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## Review Article

# Review of Local Non-Gaussianity from Multifield Inflation

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We review models which generate a large non-Gaussianity of the local form. We first briefly consider three models which generate the non-Gaussianity either at or after the end of inflation; the curvaton scenario, modulated (p)reheating, and an inhomogeneous end of inflation. We then focus on ways of generating the non-Gaussianity during inflation. We derive general conditions which a product or sum separable potential must satisfy in order to generate a large local bispectrum during slow-roll inflation. As an application, we consider two-field hybrid inflation. We then derive a formalism not based on slow roll which can be applied to models in which the slow-roll parameters become large before inflation ends. An exactly soluble two-field model is given in which this happens. Finally, we also consider further non-Gaussian observables, a scale dependence of  $f_{\text{NL}}$  and the trispectrum.

## 1. Introduction

There are many models of the universe which can predict a large non-Gaussianity. However the predicted amplitude and the shape of the non-Gaussianity are different among different classes of models. One category is those which generate the non-Gaussianity due to nontrivial classical dynamics on superhorizon scales. These models predict the shape of the bispectrum to be of the so-called “local type”, which can be expressed as an expansion of the Bardeen potential [1]

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{\text{NL}}(\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle), \quad (1)$$

where  $\Phi$  is the curvature perturbation on a Newtonian slice and  $\Phi_L$  is its linear and Gaussian part.  $\langle \Phi_L^2(\mathbf{x}) \rangle$  denotes the ensemble average in a statistically homogeneous distribution. The current limit on the local type of the nonlinearity parameter  $f_{\text{NL}}$  from seven years of WMAP data [2] is  $-10 < f_{\text{NL}} < 74$  at the 95% confidence level. Constraints are expected to improve rapidly and significantly, first with Planck data and later using large scale structure data, see the recent reviews [3–5]. The Bardeen potential is related to the

primordial curvature perturbation of  $\zeta$  on large scales and in the matter dominated era by  $\Phi = (3/5)\zeta$ .

The curvature perturbation at horizon exit is determined by the classical perturbations of the scalar fields,  $\delta\phi_i(\mathbf{x})$ . The subsequent evolution of  $\zeta$  can be conveniently described by the  $\delta N$  formalism [6–11]. The curvature perturbation is given by up to quadratic terms [11]

$$\zeta = \delta N = \sum_I N_{,I} \delta\varphi_{I*} + \frac{1}{2} \sum_{IJ} N_{,IJ} \delta\varphi_{I*} \delta\varphi_{J*} + \dots, \quad (2)$$

where  $N(\mathbf{x}, t)$  is the e-folding number evaluated in an unperturbed Universe, from the epoch of horizon exit to later epoch of uniform energy density hypersurface (for an extension to include gradient terms, see [12]). The power spectrum  $\mathcal{P}_\zeta$  and the bispectrum  $B_\zeta$  are defined by

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) \frac{2\pi^2}{k_1^3} \mathcal{P}_\zeta(k_1), \quad (3)$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3).$$

From this, we can define the observable quantities, the spectral index, the tensor-to-scalar ratio, and the nonlinearity parameter

$$n_\zeta - 1 \equiv \frac{\partial \log \mathcal{P}_\zeta}{\partial \log k}, \quad (4)$$

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = \frac{8\mathcal{P}_*}{M_p^2 \mathcal{P}_\zeta}, \quad (5)$$

$$f_{\text{NL}} = \frac{5}{6} \frac{k_1^3 k_2^3 k_3^3}{k_1^3 + k_2^3 + k_3^3} \frac{B_\zeta(k_1, k_2, k_3)}{4\pi^4 \mathcal{P}_\zeta^2}, \quad (6)$$

where  $\mathcal{P}_T = 8\mathcal{P}_*/M_p^2 = 8H_*^2/(4\pi^2 M_p^2)$  is the power spectrum of the tensor metric fluctuations. It is well known that single-field inflation does not lead to a detectably large non-Gaussianity, in fact  $f_{\text{NL}}$  is suppressed by slow-roll parameters [13]. Observably large non-Gaussianity can be obtained by breaking the slow-roll conditions during inflation [14], using extended kinetic terms [15–25], see also the reviews [26, 27], or going beyond models of single-field inflation [11, 28–34]

It is natural to consider multiple scalar field since they are ubiquitous in many beyond the standard model of particle physics, such as supersymmetry and string theory. These scalar fields generate nonadiabatic perturbations during inflation and change the evolution of the curvature perturbation after horizon exit. The residual isocurvature perturbation may be present in the primordial density fluctuation and can be correlated with the curvature perturbation or may be responsible for an observably large non-Gaussianity in the cosmic microwave background and large scale structure, for observational limits on isocurvature perturbations see [1, 35, 36]. In this paper, we will only consider models with adiabatic primordial perturbations, in which the isocurvature perturbation present during inflation is converted into an adiabatic perturbation. We also neglect the secondary non-Gaussianities generated at later times, for example see [37, 38].

There are popular multifield models [39] which may generate observably large non-Gaussianity. These include the curvaton scenario, modulated (p)reheating, and an inhomogeneous end of inflation, see Section 2. In these scenarios, large non-Gaussianity is generated either by the means of ending inflation, or after inflation. It was shown recently that it also possible to generate large non-Gaussianity during the evolution of slow-roll multifield inflation, see Section 3.

All of these models generate the large non-Gaussianity after horizon exit, such as after reheating, at the end of inflation, at the phase transition, or during inflation after horizon exit and involves the perturbation of the nonadiabatic mode. Therefore, the non-Gaussianity of these models is of the local type which is distinguishable from other shapes of non-Gaussianity (for a list of possibilities see, e.g., [40–42]), in which the non-Gaussianity is generated intrinsically from the quantum fluctuations, or during horizon exit.

In Section 2, we summarise the aforementioned three models, which are popular methods of generating a large non-Gaussianity. In Section 3, we review the possibility of

generating a large non-Gaussianity from multifield slow-roll inflation, and in Section 4, we consider hybrid inflation with two inflaton fields as an application. Then in Section 5, we discuss multifield models of inflation without assuming the slow-roll conditions and present an exact solution. Non-Gaussian observables beyond  $f_{\text{NL}}$ , such as its scale dependence and the trispectrum are introduced in Section 6. Finally, we conclude in Section 7.

## 2. Summary of Popular Models Generating a Large Local Non-Gaussianity

Multifield inflationary models [39] can lead to an observable signature of non-Gaussianity. These include models in which the large non-Gaussianity is generated either by the means of ending inflation, or after inflation. We review the three most popular of these models in this section, for a discussion on how these models are related see [43]. In the next section, we review the possibility of generating large non-Gaussianity during multifield slow-roll inflation. Later in Section 5, we discuss multifield models of inflation without assuming the slow-roll conditions. This does not exhaust all of the possible ways of generating a large local non-Gaussianity from inflation, see also the reviews [44–47] and the papers [48, 49]. Non-Gaussianity of the local form can also be generated in the ekpyrotic scenario [50, 51], although in the simplest case  $f_{\text{NL}}$  is large and negative [52], which is observationally ruled out.

*2.1. Curvaton Scenario.* In this scenario, there is a light weakly-interacting inhomogeneous scalar field, the curvaton  $\chi$ , in addition to the inflaton field,  $\phi$ . The additional scalar field is completely subdominant during inflation however, it can dominate the energy density of the Universe later since the time-averaged equation of state becomes that of the pressureless matter ( $P = 0$ ), whose energy density decreases more slowly than that of the radiation which the inflaton field has decayed into. After decay, the inhomogeneity of the scalar field leads to the density perturbation of radiation [53, 54]. This is the basis of the curvaton scenario [55–59] which liberates the inflaton field from being required to generate the observed spectrum of perturbations (although there is instead an upper bound on their allowed magnitude).

The significant non-Gaussianity of the local type can be generated in the curvaton model [60]. A study of non-Gaussianity in the curvaton scenario using second-order cosmological perturbation theory was done by Bartolo et al. [31] with a sudden decay approximation and also by Malik and Lyth [32] using a fully numeric approach, which accounted for both the sudden decay and nonsudden decay approximations. Lyth and Rodríguez [11] used the  $\delta N$  formalism including the quadratic terms (second order terms in (2)) to calculate the non-Gaussianity. The nonlinear generalisation of the curvature perturbation and a numerical study was done to give a full probability distribution function in [61]. This simple curvaton model was extended into the mixed curvaton-inflaton scenario in which fluctuations of both the inflaton and a curvaton contribute to the primordial

density perturbation studying the isocurvature perturbation in [62]. The multiple curvaton fields which contribute to the primordial density perturbation was studied in [63–65]. Observational constraints on curvaton models with isocurvature perturbation and non-Gaussianity can be found in [66–68].

The generation of large non-Gaussianity in the curvaton scenario can be easily understood using the result of the nonlinear  $\delta N$  formalism [61, 62]. Assuming the sudden decay of the curvaton field on the  $H = \Gamma$  uniform energy-density hypersurface, that is, when the local Hubble rate equals the decay rate for the curvaton, leads to a nonlinear relation between the local curvaton density and the radiation density before and after the decay

$$\Omega_r e^{4(\zeta_{\text{inf}} - \zeta)} + \Omega_\chi e^{3(\zeta_\chi - \zeta)} = 1, \quad (7)$$

where  $\Omega_r + \Omega_\chi = 1$ . Here,  $\zeta$  is the primordial curvature perturbation which remains constant on large scales in the radiation-dominated era after the curvaton decays and  $\zeta_\chi$  is the curvaton perturbation. Expanding this equation order by order yields up to second order [61]

$$\begin{aligned} \zeta = & (1 - r_\chi) \zeta_{\text{inf}} + r_\chi \zeta_\chi + \frac{r_\chi (1 - r_\chi) (3 + r_\chi)}{2} (\zeta_\chi - \zeta_{\text{inf}})^2 \\ & + \text{higher-order terms,} \end{aligned} \quad (8)$$

where  $r_\chi \equiv 3\Omega_\chi/(4 - \Omega_\chi)$  at the time of the curvaton decay. The curvature perturbation of curvaton field [61]

$$\zeta_\chi = \frac{2}{3} \frac{\delta_1 \chi}{\chi_0} - \frac{1}{3} \left( \frac{\delta_1 \chi}{\chi_0} \right)^2 + \frac{2}{9} \left( \frac{\delta_1 \chi}{\chi_0} \right)^3, \quad (9)$$

is almost constant before the curvaton decays from the start of curvaton oscillation. Note that the intrinsic non-Gaussianity in the curvaton field during oscillation is  $f_{\text{NL},\chi} = -5/4$ , which comes from the nonlinear relation between  $\delta_1 \chi$  and  $\zeta_\chi$ . After the curvaton decays into radiation, the curvature perturbation of radiation has the following non-Gaussianity parameters: [11, 31, 61]

$$f_{\text{NL}} = \frac{\tilde{r}^2}{(1 + \tilde{r})^2} \left[ \frac{5}{4r_\chi} - \frac{5}{3} - \frac{5r_\chi}{6} \right], \quad (10)$$

where  $r_\chi$  is evaluated when the curvaton decays based on the sudden decay approximation but gives good agreement with a full numerical study [32, 61], and  $\tilde{r}$  is defined as

$$\tilde{r} \equiv \frac{|r_\chi \zeta_\chi|^2}{|(1 - r_\chi) \zeta_{\text{inf}}|^2}. \quad (11)$$

In the limit of the pure curvaton scenario,  $\zeta_{\text{inf}} \rightarrow 0$ , it recovers the usual normalization for  $f_{\text{NL}}$ , that is,  $\tilde{r}^2/(1 + \tilde{r})^2 \rightarrow 1$ . Therefore, the large  $f_{\text{NL}}$  is obtained for small  $r_\chi$ . This large  $f_{\text{NL}}$ , in spite of the order of unity non-Gaussianity in  $\zeta_\chi$ , has the origin in the nonlinear relation between  $\zeta_\chi$  (or  $\delta_1 \chi$ ) and  $\zeta$  in (8).

