The former corresponds to weights on the orange lines that point to the center: two players each have a plurality of votes. The latter class involves just one plurality player.

For non-dictatorial weight configurations, plurality rule is more sensitive to the configuration of seats or voting rights than Copeland rule. This becomes more pronounced the more players are involved: Table 4.4 shows that there are about four and 32 times more structurally different committees with plurality than Copeland rule for $n = 4$ and 5, respectively; we conjecture this factor exceeds 1 000 for $n = 6$.

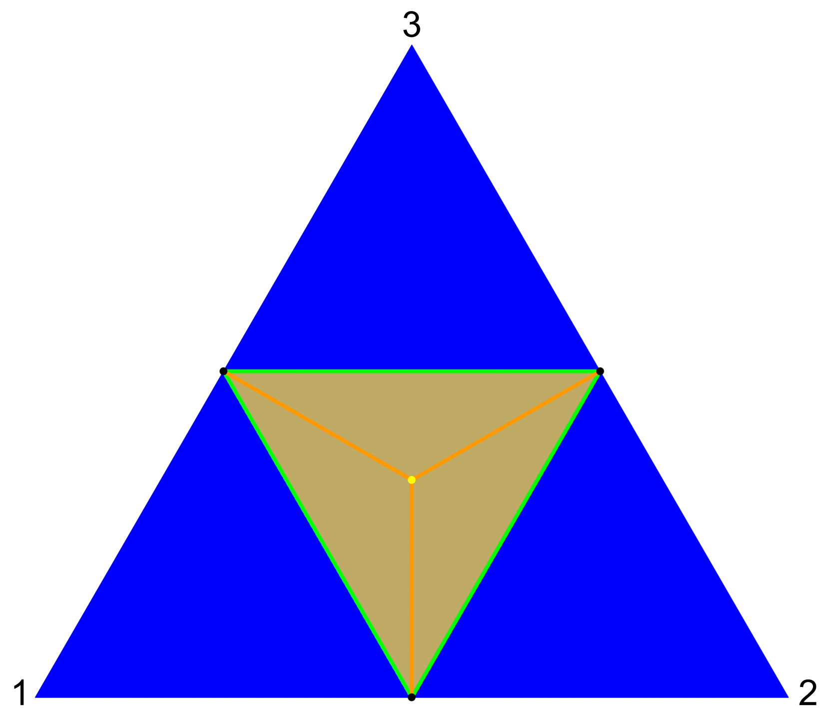

Figure 4.3 The six plurality equivalence classes for $m \geq 3$

4.5.2.3 Antiplurality Committees

In Figure 4.4, the dark blue triangles that reflected existence of a dictator player under r^C and r^P in Figures 4.2 and 4.3 shrink to the three vertices for antiplurality rule. Only the degenerate case, in which no one else has positive weight, has outcomes determined by one player's preferences alone. Otherwise, even a single vote may veto and disqualify an alternative under r^A.

Equivalence classes $\mathcal{E}^{r^A}_{\bar{\mathbf{w}},3}$ with $\bar{\mathbf{w}} \in \{(1,0,0),(1,1,0),(1,1,1),(2,1,1),(2,2,1)\}$ differ according to whether one (blue vertices), two (dark green edges), or all three players have positive weight. The latter case comes with the possibility that none (yellow center), one (orange lines), or two of them (light green triangles) have greater weight than others and hence elevated roles if the players each vote against a different alternative. For $m = 4$, this distinction becomes obsolete because there is always at least one alternative not disapproved by anyone (Proposition 4.6). Then there are just three classes $\mathcal{E}^{r^A}_{\bar{\mathbf{w}},4}$ with $\bar{\mathbf{w}} \in \{(1,0,0),(1,1,0),(1,1,1)\}$.

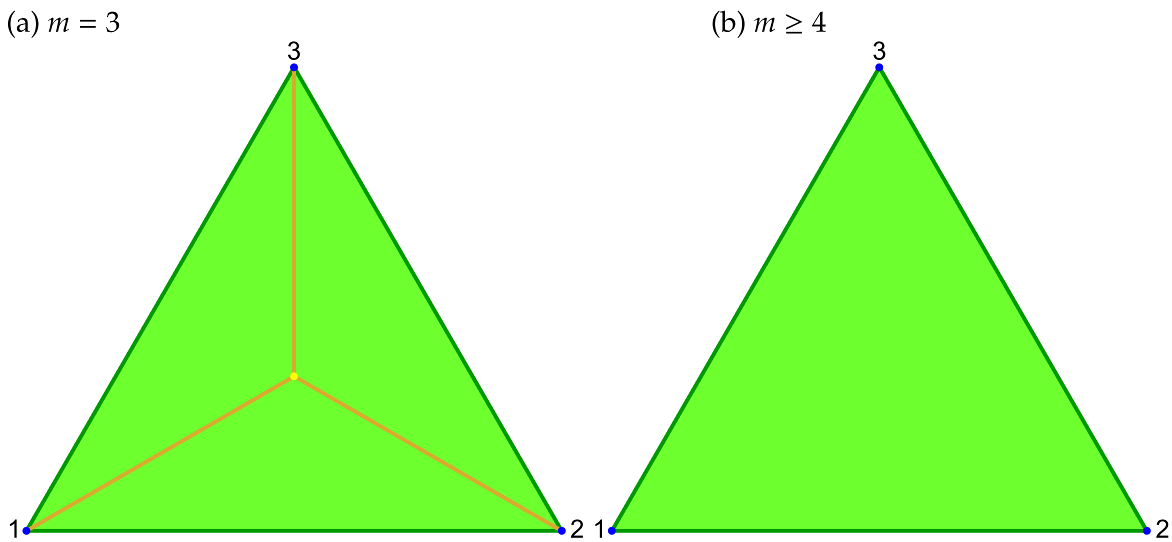

Figure 4.4 The five or three antiplurality equivalence classes

4.5.2.4 Borda Committees

Visually the most interesting geometry of committee games is induced by Borda rule. Figures 4.2 and 4.5–4.7 illustrate the quick increase in equivalence classes as the number of alternatives rises. (Recall that Figure 4.2 captures the case of $m = 2$ for all rules $r \in \{r^A, r^B, r^C, r^P\}$ by Proposition 4.1.)

The pictures indicate how sensitive Borda decision structures are to the underlying vote distribution – the more alternatives, the higher the sensitivity. This need not make a big difference in practice. Incidences of $r^B|\mathbf{w}(\mathbf{P}) \neq r^B|\mathbf{w}'(\mathbf{P})$ for similar $\mathbf{w}, \mathbf{w}'$ imply that the respective committee games differ; but depending on the context at hand, corresponding preference profiles $\mathbf{P}$ may have zero or smaller probability than

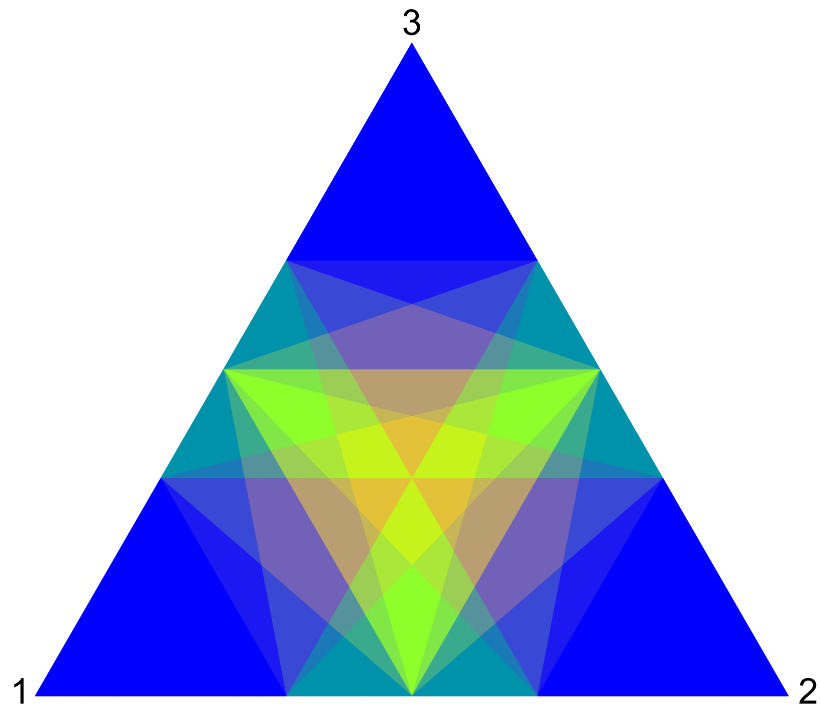

Figure 4.5 The 51 Borda equivalence classes for $m = 3$

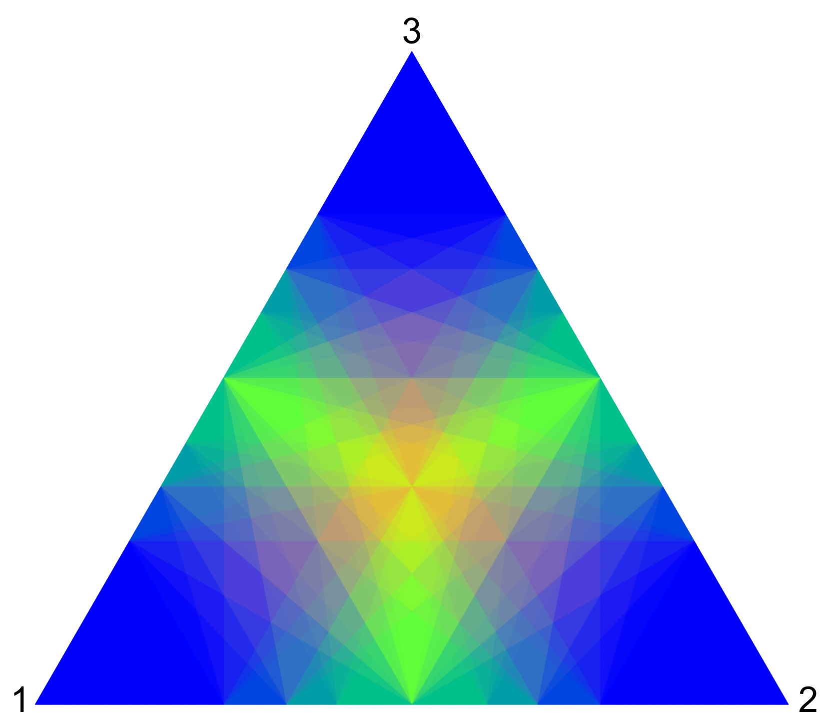

Figure 4.6 The 505 Borda equivalence classes for $m = 4$

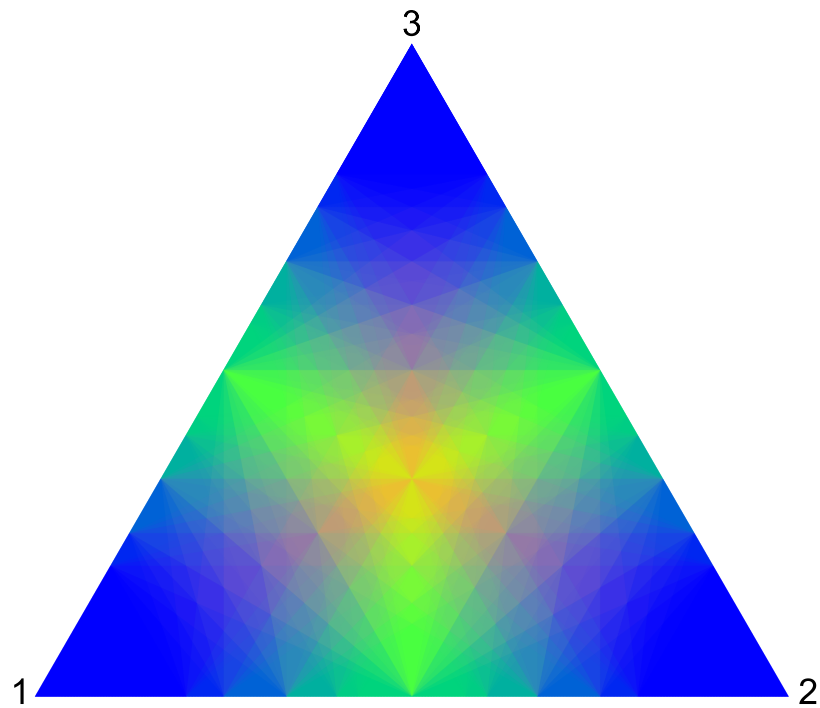

Figure 4.7 At least 2251 Borda equivalence classes for $m = 5$

profiles $\mathbf{P}'$ such that $r^B|\mathbf{w}(\mathbf{P}') = r^B|\mathbf{w}'(\mathbf{P}')$.[12] Still, from an a priori perspective the three other considered rules, r^A, r^C and r^P, involve less scope than r^B for changes in the distribution of seats or voting rights to induce different decisions.

The dark blue triangles in the corners of Figures 4.5–4.7 are smaller than those in Figures 4.2–4.3 for Copeland and plurality rule. This attests to the fact that the minimal weight w_1 required to make player 1 a dictator and players 2 and 3 null players is bigger: having 50% plus one vote suffices to win all pairwise comparisons and plurality votes while more than two thirds are needed to secure that one's top-ranked alternative is the Borda winner. The required weight increases in m.[13]

[12] Our color choices provide a rough guide to how much two mappings $r|\mathbf{w}$ and $r|\mathbf{w}'$ differ: points of similar color correspond to committees whose decisions differ for few profiles.

[13] Player 1's relative weight must exceed $(m-1)/m$ to be a Borda dictator. This was already observed by Borda in 1781 and follows from the condition that unanimous players $2, \ldots, n$ cannot make 1's second choice the winner. Moulin (1982) studies a more nuanced concept of veto power for Borda and Copeland rule, which translates to lighter colors in our figures.

4.6 Concluding Remarks

Equivalence of different distributions of seats, drawing rights, voting stock, etc. depends highly on whether decisions involve two, three, or more alternatives. Weight distributions such as $(6, 5, 2)$, $(5, 5, 5)$, or $(48\%, 24\%, 28\%)$ are equivalent for binary majority choices but not more generally. Scope for weight differences to matter has been formalized and compared across rules in this chapter.

The only Condorcet method that we featured here, i.e., Copeland rule, behaves somewhat at odds with the others: it extends the equivalences known for dichotomous choice problems to arbitrarily many options (Proposition 4.3). This might not feel surprising because winners in Copeland games are selected by binary comparisons. Is it, therefore, okay to apply insights and tools for binary voting, such as the Shapley-Shubik or Penrose-Banzhaf power indices, also to voting bodies that face non-binary options as long as the pertinent rules satisfy the Condorcet winner criterion?

This conjecture is wrong. The Copeland method is special insofar that it invokes ordinal evaluations only; most other Condorcet methods also use information on victory margins, rank positions, or distances. More alternatives then generate more scope for decisions to be sensitive to the seat distribution. Proposition 4.3 fails to generalize, for example, to Black committee games. The *Black rule* selects the Condorcet winner if one exists and otherwise uses Borda scores to break cyclical majorities. Weight distributions of $(6, 4, 3)$ and $(4, 4, 2)$ are equivalent for $m = 2$ and give rise to a cycle $a \succ_M^{\mathbf{P}} b \succ_M^{\mathbf{P}} c \succ_M^{\mathbf{P}} a$ for profile $\mathbf{P} = (cab, abc, bca)$. The Black winner is c for the former weight distribution, with a score of 15; but a wins with a score of 12 for the latter. Hence they are non-equivalent for $m = 3$. The same applies to *Kemeny–Young rule*, which minimizes total pairwise disagreements (Kemeny distances) between the rankings in profile $\mathbf{P}$ and the collective ranking; or *maximin rule*, where a winner must maximize the minimum support across all pairwise comparisons. There are more distinct Black, Kemeny–Young, or maximin committee games than Copeland ones although all involve Condorcet methods.

There is ample choice for extending the analysis. The list of sensible single-winner voting procedures that could be used by a committee is long (see, e.g., Aleskerov and Kurbanov 1999; Nurmi 2006, Ch. 7; or Laslier 2012). We have tentatively tried to identify the number of distinct committee games that involve scoring rules based on arbitrary $\mathbf{s} = (1, s_2, 0) \in \mathbb{Q}^3$ for $n = m = 3$. The numbers of structurally distinct games are roughly M-shaped: they increase from 6 plurality committees to more than 160 for $s_2 = 0.25$, fall to 51 Borda committees for $s_2 = 0.5$, increase again to at least 229 for $s_2 = 0.9$ and then drop sharply to just 5 antiplurality committees for $s_2 = 1$.[14]

The equivalence of seemingly different committee games is of theoretical and applied interest. It is relevant for the design of actual voting bodies such as the IMF's Executive Board, councils of non-governmental organizations, boards of private companies, and possibly even for empirical analysis and forecasting: sampling errors in opinion poll data should matter less, for instance, when population shares of the relevant groups fall into the middle of a big equivalence class of the applicable election rule than for a boundary point.

Whether sensitivity of collective decisions to weight differences is (un)desirable from an institutional perspective depends on context and objectives. Higher sensitivity can give bigger incentives for parties to campaign or for investment into voting stock. However, this needs to be weighed against other properties of the applicable voting methods. Links between the weight distribution and decisions are just one aspect of voting among many – but one that matters beyond binary options.

[14] Illustrations of their geometry for s_2 starting at 0.05 and increasing in steps of 0.05 until $s_2 = 0.95$ are provided in Appendix C. Some reminded us of paintings, e.g., by Bauhaus artists Paul Klee and Johannes Itten.

4.7 Appendix A Proofs

Proof of Lemma 4.1

Consider $\mathbf{w} \neq \mathbf{0} \in \mathbb{N}_0^n$ and the unanimous profile $\mathbf{P} = (P, \ldots, P) \in \mathcal{P}(A)^n$ with $a_2 P a_3 P \ldots P a_m P a_1$. Then $r|\mathbf{0}(\mathbf{P}) = a_1$ but $r|\mathbf{w}(\mathbf{P}) = a_2$ for any $r \in \{r^A, r^B, r^C, r^P\}$.

Proof of Proposition 4.1

For $A = \{a_1, a_2\}$ and arbitrary fixed $\mathbf{w} \neq \mathbf{0} \in \mathbb{N}_0^n$

$$r|\mathbf{w}(\mathbf{P}) = \begin{cases} a_2 & \text{if } \sum_{i:\, a_2 P_i a_1} w_i > \sum_{j:\, a_1 P_j a_2} w_j, \\ a_1 & \text{otherwise} \end{cases}$$

for any $r \in \{r^A, r^B, r^C, r^P\}$. So antiplurality, Borda, Copeland and plurality rule are equivalent and hence have the same equivalence classes.

Proof of Proposition 4.2

Define $w(T) := \sum_{i \in T} w_i$ for $T \subseteq N$. If $w(S) \geq w(N \setminus S)$ then $r^P|\mathbf{w}(\mathbf{P}) = a_1$ and $v(S) = 1$. If $w(S) < w(N \setminus S)$ then $r^P|\mathbf{w}(\mathbf{P}) = a_2$ and $v(S) = 0$. Proposition 4.1 extends this to $r \in \{r^A, r^B, r^C\}$.

Proof of Proposition 4.3

For a given set of alternatives $A = \{a_1, \ldots, a_m\}$ and any subset $A' \subseteq A$ that preserves the order of the alternatives, we denote the *projection* of preference profile $\mathbf{P} \in \mathcal{P}(A)^n$ to A' by $\mathbf{P}\!\downarrow_{A'}$ with $a_k \, P_i \!\downarrow_{A'} \, a_l \;:\Leftrightarrow\; [a_k P_i a_l$ and $a_k, a_l \in A']$. For instance, for $\mathbf{P} = (a_1a_2a_3, a_3a_1a_2, a_2a_3a_1)$ and $A' = \{a_1, a_3\}$ we have $\mathbf{P}\!\downarrow_{A'} = (a_1a_3, a_3a_1, a_3a_1)$. Conversely, if $A' \supseteq A$ is a superset of A with $A' \setminus A = \{a_{m+1}, \ldots, a_{m'}\}$ we define the *lifting* $\mathbf{P}\!\uparrow^{A'}$ of $\mathbf{P} \in \mathcal{P}(A)^n$ to A' by appending alternatives $a_{m+1}, \ldots, a_{m'}$ to each ordering P_i below the lowest-ranked alternative from A. That is, for $\mathbf{P} = (a_1a_2a_3, a_3a_1a_2, a_2a_3a_1)$ and $A' = \{a_1, a_2, a_3, a_4\}$ we have $\mathbf{P}\!\uparrow^{A'} = (a_1a_2a_3a_4, a_3a_1a_2a_4, a_2a_3a_1a_4)$. We let ρ or r refer to whole families of mappings and, for instance, write $\rho(\mathbf{P}) = \rho(\mathbf{P}\!\downarrow_{A'})$ if the same alternative $a^* \in A' \subset A$ happens to win for both A and the smaller set A'.