In fact (7), and thus (8), can be used “at any time” before the curvaton decay with  $r_\chi$  evaluated at that time to find the total curvature perturbation at that moment, as well as at the epoch of curvaton decay. This enables us to understand the evolution of the curvature perturbation  $\zeta$  on the uniform energy density hypersurface before the curvaton decay. Initially when the primordial radiation was generated, for example, from the inflaton decay,  $r_\chi$  is negligibly small so we cannot ignore the perturbations of the inflaton field. At this early time,  $\zeta = \zeta_{\text{inf}}$  and  $f_{\text{NL}} = 0$  since  $\tilde{r} \propto r_\chi^2$  which makes  $f_{\text{NL}}$  negligible in (10). As time goes on,  $r_\chi$  increases while  $\zeta_{\text{inf}}$  and  $\zeta_\chi$  are almost constant since there is no energy flow between two fluids satisfying adiabatic condition except around the decay time, thus the curvature perturbation evolves *gradually*. If the curvaton survives long enough,  $r_\chi$  reaches a value with which the  $r_\chi \zeta_\chi$  dominates the first term,  $\zeta_{\text{inf}}$ , and the curvaton perturbation is then responsible for the primordial inhomogeneity of the Universe. The large non-Gaussianity is possible only when  $r_\chi$  at the decay time is small that is, the energy density of curvaton is subdominant compared to that of radiation. This is because when  $r_\chi = 1$ , the curvaton energy density dominates over the radiation component before decay, then  $\zeta$  is effectively the curvature perturbation of a single field (curvaton),  $\zeta_\chi$ , and therefore non-Gaussianity  $f_{\text{NL}} = f_{\text{NL},\chi} = -5/4$ . We will also see this phenomena in the multifield slow-roll inflation later, in Section 3.

**2.2. Modulated (P)reheating.** After inflation, the energy density in the inflaton field must be transferred into radiation. In the simplest case of adiabatic perturbations, this process does not affect the primordial curvature perturbation on scales which are observable today, because these scale were much larger than the horizon at the time of reheating. However, in a spirit similar to the curvaton model, there may be a subdominant light scalar field present during inflation which modulates the efficiency of reheating. This makes the efficiency of reheating a spatially dependent process. The quasiscale invariant perturbations in this field, which during inflation are an isocurvature perturbation, may be converted into the primordial curvature perturbation during this process. For a review of reheating after inflation, see for example [69, 70].

As a simple illustration of this, in the “old” models of perturbative reheating, the decay rate of the inflaton is given by  $\Gamma \sim \lambda^2 m$ , where  $\lambda$  is the strength of the coupling between the inflaton and reheating fields and  $m$  is the mass of the inflaton. Provided that  $\Gamma$  is much less than the Hubble rate at the end of inflation, so that reheating takes place slowly, the reheating temperature is given by

$$T_{\text{reh}} \sim \sqrt{\Gamma M_P} \sim \lambda \sqrt{m M_P}. \quad (12)$$

If the coupling strength is a function of the local value of a second scalar field  $\chi$  then this will give rise to a perturbation in the decay rate of the inflaton field, and thus in the reheating temperature which is responsible for the density

perturbation after reheating. Provided that the  $\chi$  field is perturbed, the curvature perturbation can be written as

$$\zeta = \alpha \frac{\delta\Gamma}{\Gamma}, \quad (13)$$

where  $\alpha$  is a parameter depending on the ratio of  $\Gamma$  to the Hubble expansion at the end of inflation, whose value increases with decreasing  $\Gamma/H$  and leads to  $1/6$  in the limit  $\Gamma \ll H_{\text{inf}}$  [33, 71]. Hence, perturbations of the light scalar field  $\chi$  can be imprinted into the radiation temperature through the reheating process [71, 72]. In a similar way, the mass of the decaying particle may be modulated [73]. The above equation can be extended to second order, which is required to calculate  $f_{\text{NL}}$ . In the simple quadratic case that  $\Gamma = \Gamma_0 + \Gamma_1(\chi/\bar{\chi})^2 = \Gamma_0 + \Gamma_1(1 + \delta\chi/\bar{\chi})^2$ , where  $\bar{\chi}$  and  $\delta\chi$  are the homogeneous background part and the perturbation of the field  $\chi$ , respectively, the result is [33, 74–76]

$$f_{\text{NL}} = \frac{5}{12} \frac{\Gamma_0 + \Gamma_1}{\Gamma_1 \alpha}. \quad (14)$$

Hence, we can see that the non-Gaussianity can easily be larger than unity in this model by two sources. One is due to the small value of  $\alpha$  which occurs when the decay rate of the inflation is not much smaller than the Hubble constant at the end of inflation. The other is when  $\Gamma_1$  is small in which  $\chi$  field only controls part of the decay channels of inflaton field.

It is now generally considered that a period of rapid and highly nonlinear preheating preceded the perturbative reheating phase [69]. In this case, one can instead consider modulated preheating, see, for example, [77–79]. A particularly rapid form of preheating, known as instant preheating [80], has attracted more attention in this area, as it is possible to find analytic estimates [81–85]. It is possible to generate the primordial curvature perturbation in these models by having either a weak [81] or a strong [83] symmetry breaking of the two-field potential during inflation which affects the efficiency of instant preheating, or by having two inflaton fields which are coupled with different strengths to the preheat field [82]. However, all of the above models of modulated instant preheating are only allowed, even at the linear level in perturbations, for a small corner of parameter space and even then only for a special choice of initial conditions [85].

**2.3. Inhomogeneous End of Inflation.** An inhomogeneous phase transition can occur also at the end of inflation between inflationary and non-inflationary expansion due to the condition of how inflation ends [34, 86]. In the case of multicomponent inflation, there is a family of inflationary trajectories. If the family of trajectories is not perpendicular to the surface on which inflation ends then there is a possibility that inflation ends on a slice which is not of uniform density and the generation of the curvature perturbation at the end of inflation is expected [34] (This was also discussed in [87–89] using the linear relation between two fields at the end of inflation and the non-Gaussianity is transferred from that of the light field which is generated from its nonlinear self-coupling). In single field inflation, the

energy density is determined solely by the single field  $\phi$ , and the end of inflation is parameterised by the unique value  $\phi_e$ . If there is another field  $\sigma$ , then  $\phi_e(\sigma)$  will depend on position through the perturbation  $\delta\sigma(\mathbf{x})$ . As a result, the change of e-folding number from a spacetime slice of uniform energy density just before the end of inflation to a spacetime slice of uniform density just after the end of inflation will have a perturbation  $\delta N_e = \zeta_e$ . This  $\zeta_e$  is the curvature perturbation generated by the end of inflation and can dominate over the contribution from inflation [34].

Using the perturbation of  $\phi_e(\sigma)$

$$\delta\phi_e = \phi_e' \delta\sigma + \frac{1}{2} \phi_e'' (\delta\sigma)^2, \quad (15)$$

the curvature perturbation generated at the end of inflation is [34]

$$\begin{aligned} \zeta_e &= N_e' \delta\phi_e + \frac{1}{2} N_e'' (\delta\phi_e)^2 \\ &= N_e' \phi_e' \delta\sigma + \frac{1}{2} [2N_e'' \phi_e'^2 + n_e' \phi_e''] (\delta\sigma)^2. \end{aligned} \quad (16)$$

When  $\zeta_e$  dominates the perturbation generated during the inflation, the power spectrum is given by

$$\mathcal{P}_e = \frac{\phi_e'^2}{2\epsilon_e} \left( \frac{H^*}{2\pi} \right)^2 \quad (17)$$

The non-Gaussianity can also be generated at the end of inflation since the perturbation at the end of inflation is not required to be Gaussian. Thus, we expect that the observable large non-Gaussianity can be generated in this process. From (16) the non-Gaussianity is [34]

$$f_{\text{NL}} = -\frac{5}{3} \sqrt{\frac{\epsilon_e}{2}} \frac{\phi_e''}{(\phi_e')^2}. \quad (18)$$

This was applied to the simple example of extended hybrid inflation using sudden end approximation [90, 91].

Recently Sasaki [92], Naruko and Sasaki [93] calculated analytically the curvature perturbation and the non-Gaussianity in the multi-brid inflation models from horizon exit to the end of inflation considering the general couplings to the waterfall field. In general, we cannot just ignore the curvature perturbation generated during inflation. Furthermore, that is also affected by the end of inflation. A deeper understanding of non-Gaussianity both from the evolution during multifield hybrid inflation and effects from the end of inflation is considered by Byrnes et al. [94, 95] which is summarised in Section 3.

### 3. Non-Gaussianity during Slow-Roll Inflation

It is well known that the single field slow-roll inflation with canonical kinetic terms generates a nonlinear parameter of the order of the spectral tilt of the spectrum and thus too small to be observed [13]. On the other hand, in multifield inflationary models the nonadiabatic perturbation may change this conclusion. Furthermore, even within slow-roll inflation an observably large non-Gaussianity can be generated if certain conditions are satisfied.



Rigopoulos et al. introduced a formalism [96, 97] to deal with nonlinearity based on a system of fully nonlinear equations for long wavelengths and applied it to a two-field inflation model. In the case of two-field quadratic inflation with two massive fields, an analytic calculation is possible with slow-roll conditions, and this shows that the nonlinear parameter is small [98, 99]. The numerical calculation confirms the analytical prediction that it is possible to generate a narrow spike of large non-Gaussianity while the inflaton trajectory turns a corner, but the non-Gaussianity decays quickly after the corner [99, 100]. This spike is due to the temporary jump in the slow-roll parameters [101, 102].

Recently, an analysis based on the general analytic formula was done by Byrnes et al. [94, 95], and they showed that it is possible to generate observable large non-Gaussianity even during slow-roll multifield inflation models and explicitly wrote the conditions as well as concrete examples (There is a claim [103] that the sizeable value of  $f_{\text{NL}}$  can arise from a loop corrections). In this section, we summarise the general conditions for two-field slow-roll inflation to generate a large non-Gaussianity and briefly give a specific example. In Section 4 we consider the multifield hybrid inflation model which has been studied in depth, and we include the effects from the end of inflation.

**3.1. General Formulas.** In the case of a potential which is either sum or product separable, it is possible to compute the curvature perturbation using the  $\delta N$ -formalism and slow-roll conditions, and an analytic formula for the nonlinearity parameter can be obtained. The general formula for the nonlinearity parameter  $f_{\text{NL}}$  was calculated in the case of a separable potential by sum [99], and this was later extended to the non-canonical kinetic terms as well as to the separable potential by product [104]. This has been further generalized to the arbitrary number of fields [105] and to the trispectrum [106].