Now consider $A = \{a_1, \ldots, a_m\}$ for $m > 2$ and any $\mathbf{w}, \mathbf{w}' \in \mathbb{N}_0^n$ such that $(r^C, \mathbf{w}) \nsim_m (r^C, \mathbf{w}')$. So there exists $\mathbf{P} \in \mathcal{P}(A)^n$ with $r^C|\mathbf{w}(\mathbf{P}) \neq r^C|\mathbf{w}'(\mathbf{P})$. The $\mathbf{w}$ and $\mathbf{w}'$-weighted versions of the majority relation differ at $\mathbf{P}$: if all pairwise comparisons produced the same winners for weights $\mathbf{w}$ and $\mathbf{w}'$, identical Copeland winners would follow. So a weak victory of some a_k over some a_l for $\mathbf{w}$ turns into a strict victory of a_l over a_k for $\mathbf{w}'$, i.e.,

$$\sum_{i\colon a_k P_i a_l} w_i \geq \sum_{j\colon a_l P_j a_k} w_j \quad \text{and} \quad \sum_{i\colon a_k P_i a_l} w_i' < \sum_{j\colon a_l P_j a_k} w_j'. \tag{4.1}$$

Then take $A' = \{a_k, a_l\} \subset A$ where $|A'| = 2$ and projection $\mathbf{P}{\downarrow_{A'}}$. (4.1) implies

$$\sum_{i\colon a_k\, P_i{\downarrow_{A'}}\, a_l} w_i \geq \sum_{j\colon a_l\, P_j{\downarrow_{A'}}\, a_k} w_j \quad \text{and} \quad \sum_{i\colon a_k\, P_i{\downarrow_{A'}}\, a_l} w_i' < \sum_{j\colon a_l\, P_j{\downarrow_{A'}}\, a_k} w_j'.$$

If both inequalities are strict or $k < l$ then $r^C|\mathbf{w}(\mathbf{P}{\downarrow_{A'}}) = a_k \neq r^C|\mathbf{w}'(\mathbf{P}{\downarrow_{A'}}) = a_l$ and hence $(r^C, \mathbf{w}) \nsim_2 (r^C, \mathbf{w}')$. If not, a_l wins also for $\mathbf{w}$ by lexicographic tie breaking but we can consider profile $\mathbf{P}' \in \mathcal{P}(A')^n$ with $a_l P_i' a_k \Leftrightarrow a_k P_i{\downarrow_{A'}}\, a_l$ for all $i \in N$. Then $r^C|\mathbf{w}(\mathbf{P}') = a_l \neq r^C|\mathbf{w}'(\mathbf{P}') = a_k$ and $(r^C, \mathbf{w}) \nsim_2 (r^C, \mathbf{w}')$.

Conversely take $A = \{a_1, a_2\}$ and $\mathbf{w}, \mathbf{w}' \in \mathbb{N}_0^n$ such that $(r^C, \mathbf{w}) \nsim_2 (r^C, \mathbf{w}')$ and $r^C|\mathbf{w}(\mathbf{P}) = a_1 \neq r^C|\mathbf{w}'(\mathbf{P}) = a_2$ for some $\mathbf{P} \in \mathcal{P}(A)^n$. Then

$$\sum_{i\colon a_1 P_i a_2} w_i \geq \sum_{j\colon a_2 P_j a_1} w_j \quad \text{and} \quad \sum_{i\colon a_1 P_i a_2} w_i' < \sum_{j\colon a_2 P_j a_1} w_j'. \tag{4.2}$$

Consider $A' = \{a_1, a_2, \ldots, a_m\} \supset A$ where $|A'| = m$ and lifting $\mathbf{P}{\uparrow^{A'}}$. (4.2) implies

$$\sum_{i\colon a_1\, P_i{\uparrow^{A'}} a_2} w_i \geq \sum_{j\colon a_2\, P_j{\uparrow^{A'}} a_1} w_j \quad \text{and} \quad \sum_{i\colon a_1\, P_i{\uparrow^{A'}} a_2} w_i' < \sum_{j\colon a_2\, P_j{\uparrow^{A'}} a_1} w_j'$$

and alternatives $a_3, \ldots, a_m$ lose all weighted majority comparisons against a_1 and a_2 by construction of $\mathbf{P}{\uparrow^{A'}}$. So $r^C|\mathbf{w}(\mathbf{P}{\uparrow^{A'}}) = a_1 \neq r^C|\mathbf{w}'(\mathbf{P}{\uparrow^{A'}}) = a_2$. Hence $(r^C, \mathbf{w}) \nsim_m (r^C, \mathbf{w}')$. In summary, $(r^C, \mathbf{w}) \nsim_2 (r^C, \mathbf{w}') \Leftrightarrow (r^C, \mathbf{w}) \nsim_m (r^C, \mathbf{w}')$ and, a fortiori, $(r^C, \mathbf{w}) \sim_2 (r^C, \mathbf{w}') \Leftrightarrow (r^C, \mathbf{w}) \sim_m (r^C, \mathbf{w}')$.

Proof of Proposition 4.4

Let $m > n$. Consider $A = \{a_1, \ldots, a_m\}$ and any $\mathbf{w}, \mathbf{w}' \in \mathbb{N}_0^n$ such that $(r^P, \mathbf{w}) \nsim_m (r^P, \mathbf{w}')$. So there exists $\mathbf{P} \in \mathcal{P}(A)^n$ with $r^P|\mathbf{w}(\mathbf{P}) = a_k \neq r^P|\mathbf{w}'(\mathbf{P}) = a_l$. For this $\mathbf{P}$ let

$$\hat{A} := \left\{a \mid \exists i \in N \colon \forall a' \neq a \colon aP_ia'\right\}$$

denote the set of all alternatives that are top-ranked by some voter. (Obviously, $a_k, a_l \in \hat{A}$.) Now define $A' \subset A$ as the union of $\hat{A}$ and some arbitrary elements of $A \setminus \hat{A}$ such that $|A'| = n$. By construction, each $a \in A'$ has the same weighted number of top positions for projection $\mathbf{P}{\downarrow_{A'}}$ as it had for $\mathbf{P}$. So $r^P|\mathbf{w}(\mathbf{P}{\downarrow_{A'}}) = a_k \neq r^P|\mathbf{w}'(\mathbf{P}{\downarrow_{A'}}) = a_l$. Hence $(r^P, \mathbf{w}) \nsim_n (r^P, \mathbf{w}')$.

Analogously, consider $A = \{a_1, \ldots, a_n\}$ and $\mathbf{w}, \mathbf{w}' \in \mathbb{N}_0^n$ such that $(r^P, \mathbf{w}) \nsim_n (r^P, \mathbf{w}')$. A profile $\mathbf{P} \in \mathcal{P}(A)^n$ with $r^P|\mathbf{w}(\mathbf{P}) = a_k \neq r^P|\mathbf{w}'(\mathbf{P}) = a_l$ can then be lifted to $A' = A \cup \{a_{n+1}, \ldots, a_m\}$. By construction, $r^P|\mathbf{w}(\mathbf{P}{\uparrow^{A'}}) = a_k \neq r^P|\mathbf{w}'(\mathbf{P}{\uparrow^{A'}}) = a_l$. Hence $(r^P, \mathbf{w}) \nsim_m (r^P, \mathbf{w}')$. Overall, we can conclude $(r^P, \mathbf{w}) \sim_m (r^P, \mathbf{w}') \Leftrightarrow (r^P, \mathbf{w}) \sim_n (r^P, \mathbf{w}')$.

Proof of Proposition 4.5

Let $k > j$ for otherwise arbitrary $j, k \in \{1, \ldots, m\}$. Consider $A = \{a_1, \ldots, a_m\}$ and any profile $\mathbf{P} \in \mathcal{P}(A)^n$ such that player 1 prefers a_2 most and ranks all remaining alternatives lexicographically while player 2 ranks a_2 in k-th position and otherwise agrees with player 1, i.e., suppose $a_2\, P_1\, a_1\, P_1\, a_3\, P_1\, a_4\, \ldots\, a_m$ and $a_1\, P_2\, a_3\, P_2\, a_4\, \ldots\, a_k\, P_2\, a_2\, P_2\, a_{k+1}\, P_2\, a_{k+2}\, \ldots\, a_m$.

The Borda score $j\cdot(m-2)+(m-1)$ of a_1 under $\tilde{\mathbf{w}}_\mathbf{j}$ is at least as big as the corresponding score $j\cdot(m-1)+(m-k)$ of a_2. Since scores of $a_3, \ldots, a_m$ are all strictly smaller than that of a_1, we have $r^B|\tilde{\mathbf{w}}_\mathbf{j}(P) = a_1$. With $\tilde{\mathbf{w}}_\mathbf{k}$, by contrast, a_1's weighted score $k \cdot (m-2) + (m-1)$ is strictly smaller than a_2's corresponding score $k \cdot (m-1) + (m-k)$. Scores of $a_3, \ldots, a_m$ remain smaller than a_1's. So $r^B|\tilde{\mathbf{w}}_\mathbf{k}(P) = a_2$. Hence $(r^B, \tilde{\mathbf{w}}_\mathbf{j}) \nsim_m (r^B, \tilde{\mathbf{w}}_\mathbf{k})$.

Proof of Proposition 4.6

The claim is obvious for $n = 1$, as each non-degenerate weight then is equivalent to $w_1 = 1$. So consider $m \geq n+1$ for $n \geq 2$. Let $A = \{a_1, \ldots, a_m\}$ and $\mathbf{P}^\mathbf{i} \in \mathcal{P}(A)^n$ be any preference profile where the first i players rank alternative a_1 last and the remaining $n-i$ players rank alternative a_2 last. Consider any $\bar{\mathbf{w}}_\mathbf{k}$ and $\bar{\mathbf{w}}_\mathbf{l}$ with $k < l$. Then $r^A|\bar{\mathbf{w}}_\mathbf{k}(\mathbf{P}^\mathbf{k}) = a_2 \neq r^A|\bar{\mathbf{w}}_\mathbf{l}(\mathbf{P}^\mathbf{k}) = a_3$. So $\mathcal{E}^{r^A}_{\bar{\mathbf{w}}_\mathbf{1},m}, \mathcal{E}^{r^A}_{\bar{\mathbf{w}}_\mathbf{2},m}, \ldots, \mathcal{E}^{r^A}_{\bar{\mathbf{w}}_\mathbf{n},m}$ all differ.

Now assume some $\mathbf{w} \in \mathbb{N}_0^n \setminus \{\mathbf{0}\}$ with $w_1 \geq w_2 \geq \ldots \geq w_n$ satisfies $(r^A, \mathbf{w}) \nsim_m (r^A, \bar{\mathbf{w}}_\mathbf{k})$ for all $k \in \{1, \ldots, n\}$. Let l denote the index such that $w_l > 0$ and $w_{l+1} = 0$. Then both $r^A|\mathbf{w}(\mathbf{P})$ and $r^A|\bar{\mathbf{w}}_\mathbf{l}(\mathbf{P})$ equal the lexicographically minimal element in set

$$Z^l(\mathbf{P}) := \left\{ a \in A \mid \forall i \in \{1, \ldots, l\} \colon \exists a' \in A \colon aP_i a' \right\}$$

that collects all alternatives not ranked last by any of the players who have positive weight. These coincide for $\mathbf{w}$ and $\bar{\mathbf{w}}_\mathbf{l}$; and $Z^l(\mathbf{P})$ is non-empty because $m \geq n+1$. This holds for arbitrary $\mathbf{P} \in \mathcal{P}(A)^n$. Hence $r^A|\mathbf{w} \equiv r^A|\bar{\mathbf{w}}_\mathbf{l}$, contradicting the assumption that $(r^A, \mathbf{w}) \nsim_m (r^A, \bar{\mathbf{w}}_\mathbf{k})$ for all $k \in \{1, \ldots, n\}$. Consequently, $\mathcal{E}^{r^A}_{\bar{\mathbf{w}}_\mathbf{1},m}, \mathcal{E}^{r^A}_{\bar{\mathbf{w}}_\mathbf{2},m}, \ldots, \mathcal{E}^{r^A}_{\bar{\mathbf{w}}_\mathbf{n},m}$ are all antiplurality classes that exist for $m \geq n+1$ (plus the degenerate $\mathcal{E}_{\mathbf{0},m}$).

4.8 Appendix B Minimal Representations of Committees

n,m	Minimal $\bar{\mathbf{w}}$ for all antiplurality classes $\mathcal{E}^{r^A}_{\bar{\mathbf{w}},m}$			
$3,3$	1. (1,0,0)	3. (1,1,1)	5. (2,2,1)	
	2. (1,1,0)	4. (2,1,1)		
$3,m \geq 4$	1. (1,0,0)	2. (1,1,0)	3. (1,1,1)	
$4,3$	1. (1,0,0,0)	6. (2,1,1,1)	11. (3,2,2,1)	16. (4,3,2,2)
	2. (1,1,0,0)	7. (2,2,1,0)	12. (3,3,1,1)	17. (4,4,2,1)
	3. (1,1,1,0)	8. (2,2,1,1)	13. (3,3,2,1)	18. (4,4,3,2)
	4. (1,1,1,1)	9. (2,2,2,1)	14. (3,3,2,2)	19. (5,4,3,2)
	5. (2,1,1,0)	10. (3,2,1,1)	15. (4,3,2,1)	
$4,4$	1. (1,0,0,0)	3. (1,1,1,0)	5. (2,1,1,1)	7. (2,2,2,1)
	2. (1,1,0,0)	4. (1,1,1,1)	6. (2,2,1,1)	
$4,m \geq 5$	1. (1,0,0,0)	2. (1,1,0,0)	3. (1,1,1,0)	4. (1,1,1,1)

Table 4.5 Minimal representations of different antiplurality committees

n,m	Minimal $\bar{\mathbf{w}}$ for all Borda classes $\mathcal{E}^{r^B}_{\bar{\mathbf{w}},3}$			
$3,3$	1. (1,0,0)	14. (3,3,2)	27. (5,4,3)	40. (8,6,3)
	2. (1,1,0)	15. (4,3,1)	28. (7,4,1)	41. (9,6,2)
	3. (1,1,1)	16. (5,2,1)	29. (6,5,2)	42. (8,7,3)
	4. (2,1,0)	17. (4,3,2)	30. (7,5,1)	43. (8,6,5)
	5. (2,1,1)	18. (5,2,2)	31. (6,5,3)	44. (10,7,2)
	6. (2,2,1)	19. (5,3,1)	32. (7,5,2)	45. (11,7,2)
	7. (3,1,1)	20. (4,3,3)	33. (8,5,1)	46. (9,7,5)
	8. (3,2,0)	21. (5,4,1)	34. (6,5,4)	47. (10,8,3)
	9. (3,2,1)	22. (6,3,1)	35. (7,5,3)	48. (11,8,2)
	10. (4,1,1)	23. (5,3,3)	36. (7,6,2)	49. (11,9,3)
	11. (3,2,2)	24. (5,4,2)	37. (8,5,2)	50. (13,8,2)
	12. (3,3,1)	25. (6,4,1)	38. (7,5,4)	51. (12,9,7)
	13. (4,2,1)	26. (7,2,2)	39. (7,6,4)	

Table 4.6 Minimal representations of different Borda committees

n	Minimal $\bar{\mathbf{w}}$ for all Copeland classes $\mathcal{E}^{r^C}_{\bar{\mathbf{w}},m}$ and for all classes $\mathcal{E}^{r}_{\bar{\mathbf{w}},2}$ when $r \in \left\{r^A, r^B, r^P\right\}$ and for all weighted voting games $[q;\mathbf{w}]$ with $q = 0.5\sum w_i$							
3	1.	(1,0,0)	2.	(1,1,0)	3.	(1,1,1)	4.	(2,1,1)
4	1.	(1,0,0,0)	4.	(1,1,1,1)	7.	(2,2,1,1)		
	2.	(1,1,0,0)	5.	(2,1,1,0)	8.	(3,1,1,1)		
	3.	(1,1,1,0)	6.	(2,1,1,1)	9.	(3,2,2,1)		
5	1.	(1,0,0,0,0)	8.	(2,1,1,1,1)	15.	(3,2,2,1,0)	22.	(4,3,2,2,1)
	2.	(1,1,0,0,0)	9.	(2,2,1,1,0)	16.	(4,1,1,1,1)	23.	(4,3,3,1,1)
	3.	(1,1,1,0,0)	10.	(3,1,1,1,0)	17.	(3,2,2,1,1)	24.	(5,2,2,2,1)
	4.	(1,1,1,1,0)	11.	(2,2,1,1,1)	18.	(3,2,2,2,1)	25.	(4,3,3,2,2)
	5.	(2,1,1,0,0)	12.	(3,1,1,1,1)	19.	(3,3,2,1,1)	26.	(5,3,3,2,1)
	6.	(1,1,1,1,1)	13.	(2,2,2,1,1)	20.	(4,2,2,1,1)	27.	(5,4,3,2,2)
	7.	(2,1,1,1,0)	14.	(3,2,1,1,1)	21.	(3,3,2,2,2)		
6	see next page . . .							

5	. . . see previous page							
6	1.	(1,0,0,0,0,0)	36.	(3,2,2,2,2,1)	71.	(5,4,3,2,1,1)	106.	(5,5,4,3,3,2)
	2.	(1,1,0,0,0,0)	37.	(3,3,2,2,1,1)	72.	(5,4,3,2,2,0)	107.	(6,4,4,3,3,2)
	3.	(1,1,1,0,0,0)	38.	(3,3,2,2,2,0)	73.	(5,4,4,1,1,1)	108.	(6,5,4,3,2,2)
	4.	(1,1,1,1,0,0)	39.	(3,3,3,1,1,1)	74.	(6,3,2,2,2,1)	109.	(6,5,4,3,3,1)
	5.	(2,1,1,0,0,0)	40.	(4,2,2,2,1,1)	75.	(6,3,3,2,1,1)	110.	(6,5,5,2,2,2)
	6.	(1,1,1,1,1,0)	41.	(4,3,2,1,1,1)	76.	(7,2,2,2,2,1)	111.	(7,4,4,3,2,2)
	7.	(2,1,1,1,0,0)	42.	(4,3,2,2,1,0)	77.	(5,4,3,2,2,1)	112.	(7,5,3,3,2,2)
	8.	(1,1,1,1,1,1)	43.	(4,3,3,1,1,0)	78.	(4,4,3,3,2,2)	113.	(7,5,4,3,2,1)
	9.	(2,1,1,1,1,0)	44.	(5,2,2,1,1,1)	79.	(4,4,3,3,3,1)	114.	(7,5,5,2,2,1)
	10.	(2,2,1,1,0,0)	45.	(5,2,2,2,1,0)	80.	(5,3,3,3,2,2)	115.	(8,4,3,3,2,2)
	11.	(3,1,1,1,0,0)	46.	(3,3,2,2,2,1)	81.	(5,4,3,2,2,2)	116.	(6,5,4,4,3,2)
	12.	(2,1,1,1,1,1)	47.	(4,3,2,2,1,1)	82.	(5,4,3,3,2,1)	117.	(6,5,5,3,3,2)
	13.	(2,2,1,1,1,0)	48.	(4,3,3,1,1,1)	83.	(5,4,4,2,2,1)	118.	(7,5,4,3,3,2)
	14.	(3,1,1,1,1,0)	49.	(5,2,2,2,1,1)	84.	(5,5,3,2,2,1)	119.	(7,5,4,4,2,2)
	15.	(2,2,1,1,1,1)	50.	(3,3,2,2,2,2)	85.	(6,3,3,2,2,2)	120.	(7,5,5,3,3,1)
	16.	(2,2,2,1,1,0)	51.	(3,3,3,2,2,1)	86.	(6,4,3,2,2,1)	121.	(7,6,4,3,2,2)
	17.	(3,1,1,1,1,1)	52.	(4,3,2,2,2,1)	87.	(6,4,3,3,1,1)	122.	(7,6,4,3,3,1)
	18.	(3,2,1,1,1,0)	53.	(4,3,3,2,1,1)	88.	(6,4,4,2,1,1)	123.	(7,6,5,2,2,2)
	19.	(3,2,2,1,0,0)	54.	(4,3,3,2,2,0)	89.	(7,3,3,2,2,1)	124.	(8,5,4,3,2,2)
	20.	(4,1,1,1,1,0)	55.	(4,4,2,2,1,1)	90.	(7,3,3,3,1,1)	125.	(8,5,5,3,2,1)
	21.	(2,2,2,1,1,1)	56.	(4,4,3,1,1,1)	91.	(5,4,3,3,3,2)	126.	(9,4,4,3,2,2)
	22.	(3,2,1,1,1,1)	57.	(5,2,2,2,2,1)	92.	(5,4,4,3,2,2)	127.	(7,5,5,4,3,2)
	23.	(3,2,2,1,1,0)	58.	(5,3,2,2,1,1)	93.	(5,4,4,3,3,1)	128.	(7,6,5,3,3,2)
	24.	(4,1,1,1,1,1)	59.	(5,3,3,1,1,1)	94.	(5,5,3,3,3,1)	129.	(8,5,5,4,2,2)
	25.	(2,2,2,2,1,1)	60.	(5,3,3,2,1,0)	95.	(5,5,4,2,2,2)	130.	(8,6,4,3,3,2)
	26.	(3,2,2,1,1,1)	61.	(6,2,2,2,1,1)	96.	(6,4,3,3,2,2)	131.	(8,6,5,3,3,1)
	27.	(3,2,2,2,1,0)	62.	(4,3,3,2,2,1)	97.	(6,4,4,3,2,1)	132.	(9,5,5,3,2,2)
	28.	(3,3,1,1,1,1)	63.	(5,3,3,2,1,1)	98.	(6,5,3,2,2,2)	133.	(7,6,5,4,4,2)
	29.	(3,3,2,1,1,0)	64.	(4,3,3,2,2,2)	99.	(6,5,3,3,2,1)	134.	(8,6,5,4,3,2)
	30.	(4,2,1,1,1,1)	65.	(4,3,3,3,2,1)	100.	(6,5,4,2,2,1)	135.	(8,7,5,3,3,2)
	31.	(4,2,2,1,1,0)	66.	(4,4,3,2,2,1)	101.	(7,3,3,3,2,2)	136.	(9,6,5,4,2,2)
	32.	(5,1,1,1,1,1)	67.	(5,3,2,2,2,2)	102.	(7,4,3,2,2,2)	137.	(9,7,5,4,3,2)
	33.	(3,2,2,2,1,1)	68.	(5,3,3,2,2,1)	103.	(7,4,4,2,2,1)	138.	(9,7,6,4,4,2)
	34.	(3,3,2,1,1,1)	69.	(5,3,3,3,1,1)	104.	(7,4,4,3,1,1)		
	35.	(4,2,2,1,1,1)	70.	(5,4,2,2,2,1)	105.	(8,3,3,3,2,1)		