For a product separable potential,  $W(\varphi, \chi) = U(\varphi)V(\chi)$ , we can find the number of  $e$ -foldings analytically in the slow-roll limit [107],

$$N(\varphi_*, \chi_*) = -\frac{1}{M_P^2} \int_*^e \frac{U}{U} d\varphi = -\frac{1}{M_P^2} \int_*^e \frac{V}{V} d\chi. \quad (19)$$

Then, the power spectrum and the nonlinear parameters can be calculated straightforwardly using the derivatives of  $e$ -folding number [99, 104, 107]. In the case of two inflatons with canonical kinetic terms, the power spectrum and spectral index are found to be [104]

$$\begin{aligned} \mathcal{P}_\zeta &= \frac{W_*}{24\pi^2 M_P^4} \left( \frac{u^2}{\epsilon_\varphi^*} + \frac{v^2}{\epsilon_\chi^*} \right), \\ n_\zeta - 1 &= -2\epsilon^* - 4 \frac{u^2(1 - \eta_{\varphi\varphi}^*/2\epsilon_\varphi^*) + v^2(1 - \eta_{\chi\chi}^*/2\epsilon_\chi^*)}{u^2/\epsilon_\varphi^* + v^2/\epsilon_\chi^*}, \\ r &= 16 \left( \frac{u^2}{\epsilon_\varphi^*} + \frac{v^2}{\epsilon_\chi^*} \right)^{-1}, \end{aligned} \quad (20)$$

where the slow-roll parameters are

$$\epsilon_\varphi = \frac{M_P^2}{2} \left( \frac{U_\varphi}{U} \right)^2 = \epsilon \cos^2 \theta, \quad \epsilon_\chi = \frac{M_P^2}{2} \left( \frac{V_\chi}{V} \right)^2 = \epsilon \sin^2 \theta, \quad (21)$$

where  $\theta$  is the angle between the adiabatic perturbation and one of the fields and

$$\eta_{\varphi\varphi} = M_P^2 \frac{V_{\varphi\varphi}}{V}, \quad \eta_{\varphi\chi} = M_P^2 \frac{U_\varphi V_\chi}{W}, \quad \eta_{\chi\chi} = M_P^2 \frac{V_{\chi\chi}}{V}. \quad (22)$$

Note that the superscript “\*” and “ $e$ ” denotes that the values are evaluated at horizon exit and at end of inflation, respectively. In the above,  $u$  and  $v$  are evaluated at the end of inflation as

$$u \equiv \frac{\epsilon_\varphi^e}{\epsilon^e} = \cos^2 \theta^e, \quad v \equiv \frac{\epsilon_\chi^e}{\epsilon^e} = \sin^2 \theta^e. \quad (23)$$

The nonlinearity parameter  $f_{\text{NL}}$  becomes [104]

$$\begin{aligned} f_{\text{NL}} &= \frac{5}{6} \frac{2}{(u^2/\epsilon_\varphi^* + v^2/\epsilon_\chi^*)^2} \\ &\times \left[ \frac{u^3}{\epsilon_\varphi^*} \left( 1 - \frac{\eta_{\varphi\varphi}^*}{2\epsilon_\varphi^*} \right) + \frac{v^3}{\epsilon_\chi^*} \left( 1 - \frac{\eta_{\chi\chi}^*}{2\epsilon_\chi^*} \right) \right. \\ &\quad \left. - \left( \frac{u}{\epsilon_\varphi^*} - \frac{v}{\epsilon_\chi^*} \right)^2 \mathcal{A}_P \right] \end{aligned} \quad (24)$$

where

$$\begin{aligned} \mathcal{A}_P &\equiv -\frac{\epsilon_\varphi^e \epsilon_\chi^e}{(\epsilon^e)^2} \left[ \hat{\eta}^e - 4 \frac{\epsilon_\varphi^e \epsilon_\chi^e}{\epsilon^e} \right], \\ \hat{\eta} &\equiv \frac{\epsilon_\chi \eta_{\varphi\varphi} + \epsilon_\varphi \eta_{\chi\chi}}{\epsilon}. \end{aligned} \quad (25)$$

Similar formulas are found in the case of a sum separable potential [99, 104].

**3.2. Conditions for Generating a Large  $f_{\text{NL}}$ .** Using an analytic formula for the nonlinear parameter  $f_{\text{NL}}$  in the case of a sum or product separable potential, it is possible to generate significant non-Gaussianity even during slow-roll inflation with Gaussian perturbations at Hubble exit and the general conditions for it can be written explicitly [94].

For the separable potential by product large nonlinear parameter  $|f_{\text{NL}}| \gtrsim 1$  is possible when (there is a symmetric region of large non-Gaussianity under the exchange of the two fields).

$$\begin{aligned} \sin^2 \theta^* &\lesssim \sin^4 \theta^e \left( \frac{1}{\sqrt{\sin^2 \theta^e \mathcal{G}_P}} - 1 \right), \\ \mathcal{G}_P &= \frac{6}{5} \left| -\eta_{\chi\chi}^* + 2\eta_{\chi\chi}^e \right|^{-1}. \end{aligned} \quad (26)$$

with this condition, the large  $f_{\text{NL}}$  is given by

$$f_{\text{NL}} \simeq \frac{5}{6} \frac{\sin^6 \theta^e}{(\sin^2 \theta^* + \sin^4 \theta^e)^2} \left[ -\eta_{\chi\chi}^* + 2\eta_{\chi\chi}^e \right]. \quad (27)$$

For large  $f_{\text{NL}}$ , the substantial increase in  $\sin \theta$  is necessary between horizon exit and the end of inflation, which corresponds to the curve of the trajectory in the field space during the slow-roll inflation. However, the background trajectory must be almost entirely in the one-field direction.

One specific toy model is the quadratic times exponential potential [94], where the potential is given by

$$W(\varphi, \chi) = \frac{1}{2} e^{-\lambda \varphi^2 / M_P^2} m^2 \chi^2. \quad (28)$$

With expansion of exponential, this potential can be understood as the quadratic chaotic inflation model which has a coupling to another light scalar field. For this potential, considering the condition for large non-Gaussianity, (26), large  $f_{\text{NL}}$  is generated when the  $\varphi$  at horizon exit is small. One example is found to give  $|f_{\text{NL}}| \sim 30$  with  $\varphi^* \sim 10^{-3} M_P$  for  $\lambda = 0.05$  and  $\chi^* = 16 M_P$ .

For a sum separable potential, we can find similar conditions for generating a large  $f_{\text{NL}}$  [94].

#### 4. Hybrid Inflation with Two Inflaton Fields

We consider a model of two field hybrid inflation, whose potential is given by

$$\begin{aligned} W(\varphi, \chi) &= W_0 \exp\left(\frac{1}{2} \eta_{\varphi\varphi} \frac{\varphi^2}{M_P^2}\right) \exp\left(\frac{1}{2} \eta_{\chi\chi} \frac{\chi^2}{M_P^2}\right) \\ &\simeq W_0 \left(1 + \frac{1}{2} \eta_{\varphi\varphi} \frac{\varphi^2}{M_P^2} + \frac{1}{2} \eta_{\chi\chi} \frac{\chi^2}{M_P^2}\right), \end{aligned} \quad (29)$$

which is vacuum dominated, that is, which satisfies  $|\eta_{\varphi\varphi} \varphi^2| \ll M_P^2$  and  $|\eta_{\chi\chi} \chi^2| \ll M_P^2$ . It is in this regime that the two ways of writing the potential given above are equivalent (at leading order in slow roll), so we can treat the potential as being both sum and product separable. We assume that inflation ends abruptly by a waterfall field which is heavy during inflation and hence does not affect the dynamics during inflation. First, we calculate observables during slow-roll inflation. We will consider the full potential including the waterfall field in Section 4.2, which incorporate the effects from the end of inflation considered in Section 2.3. We will see that this can lead to a change in observables on the surface where the waterfall field is destabilised.

In the vacuum-dominated regime, the slow-roll solutions are

$$\varphi(N) = \varphi_* e^{-\eta_{\varphi\varphi} N}, \quad \chi(N) = \chi_* e^{-\eta_{\chi\chi} N}, \quad (30)$$

where “\*” denotes the value at the horizon exit. Throughout this section, whenever we write a quantity without making it explicit at which time it should be evaluated, we mean the equation to be valid at any time  $N$   $e$ -foldings after Hubble exit and while slow roll is valid. Generally, we will

be interested in quantities at the end of inflation, in which case we take  $N = 60$ .

The slow-roll parameters are

$$\epsilon_\varphi = \frac{1}{2} \eta_{\varphi\varphi}^2 \frac{\varphi^2}{M_P^2}, \quad \epsilon_\chi = \frac{1}{2} \eta_{\chi\chi}^2 \frac{\chi^2}{M_P^2}, \quad \epsilon = \epsilon_\varphi + \epsilon_\chi. \quad (31)$$

We note that the dominant slow-roll parameters  $\eta_{\varphi\varphi}$  and  $\eta_{\chi\chi}$  are constants during inflation in the vacuum dominated regime and that they are much larger than the slow-roll parameters  $\epsilon_\varphi$  and  $\epsilon_\chi$  throughout inflation.

From the previous section, large non-Gaussianity can be realised in either of two regions

$$\begin{aligned} \cos^2 \theta &\equiv \frac{\dot{\varphi}^2}{\dot{\varphi}^2 + \dot{\chi}^2} \simeq \frac{\epsilon_\varphi}{\epsilon_\varphi + \epsilon_\chi} \ll 1, \quad \text{or} \\ \sin^2 \theta &\equiv \frac{\dot{\chi}^2}{\dot{\varphi}^2 + \dot{\chi}^2} \simeq \frac{\epsilon_\chi}{\epsilon_\varphi + \epsilon_\chi} \ll 1. \end{aligned} \quad (32)$$

Since the two regions are symmetrical [94] (before specifying the values of  $\eta_{\varphi\varphi}$  and  $\eta_{\chi\chi}$ ), in the rest of this section, we will focus on the second region. In this region where  $\epsilon_\varphi \gg \epsilon_\chi$ ,  $|f_{\text{NL}}| > 1$  is fulfilled by the condition

$$\sin^2 \theta^* \lesssim \sin^4 \theta \left( \sqrt{\frac{5 |\eta_{\chi\chi}|}{6 \sin^2 \theta}} - 1 \right), \quad (33)$$

in other words,

$$|\eta_{\chi\chi}|^{-1} e^{-4(\eta_{\varphi\varphi} - \eta_{\chi\chi})N} \lesssim \sin^2 \theta \simeq \frac{\epsilon_\chi}{\epsilon_\varphi} \lesssim |\eta_{\chi\chi}|. \quad (34)$$

This implies three inequalities on  $\theta$

$$\begin{aligned} \sin^2 \theta^* &< \frac{1}{3} \left(\frac{5}{6}\right)^2 \left(\frac{3}{4}\right)^4 |\eta_{\chi\chi}|^2, \\ \sin^2 \theta &< \frac{5}{6} |\eta_{\chi\chi}|, \quad \frac{\sin^2 \theta}{\sin^2 \theta^*} > \frac{24}{5} \frac{1}{|\eta_{\chi\chi}|}. \end{aligned} \quad (35)$$

Note that in this region  $\sin \theta \simeq \eta_{\chi\chi} \chi / (\eta_{\varphi\varphi} \varphi)$  from (30), we require  $N(\eta_{\varphi\varphi} - \eta_{\chi\chi}) > 1$  so that  $\sin^2 \theta$  grows significantly during inflation.

*4.1. Simplified Formula for the Observables When  $f_{\text{NL}}$  Is Large.* We can substantially simplify all of the above formula in the case where  $f_{\text{NL}}$  is large. We define the quantity

$$\tilde{r} \equiv \frac{(\partial N / \partial \chi_*)^2}{(\partial N / \partial \varphi_*)^2} = \frac{\epsilon_\chi}{\epsilon_\varphi} e^{2(\eta_{\varphi\varphi} - \eta_{\chi\chi})N}. \quad (36)$$

In the region, we are considering where  $f_{\text{NL}}$  is large, this is approximately given by the initial and final angles of the background trajectory with different exponents

$$\tilde{r} \simeq \frac{\sin^4 \theta}{\sin^2 \theta^*}. \quad (37)$$

In the case of large non-Gaussianity it follows that

$$\mathcal{P}_\zeta \simeq \frac{W_*}{24\pi^2 M_p^4 \epsilon_*} \left( 1 + \frac{\epsilon_\chi}{\epsilon_\varphi} e^{2(\eta_{\varphi\varphi} - \eta_{\chi\chi})N} \right) = \frac{8}{r} \left( \frac{H_*}{2\pi} \right)^2, \quad (38)$$

$$n_\zeta - 1 \simeq 2 \frac{\eta_{\varphi\varphi} + \tilde{r}\eta_{\chi\chi}}{1 + \tilde{r}}, \quad (39)$$

$$r \simeq \frac{16\epsilon_*}{1 + \tilde{r}}, \quad (40)$$

$$f_{\text{NL}} \simeq \frac{5}{6} \frac{\sin^6 \theta^e}{(\sin^2 \theta^* + \sin^4 \theta^e)^2} \eta_{\chi\chi} = \frac{5}{6} \frac{\tilde{r}}{(1 + \tilde{r})^2} \eta_{\chi\chi} e^{2(\eta_{\varphi\varphi} - \eta_{\chi\chi})N}. \quad (41)$$

The first condition in (35) implies that

$$\chi_* \ll \varphi_*. \quad (42)$$

We, therefore, require a very small value of  $\chi_*$  in order to have a large non-Gaussianity. While this may be a fine tuning, we comment that requiring a large ratio of the initial field values is common in models where the isocurvature perturbation generates a significant part of the primordial adiabatic perturbation, see the discussion in the conclusion of [85]. If the inflaton perturbations are neglected a priori, such as in the pure (i.e., not mixed) curvaton scenario, this remains implicitly true. For an explicit discussion of this, in the simple case that both the inflaton and curvaton fields have a quadratic potential, see [108].

The sign of  $f_{\text{NL}}$  is determined by the sign of  $\eta_{\chi\chi}$ . The amplitude of  $f_{\text{NL}}$  depends exponentially on the difference of the slow-roll parameters,  $\eta_{\varphi\varphi} - \eta_{\chi\chi}$ , which we require to be positive to be in the branch of large non-Gaussianity where  $\sin^2 \theta \ll 1$ , while the spectral index depends on a weighted sum of the slow-roll parameters, so it is possible to have a large non-Gaussianity and a scale invariant spectrum. However, it is not possible to have a large and positive  $f_{\text{NL}}$  and a red spectrum of perturbations. We will see in Section 4.2.2 that by including the effect of the waterfall field this conclusion may change, depending on the values of the coupling constants between the two inflaton fields and the waterfall field.

In Table 1, we give some explicit examples of values of  $\eta_{\varphi\varphi}$ ,  $\eta_{\chi\chi}$ ,  $\varphi_*$  and  $\chi_*$  which lead to a large non-Gaussianity. Using (4), we also calculate the spectral index. The first example in Table 1 shows that it is possible to have  $|f_{\text{NL}}| \simeq 100$  and a scale invariant spectrum. We also see that it is possible to generate a large non-Gaussianity during slow roll with  $\eta_{\varphi\varphi}$  and  $\eta_{\chi\chi}$  both positive or both negative, or when one is positive and the other negative corresponding to a saddle point. The results for this model were verified using a novel calculational method of momentum transport equations in [109].

**4.2. Effect of the Waterfall Field and Further Evolution after Inflation.** In this section, we include the effects of the waterfall field  $\rho$  which is required to end hybrid inflation. Inflation ends when the waterfall field is destabilised that is, when its effective mass becomes negative. During inflation,

the waterfall field is heavy, and it is trapped with a vacuum expectation value of zero, so we can neglect it during inflation. The end of inflation occurs when the effective mass of the waterfall field is zero, which occurs on a hypersurface defined in general by [92, 93]

$$\sigma^2 = G(\varphi, \chi) \equiv g_1^2 \varphi^2 + g_2^2 \chi^2, \quad (43)$$

which is realised by the potential  $W(\varphi, \chi)$ , defined by (29), where  $W_0$  is given by

$$W_0 = \frac{1}{2} G(\varphi, \chi) \rho^2 + \frac{\lambda}{4} \left( \rho^2 - \frac{\sigma^2}{\lambda} \right)^2. \quad (44)$$

Here,  $g_1$  ( $g_2$ ) is the coupling between the  $\varphi$  ( $\chi$ ) field and  $\rho$  is the waterfall field with self-coupling  $\lambda$ . In general, the hypersurface defined by this end condition is not a surface of uniform energy density.

As discussed earlier, this is an example of a model with an inhomogeneous end of inflation, that is, where inflation ends at slightly different times in different places. It has also been shown for the hybrid potential we are considering that this can be used to generate a large amount of non-Gaussianity, for certain parameters values and fine tuning of the parameters [90, 93]. However, these papers concern the large non-Gaussianity generated at the end of inflation rather than during slow-roll inflation, by having a very large ratio of couplings  $g_1/g_2 \ll 1$ . Here, we consider the case where  $g_1$  and  $g_2$  have the same order of magnitude with  $g_1^2/g_2^2 = \eta_{\varphi\varphi}/\eta_{\chi\chi}$  in Section 4.2.1 and with  $g_1^2 = g_2^2$  in Section 4.2.2.

**4.2.1.  $g_1^2/g_2^2 = \eta_{\varphi\varphi}/\eta_{\chi\chi}$ .** In this case, we have chosen the coupling constants (which can satisfy  $g_1^2 < 0$  and/or  $g_2^2 < 0$ ) such that the surface where the waterfall field is destabilised corresponds to a surface of uniform energy density. This is because the end condition can be rewritten as  $\sigma^2 = 2g_2^2/\eta_{\chi\chi}((1/2)\eta_{\varphi\varphi}\varphi^2 + (1/2)\eta_{\chi\chi}\chi^2) \propto W$ . In this case, the value of all observable quantities such as the power spectrum and non-Gaussianity are the same as those we calculated previously which were valid at the final hypersurface of uniform energy density during inflation.

**4.2.2.  $g_1^2 = g_2^2$ .** In this case, the end of inflation given by the condition in (43) does not occur on a uniform energy density hypersurface [93]. We will show how the non-Gaussianity is modified by the condition at the end of inflation in this example. In general, we expect there to be some modification to non-Gaussianity from the end of inflation, except in the special case we considered in Section 4.2.1. We have checked in [95] that the correction from the extra expansion which occurs from the surface on which inflation ends up to a surface of uniform energy density is small, and therefore one can still use the  $\delta N$  formalism in this case.

TABLE 1: Table showing some initial conditions for the hybrid inflation model that lead to large levels of non-Gaussianity. The table shows the parameter  $\tilde{r}$ , the bispectrum and the trispectrum nonlinearity parameters, the spectral index and tensor-to-scalar ratio. They are evaluated when the number of  $e$ -foldings from the end of inflation is  $N_k = 60$ . The trispectrum parameters  $\tau_{\text{NL}}$  and  $g_{\text{NL}}$  are defined and calculated in Section 6.1.

$\eta_{\varphi\varphi}$	$\eta_{\chi\chi}$	$\varphi^*$	$\chi^*$	$\tilde{r}$	$f_{\text{NL}}$	$\tau_{\text{NL}}$	$g_{\text{NL}}$	$n_\zeta - 1$	$r$
0.04	-0.04	1	$6.8 \times 10^{-5}$	1	-123	$4.4 \times 10^4$	-33	0	0.006
0.04	-0.04	1	$1.5 \times 10^{-4}$	5	-68	$8 \times 10^3$	-24	-0.05	0.002
0.08	0.01	1	0.0018	1	9.27	247	0.77	0.09	0.026
0.02	-0.04	1	0.00037	1	-11.1	357	-2.6	-0.02	0.002
-0.01	-0.09	1	$3 \times 10^{-6}$	0.16	-132	$1.8 \times 10^5$	-44	-0.04	0.0007
0.06	-0.01	1	$4.3 \times 10^{-4}$	0.1	-3	148	-0.2	0.11	0.026
0.01	-0.06	1	$7.5 \times 10^{-6}$	0.04	-8	$2.5 \times 10^3$	-2	0.01	0.0008

In this case, the power spectrum and  $f_{\text{NL}}$  in the limit of large non-Gaussianity are [93]

$$\mathcal{P}_\zeta \simeq \frac{W_*}{24\pi^2 M_P^4 \epsilon_*} \left( 1 + \frac{\eta_{\varphi\varphi}^2 \epsilon_\chi}{\eta_{\chi\chi}^2 \epsilon_\varphi} e^{2(\eta_{\varphi\varphi} - \eta_{\chi\chi})N} \right) = \frac{8}{\tilde{r}} \left( \frac{H_*}{2\pi} \right)^2, \quad (45)$$

$$n_\zeta - 1 \simeq 2 \frac{\eta_{\varphi\varphi} + (\eta_{\varphi\varphi}^2/\eta_{\chi\chi}^2) \tilde{r} \eta_{\chi\chi}}{1 + (\eta_{\varphi\varphi}^2/\eta_{\chi\chi}^2) \tilde{r}}, \quad (46)$$

$$r \simeq 16\epsilon_* \left( 1 + \frac{\eta_{\varphi\varphi}^2 \tilde{r}}{\eta_{\chi\chi}^2} \right)^{-1}, \quad (47)$$

$$f_{\text{NL}} \simeq \frac{5}{6} \frac{(\eta_{\varphi\varphi}^2/\eta_{\chi\chi}^2) \tilde{r}}{\left( 1 + (\eta_{\varphi\varphi}^2/\eta_{\chi\chi}^2) \tilde{r} \right)^2} \eta_{\varphi\varphi} e^{2(\eta_{\varphi\varphi} - \eta_{\chi\chi})N}. \quad (48)$$

We can see that the observables at the end of inflation are changed by the additional ratio of  $\eta_{\varphi\varphi}^2/\eta_{\chi\chi}^2$  before  $\tilde{r}$ . Furthermore,  $f_{\text{NL}}$  has an additional factor of  $\eta_{\varphi\varphi}/\eta_{\chi\chi}$ .