Table 4.7 Minimal representation of different Copeland committees for $m \geq 2$, and of different antiplurality, Borda and plurality committees for $m = 2$, and of different weighted voting games with a simple majority

n, m	Minimal $\bar{\mathbf{w}}$ for all plurality classes $\mathcal{E}^{r^P}_{\bar{\mathbf{w}},m}$							
$3, m \geq 3$	1.	(1,0,0)	3.	(1,1,1)	5.	(2,2,1)		
	2.	(1,1,0)	4.	(2,1,1)	6.	(3,2,2)		
$4, 3$	1.	(1,0,0,0)	10.	(2,2,2,1)	19.	(4,3,2,1)	28.	(5,4,3,1)
	2.	(1,1,0,0)	11.	(3,2,1,1)	20.	(4,3,2,2)	29.	(5,4,3,2)
	3.	(1,1,1,0)	12.	(3,2,2,0)	21.	(4,3,3,1)	30.	(6,4,3,2)
	4.	(1,1,1,1)	13.	(3,2,2,1)	22.	(4,4,2,1)	31.	(6,5,3,2)
	5.	(2,1,1,0)	14.	(3,3,1,1)	23.	(5,2,2,2)	32.	(6,5,4,2)
	6.	(2,1,1,1)	15.	(3,2,2,2)	24.	(4,3,3,2)	33.	(7,4,4,2)
	7.	(2,2,1,0)	16.	(3,3,2,1)	25.	(5,3,3,1)	34.	(7,6,4,2)
	8.	(2,2,1,1)	17.	(4,2,2,1)	26.	(5,3,3,2)		
	9.	(3,1,1,1)	18.	(3,3,2,2)	27.	(5,4,2,2)		
$4, m \geq 4$	1.	(1,0,0,0)	10.	(2,2,2,1)	19.	(4,3,2,1)	28.	(5,4,2,2)
	2.	(1,1,0,0)	11.	(3,2,1,1)	20.	(4,3,2,2)	29.	(5,4,3,1)
	3.	(1,1,1,0)	12.	(3,2,2,0)	21.	(4,3,3,1)	30.	(5,4,3,2)
	4.	(1,1,1,1)	13.	(3,2,2,1)	22.	(4,4,2,1)	31.	(5,4,4,2)
	5.	(2,1,1,0)	14.	(3,3,1,1)	23.	(5,2,2,2)	32.	(6,4,3,2)
	6.	(2,1,1,1)	15.	(3,2,2,2)	24.	(4,3,3,2)	33.	(6,5,3,2)
	7.	(2,2,1,0)	16.	(3,3,2,1)	25.	(5,3,3,1)	34.	(6,5,4,2)
	8.	(2,2,1,1)	17.	(4,2,2,1)	26.	(4,4,3,2)	35.	(7,4,4,2)
	9.	(3,1,1,1)	18.	(3,3,2,2)	27.	(5,3,3,2)	36.	(7,6,4,2)