In Table 2, we show the values of  $f_{\text{NL}}$ ,  $n_\zeta - 1$  and  $r$  with end condition  $g_1^2 = g_2^2$  for the same parameter values that we used in Table 1, where the inflation ends on the uniform energy density hypersurface. The first two examples show that if  $\eta_{\varphi\varphi} = -\eta_{\chi\chi}$ , then the observables are unchanged for two different end conditions except that the sign of  $f_{\text{NL}}$  is switched. The second example in the table shows that in this case it is possible to have a red spectral index and a positive value of  $f_{\text{NL}}$ . For many values of the initial parameters when  $|\eta_{\varphi\varphi}/\eta_{\chi\chi}| \neq 1$ , the magnitude of  $f_{\text{NL}}$  decreases compared to Table 1, but the reverse can also happen, an example of this where  $|f_{\text{NL}}|$  grows by more than an order of magnitude is shown in the final row of Tables 1 and 2.

**4.2.3. Further Evolution after Inflation.** So far, we have assumed a quick transition to the radiation epoch at the end of inflation, thereby neglecting the dynamics of the waterfall field. However, if we consider the role of the waterfall field, then after the waterfall field is destabilised, there may be a further evolution of the primordial curvature perturbation, which will lead to a change of the observable parameters. This applies to any model with an inhomogeneous end of inflation, since there are isocurvature perturbations still

present after the waterfall field is destabilised and inflation has ended. Further evolution will depend on the details of reheating in a model dependent way. To the best of our knowledge, this issue has not been considered in depth in any paper. If we assume an instantaneous transition to radiation domination (so a completely efficient and immediate decay of the waterfall and inflaton fields), then there will be no further change to the observables as we have argued in the previous section. However, this is clearly an idealised case.

In the special case where the waterfall field is also light during inflation, Barnaby and Cline [110] have shown there is the possibility of generating a large non-Gaussianity during preheating for certain parameter values. This is possible even if there is only one inflaton field and the waterfall field present. However, in this case, inflation does not end abruptly when the waterfall field is destabilised so this is not the scenario we have considered in this paper. A recent claim that even a heavy waterfall field may lead to a scale invariant local non-Gaussianity has been withdrawn [111].

## 5. Multiple-Field Inflation without Slow Roll

In this section, we demonstrate a new method that provides, for certain classes of models, analytical expressions for  $f_{\text{NL}}$  valid in regimes *beyond* a slow-roll approximation [112]. This method is based on the first-order Hamilton-Jacobi formalism developed by Salopek and Bond [113], which allows us to express inflationary observables in the multifield case, without having to focus on a slow-roll regime (see [114] for a similar application of this formalism to the single-field case).

The main idea on which our arguments are based is to focus on inflationary multifield trajectories in which the *Hubble rate*, and not the potential, is separable. This approach allows us to analytically study cases where the non-Gaussian parameter  $f_{\text{NL}}$  becomes large, in regimes in which the slow-roll parameters are enhanced after Hubble exit, but before inflation ends. In particular, we provide analytical equations that express the nonlinearity parameter  $f_{\text{NL}}$ , in terms of quantities that generalise the slow-roll parameters during inflation. Using this method, we demonstrate in the next subsection an exact solution of multifield inflation



TABLE 2: Same as Table 1, but with different end condition,  $g_1^2 = g_2^2$  as used in Section 4.2.2.

$\eta_{\varphi\varphi}$	$\eta_{\chi\chi}$	$\varphi_*$	$\chi_*$	$\tilde{r}$	$f_{\text{NL}}$	$n_\zeta - 1$	$r$
0.04	-0.04	1	$6.8 \times 10^{-5}$	1	123	0	0.006
0.04	-0.04	1	$1.5 \times 10^{-4}$	5	68	-0.05	0.002
0.08	0.01	1	0.0018	1	4.59	0.02	0.0008
0.02	-0.04	1	0.00037	1	3.5	0.02	0.026
-0.01	-0.09	1	$3 \times 10^{-6}$	0.16	-0.2	-0.02	0.0008
0.06	-0.01	1	$4.3 \times 10^{-4}$	0.1	38	0.01	0.006

which can give rise to a large non-Gaussianity due to a strong break down in slow roll shortly before inflation ends.

The first-order Hamilton-Jacobi equations of motion are

$$H^2 = \frac{1}{3}W(\phi, \chi) + \frac{2}{3} \left[ \left( \frac{\partial H}{\partial \phi} \right)^2 + \left( \frac{\partial H}{\partial \chi} \right)^2 \right], \quad (49)$$

$$\dot{\phi} = -2 \frac{\partial H}{\partial \phi}, \quad \dot{\chi} = -2 \frac{\partial H}{\partial \chi}.$$

We concentrate on solutions satisfying the following Ansatz for the Hubble parameter  $H(\phi, \chi)$

$$H(\phi, \chi) = H^{(1)}(\phi) + H^{(2)}(\chi), \quad (50)$$

that is, we demand that it can be split as a sum (hence, we follow an analysis related to [99]. In an analogous way, one could also consider a situation in which the Hubble parameter splits as a *product* of single-field pieces [104]. The formula's in this section were extended to an arbitrary number of fields with a sum separable Hubble parameter in [115]) of two pieces, each one depending on a single field. Notice that this is different with respect to the requirement of separable potentials usually done in the literature and earlier in this paper.

It is convenient to define the following quantities:

$$\delta^\phi = \left( \frac{H_{,\phi}^{(1)}}{H} \right)^2, \quad \delta^\chi = \left( \frac{H_{,\chi}^{(2)}}{H} \right)^2, \quad (51)$$

$$\gamma^\phi = \frac{H_{,\phi\phi}^{(1)}}{H}, \quad \gamma^\chi = \frac{H_{,\chi\chi}^{(2)}}{H},$$

and  $\delta = \delta^\phi + \delta^\chi$ . Although their definition resembles the corresponding one for the usual slow-roll parameters  $\epsilon$  and  $\eta$ , they do *not* coincide with them when taking a slow roll limit (it is nevertheless simple to work out the relation with the slow roll parameters, defined by (21) and (22)). In a slow-roll regime, one finds  $\delta^\phi \simeq \epsilon^\phi/2$ ,  $\gamma^\phi \simeq (\eta^{\phi\phi} - \epsilon^\phi)/2$ . We assume that the quantities  $\delta$  and  $\gamma$  are much smaller than unity at Hubble exit  $t = t_*$ , in order that we can use the  $\delta N$  formalism. During inflation, by definition, we have to ensure that the quantity  $\epsilon_H \equiv -\dot{H}/H^2 = 2\delta < 1$ . The quantities  $\gamma$  can, however, become much larger than unity during inflation.

Using similar techniques to those applied to a separable potential, one can calculate the spectral index and  $f_{\text{NL}}$ , the details were given in [112] and the results are

$$n_\zeta - 1 = -4\delta_* - 4 \frac{u_H (1 - (\gamma_*^\phi/\delta_*^\phi) u_H)}{u_H^2/\delta_*^\phi + v_H^2/\delta_*^\chi} + (-4) \frac{v_H (1 - (\gamma_*^\chi/\delta_*^\chi) v_H)}{u_H^2/\delta_*^\phi + v_H^2/\delta_*^\chi} \quad (52)$$

$$\frac{6}{5} f_{\text{NL}} = 2 \frac{(u_H^2/\delta_*^\phi) (1 - (\gamma_*^\phi/\delta_*^\phi) u_H)}{(u_H^2/\delta_*^\phi + v_H^2/\delta_*^\chi)^2} + 2 \frac{(v_H^2/\delta_*^\chi) (1 - (\gamma_*^\chi/\delta_*^\chi) v_H)}{(u_H^2/\delta_*^\phi + v_H^2/\delta_*^\chi)^2} + 2 \frac{(u_H/\delta_*^\phi - v_H/\delta_*^\chi)^2 \mathcal{A}_H}{(u_H^2/\delta_*^\phi + v_H^2/\delta_*^\chi)^2}, \quad (53)$$

where the new symbols are defined by

$$u_H \equiv \frac{H_*^{(1)} + Z_e}{H_*}, \quad v_H \equiv \frac{H_*^{(2)} - Z_e}{H_*},$$

$$Z_e = \frac{(H_e^{(2)} \delta_e^\phi - H_e^{(1)} \delta_e^\chi)}{\delta_e}, \quad (54)$$

$$\mathcal{A}_H = -\frac{H_e^2}{H_*^2} \frac{\delta_e^\phi \delta_e^\chi}{\delta_e} \left( \frac{1}{2} - \frac{\gamma_e^{ss}}{\delta_e} \right),$$

$$\gamma^{ss} = \frac{(\delta^\chi \gamma^\phi + \delta^\phi \gamma^\chi)}{\delta}.$$

We reiterate that the formula for  $f_{\text{NL}}$  is exact (apart from neglecting a slow-roll suppressed contribution due to the non-Gaussianity of the fields at Hubble exit) and is not based on a slow-roll expansion. Although  $\delta_*$  and  $\gamma_*$  are necessarily small, the quantity  $\mathcal{A}$  is not suppressed by these parameters and can assume large values during inflation, enhancing  $f_{\text{NL}}$ . We note in agreement with [99] that if one of the fields has reached a minimum so  $\dot{\phi} = 0$  or  $\dot{\chi} = 0$  at the end of inflation then  $\mathcal{A}_H = 0$  and the non-Gaussianity at the end of inflation will be very small. Therefore, any model of inflation with a separable potential or separable Hubble factor with a large non-Gaussianity present at the end of inflation must

have both fields still evolving, and therefore the presence of isocurvature modes. It would, therefore, also be interesting to study the evolution of the perturbations after inflation, to see if this leads to an important change in the observables [116].

*5.1. Exact Solution.* Very few exact solutions in multiple-field inflation are known [117]. Here, we present an exact solution which can give rise to a very large non-Gaussianity at the end of inflation. A potential with a similar form may be motivated in string theory, in the context of Kähler moduli inflation [118–125].

The potential we consider is [112]

$$W(\phi, \chi) = U_0 \left( -A_1 e^{-\alpha\phi} + A_2 e^{-2\alpha\phi} - B_1 e^{-\beta\chi} + B_2 e^{-2\beta\chi} + \frac{A_1 B_1}{2} e^{-\alpha\phi - \beta\chi} \right), \quad (55)$$

where the parameters  $\alpha$  and  $\beta$  satisfy

$$\alpha^2 = \frac{3}{2} - \frac{6A_2}{A_1^2}, \quad \beta^2 = \frac{3}{2} - \frac{6B_2}{B_1^2}. \quad (56)$$

The Hamilton-Jacobi equations have the following exact solution for  $H$  and the fields:

$$\begin{aligned} H &= H_0 \left( 1 - \frac{A_1}{2} e^{-\alpha\phi} - \frac{B_1}{2} e^{-\beta\chi} \right), \\ \phi &= \frac{1}{\alpha} \ln \left[ e^{\alpha\phi_*} - A_1 \alpha^2 H_0 t \right], \\ \chi &= \frac{1}{\beta} \ln \left[ e^{\beta\chi_*} - B_1 \beta^2 H_0 t \right], \end{aligned} \quad (57)$$

where we have defined  $H_0 = \sqrt{U_0/3}$ ,  $t$  is cosmic time, and we have set  $t_* = 0$ . The overall factor to the potential  $U_0$  can be freely chosen so that the amplitude of the scalar power spectrum matches the observed amplitude of perturbations in the CMB. The scale factor results

$$a(t) = a_0 \left( e^{\alpha\phi_*} - \alpha^2 A_1 H_0 t \right)^{1/2\alpha^2} \left( e^{\beta\chi_*} - \beta^2 B_1 H_0 t \right)^{1/2\beta^2} e^{H_0 t}. \quad (58)$$

Notice that the solution becomes singular at late times, when the scale factor vanishes and the field values diverge. This singularity occurs well after inflation ends, and we will not need to discuss it in our analysis. From the relations

$$\gamma^\phi = -\alpha\sqrt{\delta^\phi}, \quad \gamma^\chi = -\beta\sqrt{\delta^\chi}, \quad (59)$$

we notice that in this example  $\gamma^\phi$  and  $\gamma^\chi$  can become much larger than unity at the end of inflation if  $|\alpha|$  and/or  $|\beta|$  are much greater than one. This corresponds to a break down in slow roll, although  $\epsilon_H$  remains smaller than unity during the inflationary era by definition.

As a simple, concrete example of parameter choice which gives rise to a large non-Gaussianity, we choose a regime in which the parameters  $|\alpha|$  and  $|\beta|$  are both large, let us say larger than some quantity  $R \gg 1$ . We note from (56) that

we are, therefore, required to have  $A_2 < 0$  and  $B_2 < 0$ . In this case, the potential does not have a minimum, but inflation still ends through  $\epsilon_H$  growing larger than unity, and we can trust our results in this regime. The potential would need modifications which apply after inflation in order for reheating to take place and these modifications may provide a minimum for the potential. We parameterise the values of the quantities  $\delta_e^\phi$  and  $\delta_e^\chi$  at the end of inflation as

$$\delta_e^\phi = \frac{1}{2m}, \quad \delta_e^\chi = \frac{1}{2n} \quad \text{such that} \quad \frac{1}{m} + \frac{1}{n} = 1, \quad (60)$$

where the final equality follows from  $\epsilon_H = 1$ . We assume that the quantities  $m$  and  $n$  are not too big. Namely, they satisfy the inequality  $m, n \ll R^2$ . This implies that  $H_0 \simeq H_* \simeq H_e$ . We choose  $|\gamma_*^\phi| = 1/40p$ ,  $|\gamma_*^\chi| = 1/40q$ , such that we can write

$$N_{\text{tot}} = 60 \simeq 10(p + q). \quad (61)$$

This fixes the initial values for the fields  $\phi_*$  and  $\chi_*$ , to the values

$$\begin{aligned} e^{\alpha\phi_*} &= 20p\alpha^2 A_1, \\ e^{\beta\chi_*} &= 20q\beta^2 B_1. \end{aligned} \quad (62)$$

The conditions (60) imply

$$\begin{aligned} H_0 t_e &= 20p - \sqrt{\frac{m}{2\alpha^2}}, \\ H_0 t_e &= 20q - \sqrt{\frac{n}{2\beta^2}}. \end{aligned} \quad (63)$$

Since  $m$  and  $n$  are much smaller than  $R^2$ , we expect  $p \simeq q$ . We hence have the simple relations

$$\delta_*^\phi \simeq \frac{1}{4N_{\text{tot}}^2 \alpha^2}, \quad \delta_*^\chi \simeq \frac{1}{4N_{\text{tot}}^2 \beta^2}, \quad \gamma_*^\phi \simeq \gamma_*^\chi \simeq -\frac{1}{2N_{\text{tot}}}. \quad (64)$$

From (52), we find

$$n_\zeta - 1 \simeq -\frac{2}{N_{\text{tot}}} \simeq -0.04. \quad (65)$$

So, we have a red spectral index in agreement with present day observations [126] and in [112] it was also shown that the tensor-to-scalar ratio for this model is negligible.

When evaluated at the end of inflation, we find

$$\frac{6}{5} f_{\text{NL}} \simeq -2\sqrt{2mn} \left[ \frac{\alpha^2 n - \beta^2 m}{\alpha^2 n^2 + \beta^2 m^2} \right]^2 \left( \frac{\alpha}{\sqrt{n}} + \frac{\beta}{\sqrt{m}} \right). \quad (66)$$

We stress that this formula provide only the dominant contributions to  $f_{\text{NL}}$ . It is valid in the case  $|\alpha|, |\beta| > R \gg 1$ , and  $m, n < R^2$ .

As an explicit case, we take  $\alpha = -100$ ,  $\beta = 20$ ,  $m = 6$ ,  $n = 6/5$  and the initial conditions to satisfy (62) with  $N_{\text{tot}} = 60$ .

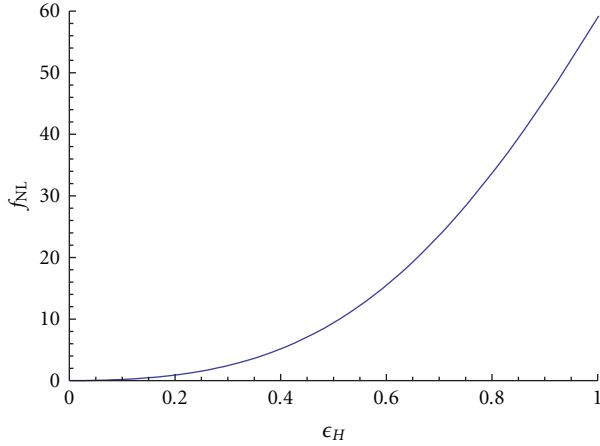


FIGURE 1: Plot showing  $f_{\text{NL}}$  as a function of  $\epsilon_H$  towards the end of inflation, for the values of the parameters given in the text. Inflation ends when  $\epsilon_H = 1$ ; for this example  $f_{\text{NL}} \approx 59$  at that time.

Then, independently of the values of  $A_1$  and  $B_1$ , we find  $f_{\text{NL}} \approx 58$  from the simplified formula (66), and  $f_{\text{NL}} \approx 53$  from the full formula (53). See Figures 1 and 2. Notice that the results for  $f_{\text{NL}}$  differ from each other by quantities of order  $1/R = 1/\beta$  as expected by the approximations we have made. At the end of inflation, the parameters are  $\gamma^\phi = 29$ ,  $\gamma^\chi = -13$  and this shows that the slow-roll approximation has been significantly broken, (it is nevertheless simple to work out the relation with the slow roll parameters, defined by (21) and (22). In a slow-roll regime, one finds  $\delta^\phi \approx \epsilon^\phi/2$ ,  $\gamma^\phi \approx (\eta^{\phi\phi} - \epsilon^\phi)/2$ ). From Figure 2, one can see that the trajectory is straight for much of inflation but it turns near the end of inflation (during the last  $e$ -folding before inflation ends), and it is during this time that  $f_{\text{NL}}$  grows larger than unity.

Notice that our requirements of final values for the quantities  $\delta_\epsilon^\phi$  and  $\delta_\epsilon^\chi$  (both much bigger than  $1/R$ ) imposes fine-tuning constraints on the initial conditions, since at leading order in  $1/R$  the values for  $\gamma_*^\phi$  and  $\gamma_*^\chi$  must coincide (see (62) and recall that  $p \approx q$ ).

We stress that the enhancement of non-Gaussianity occurs towards the end of inflation. What happens just *after* inflation is a model-dependent issue; we cannot address this question within the approximations used in this concrete model. For our choice of parameters, we notice that  $f_{\text{NL}}$  is still increasing at the end of inflation, see Figure 1, but for other choices it may start to decrease before inflation ends, see [115]. We stress that in the regime where  $|\alpha|, |\beta| \gg 1$  the slow-roll parameters will necessarily become much greater than unity by the end of inflation, which may correspond to  $|\gamma^{\text{ss}}| \gg 1$ : this is one reason by which  $f_{\text{NL}}$  can become large. Our formalism is at least in principle suitable to study the evolution of non-Gaussianity after inflation, when the parameter  $\epsilon_H$  becomes larger than unity (although we restate that the specific potential being considered here anyway needs to be modified after inflation ends).

## 6. Higher Order Non-Gaussian Observables

Non-Gaussianity is most popularly parameterised in terms of one nonlinearity parameter,  $f_{\text{NL}}$ . There are several different  $f_{\text{NL}}$ 's used in the literature which parameterise different shape dependences of the bispectrum [41, 42], the most popular being the local model discussed in this paper and the so-called equilateral model which can arise from models of inflation with noncanonical kinetic terms (a popular example is DBI inflation [15–25]). But can we learn more than one number from an observation of non-Gaussianity? And if  $f_{\text{NL}}^{\text{local}}$  is detected how can we hope to distinguish between the models which can generate this? Fortunately, observations of non-Gaussianity have the possibility of teaching us much more than the value of one parameter. Two examples of further non-Gaussian observables are a scale dependence of  $f_{\text{NL}}$  and the trispectrum.

**6.1. Trispectrum.** The four point function of the primordial curvature perturbation is defined by

$$\begin{aligned} & \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_e \\ & \equiv T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4), \end{aligned} \quad (67)$$

which using the  $\delta N$  formalism, and assuming that the fields have a Gaussian distribution at Hubble exit, is given by

$$\begin{aligned} & T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \\ & = \tau_{\text{NL}} [P_\zeta(k_{13})P_\zeta(k_3)P_\zeta(k_4) + (11 \text{ perms})] \\ & \quad + \frac{54}{25} g_{\text{NL}} [P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + (3 \text{ perms})], \end{aligned} \quad (68)$$

where  $k_{13} \equiv |\mathbf{k}_1 + \mathbf{k}_3|$ , and the trispectrum nonlinearity parameters can be calculated using [98, 106, 127]

$$\begin{aligned} \tau_{\text{NL}} & = \frac{N_{AB}N^ACN^BN^C}{(N_D N^D)^3}, \\ g_{\text{NL}} & = \frac{25 N_{ABC}N^AN^BN^C}{54 (N_D N^D)^3}. \end{aligned} \quad (69)$$

Hence, we see that the trispectrum depends on two nonlinearity parameters (as opposed to one,  $f_{\text{NL}}$ , for the bispectrum), and they may be observationally distinguishable since they are prefactors of terms with different shape dependences (68). The current observational bound on the local type of the bispectrum from seven years of WMAP data is  $-10 < f_{\text{NL}} < 74$  at the  $2\sigma$  level [2]. Recently, there has been a first constraint on both  $\tau_{\text{NL}}$  and  $g_{\text{NL}}$ , which came from using WMAP5 data, with the bounds  $-7.4 < g_{\text{NL}}/10^5 < 8.2$  and  $-0.6 < \tau_{\text{NL}}/10^4 < 3.3$  at 95% confidence [128]. There have been two other observational constraints on the trispectrum through  $g_{\text{NL}}$ , setting  $\tau_{\text{NL}} = 0$ . The bounds, which in both cases are roughly  $|g_{\text{NL}}| \lesssim 10^5$ – $10^6$ , come from large scale structure [129], and the CMB [130]. If there is no detection of non-Gaussianity it is expected that with Planck data the bounds will be reduced to about  $|f_{\text{NL}}| \lesssim 10$ ,  $\tau_{\text{NL}} \lesssim 10^3$  and  $g_{\text{NL}} \lesssim 10^5$  at the  $2\sigma$  level and future observations