Table 4.8 Minimal representations of different plurality committees

4.9 Appendix C
Geometry of General Scoring Rules for $m = 3$

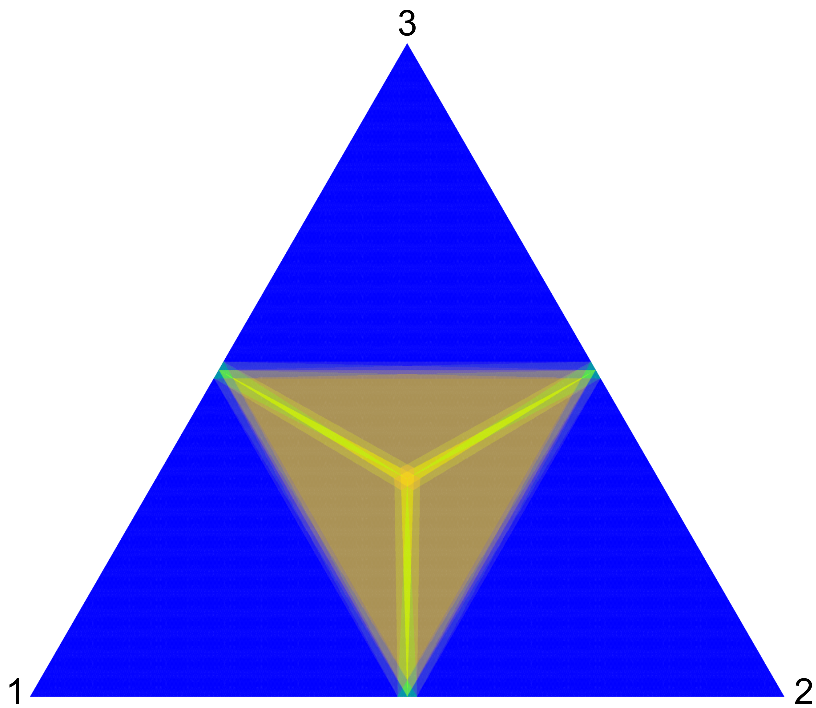

Figure 4.8 At least 162 equivalence classes for $s_2 = 0.05$

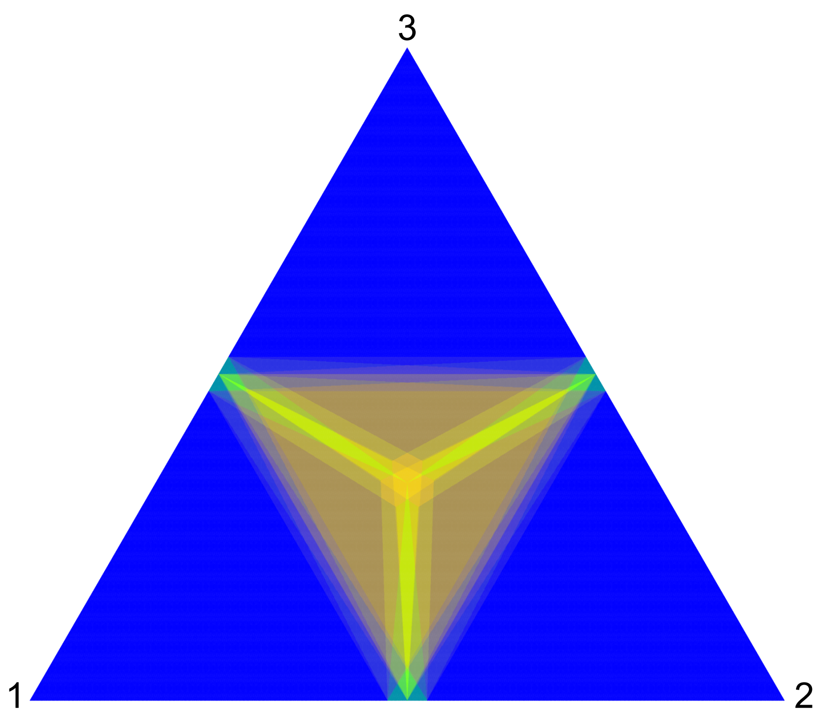

Figure 4.9 At least 163 equivalence classes for $s_2 = 0.10$

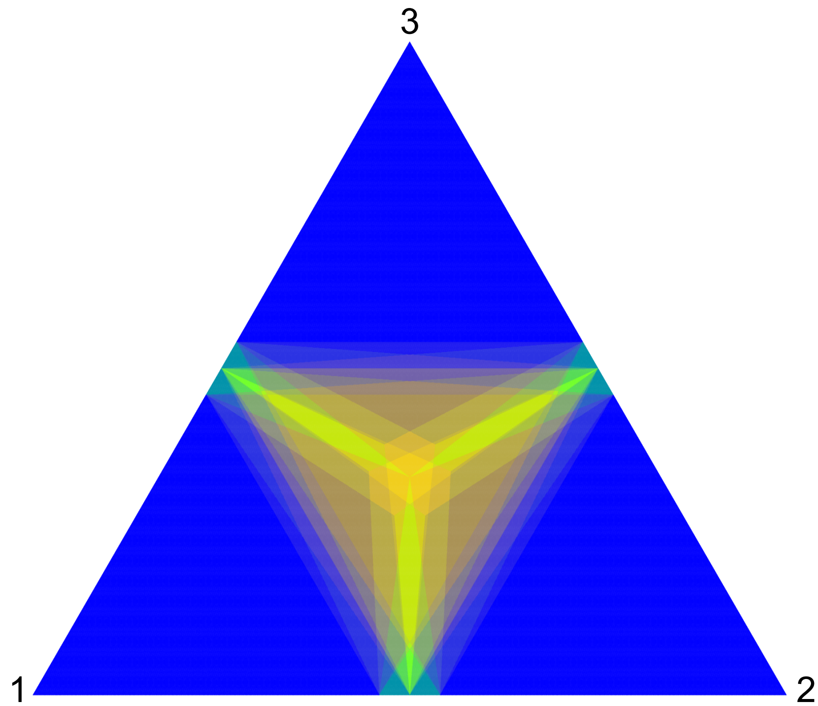

Figure 4.10 At least 162 equivalence classes for $s_2 = 0.15$

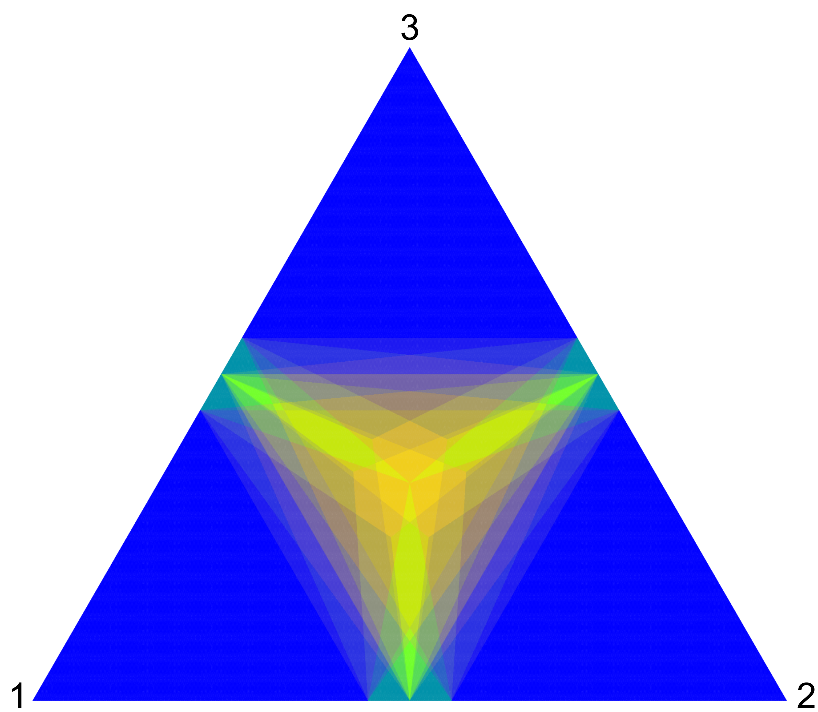

Figure 4.11 At least 163 equivalence classes for $s_2 = 0.20$

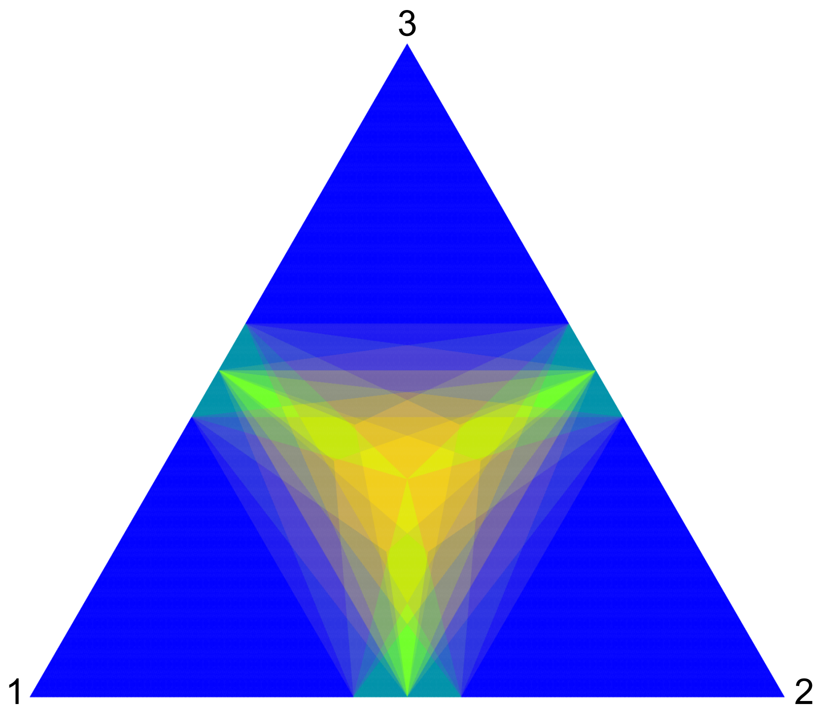

Figure 4.12 At least 163 equivalence classes for $s_2 = 0.25$

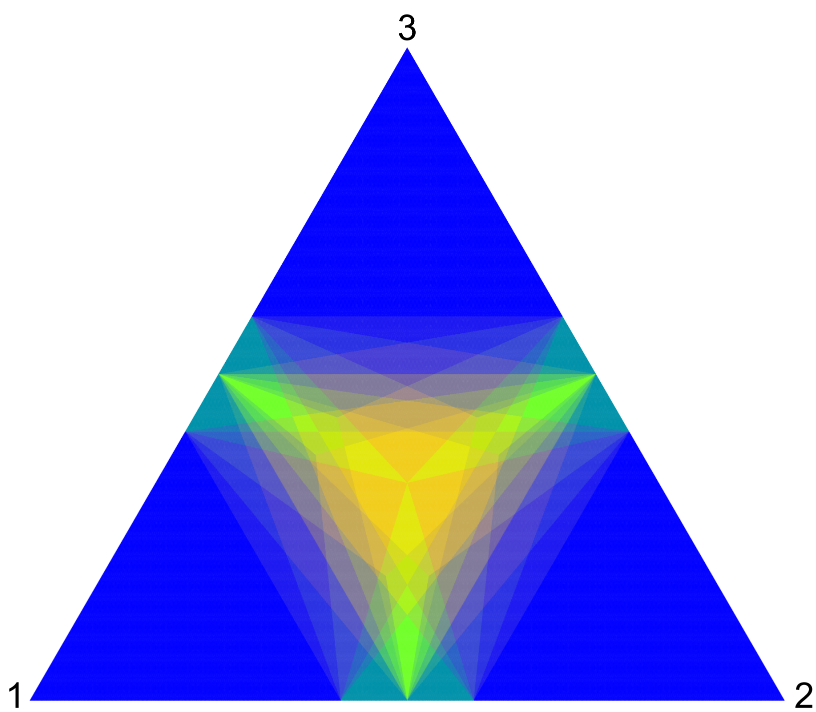

Figure 4.13 At least 154 equivalence classes for $s_2 = 0.30$

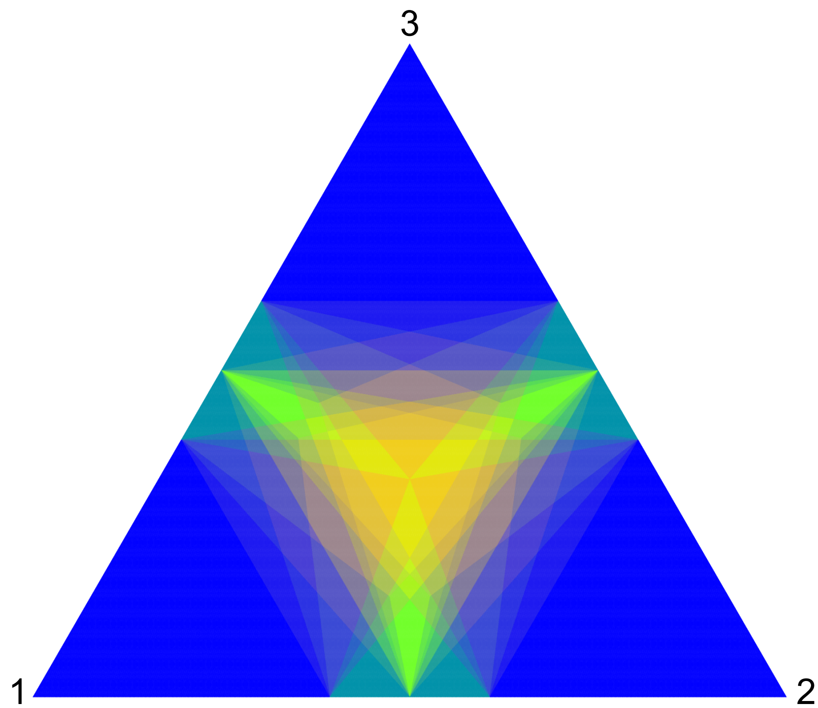

Figure 4.14 At least 146 equivalence classes for $s_2 = 0.35$

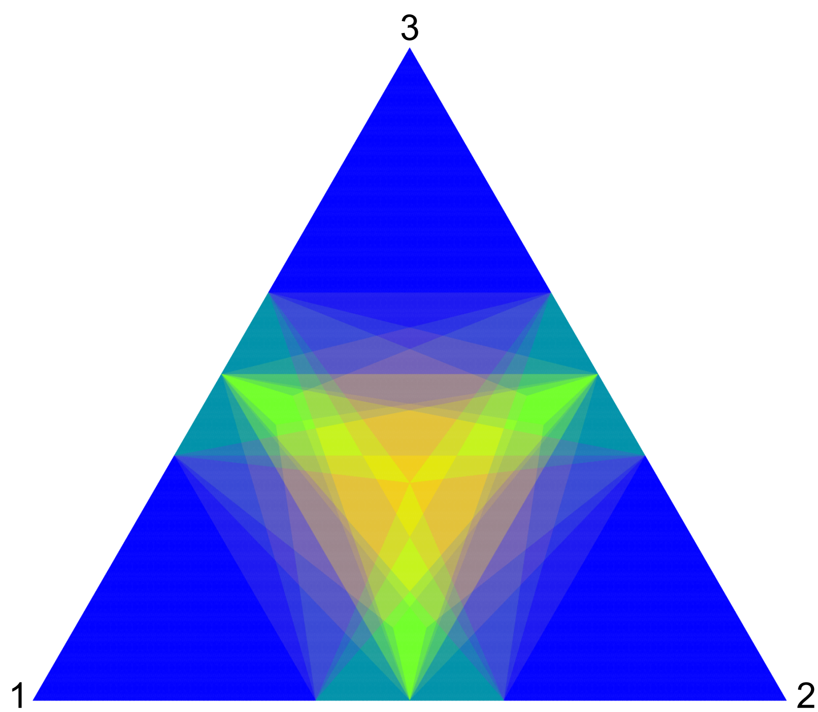

Figure 4.15 At least 147 equivalence classes for $s_2 = 0.40$

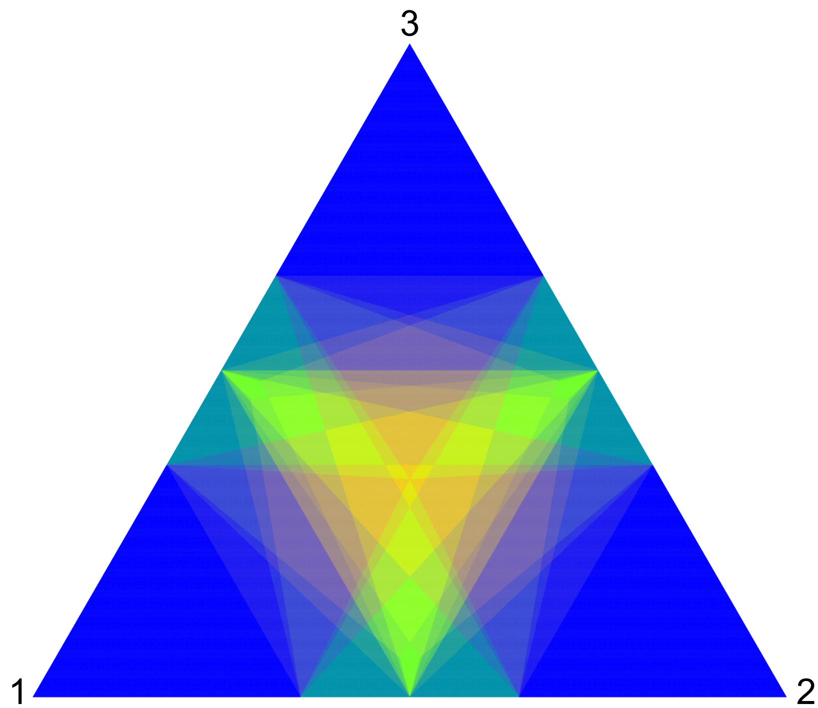

Figure 4.16 At least 146 equivalence classes for $s_2 = 0.45$

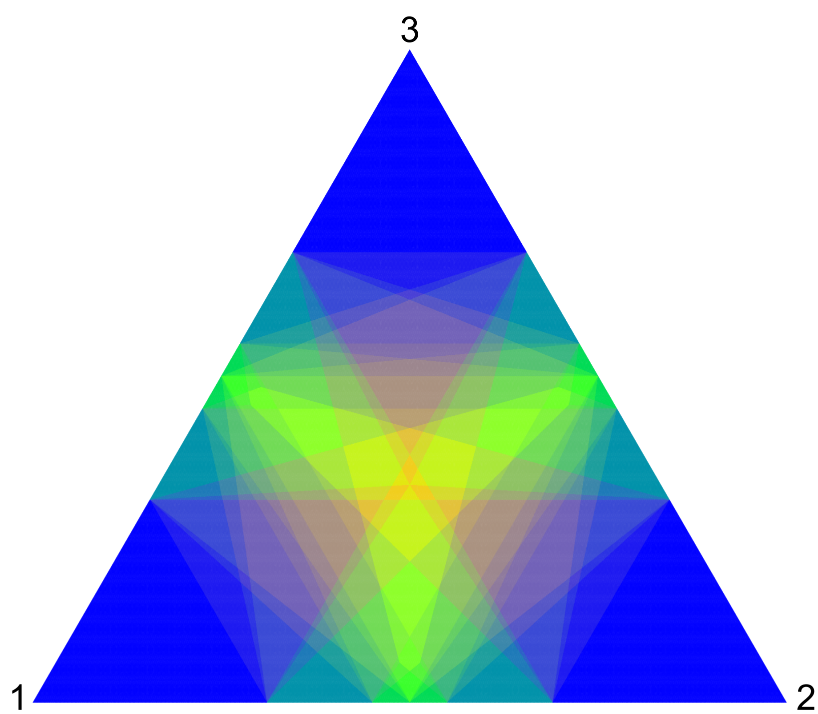

Figure 4.17 At least 216 equivalence classes for $s_2 = 0.55$

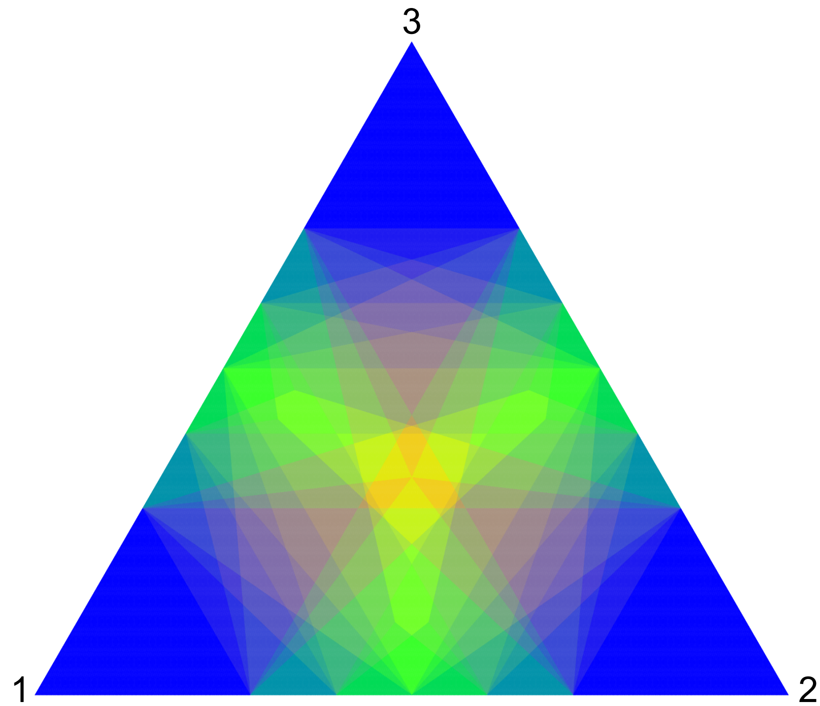

Figure 4.18 At least 216 equivalence classes for $s_2 = 0.60$

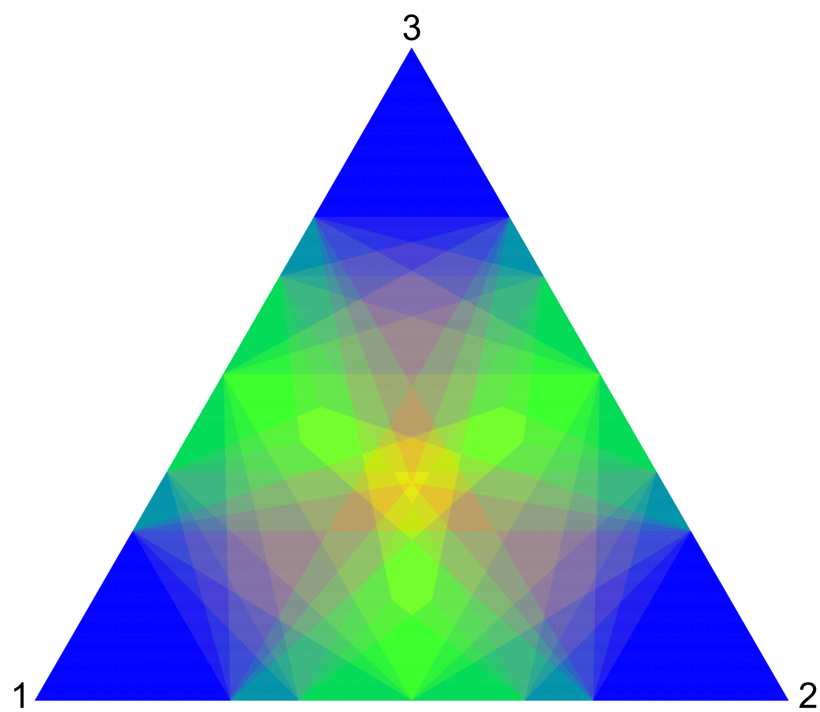

Figure 4.19 At least 216 equivalence classes for $s_2 = 0.65$

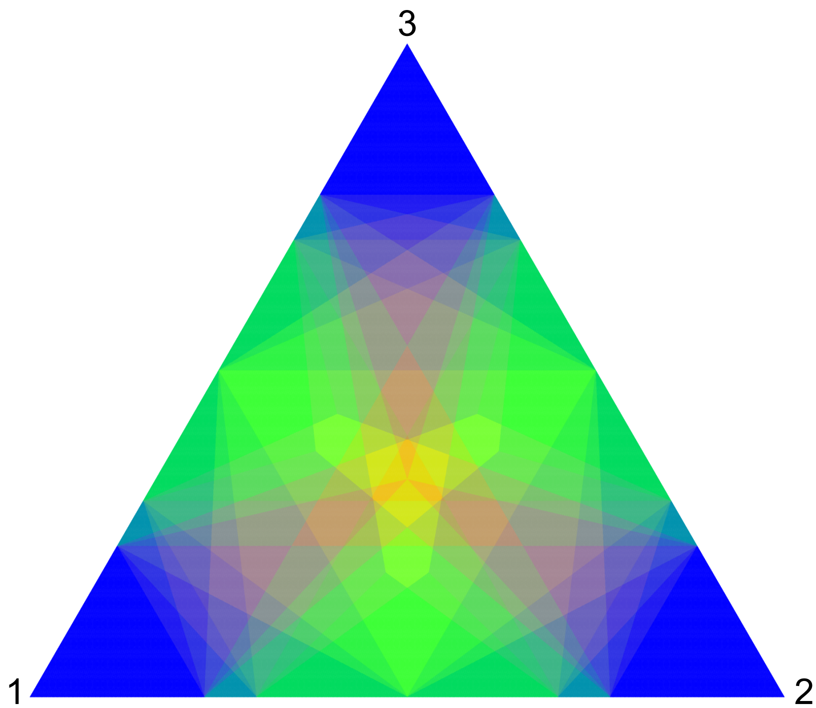

Figure 4.20 At least 220 equivalence classes for $s_2 = 0.70$

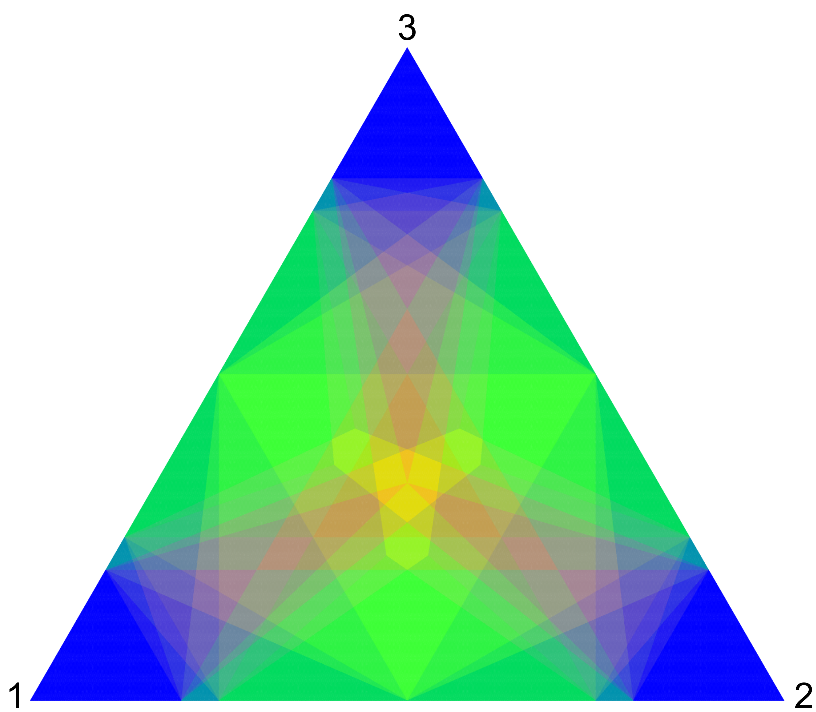

Figure 4.21 At least 211 equivalence classes for $s_2 = 0.75$

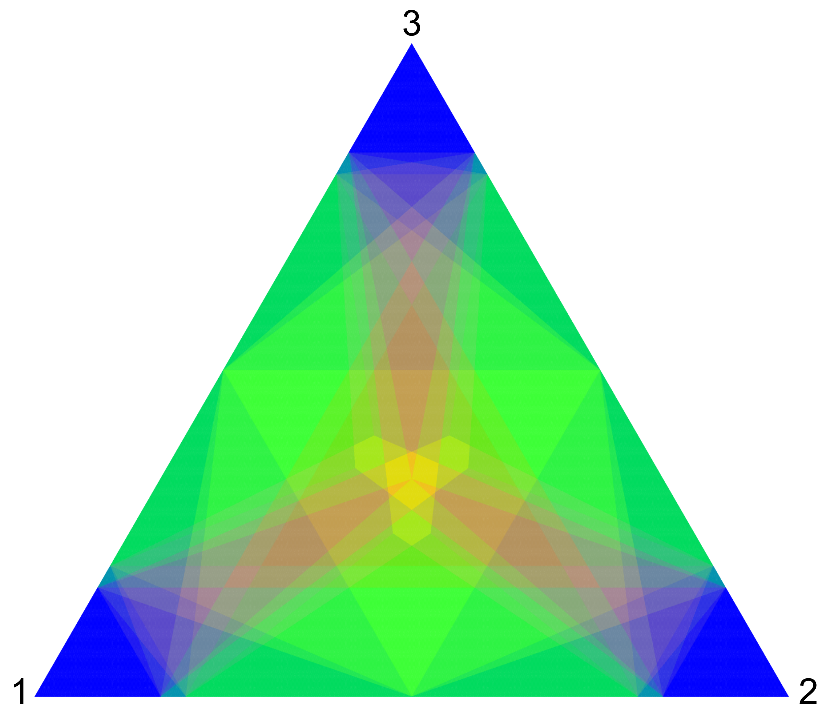

Figure 4.22 At least 217 equivalence classes for $s_2 = 0.80$

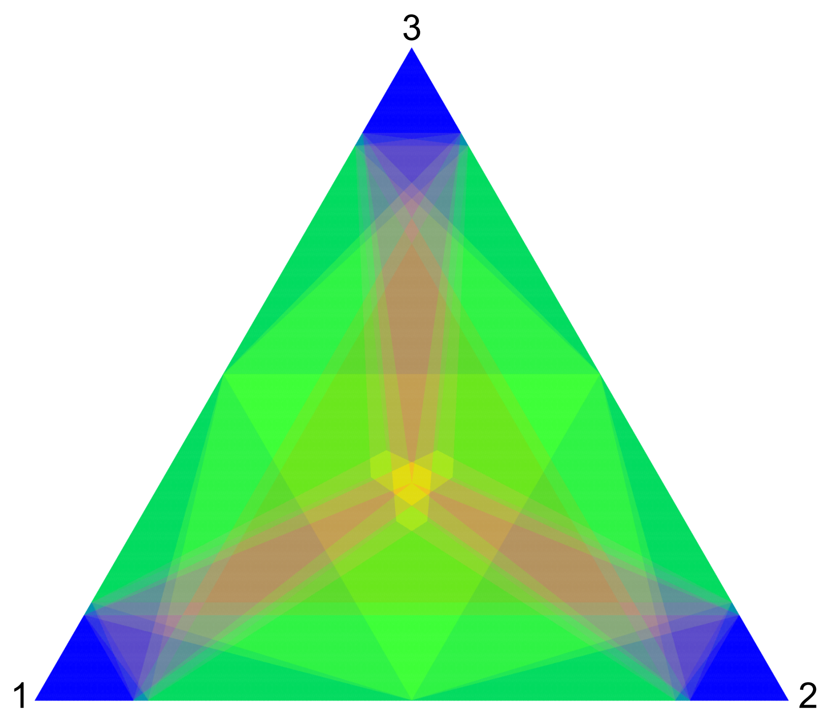

Figure 4.23 At least 228 equivalence classes for $s_2 = 0.85$

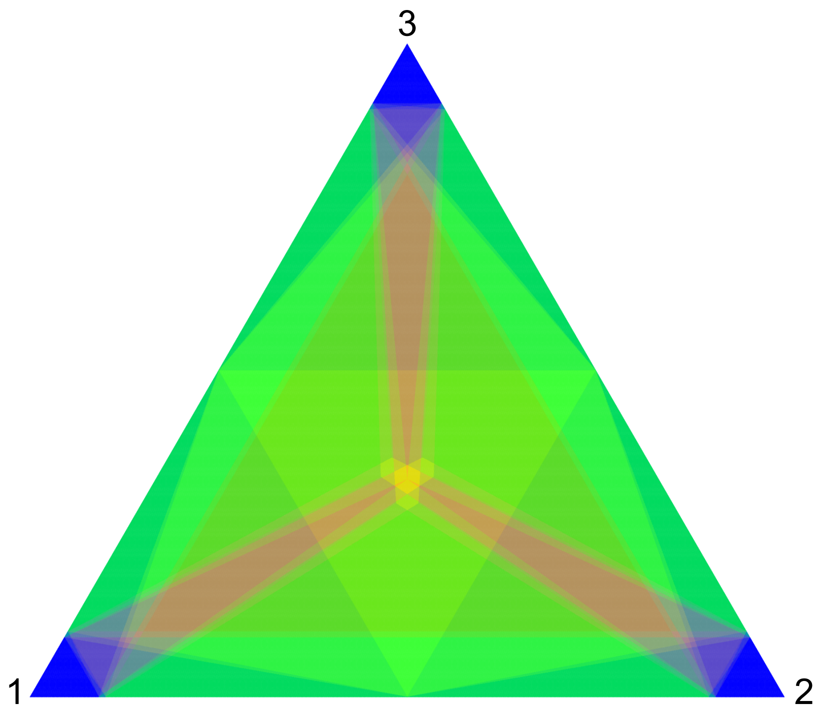

Figure 4.24 At least 229 equivalence classes for $s_2 = 0.90$

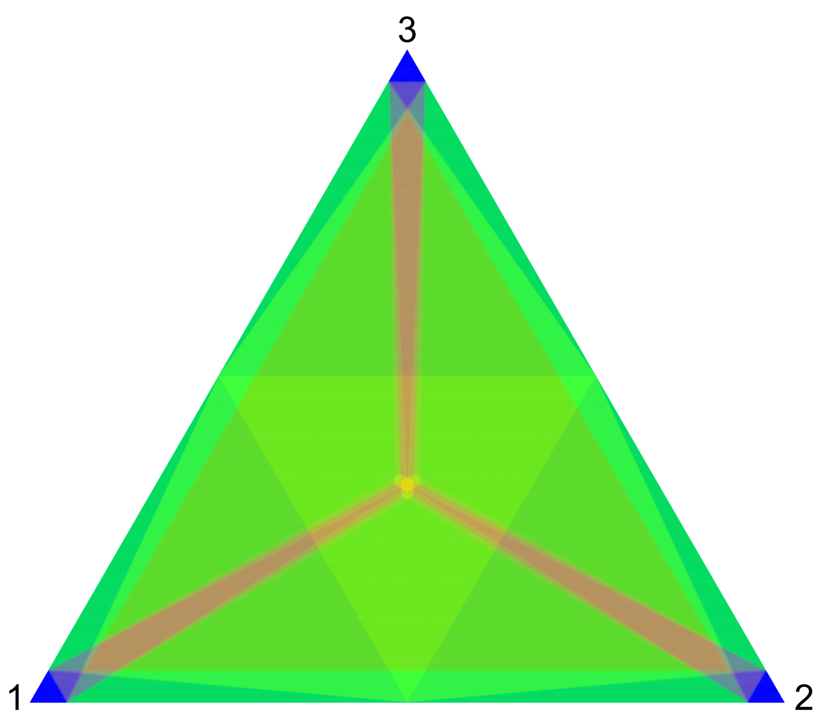

Figure 4.25 At least 228 equivalence classes for $s_2 = 0.95$

Chapter 5

Influence in Weighted Committee Games

The aggregation of individual preference orderings to a collective choice is a necessary element of business, politics, and beyond. Members of a board, council, or committee are rarely aware of how sensitive their collective decision is to the adopted aggregation rule. Consider, for illustration, a city council with 14 members who need to fill the position of the city manager with a job candidate from set $A = \{a, b, c, d, e\}$. Suppose the committee members are divided into three groups with homogeneous preferences: 6 from the mayor's party (group 1), 5 from the main opposition party (group 2), and 3 independents (group 3). Assume that the preferences of the groups can be described as follows (from top rank to bottom):

Group 1	Group 2	Group 3
a	b	c
d	c	e
e	d	d
c	e	b
b	a	a

Table 5.1 Preferences in the city council example

Suppose members of the council are constrained to voting sincerely or lack the information about each other's preferences to make strategic voting worthwhile. If then everybody votes for his or her highest ranked candidate, a is the winner under standard *plurality voting*. However, no candidate secures a 50 %-majority. So the committee might do a runoff between the two candidates with most votes, a and b. Such a *plurality with runoff* procedure is used, e.g., for selecting the French president. Then candidate b wins. Alternatively, the committee might conduct pairwise votes between all candidates. In this case, c emerges as the so-called *Condorcet winner*: it gets a majority against every other candidate. The committee might also apply the *Borda rule*: each voter gives 0 points to his or her lowest-ranked candidate, 1 point to the second-lowest, 2 points to the third-lowest, etc. Candidate d's total score is 34 and the highest; so then d would be hired. Yet another prominent procedure, known as *approval voting*, can make e the winner.

The lesson of this example is: be careful when you adopt a particular aggregation method in a committee. The outcome may not only depend on preferences but can be highly sensitive to the voting procedure. If the largest group wants to get its favored candidate, it should try to impose plurality rule. While, for the configuration at hand, the smallest group should, e.g., have argued for doing pairwise comparisons.

This chapter investigates whether such recommendations are also possible without the benefit of hindsight, i.e., when preferences of the committee members are not yet known. Can anything be said *a priori* about which groups benefit or suffer if a specific voting rule is adopted? Knowing only the partition of a committee into differently sized groups and possibly the number of considered alternatives, are there robust reasons to prefer a scoring method, like Borda's rule, over binary comparisons à la Condorcet, or a runoff procedure if you belong to a particular group?

Answers will arguably depend on which metric is used for evaluating different aggregation rules. Seeking *success* in the sense of obtaining outcomes that are, on average, ranked highly by the group may have different implications from seeking *influence* in the sense of outcomes being, on average, very sensitive to potential perturbations of the group's preferences. Influence, i.e., the expected effect of a group's preferences on the collective decision is the focus of this chapter. It has a long tradition of being studied with the help of *voting power indices*.[1] These – most prominently the *Penrose-Banzhaf index* (Penrose 1946; Banzhaf 1965) and the *Shapley-Shubik index* (Shapley and Shubik 1954) – are usually defined for binary decision environments. Binary *weighted voting games* have received wide attention since von Neumann and Morgenstern (1953, Ch. 10) first formalized them. See, e.g., Mann and Shapley (1962), Riker and Shapley (1968), Owen (1975) or Brams (1978) for classical contributions and, more recently, Barberà and Jackson (2006), Felsenthal and Machover (2013), Koriyama et al. (2013).

The key contribution of this chapter is to extend the pertinent methods from weighted voting on binary options to several, practically relevant decision proce-

[1] The terms "influence" and "power" are often used as synonyms in the literature.

dures for three or more alternatives. We make, to the best of our knowledge, the first attempt to compare the power implications of adopting, say, Borda rule vs. doing pairwise comparisons vs. having a sequential runoff.

We rely on the formalization of weighted committee decisions by Kurz, Mayer, and Napel (2018) in our analysis (cf. Chapter 4). They define a *weighted committee game* as a combination of a set N of players, a set A of alternatives, a vector $\mathbf{w}$ of voting weights and an anonymous aggregation method r (such as Borda rule, plurality rule, etc.) We will briefly review the suggested framework in Section 5.2.2. Their analysis focuses on the structural investigation of the corresponding games: identifying equivalences and minimal representations analogous to those known for binary simple games and weighted voting games (see, e.g., Houy and Zwicker 2014 or Freixas et al. 2017). We focus on measuring influence in weighted committee games and try to evaluate voting rules for multiple alternatives from a given player's power perspective.

Power evaluations of different voting procedures complement comparisons in social choice theory that investigate properties like Condorcet efficiency, various monotonicity and no-show paradoxes, or manipulability (see, e.g., Nurmi 1987 and Felsenthal and Nurmi 2017, 2018). Our findings pertain to shareholders' meetings of corporations, where voting weights reflect participants' shareholdings, as well as to various types of boards with a regional or divisional structure, where voting weights reflect seat numbers of voter blocs.

The Executive Board of the International Monetary Fund (IMF) can be considered as a prominent example of weighted committee voting. At the urging of emerging market economies, voting weights among the 24 Directors as well as rules for the choice of the IMF's Managing Director were reformed in 2016. The Managing Director was historically always a European selected by consensus with the US, but things shall be more competitive in the future. The winner will be adopted from a shortlist of at most three candidates by "a majority of the votes cast" (IMF Press Release 2016/19). There is scope to interpret the provision, since the IMF has not officially defined yet what "majority" refers to in case of more than two candidates. We consider several possibilities in Section 5.4 and investigate (*i*) if the reform of the voting weights makes real rather than only a cosmetic difference and (*ii*) whether the choice of the voting rule (i.e., the interpretation of the term "majority") can have practical relevance in the next election.

We focus on five standard single-winner procedures: plurality, plurality runoff, Borda, Copeland and Schulze rule (cf. Section 5.2.1). For a given voting rule and partition of a committee, the *a priori voting power* of group i is quantified as the expected sensitivity of the outcome to changes in this group's strict preference ordering. We determine whether the outcome at a given preference profile would change in response to a change of group i's preferences. The latter might be a spontaneous change of mind or brought about by a well-endowed lobbyist. We evaluate the average sensitivity of the outcome with respect to preference perturbations at any given profile assuming all conceivable profiles of preferences to be equally likely (just as the Penrose-Banzhaf index for binary alternatives). A formal definition is given in Section 5.3 and illustrated with a toy example. In Section 5.4, we apply the new measure to the IMF Executive Board.

Extensive comparisons of the indicated rules show how group sizes and voting procedures jointly determine the distribution of influence between large, medium and small groups of voters for $m \geq 3$ alternatives. We identify tentative "rules of thumb" regarding the beneficiaries of particular voting rules (cf. Section 5.5). Though our influence measure coincides with the Penrose-Banzhaf index for $m = 2$, it is in general unwarranted to infer the balance of power for decisions on three or more alternatives from standard measures of binary voting power.

5.1 Related Literature

Power index analysis provides analytical tools that quantify how a decision rule allocates influence over the collective decision to the respective players. Although many decisions involve more than two alternatives, most of the analysis of voting power has focused on the binary case. The latter is usually modeled as a simple or weighted voting game. The literature is surveyed, e.g., in Felsenthal and Machover (1998), Laruelle and Valenciano (2008) and Napel (2018).

Since many real-world committee decisions allow for more than two elements in the input and output domain, the standard dichotomous framework is restrictive. Even for simple "yes" or "no" decisions, voters usually have more than just two options. For example, they can abstain or not even participate in a vote or election. This has been formalized by means of *ternary voting games* (Felsenthal and Machover 1997a, Tchantcho et al. 2008, Parker 2012) with the option to support a proposal, to reject it, or to abstain; and by *quaternary voting games* (Laruelle and Valenciano 2012) with the additional option not to participate in a ballot. Moreover, players could also be allowed to express different intensities of support: so-called (j,k)-

games, introduced by Hsiao and Raghavan (1993) and Freixas and Zwicker (2003, 2009), give each player the possibility to express one of j ordered levels of approval. The resulting j-partitions of players are mapped to one of k ordered output levels. $(3,2)$-games correspond to ternary voting games; $(4,2)$-games are quaternary voting games for which abstention and absence have an ordered impact on passage of a proposal.[2] Power indices for (j,k)-games were introduced by Freixas (2005a, 2005b). A general framework for quantifying influence in settings with richer than binary actions, and therefore involving strategic considerations, has been proposed by Napel and Widgrén (2004).

The linear orderings of all players' actions and of the feasible outcomes required by (j,k)-games are naturally given in many applications. For instance, school or university committees who have to agree on distinctions and discrete grades may be modeled as (j,k)-games; so do committees which decide on the scale or intensity of a specific policy intervention. The assumption of ordered actions and outcomes is, however, problematic when options have multidimensional attributes – for instance, if a job applicant, policy program, location of a facility, etc. is to be selected. Pertinent extensions of simple games, along with corresponding power measures, have been introduced as *multicandidate voting games* by Bolger (1986) and taken up as *simple r-games* by Amer et al. (1998) and as *weighted plurality games* by Chua et al. (2002). They allow each player to vote for a single candidate. This results in partitions of player set N which, in contrast to (j,k)-games, are mapped to a winning candidate without ordering restrictions.

A second branch of the literature we connect to is the vulnerability of voting systems to strategic behavior. Ever since Gibbard's (1973) and Satterthwaite's (1975) seminal work, it is well-known that every surjective, resolute and non-dictatorial social choice function defined on the universal domain must be manipulable. Although this is damaging to the view that the outcome of a vote should reflect "the will of the people", one has to put it into context. Every reasonable social choice function gives rise to at least one preference profile at which individual strategic misrepresentation is possible; but the Gibbard-Satterthwaite theorem does not say anything on the prevalence of manipulation possibilities. If the proportion of manipulable profiles is low, strategic voting might be of little concern.

This has led to the investigation of the rule-specific extent of manipulability. A

[2] The requirement of a player-independent ordering for the distinct individual inputs to the decision mechanism is not satisfied for all quaternary voting games, i.e., that class of games is a superset of the class of $(4,2)$-games.

first straightforward approach is to simply count the number of profiles at which at least one voter can successfully manipulate. This has been done in computational studies by, e.g., Nitzan (1985) and Kelly (1993).

A relative of our power index has already featured in Nitzan's work. He looked at the aggregate sensitivity of anonymous scoring rules before investigating opportunities for strategic misrepresentation. Nitzan checked if the outcome can be affected by *some* individual preference variation, i.e., a change in preferences that is not necessarily driven by strategic concerns but just by some naive changes in the decision-making process. The key difference to our measure of influence is that we consider the effect of a *given* voter's preferences on outcomes. We thus obtain a measure of each voter's a priori influence at the individual level while Nitzan's analysis concerned the aggregate level, i.e., addresses the question how likely the voting rule is sensitive to some voter's preference variation.

Several extensions of Nitzan's approach have been proposed (see Aleskerov and Kurbanov 1999 or Smith 1999). Instead of just enumerating the manipulable profiles, one can count the number of voters who can manipulate on average; or determine the average improvement – in terms of ranks – due to successful strategic misrepresentation at a given profile; or determine the highest possible improvement; etc.

5.2 Preliminaries

5.2.1 Anonymous Voting Rules

We consider finite sets $N = \{1, \dots, n\}$ of n players or voters such that each voter $i \in N$ has strict preferences P_i over a set $A = \{a_1, \dots, a_m\}$ of $m \geq 2$ alternatives. We will write abc in abbreviation of aP_ibP_ic. The set of all $m!$ strict preference orderings on A is denoted by $\mathcal{P}(A)$. A *(resolute) voting rule* $r\colon \mathcal{P}(A)^n \to A$ maps each preference profile $\mathbf{P} = (P_1, \dots, P_n)$ to a single winning alternative $a^* = r(\mathbf{P})$.

Throughout our analysis r will be defined by truthful voting.[3] We will consider the following five anonymous rules that treat all voters $i \in N$ symmetrically:[4]

[3]Our assumption of truthful voting might, e.g., be justified by players being political agents to principals (constituents) that value truth telling. There is also experimental evidence that players often lack or ignore the information required to make manipulation profitable (cf. Kube and Puppe 2009). Even if they are aware of the other voters' preferences, they might face a hard (NP-complete) problem in identifying profitable deviations. See Nurmi (2016).

[4]Formally, construct $\mathbf{P}'$ by applying a permutation $\pi\colon N \to N$ to $\mathbf{P}$, so that $\mathbf{P}' = (P_{\pi(1)}, \dots, P_{\pi(n)})$. Then r is *anonymous* if for all such $\mathbf{P}, \mathbf{P}'$ the winning alternative $a^* = r(\mathbf{P}) = r(\mathbf{P}')$ is the same.

Plurality

The most simple voting method is *plurality rule* r^P: each voter endorses his or her top-ranked alternative; then the alternative which is ranked first by the most voters is chosen. That is, $a^* = r^P(\mathbf{P})$ implies

$$a^* \in \arg\max_{a \in A} \left| \{ i \in N \mid \forall a' \neq a \in A \colon aP_i a' \} \right|. \tag{5.1}$$

Plurality Runoff

Similarly, *plurality runoff rule* r^{PR} asks each voter to name his or her top ranked alternative. If an alternative is ranked first by more than half of the voters, it is chosen. Otherwise a runoff is held between the alternatives $a_{(1)}$ and $a_{(2)}$ which are lexicographically minimal among the alternatives that received the (second-)most votes. Formally, conceive of $a_{(1)}, a_{(2)}, \ldots, a_{(m)}$ as a permutation of the alternatives in A such that they are ordered according to decreasing numbers of votes and, in case of ties, in decreasing lexicographic order.

Then, $a^* = r^{PR}(\mathbf{P})$ implies

$$a^* \in \begin{cases} \arg\max_{a \in A} \left| \{ i \in N \mid \forall a' \neq a \in A \colon aP_i a' \} \right| & \text{if } \max_{a \in A} \left| \{ i \in N \mid \forall a' \neq a \in A \colon aP_i a' \} \right| > \dfrac{n}{2} \\ \arg\max_{a \in \{a_{(1)}, a_{(2)}\}} \left| \{ i \in N \mid \forall a' \neq a \in \{a_{(1)}, a_{(2)}\} \colon aP_i a' \} \right| & \text{otherwise.} \end{cases} \tag{5.2}$$

Borda

Our third benchmark is *Borda rule* r^B. It requires each voter i to give $m-1$, $m-2$, $\ldots$, 0 points to the alternative that is ranked first, second, $\ldots$, and last according to P_i. Then it selects the alternative with the highest total number of points (known as *Borda score*). Formally, let

$$b_i(a, \mathbf{P}) := \left| \{ a' \in A \mid aP_i a' \} \right|$$

be the number of alternatives ranked below a according to i's preferences. Then $a^* = r^B(\mathbf{P})$ implies

$$a^* \in \arg\max_{a \in A} \sum_{i \in N} b_i(a, \mathbf{P}). \tag{5.3}$$

Copeland

The fourth benchmark is *Copeland rule* r^C. Pairwise majority votes are held between all alternatives; the alternative that beats the most others is selected. Formally, let the *majority relation* $\succ^{\mathbf{P}}_M$ be defined by

$$a \succ^{\mathbf{P}}_M a' \;:\Leftrightarrow\; \left|\{i \in N \mid aP_i a'\}\right| > \left|\{i \in N \mid a'P_i a\}\right|.$$

Then $a^* = r^C(\mathbf{P})$ implies

$$a^* \in \arg\max_{a \in A} \left|\{a' \in A \mid a \succ^{\mathbf{P}}_M a'\}\right|. \tag{5.4}$$

Rule r^C is a Condorcet method: if some alternative a is a Condorcet winner, i.e., beats all others in pairwise majority comparisons, then $r^C(\mathbf{P}) = a$.

Schulze

A fairly recent voting method is *Schulze rule* r^S, as introduced by Schulze (2011). It is also a Condorcet method and based on indirect comparisons of alternatives a_i and a_j by defining a *path* $\mathbf{a_{i\to j}} := a(1), \ldots, a(k)$ from a_i to a_j with the following properties:

1. $a(1) = a_i$
2. $a(k) = a_j$
3. For all $i = 1, \ldots, (k-1)$: $a(i) \succ^{\mathbf{P}}_M a(i+1)$

Every alternative $a(i)$ in the sequence $a(1), \ldots, a(k)$ wins the binary comparison against its successor $a(i+1)$. The *strength* $s(\mathbf{a_{i\to j}})$ of a path $\mathbf{a_{i\to j}} = a(1), \ldots, a(k)$ is defined as the strength of its weakest link, i.e.,

$$s(\mathbf{a_{i\to j}}) := \min\left\{d[a(i), a(i+1)] \mid i = 1, \ldots, (k-1)\right\}$$

where $d[a(i), a(i+1)] = \left|\{i \in N \mid a(i)\, P_i\, a(i+1)\}\right|$. The *strength of the strongest path* from a_i to a_j is then

$$p[a_i, a_j] := \max\left\{s(\mathbf{a_{i\to j}}) \mid \mathbf{a_{i\to j}} \text{ is a path from } a_i \text{ to } a_j\right\}.$$

That is, one calculates the strength of every possible path from a_i to a_j and then selects the path with the highest strength. If there is no path from a_i to a_j, $p[a_i, a_j] := 0$.

Thus, $a^* = r^S(\mathbf{P})$ implies

$$a^* \in \Big\{a \in A \mid p[a,a'] \geq p[a',a] \;\forall\, a' \neq a \in A\Big\}. \tag{5.5}$$

Several private organizations adopted the Schulze rule for internal elections in recent years. Among them are the Pirate Parties of Austria, Germany and Sweden, Debian, Ubunto and the Wikimedia Foundation. We remark that the Schulze method is closely related to the *Simpson-Kramer rule*, which is also known as *maximin rule*.[5]

We assume that whenever there is a non-singleton set $A^* = \{a^*_{i_1}, \ldots, a^*_{i_k}\}$ of optimizers in eqs. (5.1)–(5.5), the alternative $a^*_{i^*} \in A^*$ with lowest index $i^* = \min\{i_1, \ldots, i_k\}$ is selected. This amounts to *lexicographic tie breaking* for $A \subset \{a, \ldots, z, aa, ab, \ldots\}$.

5.2.2 Weighted Committee Games

Real committees are more likely than not to involve a non-anonymous voting rule. This can be because designated members like a chairperson or agenda-setter have veto rights or procedural privileges. Or an anonymous decision rule r applies not at the voter level but at the level of their respective votes, shareholdings, etc. We could also think of the relevant players $i \in N$ in a committee to be differently sized but well-disciplined parties or interest groups with homogeneous preferences. It is then obvious that the introduction of voting weights implies non-anonymity at the level of voter blocks – even if the utilized voting rule is anonymous at the level of individual voters.

Single voters who differ in their numbers of votes and homogeneous groups of voters are formally equivalent: the committee's voting rule amounts to the *combination of an anonymous baseline voting rule r with voting weights $w_1, \ldots, w_n$* associated to the relevant players.

Following Chapter 4, we conceive of a rule r as representing the entire associated family of mappings from n-tuples of strict preferences over $A = \{a_1, \ldots, a_m\}$ to a winner $a^* \in A$ for arbitrary n. Then, the indicated combination is simply a *replication operation*. It defines the weighted voting rule $r|\mathbf{w}\colon \mathcal{P}(A)^{w_\Sigma} \to A$ by

$$r|\mathbf{w}(\mathbf{P}) := r(\underbrace{P_1, \ldots, P_1}_{w_1 \text{ times}}, \underbrace{P_2, \ldots, P_2}_{w_2 \text{ times}}, \ldots, \underbrace{P_n, \ldots, P_n}_{w_n \text{ times}})$$

[5]For $m = 3$, the Schulze winner and the Simpson-Kramer winner are identical; for larger m they coincide in about 99 % of all instances. These small differences notwithstanding, the Schulze rule fixes the most common flaws of the Simpson-Kramer rule. See Schulze (2011) for an elaborate discussion.

for a given anonymous rule r and a non-negative, non-degenerate weight vector $\mathbf{w} = (w_1, \ldots, w_n) \in \mathbb{N}_0^n$ with $w_\Sigma := \sum_{i=1}^n w_i > 0$. Moreover, we assume that $r|\mathbf{0}(\mathbf{P}) := a_1$ in the degenerate case $\mathbf{w} = (0, \ldots, 0)$.

The combination $(N, A, r|\mathbf{w})$ of a set of voters, a set of alternatives and a particular weighted voting rule is called a *weighted committee game*. Such games were introduced and their structure was extensively studied in Chapter 4.

When the underlying anonymous rule is plurality rule r^P, we call $(N, A, r^P|\mathbf{w})$ a *(weighted) plurality committee*. Similarly, $(N, A, r^{PR}|\mathbf{w})$, $(N, A, r^B|\mathbf{w})$, $(N, A, r^C|\mathbf{w})$ and $(N, A, r^S|\mathbf{w})$ are respectively referred to as a *plurality runoff committee*, *Borda committee*, *Copeland committee*, and *Schulze committee*.

Weighted representations of given committee games are far from unique. For instance consider the weighted plurality committee $(N, A, r^P|\mathbf{w})$. Since having more than half of the total voting weight renders player j a dictator, all weighted committee games $(N, A, r^P|\mathbf{w})$ with $\mathbf{w} \in \mathcal{E} = \left\{\mathbf{w} \in \mathbb{N}_0^n \mid w_j > \sum_{i \neq j} w_i\right\}$ are equivalent. That is, $r^P|\mathbf{w}(\mathbf{P}) = r^P|\mathbf{w}'(\mathbf{P})$ for all $\mathbf{P} \in \mathcal{P}(A)^n$ and all $\mathbf{w}, \mathbf{w}' \in \mathcal{E}$.

In general, two weighted committee games $(N, A, r|\mathbf{w})$ and $(N', A', r'|\mathbf{w}')$ are *equivalent* if $N = N'$, $A = A'$, and $r \neq r'$ or $\mathbf{w} \neq \mathbf{w}'$ but the respective mappings from preference profiles to outcomes a^* are the same; that is, when $r|\mathbf{w}(\mathbf{P}) = r'|\mathbf{w}'(\mathbf{P})$ for all $\mathbf{P} \in \mathcal{P}(A)^n$. Chapter 4 investigates situations where $r = r'$ and tries to capture *structural equivalence* in the general sense that $(N, A, r|\mathbf{w})$ and $(N', A', r|\mathbf{w}')$ *reflect the same decision environment* even though weights and labels of players or alternatives may differ.[6] For instance, it turns out that when $n = 3$ voter groups decide on $m = 3$ alternatives with Borda rule, one can distinguish 51 equivalence classes of committees, i.e., families of weights or group sizes that induce the same mapping from preference profiles to outcomes; in contrast, there are only 4 classes for binary voting. There are also just 4 with Copeland rule for $m = 3$: either the largest group has dictatorial power, the largest two share power equally, or all three share power either equally or with an advantage to the largest.

We here take N, A and the distribution of voting weights $\mathbf{w}$ as given and seek to identify the implications of choosing $r \in \{r^P, r^{PR}, r^B, r^C, r^S\}$ on players' possibilities to influence the outcome. To do so, we first try to quantify influence on the outcome in a suitable way.

[6]In particular this means that, e.g., the three games $(N, A, r|\mathbf{w})$ with $\mathbf{w} \in \left\{(6, 5, 3), (5, 3, 6), (3, 6, 5)\right\}$ are treated as equivalent. Although they obviously differ from the perspectives of players 1, 2 and 3, they do not from the perspective of the large, medium and small player, i.e., the decision environment stays the same.