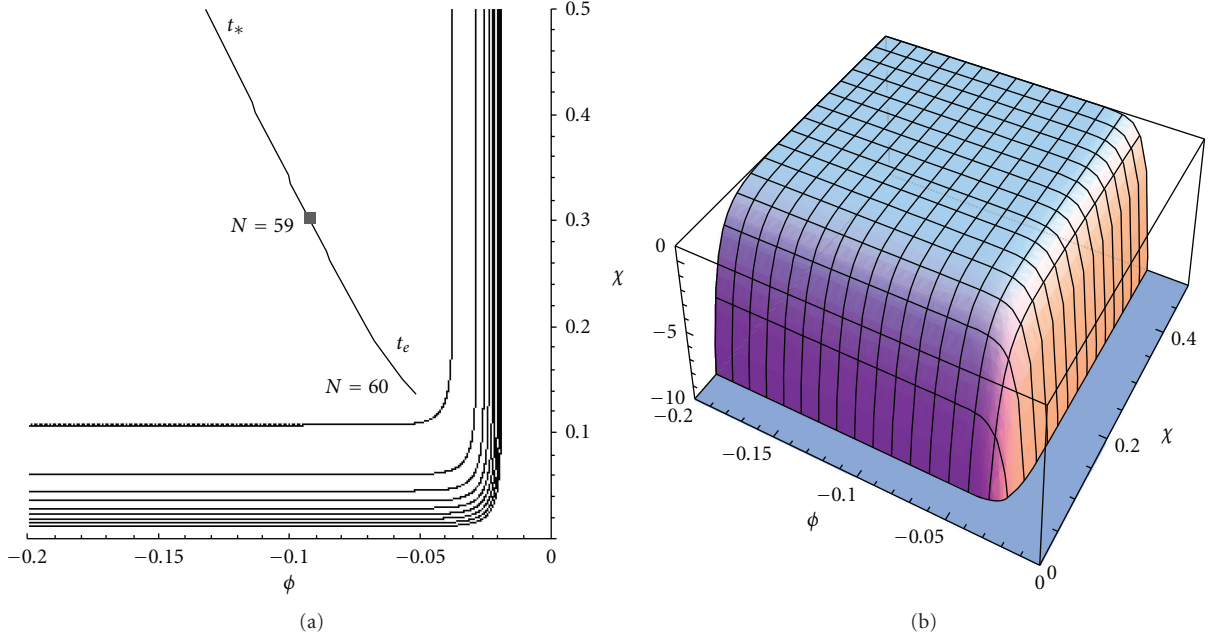


FIGURE 2: (a) shows the trajectory considered for the parameters given after (66) superimposed on a contour plot of the potential. The trajectory is shown from the time of horizon crossing at  $t_*$  and runs until the end of inflation at  $N = 60$ . The square on the trajectory, labelled with  $N = 59$ , indicates a point along the trajectory one  $e$ -folding before inflation ends as  $\phi$  and  $\chi$  roll towards zero. This shows that the fields roll much more quickly during the final stage of inflation, and the trajectory curves near the end. (b) shows the potential for the same parameter values. Notice that inflation ends on the plateau long before the potential becomes negative.

may become even tighter [131–135]. Note that the expected future constraint on  $g_{\text{NL}}$  is about two orders of magnitude weaker than that on  $\tau_{\text{NL}}$  [135]. We will see that it is also possible for some models and parameter ranges that the first observational signature will come through the trispectrum rather than the bispectrum.

**6.1.1. Trispectrum in Two-Field Hybrid Inflation.** In the regime where  $|g_{\text{NL}}|$  and  $\tau_{\text{NL}}$  are greater than unity, they are given by

$$g_{\text{NL}} = \frac{10}{3} \frac{\tilde{r}(\eta_{\phi\phi} - 2\eta_{\chi\chi}) - \eta_{\chi\chi}}{1 + \tilde{r}} f_{\text{NL}}, \quad (70)$$

$$\tau_{\text{NL}} = \frac{\tilde{r}}{(1 + \tilde{r})^3} \eta_{\chi\chi}^2 e^{4N(\eta_{\phi\phi} - \eta_{\chi\chi})} = \frac{1 + \tilde{r}}{\tilde{r}} \left(\frac{6}{5} f_{\text{NL}}\right)^2. \quad (71)$$

We see that  $g_{\text{NL}}$  is subdominant to  $f_{\text{NL}}$  and hence will not provide a competitive observational signature. It follows from (71) that  $\tau_{\text{NL}} > (6f_{\text{NL}}/5)^2$ , so  $\tau_{\text{NL}}$  may be large and provide an extra observable parameter for this model. This inequality between  $\tau_{\text{NL}}$  and  $f_{\text{NL}}$  is true in general [75], and equality is reached whenever a single field direction during inflation generates the primordial curvature perturbation. However, it is usually assumed that  $\tau_{\text{NL}} \sim f_{\text{NL}}^2$  since both arise from second derivatives in the  $\delta N$  formalism. In fact, for our model it is possible to have a small  $f_{\text{NL}}$  (and hence also a small  $g_{\text{NL}}$ ) but a large and potentially observable  $\tau_{\text{NL}}$ . For this, we require that  $\tilde{r} \ll 1$ , although in practice if we make it too

small it may no longer be possible to satisfy a constraint on a minimum possible field velocity consistent with a classical slow-roll trajectory, as discussed in [95] (see also [136] and a discussion which reaches a different conclusion is given in [137]). In the final example in Table 1, we give an explicit example of parameter values which give rise to an  $f_{\text{NL}}$  which is probably too small to be detected with Planck but with a very large trispectrum through  $\tau_{\text{NL}} > 10^3$  that should be detectable at a high significance. For another example with  $f_{\text{NL}}, g_{\text{NL}} \lesssim O(1)$  but  $\tau_{\text{NL}} \gg 1$ , see [76]. In contrast, it has been shown in several papers [127, 138–142] that in the curvaton scenario where the curvaton has a nonquadratic potential it is possible to realise  $|g_{\text{NL}}| \gg 1$  while  $\tau_{\text{NL}} = (6f_{\text{NL}}/5)^2$  is small with some tuning of parameters. This is also possible in the exact solution which we presented in Section 5 and we consider this next.

**6.1.2. Trispectrum in the Exact Solution.** In this regime, where  $|\alpha| \gg |\beta| > R$  and  $m, n > 1/R^2$ , it is also possible to give compact expressions for the trispectrum (4-point function) nonlinearity parameters, in terms of  $f_{\text{NL}}^2$  as

$$\tau_{\text{NL}} \simeq \left(\frac{6}{5} f_{\text{NL}}\right)^2, \quad \frac{54}{25} g_{\text{NL}} \simeq -\frac{3}{2} \frac{n-m}{m} \left(\frac{6}{5} f_{\text{NL}}\right)^2. \quad (72)$$

The complete result was given in [115]. Hence, both of the trispectrum nonlinearity parameters are generally large whenever  $f_{\text{NL}}$  is. We note that if  $m \simeq 1$  then from (60)  $n \gg 1$ , so the trispectrum through  $g_{\text{NL}}$  will give the dominant signal of non-Gaussianity through a large, negative  $g_{\text{NL}}$ .



6.2. *Scale Dependence of  $f_{\text{NL}}$ .* In its simplest form, the local form of  $f_{\text{NL}}$  defined by (1) is a constant parameter, independent of both position and scale. However, in realistic models  $f_{\text{NL}}$  is likely to be mildly scale-dependent. This may happen in two ways. In general, single-field models (such as the curvaton scenario) where the scalar field which generates the primordial curvature perturbation has a nonquadratic potential the non-linearities this generates will give rise to scale dependence of  $f_{\text{NL}}$ . Alternatively, even in models where all of the fields have a quadratic potential; if the primordial curvature perturbation has contributions from more than one field, and the fields do not all have the same mass, then  $f_{\text{NL}}$  will again have a scale dependence. This is because the correlation between the first and second order terms of  $\zeta$ , which the bispectrum depends on, will become scale-dependent. This is indeed what happens in the two-field hybrid model presented earlier. For details about a scale dependence of local  $f_{\text{NL}}$ , see Byrnes et al. [143].

We define a second observable parameter derived from the bispectrum

$$n_{f_{\text{NL}}} = \frac{d \ln |f_{\text{NL}}|}{d \ln k}, \quad (73)$$

which is analogous to the scale dependence of the power spectrum. There is a subtlety here, which is that in general  $f_{\text{NL}}$  may depend on three independent parameters,  $k_1, k_2$ , and  $k_3$ . However, it was shown in [143] that provided one takes the derivative while keeping the ratio of the three  $k$  vectors fixed then  $n_{f_{\text{NL}}}$  is independent of the shape of the triangle described by the three  $k$  vectors, which makes this a well defined quantity. Observational prospects for this quantity were considered in [144], who showed that the Planck satellite is sensitive to  $n_{f_{\text{NL}}} \simeq 0.1$ , assuming a fiducial value of  $f_{\text{NL}} = 50$ . This observational sensitivity to  $n_{f_{\text{NL}}}$  is about a factor of two larger than the current preferred value of the power spectrum's spectral index and may therefore provide an interesting extra constraint on non-Gaussian models. The scale dependence of an equilateral form of  $f_{\text{NL}}$  has also been considered from both a theoretical and observational perspective, see for example [144–147].

In the example of two-field hybrid inflation,  $f_{\text{NL}}$  has a scale dependence both because of the exponential term in  $f_{\text{NL}}$ , (41), and because  $\tilde{r}$  will vary through the change of the initial value of  $\sin^2 \theta^*$ . We find

$$\frac{\partial \ln \tilde{r}}{\partial \ln k} = \frac{\partial \ln e^{2N(\eta_{\phi\phi} - \eta_{\chi\chi})}}{\partial \ln k} = -2(\eta_{\phi\phi} - \eta_{\chi\chi}). \quad (74)$$

Using this we find from (41) that

$$n_{f_{\text{NL}}} \equiv \frac{d \log f_{\text{NL}}}{d \log k} = -4 \frac{\eta_{\phi\phi} - \eta_{\chi\chi}}{1 + \tilde{r}}. \quad (75)$$

In the case that we include the effect from the surface where the waterfall field is destabilised and  $g_1^2 = g_2^2$ , we find from (48) that

$$n_{f_{\text{NL}}} = -4 \frac{\eta_{\phi\phi} - \eta_{\chi\chi}}{1 + (\eta_{\phi\phi}/\eta_{\chi\chi})^2 \tilde{r}}. \quad (76)$$

For both cases, the spectral index of  $f_{\text{NL}}$  satisfies

$$-4(\eta_{\phi\phi} - \eta_{\chi\chi}) < n_{f_{\text{NL}}} < 0, \quad (77)$$

for any value of  $\tilde{r}$  and hence  $f_{\text{NL}}$  will be smaller on small scales.

Because we require a relatively large value of  $\eta_{\phi\phi} - \eta_{\chi\chi} > 1/N$  for our model to generate a large non-Gaussianity it is quite possible for our model to generate a relatively significant scale dependence of  $f_{\text{NL}}$ . However, the amount also depends on  $\tilde{r}$ , and when this is large, then the  $\chi$  field is almost solely responsible for generating  $\zeta$  at both first and second order and  $n_{f_{\text{NL}}}$  is suppressed. We can also see in agreement with the statement at the beginning of this subsection that in this case, if the mass of the two fields are equal, then the two fields have the same scale dependence and  $n_{f_{\text{NL}}} = 0$ .