5.3 Measuring Influence in Weighted Committee Games

5.3.1 General Idea

The basic idea is to operationalize power of group i with weight w_i under rule r as the expected sensitivity of $r|\mathbf{w}(\mathbf{P})$ to changes of P_i. We assume that all groups' preferences are drawn independently from the uniform distribution over $\mathcal{P}(A)$, i.e., all profiles $\mathbf{P}$ are equally likely.[7] Then we allow P_i to switch to arbitrary $P'_i \neq P_i \in \mathcal{P}(A)$ and check if this changes the outcome. A different outcome will result only at some profiles $\mathbf{P}$ and for a few of the $m!-1$ orderings P'_i to which group i's preferences might possibly switch. But the more often $r|\mathbf{w}(\mathbf{P})$ is sensitive to P_i, the greater is group i's a priori influence on the outcome.[8]

A preference variation might directly reflect a change in individual tastes. Alternatively, such variations could reflect intentional preference misrepresentation arising, e.g., because a lobbyist might have successfully bribed voters. Such variations might also be the result of mistakes occurring in some stage of the decision-making procedure. Or a voter gets new last-minute information, positive or negative, about one of the candidates. The more influential the voter is, the more likely are such variations to affect the voting outcome.

Formally, sensitivity of outcome $r|\mathbf{w}(\mathbf{P})$ at profile $\mathbf{P} = (P_1, \ldots, P_{i-1}, P_i, P_{i+1}, \ldots, P_n)$ with respect to a change in player i's preference ordering from P_i to P'_i is captured by indicator function

$$\Delta r|\mathbf{w}(\mathbf{P}, \mathbf{P}') = \begin{cases} 1 & \text{if} \quad r|\mathbf{w}(\mathbf{P}) \neq r|\mathbf{w}(\mathbf{P}') \\ 0 & \text{if} \quad r|\mathbf{w}(\mathbf{P}) = r|\mathbf{w}(\mathbf{P}') \end{cases}$$

where $\mathbf{P}' = (P_1, \ldots, P_{i-1}, P'_i, P_{i+1}, \ldots, P_n)$. The average sensitivity of outcome $r|\mathbf{w}(\mathbf{P})$ at profile $\mathbf{P}$ with respect all $m! - 1$ possible preference perturbations of player i is then given by $\frac{1}{(m!-1)} \sum_{P'_i \neq P_i \in \mathcal{P}(A)} \Delta r|\mathbf{w}(\mathbf{P}, \mathbf{P}')$.

Averaging this over all $(m!)^n$ conceivable preference profiles, we get a measure of

[7] This probabilistic assumption about preferences is also known as the *Impartial Culture* framework. Although we are aware of its limited empirical support (see, e.g., Regenwetter et al. 2012, Ch. 1), we take it as a starting point and will consider more restrictive assumptions like single-peakedness in future research.

[8] One could also be more restrictive and only consider local preference variations, that is, only allow for changes to adjacent orderings. So when $m = 3$ and $P_i = abc$, voter i might only be allowed to switch to $P'_i = acb$ and $P''_i = bac$. This affects our findings on influence in moderate quantitative ways but results in very similar qualitative findings.

player i's *a priori influence* – i.e., evaluated at a stage when own and others' preferences are not yet known – in the weighted committee game $(N, A, r|\mathbf{w})$, denoted by

$$\widehat{\mathcal{I}}_i(r|\mathbf{w}) = \frac{\sum_{\mathbf{P}\in\mathcal{P}(A)^n} \sum_{P_i'\neq P_i\in\mathcal{P}(A)} \Delta r|\mathbf{w}(\mathbf{P}, \mathbf{P}')}{(m!-1)(m!)^n}, \quad i \in N.$$

This measure equals the probability that a change of player i's preferences from P_i to a random $P_i' \neq P_i$ at a randomly drawn profile $\mathbf{P}$ affects the outcome. For example, $\widehat{\mathcal{I}}_i(r|\mathbf{w}) = 0.5$ means that 50 % of player i's preference variations change the outcome, treating all profiles and all variations as equally likely.

$\widehat{\mathcal{I}}_i(r|\mathbf{w})$ is zero if player i is a *null player*. It is smaller than one for all non-null players (i.e., even for a dictator). In particular, the expected sensitivity of the outcome at a given profile $\mathbf{P}$ with respect to a dictator's preference perturbations equals $(m! - (m-1)!)/(m! - 1)$. The intuition behind this is straightforward: for a dictator, the $(m-1)! - 1$ preference perturbations that leave the top-choice unchanged cannot have an effect on the outcome for any Pareto-efficient voting rule like the rules considered here. So from the $m! - 1$ orderings different from a given P_i, this leaves $m! - (m-1)!$ alternative orderings that will alter the outcome under dictatorship. We therefore propose

$$\mathcal{I}_i(r|\mathbf{w}) = \frac{\widehat{\mathcal{I}}_i(r|\mathbf{w})}{(m! - (m-1)!)/(m! - 1)}, \quad i \in N$$

as a dictator-normalized measure of player i's a priori influence in game $(N, A, r|\mathbf{w})$. This normalization destroys the probabilistic interpretation of $\widehat{\mathcal{I}}_i(r|\mathbf{w})$. It rather gives an intuitive assessment of how close player i is to being a dictator (and how far from being a null player). A value of $\mathcal{I}_i(r|\mathbf{w}) = 0.5$, for instance, means that i's influence on outcomes is halfway between that of a null player and a dictator. In other words, the probability to affect the outcome with a random last-minute change of mind is 50 % that of a dictator.

5.3.2 Relationship to the Penrose-Banzhaf Index

Before we illustrate analysis based on $\mathcal{I}(r|\mathbf{w})$ with a simple example in Section 5.3.3, it is worth pointing out that our non-normalized measure $\widehat{\mathcal{I}}(r|\mathbf{w})$ coincides with the well-known Penrose-Banzhaf index in case of $m = 2$. To see this, first note that our rules r^P, r^{PR}, r^B, r^C and r^S are equivalent for $m = 2$, i.e., they always select the same winning alternative. See Proposition 4.1 in this thesis; the argument immediately extends to plurality runoff and Schulze rule. Also note that for $m = 2$, every player has just one preference variation and the number of coalitions in the theory of simple voting games equals the number of profiles in our setting.

Specifically, in the binary case, weighted committee games reduce to simple or weighted voting games, which are a special case of cooperative games (N, v). Instead of working with voting rule r, one considers the *coalitional function* $v\colon 2^N \to \{0, 1\}$ such that $v(S) = 1$ if $1\,P_i\,0$ for all $i \in S$ implies $r(\mathbf{P}) = 1$. A coalition S is referred to as *winning* if $v(S) = 1$ and as *losing* if $v(S) = 0$. One usually denotes the set of winning coalitions by $\mathcal{W}$ and the set of losing coalitions by $\mathcal{L}$. If $i \notin S$ and $S \in \mathcal{L}$ but $S \cup \{i\} \in \mathcal{W}$, player i is called *critical outside* S. Similarly, player i is *critical inside* S if $i \in S$ and $S \in \mathcal{W}$ but $S \setminus \{i\} \in \mathcal{L}$.

A simple voting game (N, v) is called weighted voting game if it can be represented by a set of non-negative weights $\mathbf{w} = (w_1, \ldots, w_n)$ and a positive quota q such that $v(S) = 1$ if and only if $\sum_{i \in S} w_i \geq q$. One then writes $[q; \mathbf{w}]$ interchangeably with (N, v), i.e., $(N, v) = [q; \mathbf{w}]$.

There is a plethora of power indices for simple simple games. One of the most popular is the Penrose-Banzhaf index (PBI) defined by

$$PBI_i(v) = \frac{1}{2^{n-1}} \sum_{S \subseteq N \setminus \{i\}} [v(S \cup \{i\}) - v(S)], \quad i \in N.$$

The PBI considers all coalitions $S \subseteq N \setminus \{i\}$ and counts how often player i is *critical outside* S, denoted by $\eta_i^- = \sum_{S \subseteq N \setminus \{i\}} [v(S \cup \{i\}) - v(S)]$. Equivalently, one could consider all coalitions $S \subseteq N$ with $i \in S$ and count the number of coalitions for which i is *critical inside*, denoted by $\eta_i^+ = \sum_{S \subseteq N, i \in S} [v(S) - v(S \setminus \{i\})]$.

Chapter 4 (Proposition 4.2) already showed that weighted committee games with $m = 2$ are in bijection to weighted voting games $[q; \mathbf{w}]$ with a 50 %-majority quota, i.e., $r|\mathbf{w}(\mathbf{P}) = a_1 \Leftrightarrow v(S) = 1$ where $A = \{a_1, a_2\}$ and $S = \{i \in N \,|\, a_1\,P_i\,a_2\} \subseteq N$ is the set of players who prefer a_1 at profile $\mathbf{P} \in \mathcal{P}(A)^n$.

With this at hand, it is easy to see that in the binary case, our non-normalized influence measure coincides with the PBI applied to the respective weighted voting game with a 50 %-majority quota. While the PBI considers $S \subseteq N \setminus \{i\}$ and identifies η_i^- (or $S \subseteq N, i \in S$ and η_i^+), we consider all coalitions $S \subseteq N$ (or, in our terminology, profiles) and count both η_i^- and η_i^+. Since $\eta_i^- = \eta_i^+$, we have that

$$\sum_{P_i' \neq P_i \in \mathcal{P}(A)} \Delta r|\mathbf{w}(\mathbf{P}, \mathbf{P}') = 2 \cdot \sum_{S \subseteq N} [v(S \cup \{i\}) - v(S)], \quad i \in N$$

where $S = \{j \in N \,|\, a_1 \, P_j \, a_2\} \subseteq N$ is the set of players who prefer a_1 at profile $\mathbf{P} \in \mathcal{P}(A)^n$.[9] For $m = 2$, using that $(m!)^n = 2 \cdot 2^{(n-1)}$, we get

$$\widehat{\mathcal{I}}(r|\mathbf{w}) = PBI(N, v)$$

with $r \in \{r^P, r^{PR}, r^B, r^C, r^S\}$ and $(N, v) = [\frac{1}{2}w_\Sigma; \mathbf{w}]$.[10]

5.3.3 Illustration

To illustrate the use of our power index, consider again the city council example with $N = \{1,2,3\}$ and voting weights of 6, 5, and 3. For the ease of exposition, assume that $A = \{a, b, c\}$ and let the committee decide by Borda rule. The corresponding weighted Borda committee is then given by $(N, A, r^B|(6, 5, 3))$. Consider the following preference profile $\mathbf{P}$:

P_1	P_2	P_3
b	a	c
c	b	b
a	c	a

Table 5.2 Preference profile $\mathbf{P}$ in the city council example

Table 5.3 shows the Borda scores at $\mathbf{P}$ for weights $(6, 5, 3)$:

	a	b	c
Group 1	0	12	6
Group 2	10	5	0
Group 3	0	3	6
Result	10	**20**	12

Table 5.3 Calculation of $r^B|(6, 5, 3)(\mathbf{P})$

Thus $r^B|(6, 5, 3)(\mathbf{P}) = b$ with a score of **20**.

Let us now check how changes in the preferences of group 1 affect, ceteris paribus, the voting outcome. Preferences can change from $P_1 = bca$ to any $P_1' \in$

[9]It is feasible to sum over all $S \subseteq N$ on the right hand side of above eq. since $S \cup \{i\} = S$ if $i \in S$.

[10]The same applies to the PBI for weighted voting games with $q = \sum w_i/2 + \epsilon$ for small $\epsilon > 0$. Weighted voting games $[q; \mathbf{w}]$ and $[q'; \mathbf{w}]$ with quota $q = \sum w_i/2$ and $q' = q + \epsilon$ for small $\epsilon > 0$ are *duals* of each other, i.e., the set of winning coalitions in one game is the set of blocking coalitions in the other. The PBI satisfies the so-called *duality property*, i.e., $PBI_i[q; \mathbf{w}] = PBI_i[q'; \mathbf{w}]$ for $i \in N$. See, e.g., Felsenthal and Machover (1998, Ch. 2 and 3) or Taylor and Zwicker (1999, Ch. 1).

$\{abc, acb, bac, cab, cba\}$. The respective points associated with the new Borda winner $r^B|(6,5,3)(\mathbf{P}')$ are printed in bold in Table 5.4. If the change from P_1 to P_1' results in a new winner, i.e., $r^B|(6,5,3)(\mathbf{P}') \neq r^B|(6,5,3)(\mathbf{P})$, the score is marked with an asterisk. Four out of five perturbations of P_1 at $\mathbf{P}$ change the outcome.

P_1'	a	b	c
abc	**22***	14	6
acb	**22***	8	12
bac	16	**20**	6
cab	16	8	**18***
cba	10	14	**18***

Table 5.4 Effects of changes of group 1's preferences from $P_1 = bca$ to P_1'

Repeating above procedure for the members of group 2 (change of $P_2 = abc$ to any $P_2' \in \{acb, bac, bca, cab, cba\}$) and group 3 (change of $P_3 = cba$ to any $P_3' \in \{abc, acb, bac, bca, cab\}$), we get the results as reported in Tables 5.5 and 5.6.

P_2'	a	b	c
acb	10	15	**17***
bac	5	**25**	12
bca	0	**25**	17
cab	5	15	**22***
cba	0	20	**22***

Table 5.5 Effects of changes of group 2's preferences from $P_2 = abc$ to P_2'

P_3'	a	b	c
abc	16	**20**	6
acb	16	**17**	9
bac	13	**23**	6
bca	10	**23**	9
cab	13	**17**	12

Table 5.6 Effects of changes of group 3's preferences from $P_3 = cba$ to P_3'

It becomes clear that the members of group 1 have, at the considered profile $\mathbf{P}$, more say than the other two groups. Only three out of five perturbations of P_2 affect the outcome. The members of group 3 have no influence at $\mathbf{P}$: there is no P_3' that alters the outcome.

Repeating the above procedure for all profiles $\mathbf{P} \in \mathcal{P}(A)^n$, the expected (dictator-normalized) influences evaluate to

$$\mathcal{I}(r^B|(6,5,3)) \approx (0.6806, 0.5972, 0.3611).$$

Table 5.7 reports the results of doing the same exercise for all of the five considered voting rules and up to five alternatives. If the large group can decide on the voting rule before knowing anything about own and others' preferences, they should insist on Borda rule if $m = 3$ and plurality rule if $m \in \{4, 5\}$. For the medium-sized and small group it is even easier. For any $m \in \{3, 4, 5\}$ the latter should argue for Copeland rule and the former for Borda rule if they seek as much influence on the voting outcome as possible.

The distribution of influence under plurality rule tends to $(1, 0, 0)$ as $m \to \infty$: the proportion of profiles at which players 2 and 3 have the same candidate top-ranked goes to zero as $m \to \infty$. At all other profiles, player 1 is as influential as a dictator. Consequently, the probability that the outcome is as sensitive to player 1 as it would be under a dictatorship goes to 1.

Let us remark that for $m = 2$, the weight distribution $(6, 5, 3)$ translates into equal influence of 0.5 for all three groups: it is easy to see that any two players could then jointly implement their preferred alternative – independently of the voting rule.[11] In contrast, the distribution of votes *and* the choice of the voting rule matter for outcomes and the distribution of influence for $m > 2$.

5.4 Application: International Monetary Fund

The previous section illustrated our approach of measuring the a priori power distribution for committee decisions on more than two alternatives with a toy example. We now go one step further and apply it to a real-world committee. In particular we are going to evaluate the IMF's recent reform of voting weights as well as the reform of the selection process for its Managing Director. The power distribution in the IMF has been investigated with binary power index analysis several times. See, e.g., Leech (2002, 2003), Aleskerov et al. (2008), or Leech and Leech (2013).

[11] Recall that all our considered voting rules are equivalent for $m = 2$.

$\mathcal{I}_i(r\|(6,5,3))$		$m=3$	$m=4$	$m=5$
r^P	$\mathcal{I}_1$	0.6667	0.7500	0.8000
	$\mathcal{I}_2$	0.4444	0.3750	0.3200
	$\mathcal{I}_3$	0.4444	0.3750	0.3200
r^{PR}	$\mathcal{I}_1$	0.5556	0.5833	0.6000
	$\mathcal{I}_2$	0.5556	0.5833	0.6000
	$\mathcal{I}_3$	0.5000	0.5000	0.5000
r^B	$\mathcal{I}_1$	0.6806	0.7372	0.7631
	$\mathcal{I}_2$	0.5972	0.6246	0.6462
	$\mathcal{I}_3$	0.3611	0.3644	0.3839
r^C	$\mathcal{I}_1$	0.5509	0.5851	0.6098
	$\mathcal{I}_2$	0.5509	0.5851	0.6098
	$\mathcal{I}_3$	0.5509	0.5851	0.6098
r^S	$\mathcal{I}_1$	0.5972	0.6584	0.7011
	$\mathcal{I}_2$	0.5278	0.5426	0.5515
	$\mathcal{I}_3$	0.5278	0.5426	0.5515

Table 5.7 $\mathcal{I}(r|(6,5,3))$ for $m \in \{3,4,5\}$ and $r \in \{r^P, r^{PR}, r^B, r^C, r^S\}$

5.4.1 Institutional Background

The International Monetary Fund dates back to 1944 when the Articles of Agreement of the IMF were adopted at the United Nations conference in Bretton Woods. Back then, the primary goal of the Fund was to further economic cooperation in order to prevent disparities in the global economy like the ones that contributed to the Great Depression and the major worldwide economic turndown in the 1930s. According to its own account, the IMF's goal nowadays is to ensure the stability of the international financial architecture, to resolve crises, to promote sustainable growth and macroeconomic stability and to reduce poverty. See IMF (2018a) for a detailed overview about the IMF's goals and main tools for carrying out its mandate.

The two main decision bodies of the IMF are the Board of Governors and the Executive Board. The former has delegated most of its powers to the latter and only remains responsible for general decisions like the allocation of special drawing rights or the admittance of new members. We will therefore focus on the Executive Board in the following.

The Executive Board consists of 24 Executive Directors. Six of them are elected by the countries with the largest quota. That is, USA, China, Japan, Germany, France and the UK. They each have their own Director. The remaining 183 countries are grouped into eighteen constituencies that represent at least four countries. Every constituency elects its own Director to represent the interests of the group members.

The IMF uses a weighted voting system in which a member's voting weight is based on its financial contribution to the Fund (measured in special drawing rights). Each Executive Director that represents a single country can cast as many votes as was allotted to his or her state.[12] Directors that represent a group of countries can cast as many votes en bloc as their group has in total. See IMF (2018b) for how the IMF is partitioned into the 24 groups. A list of the corresponding voting weights for every country is available upon request.

5.4.2 IMF Voting Weights Reform 2016

The IMF's governance structure today still largely reflects the world at the end of World War II, when it was established. In particular, high-income countries like the USA, Japan, Germany, France and the UK have a solid majority and can thus effectively run the organization. Low- and middle-income countries hardly have significant say. Emerging market economies like China and India have raised this issue repeatedly and keep seeking more influence for the developing world in the global financial architecture.

Therefore, there have been several pushes to reform the governance structure of the IMF. First, the so-called Singapore reforms of 2006 increased China's vote share from 2.9 % to 3.6 % and that of the BRIC countries plus Mexico from 10.1 % to 11.1 %. The most recent reform was agreed in 2010 and has started to be implemented in 2016. A significant percentage of vote shares is shifting from the USA and European countries to emerging and developing countries.[13] For instance, China's vote share went up to 6.1 % (compared to 3.8 % before). India's share increased to 2.6 % (2.3 %), Russia's to 2.6 % (2.4 %), Brazil's to 2.2 % (1.7 %) and Mexico's to 1.8 % (1.5 %). This brings India and Brazil into the list of the top ten IMF members in terms of vote shares.

In addition to reforming voting weights, the IMF also changed the selection process for its Managing Director. Prior to the reform, the process was quite intransparent and criticized as undemocratic: the Managing Director always was a European

[12]Each member gets one vote for every 100 000 special drawing rights plus a fixed amount of basic votes.

[13]The voting share of the USA still dwarfs all other members. The USA were nevertheless one of the last countries to ratify the reform.

selected by consensus with the USA. In contrast, the new process is advertised as "open, merit based, and transparent" (IMF Press Release 16/19). Every Governor or Executive Director may nominate a candidate. If the number of nominees exceeds three, the Executive Board draws up a shortlist of three candidates. From this list, the new Managing Director will be selected "by a majority of the votes cast" by the Executive Board. What "majority" exactly means if the number of candidates exceeds two leaves room for interpretation.

We take this as an opportunity to apply our approach of measuring influence in weighted committee games. We take it to be reasonable to interpret the new selection requirement as (*i*) receiving the most votes (plurality rule). However, one could also read the declaration as demanding (*ii*) a two-candidate runoff if nobody gets an outright majority (plurality runoff rule). Another conceivable interpretation would be to ask (*iii*) the winner to secure the highest number of pairwise majority wins against the other two competitors (Copeland rule).

5.4.3 Influence in the IMF Executive Board

We, first, seek to identify if recent changes of the distribution of IMF drawing rights, hence votes, can make a difference in future voting practice and, second, to test if different interpretations of the selection process matter for the distribution of influence in the IMF Executive Board. Since the large number of preference profiles (6^{24}) prevents a direct calculation of $\mathcal{I}(r|\mathbf{w})$, our results are based on Monte Carlo simulation. Table 5.8 reports the estimated influence numbers; respective 95 %-confidence intervals are provided in Appendix A (Tables 5.9 to 5.11).

As one can immediately see from the influence numbers reported in Table 5.8, the aim of increasing the voting power of emerging market economies by the 2016 reform was achieved: higher vote shares translate into more influence in the Executive Board.[14] This is independent of which voting rule interpretation (*i*)–(*iii*) we consider. The rise in influence is particularly pronounced for China, with an increase by more than 50 %. Other emerging market economies also have a greater say than before the 2016 reform. The influence of the groups that include Brazil and Turkey increased by about 17 % and 11 %, respectively; that of the Indonesian group by about 10 %; and Russia and the Indian and Spanish group (that, e.g., includes Mexico) each gained about 9 %. Intended or not, many African countries are among the greatest losers: the South African group lost about 10 % of its influence. The greatest loser, however,

[14] We remark that this also holds for $m = 2$.

	Vote share (%)		$\mathcal{I}(r^P\|\mathbf{w})$		$\mathcal{I}(r^{PR}\|\mathbf{w})$		$\mathcal{I}(r^C\|\mathbf{w})$	
	$\mathbf{w}_{\text{pre}}$	$\mathbf{w}_{\text{post}}$	$\mathbf{w}_{\text{pre}}$	$\mathbf{w}_{\text{post}}$	$\mathbf{w}_{\text{pre}}$	$\mathbf{w}_{\text{post}}$	$\mathbf{w}_{\text{pre}}$	$\mathbf{w}_{\text{post}}$
USA	16.72	16.55	0.7121	0.7048	0.6735	0.6669	0.6875	0.6807
Japan	6.22	6.16	0.1986	0.1988	0.2239	0.2234	0.2164	0.2159
China	3.80	6.10	0.1216	0.1966	0.1404	0.2210	0.1340	0.2159
Netherlands*	6.56	5.40	0.2092	0.1742	0.2350	0.1972	0.2277	0.1899
Germany	5.80	5.33	0.1851	0.1719	0.2097	0.1949	0.2023	0.1876
Spain*	4.90	5.30	0.1567	0.1711	0.1788	0.1939	0.1717	0.1865
Indonesia*	3.93	4.32	0.1254	0.1392	0.1448	0.1595	0.1381	0.1526
Italy*	4.22	4.12	0.1349	0.1331	0.1551	0.1529	0.1483	0.1461
France	4.28	4.04	0.1370	0.1305	0.1575	0.1499	0.1507	0.1432
United Kingdom	4.28	4.04	0.1370	0.1303	0.1574	0.1499	0.1506	0.1431
Korea*	3.62	3.89	0.1158	0.1257	0.1340	0.1445	0.1277	0.1380
Canada*	3.59	3.36	0.1150	0.1082	0.1332	0.1255	0.1268	0.1194
Sweden*	3.39	3.27	0.1085	0.1056	0.1259	0.1224	0.1198	0.1164
Turkey*	2.91	3.21	0.0933	0.1037	0.1087	0.1203	0.1031	0.1143
Utd. Arab Emirates*	3.21	3.09	0.1027	0.0998	0.1195	0.1159	0.1134	0.1101
Brazil*	2.61	3.05	0.0835	0.0984	0.0979	0.1144	0.0927	0.1087
India*	2.80	3.05	0.0897	0.0984	0.1048	0.1145	0.0993	0.1087
South Africa*	3.41	3.04	0.1091	0.0981	0.1267	0.1141	0.1205	0.1083
Switzerland*	2.80	2.73	0.0896	0.0883	0.1047	0.1030	0.0992	0.0976
Russian Federation	2.39	2.60	0.0763	0.0838	0.0896	0.0979	0.0847	0.0926
Iran*	2.26	2.19	0.0722	0.0707	0.0851	0.0831	0.0804	0.0785
Saudi Arabia	2.80	2.02	0.0896	0.0650	0.1047	0.0766	0.0992	0.0723
Argentina*	1.84	1.58	0.0587	0.0511	0.0695	0.0603	0.0655	0.0567
Dem. Rep. Congo*	1.46	1.57	0.0466	0.0505	0.0554	0.0598	0.0520	0.0562

Table 5.8 Distribution of influence in the IMF Executive Board $(N, A, r|\mathbf{w})$ before ($\mathbf{w}_{\text{pre}}$) and after implementation of the 2016 reform ($\mathbf{w}_{\text{post}}$) under plurality, plurality runoff and Copeland rule for $m = 3$. Member states that represent a group of members are marked by an asterisk. We selected the largest member of each group to represent the group. The pre-reform scenario does not include Nauru, which joined the IMF in April 2016.

is Saudi Arabia with a decrease in a priori voting power of about 27 %. Among the European countries, Germany, France and UK each lose between 5 % and 7 %. The USA remain largely unaffected and lose only about 1 % of their influence.

Table 5.8 also shows whether adopting one or the other of the possible interpretations of the selection process for the Managing Director makes a difference. It turns out that they really differ for countries' chances to influence the outcome. There is a simple pattern: while the USA should prefer plurality rule over both Copeland and plurality runoff rule, it is the opposite for *all* other (groups of) countries. They all have more influence under plurality runoff rule than under Copeland rule than under plurality rule. This is in line with our findings from Section 5.3.3: if three alternatives are on the table, the largest group should push for plurality rule while medium and small groups are better off under plurality runoff rule and Copeland rule.

5.5 Towards a More General Comparison

The examples provided in the previous sections compared voting rules for a specific weight distribution. It is worthwhile to check whether observations such as the largest group benefitting from plurality rule or the smallest from Copeland rule, can be generalized. Can recommendations still be given if – in addition to individual preferences – the exact distribution of voting weights is unknown yet? Ideally, we would like to move beyond isolated examples and identify possible general biases of the considered rules for, say, large or small shareholders in a corporation, delegation of big or small size in a representative body, etc.

As a first step in this direction, we make use of the geometric approach proposed in Chapter 4, where all structurally distinct weighted committee games for $n = 3$ players were illustrated. In each of the subsequent triangles (Figure 5.2, and Figures 5.4 to 5.11 in Appendix B) we compare two voting rules. Similar to Chapter 4, the points in our triangles correspond to voting weight distributions. We use the standard projection of the 3-dimensional unit simplex of relative weights to the plane, illustrated in Figure 5.1. The weight axes are suppressed in subsequent figures. The color choices indicate which voting rule gives greater influence to *player 1*. Figure 5.2, e.g., compares Borda and plurality rule. Weight distributions colored in green indicate that player 1 has more influence under Borda rule than under plurality rule. For weight distributions colored in red it is the other way round. Distributions for which player 1 has equal influence under both rules are colored in yellow. We thus partition all non-degenerate weight distributions $\mathbf{w} \in \mathbb{N}_0^3$ into three categories: green areas indicate that player 1 should go for Borda rule and red areas that he should insist on plurality rule (assuming a procedural decision between the two). Yellow areas come with identical influence for player 1 under either rule. Similar interpretations apply to all other binary comparisons in Figures 5.4 to 5.11 provided in Appendix B.

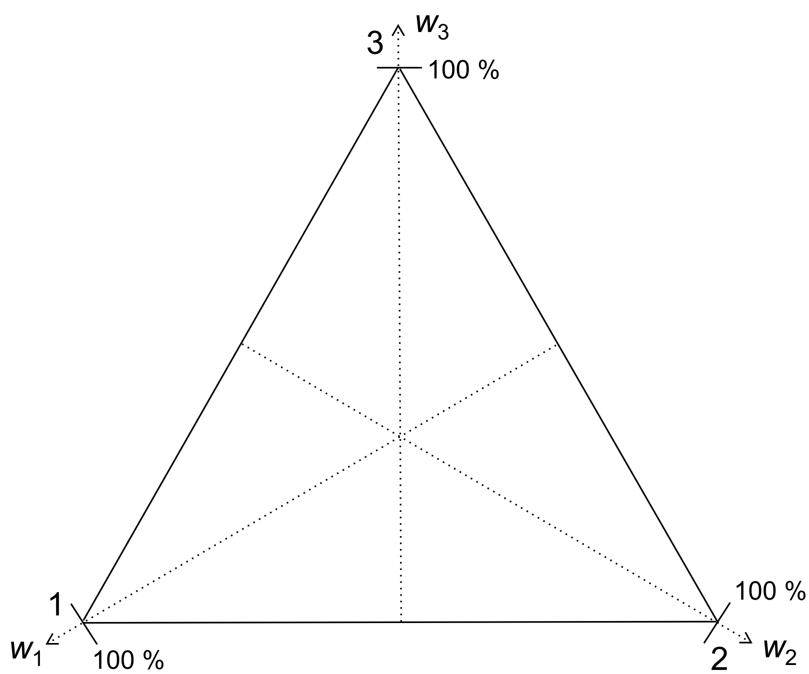

Figure 5.1 Simplex of all distributions of relative voting weights for $n = 3$

Since the outcome under Borda rule is known to be highly sensitive to changes in the weight distribution (cf. Figures 4.5–4.7 in Chapter 4), we use different intensities of red and green in comparisons that involve Borda rule. Darker tones of green in Figure 5.2 and Figures 5.4 to 5.7 indicate a larger influence advantage for Borda, respectively, than lighter tones of green. Similarly, darker red indicates a more pronounced advantage for the respective other rule over Borda rule. For all comparisons not involving Borda, we stick to a simple partition in green, red and yellow; these rules are much less sensitive to changes in the weight distribution.[15] Note that visibility of the color indications is restricted to areas instead of isolated points or line segments.

[15]Corresponding pictures with different – compared to Borda rule much less variable – color intensities are available upon request.

5.5.1 Illustration: Borda Rule versus Plurality Rule

Let us illustrate some of the inferences that can be drawn from the comparison of Borda vs. plurality committees in Figure 5.2.[16] We focus on generic cases where $w_1 \neq w_2 \neq w_3$.

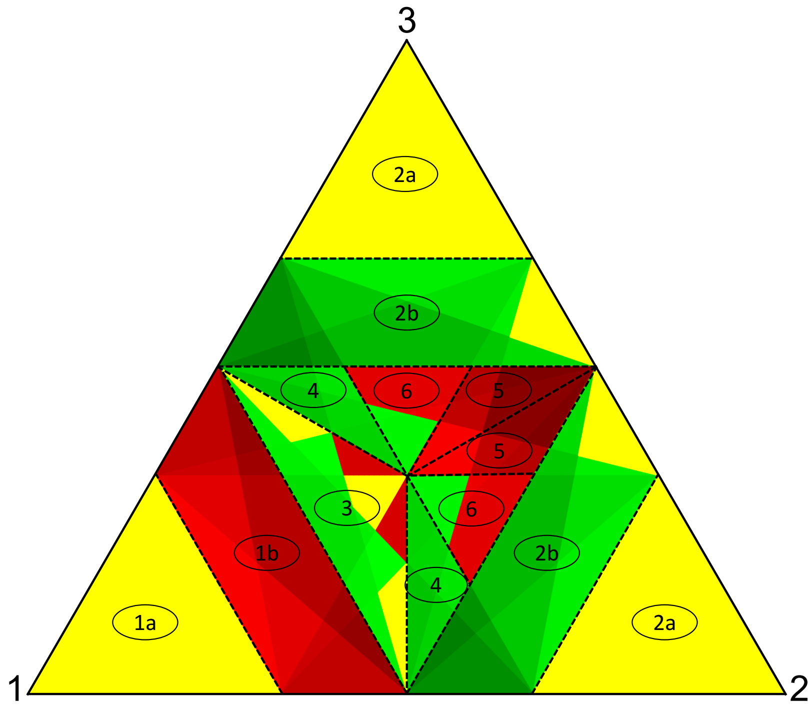

Figure 5.2 Borda vs. plurality for $m = 3$. Regions colored in (green/yellow/red) indicate that Borda influence is (greater than/equal to/smaller than) plurality influence

Assume you are player 1 and you can impose either Borda rule or plurality rule at a constitutional stage at which the committee is set up and endowed with procedural rules; later committee decisions will be about selecting a winner from three candidates. If your objective is to have the greatest possible influence, then the following "rules of thumb" might be useful:

- You should impose plurality rule if you will later have a majority of votes (region 1): either Borda rule and plurality rule both give you equal influence (in case you are a plurality and a Borda dictator in region 1a); or plurality rule is strictly better (in case you have more than half but at most two thirds

[16]We highlighted "key" areas for the analysis. The same figure without numbering these areas is again provided in Appendix B (Figure 5.4).

of total votes in region 1b).[17]

- By similar reasoning, you should go for Borda rule if one of the other two players will have a majority (regions 2a and 2b).
- If nobody will wield a majority but you will at least have a plurality, then it is safe to go for Borda rule in most cases (region 3). The only exception are two small red triangles in region 3. They indicate situations in which players' weights are relatively similar and exactly one of the other two players has more than one third of total votes (e.g. player 2 for $\mathbf{w} = (35, 34, 31)$).
- If you know that you will only be the second largest player but have more than one third of total votes (and nobody has a majority), then you should impose Borda rule (region 4).
- If you know that you will be the smallest player and the other two each have more than one third of total voting weight, plurality rule gives you more influence (region 5).
- The remaining case involves one other player and you each having less than one third of total votes (region 6). Then it depends: if all three players are relatively close to each other, Borda rule gives you greater influence a priori; if, however, the largest player's share is close to 50 %, then you should better pick plurality rule.

5.5.2 Further Comparisons

Similar reasoning also applies to all other comparisons. The corresponding figures are provided in Appendix B. For comparisons not involving Borda rule, things are rather simple. For instance, when plurality rule is compared to either Copeland, plurality runoff, or Schulze rule (Figure 5.8), the recommendation always is the same: you should impose plurality rule if you have the most votes. If someone else has the most votes, you are better off under the respective other rule (you are either equally influential, or the respective other rule makes you more influential).

The same applies to Copeland rule versus Schulze rule (Figure 5.9). If you wield at least a plurality of votes, Schulze rule entails greater influence than Copeland; in case someone else has the most votes, it is the opposite.

[17] While a player needs more than $(m-1)/m$ of total weight to be a Borda dictator, it is sufficient to have more than 50 % to be a plurality dictator.

Regarding Copeland rule versus plurality runoff rule (Figure 5.10), you would always chose the former if you have at least the second-most votes; otherwise you go for Copeland rule. If you can chose between plurality runoff rule and Schulze rule (Figure 5.11), insist on Schulze rule if you are the largest or the smallest player; otherwise it is better to install a plurality runoff rule.

5.5.3 The "Best" Voting Rule

We can summarize all ten binary influence comparisons of the five considered voting rules in *one* picture. It indicates the "best" voting rule for any weight distribution, i.e., the rule which maximizes the a priori influence of player 1 for a given weighted committee game. We do so in Figure 5.3. Voting weight configurations of the same color share the same influence-maximizing voting rule(s). Figure 5.3 provides a "map" that reveals which of our five voting rules maximize the a priori influence of player 1 on decisions over $m = 3$ alternatives for all different distributions of voting weights. When areas correspond to line segments or single points in the simplex, we have manually enlarged these in order to improve visibility.

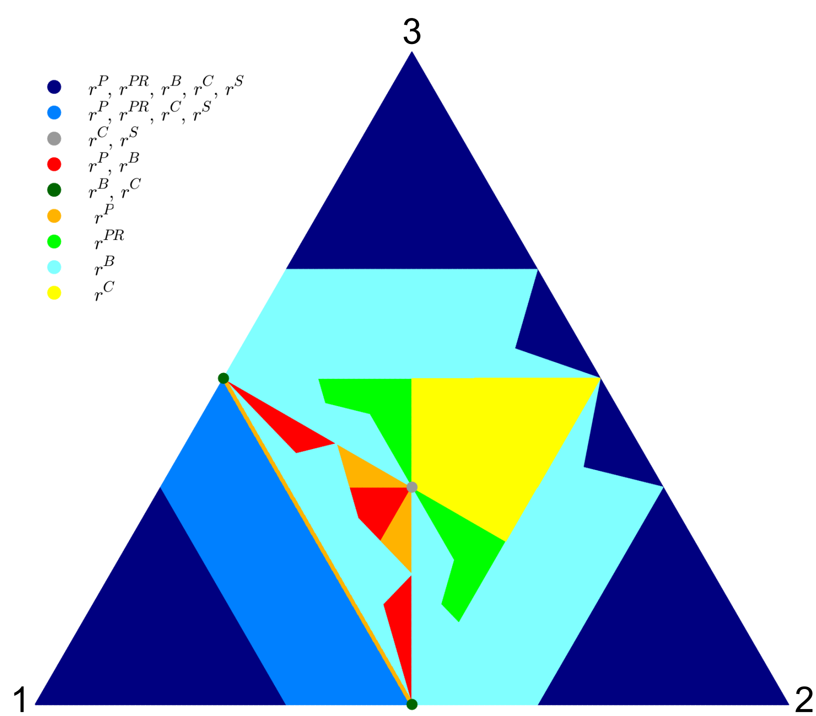

Figure 5.3 A map of the influence maximizing voting rules for $m = 3$.

5.6 Concluding Remarks

This chapter has proposed a framework for analyzing the a priori implications of different voting rules for the influence (voting power) of homogeneous voter groups that are differently sized or players who wield asymmetric voting weights in committees. For $m > 2$ alternatives, the distribution of influence depends not only on vote shares but also on which of the various existing aggregation methods – from simple plurality to complicated Schulze rule (among others) – are used. The existing indices of binary voting power need to be complemented by power indices for three or more alternatives. We provide one such index. It is based on evaluating whether preference changes by an individual player translate into a change of the outcome. The index rescales the probability that a random preference variation leads to an outcome change such that a dictator player has a power of one and a null player a power of zero (assuming preferences are independent and uniformly distributed a priori).

Although the specific distribution of voting weights matters for players' influence, one can nevertheless infer some general "rules of thumb" on procedural biases by considering all different weight distributions, at least for a small number of players. For the five rules in our focus, except for Borda rule, this is surprisingly easy and does not even require much knowledge of the exact distribution of voting weights. That is, the indicated recommendations in Section 5.5 are quite robust with respect to the specific distribution of weights. It gets more complicated when Borda rule is part of the menu. But even then it is possible to derive comprehensible (if admittedly less robust) recommendations.

Our results matter for the design of institutions and all committees whose decisions are not restricted to purely binary choices. In Section 5.4 we showed that the choice of the voting rule will indeed affect what individual members of the IMF's Executive Board can achieve if, e.g., it is to decide on three candidates for its next Managing Director. Obviously, there is ample choice for analyzing other real-world committees that use a weighted voting system and that have to choose, at least sometimes, between more than two alternatives. For instance, one could look at the newly constituted FIFA Council and the FIFA Congress. Both consist of (more or less) homogeneous but differently sized groups, i.e., representatives from UEFA, CAF, AFC, etc. It might be of interest to determine how much influence the election rules for the next FIFA president or World Cup location give to the respective regions in order to assess biases or attractiveness to outside influence.

There is also ample choice of other single-winner rules that could be analyzed and added to the influence map in Figure 5.3 (see, e.g., Aleskerov and Kurbanov 1999; Nurmi 2006, Ch. 7; or Laslier 2012). Moreover, nothing in principle precludes similar analysis for multi-winner rules (see, e.g., Elkind et al. 2017) and strategic voting equilibria. A first step towards analysis of the latter might focus on single-peaked preferences so as to exclude the complications of sophisticated voters (at least for Condorcet consistent rules).

Our voting power analysis is inspired by that of Penrose, Banzhaf, Shapley and others for the case of binary alternatives. It is therefore natural to ask whether there are analogues in our setting to well-known normative benchmarks in the binary case such as Penrose's square root rule or the Shapley linear rule (see, e.g, Felsenthal and Machover 1998, Ch. 3.4, for the former, and Kurz, Maaser, and Napel 2018 on the latter). We leave this for future research.

5.7 Appendix A Confidence Intervals for the IMF Example

	95 %-confidence interval $\mathcal{I}_i(r^P \vert \mathbf{w}_{\text{pre}})$	95 %-confidence interval $\mathcal{I}_i(r^P \vert \mathbf{w}_{\text{post}})$
USA	[0.7119; 0.7124]	[0.7046; 0.7051]
Japan	[0.1983; 0.1988]	[0.1986; 0.1990]
China	[0.1214; 0.1218]	[0.1964; 0.1969]
Netherlands*	[0.2089; 0.2094]	[0.1740; 0.1744]
Germany	[0.1849; 0.1853]	[0.1717; 0.1721]
Spain*	[0.1565; 0.1569]	[0.1709; 0.1713]
Indonesia*	[0.1252; 0.1255]	[0.1390; 0.1394]
Italy*	[0.1347; 0.1351]	[0.1329; 0.1333]
France	[0.1368; 0.1372]	[0.1303; 0.1307]
United Kingdom	[0.1368; 0.1372]	[0.1301; 0.1305]
Korea*	[0.1156; 0.1160]	[0.1255; 0.1259]
Canada*	[0.1148; 0.1152]	[0.1081; 0.1084]
Sweden*	[0.1084; 0.1087]	[0.1055; 0.1058]
Turkey*	[0.0931; 0.0934]	[0.1035; 0.1039]
Utd. Arab Emirates*	[0.1025; 0.1028]	[0.0997; 0.1000]
Brazil*	[0.0833; 0.0836]	[0.0982; 0.0986]
India*	[0.0895; 0.0898]	[0.0983; 0.0986]
South Africa*	[0.1089; 0,1092]	[0.0980; 0.0983]
Switzerland*	[0.0895; 0.0898]	[0.0882; 0.0885]
Russian Federation	[0.0761; 0.0764]	[0.0837; 0.0840]
Iran*	[0.0721; 0.0724]	[0.0705; 0.0708]
Saudi Arabia	[0.0894; 0.0897]	[0.0649; 0.0652]
Argentina*	[0.0586; 0.0588]	[0.0510; 0.0512]
Dem. Rep. Congo*	[0.0465; 0.0467]	[0.0504; 0.0506]

Table 5.9 95 %-confidence intervals for Monte-Carlo estimations of $\mathcal{I}_i(r^P|\mathbf{w}_{\text{pre}})$ and $\mathcal{I}_i(r^P|\mathbf{w}_{\text{post}})$

	95 %-confidence interval $\mathcal{I}_i(r^{PR}\vert\mathbf{w}_{\text{pre}})$	95 %-confidence interval $\mathcal{I}_i(r^{PR}\vert\mathbf{w}_{\text{post}})$
USA	[0.6732; 0.6737]	[0.6667; 0.6671]
Japan	[0.2237; 0.2241]	[0.2231; 0.2236]
China	[0.1402; 0.1406]	[0.2208; 0.2213]
Netherlands*	[0.2348; 0.2353]	[0.1970; 0.1974]
Germany	[0.2095; 0.2100]	[0.1947; 0.1952]
Spain*	[0.1786; 0.1790]	[0.1937; 0.1941]
Indonesia*	[0.1446; 0.1450]	[0.1593; 0.1597]
Italy*	[0.1549; 0.1553]	[0.1527; 0.1531]
France	[0.1573; 0.1577]	[0.1497; 0.1501]
United Kingdom	[0.1572; 0.1576]	[0.1497; 0.1501]
Korea*	[0.1338; 0.1342]	[0.1443; 0.1447]
Canada*	[0.1330; 0.1333]	[0.1253; 0.1256]
Sweden*	[0.1257; 0.1261]	[0.1223; 0.1226]
Turkey*	[0.1085; 0.1089]	[0.1201; 0.1204]
Utd. Arab Emirates*	[0.1193; 0.1196]	[0.1158; 0.1161]
Brazil*	[0.0977; 0.0981]	[0.1142; 0.1146]
India*	[0.1046; 0.1049]	[0.1143; 0.1146]
South Africa*	[0.1265; 0.1268]	[0.1139; 0.1142]
Switzerland*	[0.1045; 0.1049]	[0.1029; 0.1032]
Russian Federation	[0.0894; 0.0897]	[0.0977; 0.0980]
Iran*	[0.0849; 0.0852]	[0.0830; 0.0833]
Saudi Arabia	[0.1045; 0.1048]	[0.0765; 0.0768]
Argentina*	[0.0693; 0.0696]	[0.0602; 0.0605]
Dem. Rep. Congo*	[0.0553; 0.0556]	[0.0597; 0.0599]

Table 5.10 95 %-confidence intervals for Monte-Carlo estimations of $\mathcal{I}_i(r^{PR}|\mathbf{w}_{\text{pre}})$ and $\mathcal{I}_i(r^{PR}|\mathbf{w}_{\text{post}})$

	95 %-confidence interval $\mathcal{I}_i(r^C\|\mathbf{w}_{\text{pre}})$	95 %-confidence interval $\mathcal{I}_i(r^C\|\mathbf{w}_{\text{post}})$
USA	[0.6873; 0.6877]	[0.6804; 0.6809]
Japan	[0.2162; 0.2166]	[0.2157; 0.2162]
China	[0.1338; 0.1342]	[0.2134; 0.2139]
Netherlands*	[0.2275; 0.2279]	[0.1897; 0.1901]
Germany	[0.2021; 0.2026]	[0.1874; 0.1878]
Spain*	[0.1715; 0.1719]	[0.1863; 0.1867]
Indonesia*	[0.1379; 0.1383]	[0.1524; 0.1528]
Italy*	[0.1481; 0.1485]	[0.1459; 0.1462]
France	[0.1505; 0.1509]	[0.1430; 0.1434]
United Kingdom	[0.1504; 0.1508]	[0.1429; 0.1433]
Korea*	[0.1275; 0.1279]	[0.1378; 0.1382]
Canada*	[0.1267; 0.1270]	[0.1192; 0.1195]
Sweden*	[0.1197; 0.1200]	[0.1163; 0.1166]
Turkey*	[0.1030; 0.1033]	[0.1141; 0.1145]
Utd. Arab Emirates*	[0.1133; 0.1136]	[0.1099; 0.1102]
Brazil*	[0.0925; 0.0928]	[0.1085; 0.1089]
India*	[0.0991; 0.0995]	[0.1086; 0.1089]
South Africa*	[0.1204; 0.1207]	[0.1081; 0.1085]
Switzerland*	[0.0991; 0.0994]	[0.0975; 0.0978]
Russian Federation	[0.0846; 0.0849]	[0.0925; 0.0928]
Iran*	[0.0802; 0.0805]	[0.0783; 0.0786]
Saudi Arabia	[0.0991; 0.0994]	[0.0721; 0.0724]
Argentina*	[0.0653; 0.0656]	[0.0566; 0.0568]
Dem. Rep. Congo*	[0.0519; 0.0521]	[0.0561; 0.0564]

Table 5.11 95 %-confidence intervals for Monte-Carlo estimations of $\mathcal{I}_i(r^C|\mathbf{w}_{\text{pre}})$ and $\mathcal{I}_i(r^C|\mathbf{w}_{\text{post}})$

5.8 Appendix B
Binary Comparisons of Voting Rules for $m = 3$

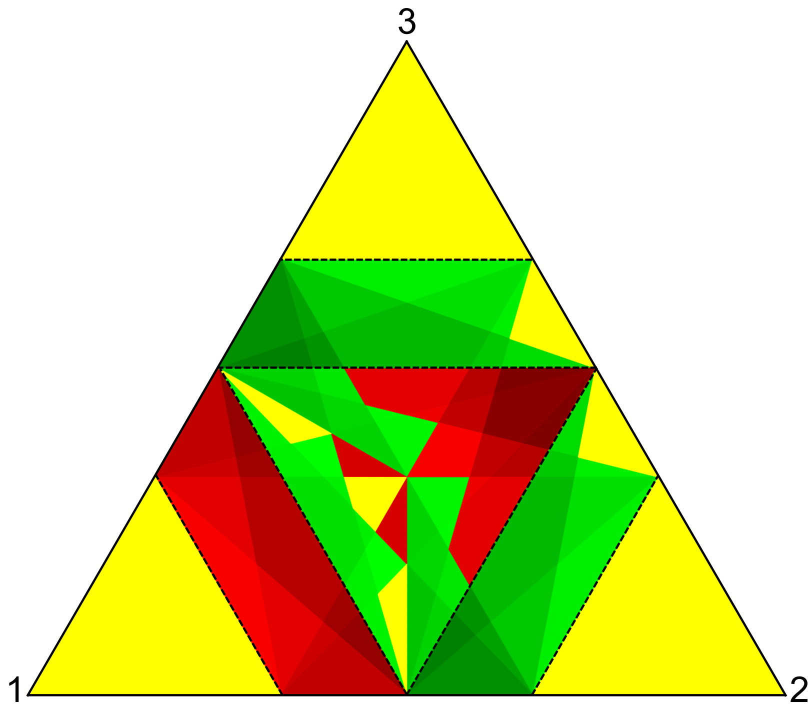

Figure 5.4 Borda vs. plurality

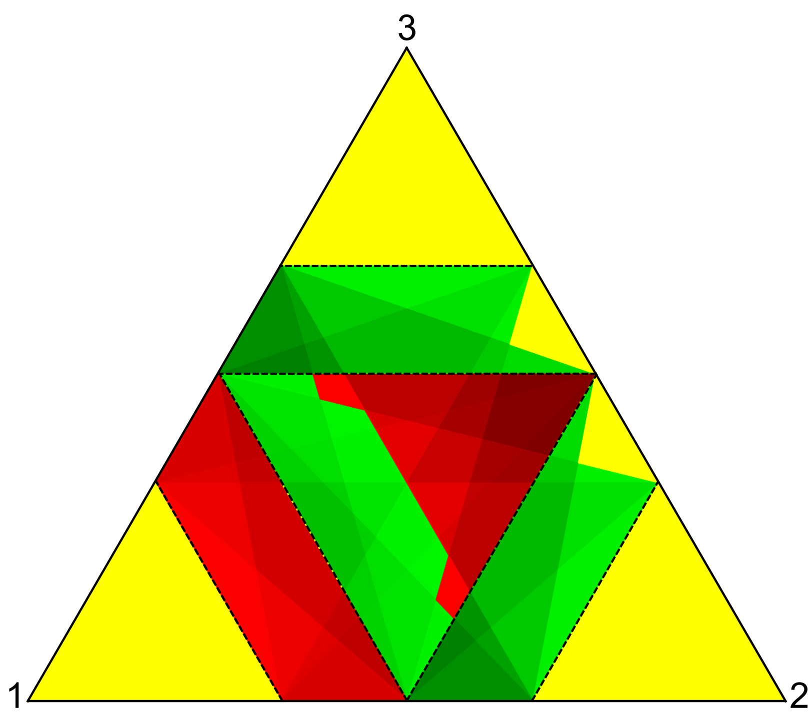

Figure 5.5 Borda vs. Copeland

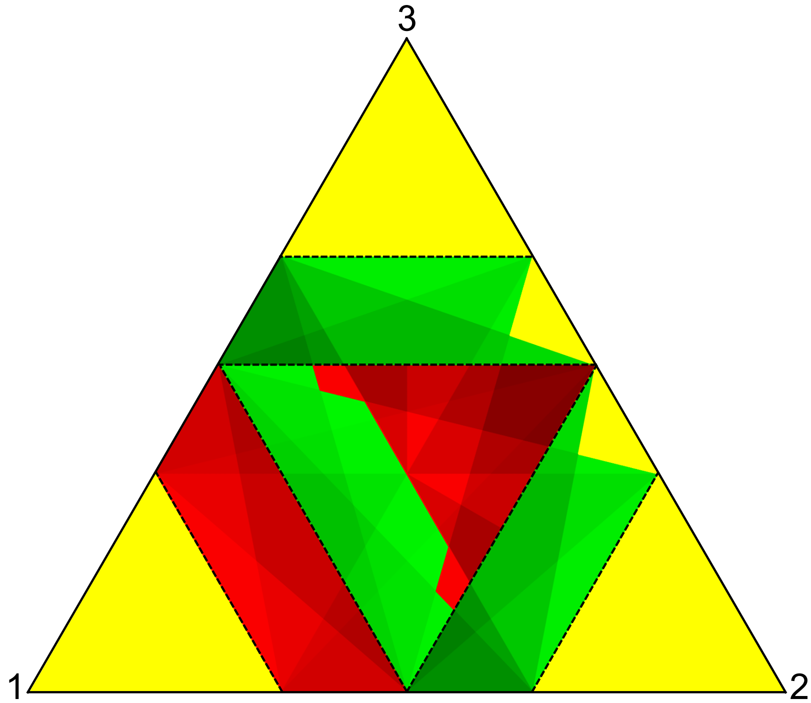

Figure 5.6 Borda vs. plurality runoff

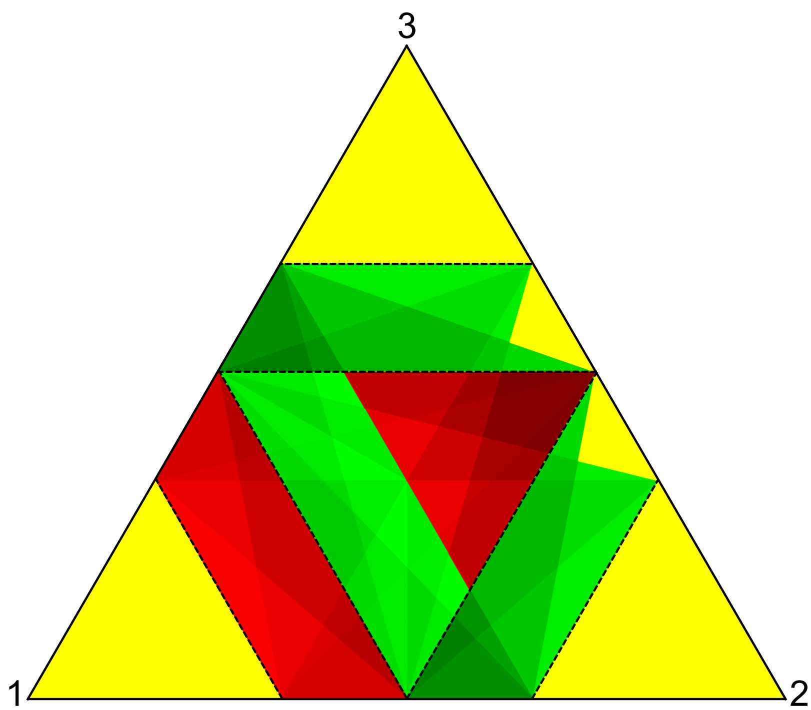

Figure 5.7 Borda vs. Schulze

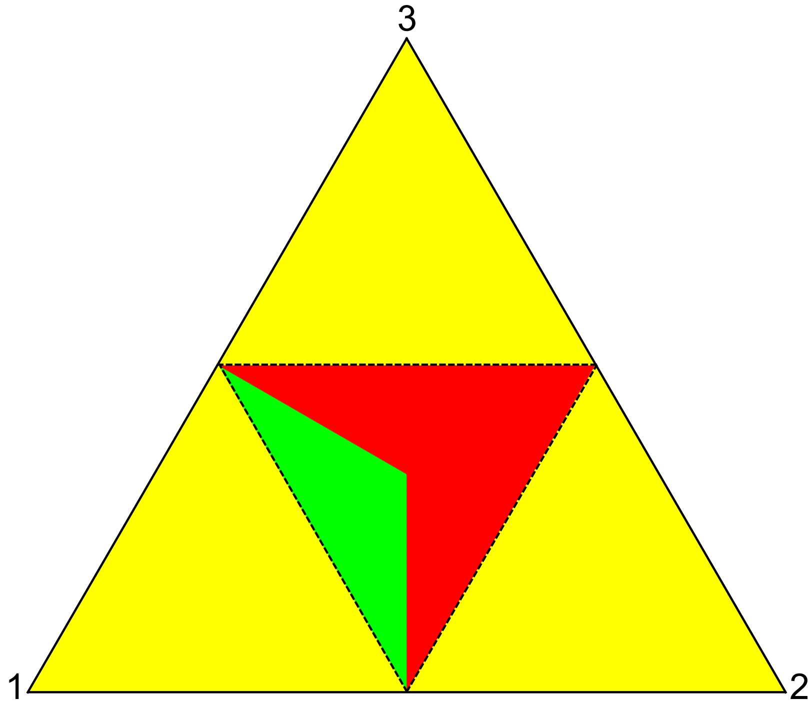

Figure 5.8 Plurality vs. plurality runoff, Copeland and Schulze

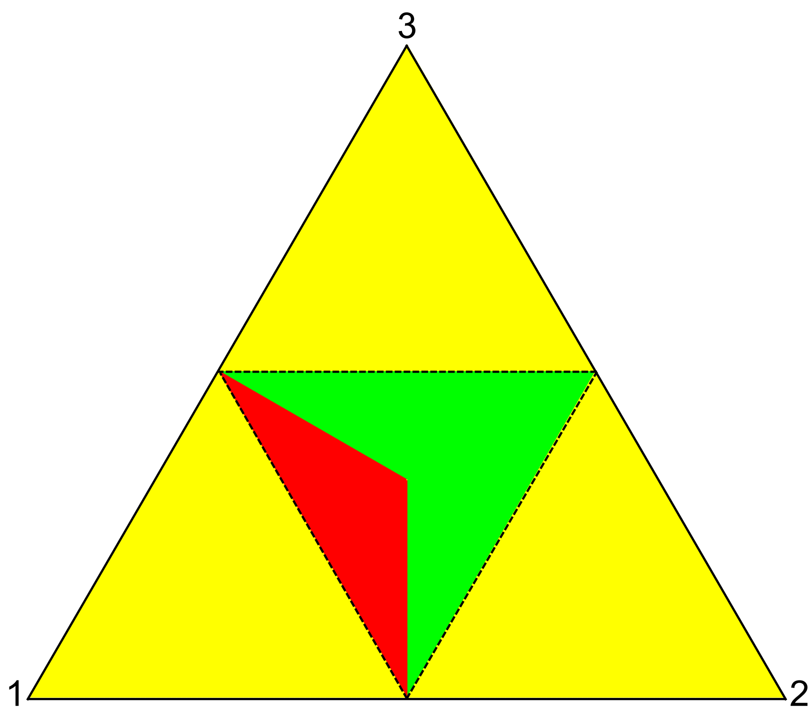

Figure 5.9 Copeland vs. Schulze

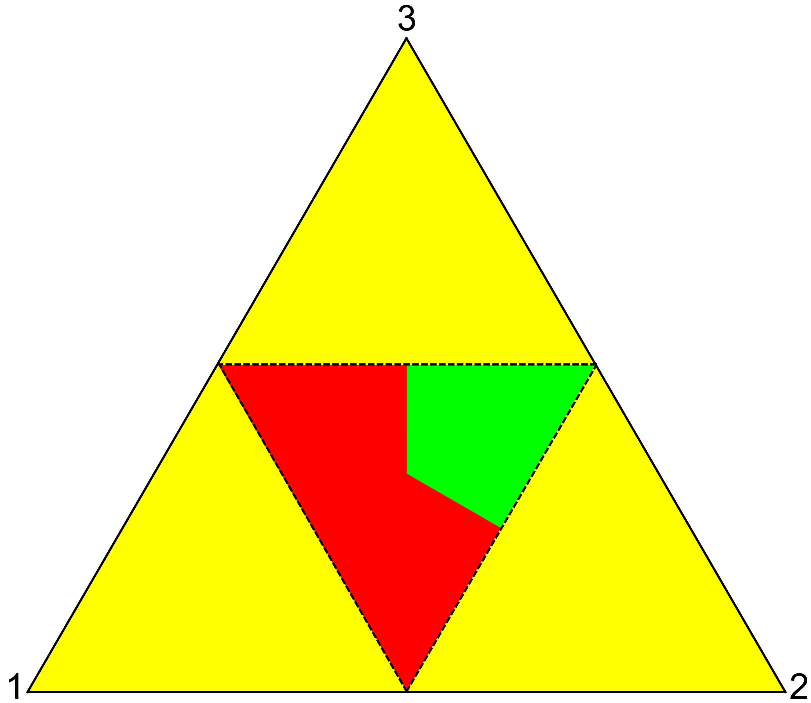

Figure 5.10 Copeland vs. plurality runoff

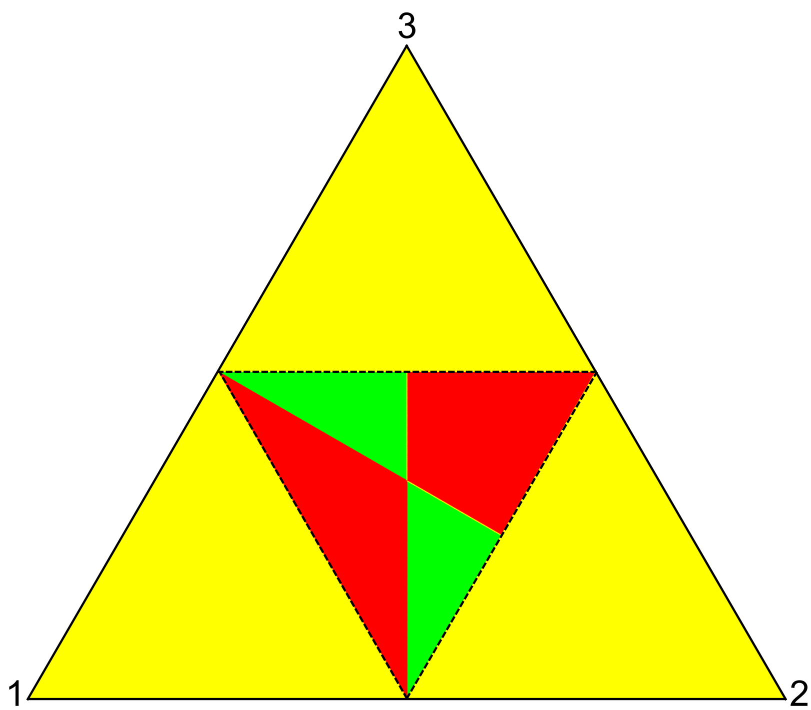

Figure 5.11 Plurality runoff vs. Schulze

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