We note that this is in contrast to the large non-Gaussianity from an inhomogeneous end of inflation found in [93]. In the specific cases, they considered to generate a large non-Gaussianity the non-Gaussianity was generated purely at the end of inflation and  $f_{\text{NL}}$  is scale-independent. In detail, we see from (4.4) and (4.24) in [93] that their formulas for  $f_{\text{NL}}$  does not depend on  $N$  or on any quantities evaluated at Hubble exit. This is also in contrast to the exact solution considered in Section 5.1. Our expression for  $f_{\text{NL}}$  in (66) depends on the initial values through  $p$  and  $q$ , and this dependence drops out at leading order in  $\alpha$ . Hence,  $f_{\text{NL}}$  is *independent* of the number of  $e$ -foldings and scale-independent.

## 7. Conclusions

We have reviewed various models which can generate a large local non-Gaussianity. A feature shared by all of these models is that they have more than one light scalar field present during inflation. This extra degree of freedom generates an isocurvature perturbation which is at least partially converted into the primordial curvature perturbation after horizon exit of the modes which are observable today. In the curvaton and modulated reheating scenarios, this conversion occurs after the end of inflation, while in the inhomogeneous end of inflation scenario, this conversion occurs on the non-uniform energy density hypersurface on which inflation ends. For these three scenarios, the light field which generates the primordial curvature perturbation after or at the end of inflation can be treated as a test field which does not affect the inflationary dynamics.

Our main focus has been on models in which a large non-Gaussianity is generated during inflation. This can occur even within slow-roll inflation for certain potentials and certain trajectories. We have shown, at least in the case of a separable potential, that the trajectory is required to be almost entirely along the direction of one field but that the orthogonal field must become more important towards the end of inflation, and hence the inflationary trajectory must curve. In absolute terms, the change to the angle of the

background trajectory is small (compared to a trajectory which turns by a right angle during inflation), but in relative terms it must grow by at least an order of magnitude. This is in contrast to the previous three scenarios.

We have reviewed two-field hybrid inflation as an explicit model of a separable potential where the conditions required to generate a large non-Gaussianity can be satisfied. The conditions can be satisfied for any possible combination of positive and negative  $\eta$  parameters, so the potential can be bowl-shaped, a hill-top, or have a saddle point. The main conditions which must be satisfied is that the difference of the two  $\eta$  parameters must not be too small,  $\eta_{\varphi\varphi} - \eta_{\chi\chi} \sim 0.1$ , and the value of the  $\chi$  field must be very subdominant to that of the  $\varphi$  field initially (or vice versa). In general, but depending on the coupling constants between the two inflaton fields and the waterfall field, there is a change to observables at the end of inflation, due to the fact that the surface on which the waterfall field is destabilised and inflation ends might not be a surface of uniform energy density. This effect is responsible for the inhomogeneous end of inflation scenario. It is then a model, dependent question whether there will be further evolution to the observables during reheating in this model, this deserves further attention.

One similarity that this hybrid inflation model has together with the quadratic curvaton scenario is that in both cases the initially subdominant (approximately isocurvature) field  $\chi$  has the ratio  $\delta\chi/\chi \sim \zeta_\chi$  approximately constant and the field fluctuations do not become more non-Gaussian with time. However, the effect of this light field on the primordial curvature perturbation grows, during inflation in the hybrid scenario and before the curvaton decay in the curvaton scenario. It is this nonlinear transfer between the field fluctuation and  $\zeta$ , described by the  $\delta N$  formalism, which can generate a large non-Gaussianity. Therefore, the non-Gaussianity in the hybrid scenario which we have studied is generated on super horizon scales during slow-roll inflation, in a similar way to which non-Gaussianity is generated over time in the curvaton scenario before the decay of the curvaton. The evolution of  $f_{\text{NL}}$  during inflation is explicitly calculated and plotted in [95]. This conclusion is somewhat different from that in [47], and we plan to elaborate on this point in a future work. For more discussion on the distinction between non-Gaussianity generated by a non-Gaussian field perturbation, and non-Gaussianity generated by a nonlinear transfer between a Gaussian field perturbation and  $\zeta$ , see [109]. An example where the subdominant fields fluctuations can become non-Gaussian due to a large self interaction was discussed by Bernardeau [45].

In order to study models where slow-roll breaks down before the end of inflation, it is clearly necessary to go beyond a formalism based on the slow-roll approximation. We have shown how this can be done in the context a separable Hubble parameter instead of a separable potential and this leads to an exact expression for (the local part of)  $f_{\text{NL}}$  in these models. As an explicit example, an exact two-field solution with an exponential potential was given. For some parameter choices, this leads to a strong break down of slow roll before the end of inflation, which may give rise to a large non-Gaussianity. Further work is also required for this model to

understand how the potential may be modified after the end of inflation in order that reheating occurs.

Non-Gaussianity is a topical field, in which observations have improved greatly over the last decade through both studies of the CMB and large-scale structure. Observations so far have heavily focused on constraining the bispectrum nonlinearity parameter  $f_{\text{NL}}$ . Currently, the tightest constraint comes from the WMAP satellite, assuming the local model of non-Gaussianity this constrains the amplitude of the non-Gaussian part of the primordial curvature perturbation to be less than about one thousandth the amplitude of the Gaussian perturbation. This constraint is likely to be tightened considerably by the Planck satellite, which is currently taking data, or instead there might be a detection. A detection of  $f_{\text{NL}}$  at this level would rule out simplest models of inflation, which are single field with a canonical kinetic term. Clearly, this would be an extremely exciting result.

However, even if we are in the fortunate position of having a detection of  $f_{\text{NL}}$  as well as improved constraints/detection of the scalar-to-tensor ratio and the spectral index, there will probably still be several viable scenarios, as detailed in this paper, which for suitable parameter choices and initial conditions can match the observations. Fortunately, non-Gaussianity is about much more than one number. The trispectrum (four-point function) depends on two nonlinearity parameters. In general,  $\tau_{\text{NL}} \geq (6f_{\text{NL}}/5)^2$ . If the current observational hints (which are not statistically significant) that  $f_{\text{NL}} \sim 40$  turn out to be true, then both the bispectrum and the trispectrum should be large enough for Planck to detect. Even if the bispectrum turns out to be much smaller, although for many models  $\tau_{\text{NL}}$  is close to the lower bound, we have seen that in the model of hybrid inflation it is possible to have  $\tau_{\text{NL}} \gg f_{\text{NL}}^2$ , so the trispectrum might even be the first observational signature of non-Gaussianity. Alternatively, the trispectrum through a large  $g_{\text{NL}}$  might give the first observational signature, as is possible in self-interacting curvaton models or the exact solution with an exponential potential. If  $f_{\text{NL}}$  is detected, it will also be possible to either constrain or detect a scale dependence of this parameter. Although it is often assumed to be constant, this is only true for certain simple models, and for example in the two-field hybrid inflation model it generally has a significant scale dependence. We have, therefore, seen that non-Gaussianity is an important and powerful method of constraining and distinguishing between the many models of inflation.

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## References

- [1] E. Komatsu and D. N. Spergel, "Acoustic signatures in the primary microwave background bispectrum," *Physical Review D*, vol. 63, no. 6, Article ID 063002, 2001.
- [2] E. Komatsu, K. M. Smith, J. Dunkley, et al., "Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological interpretation," <http://arxiv.org/abs/1001.4538>.
- [3] E. Komatsu, "Hunting for primordial non-Gaussianity in the cosmic microwave background," *Classical and Quantum Gravity*, vol. 27, no. 12, Article ID 124010, 2010.
- [4] M. Liguori, E. Sefusatti, J. R. Fergusson, and E. P. S. Shellard, "Primordial non-Gaussianity and bispectrum measurements in the cosmic microwave background and large-scale structure," <http://arxiv.org/abs/1001.4707>.
- [5] L. Verde, "Non-Gaussianity from large-scale structure surveys," *Advances in Astronomy, Special issue "Testing the Gaussianity and Statistical Isotropy of the Universe"*, vol. 2010, Article ID 768675, 15 pages, 2010, <http://arxiv.org/abs/1001.5217>.
- [6] A. A. Starobinsky, "Multicomponent de Sitter (inflationary) stages and the generation of perturbations," *JETP Letters*, vol. 42, no. 3, pp. 152–155, 1985.
- [7] A. A. Starobinsky, "Mnogokomponentnye de-sitterovskie (inflyatsionnye) stadii i generatsiya vozmushchenii," *Pisma Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, vol. 42, no. 3, pp. 124–127, 1985.
- [8] M. Sasaki and E. D. Stewart, "A general analytic formula for the spectral index of the density perturbations produced during inflation," *Progress of Theoretical Physics*, vol. 95, no. 1, pp. 71–78, 1996.
- [9] M. Sasaki and T. Tanaka, "Super-horizon scale dynamics of multi-scalar inflation," *Progress of Theoretical Physics*, vol. 99, no. 5, pp. 763–781, 1998.
- [10] D. H. Lyth, K. A. Malik, and M. Sasaki, "A general proof of the conservation of the curvature perturbation," *Journal of Cosmology and Astroparticle Physics*, vol. 2005, no. 5, article 004, 2005.
- [11] D. H. Lyth and Y. Rodríguez, "Inflationary prediction for primordial non-Gaussianity," *Physical Review Letters*, vol. 95, no. 12, Article ID 121302, 4 pages, 2005.
- [12] Y.-I. Takamizu, S. Mukohyama, M. Sasaki, and Y. Tanaka, "Non-Gaussianity of superhorizon curvature perturbations beyond  $\delta N$  formalism," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 6, article 019, 2010.
- [13] J. Maldacena, "Non-Gaussian features of primordial fluctuations in single field inflationary models," *Journal of High Energy Physics*, vol. 2003, no. 5, article 013, 2003.
- [14] X. Chen, R. Easther, and E. A. Lim, "Large non-Gaussianities in single-field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 6, article 023, 2007.
- [15] E. Silverstein and D. Tong, "Scalar speed limits and cosmology: acceleration from D-cceleration," *Physical Review D*, vol. 70, no. 10, Article ID 103505, 18 pages, 2004.
- [16] M. Alishahiha, E. Silverstein, and D. Tong, "DBI in the sky: non-Gaussianity from inflation with a speed limit," *Physical Review D*, vol. 70, no. 12, Article ID 123505, 15 pages, 2004.
- [17] X. Chen, M.-X. Huang, S. Kachru, and G. Shiu, "Observational signatures and non-Gaussianities of general single-field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 1, article 002, 2007.
- [18] M.-X. Huang and G. Shiu, "Inflationary trispectrum for models with large non-Gaussianities," *Physical Review D*, vol. 74, no. 12, Article ID 121301, 2006.
- [19] D. Langlois, S. Renaux-Petel, D. A. Steer, and T. Tanaka, "Primordial perturbations and non-Gaussianities in DBI and general multi-field inflation," *Physical Review D*, vol. 78, Article ID 063523, 15 pages, 2008.
- [20] F. Arroja, S. Mizuno, and K. Koyama, "Non-Gaussianity from the bispectrum in general multiple field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 8, article 015, 2008.
- [21] X. Gao and B. Hu, "Primordial trispectrum from entropy perturbations in multifield DBI model," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 8, article 012, 2009.
- [22] Y.-F. Cai and H.-Y. Xia, "Inflation with multiple sound speeds: a model of multiple DBI type actions and non-Gaussianities," *Physics Letters B*, vol. 677, no. 5, pp. 226–234, 2009.
- [23] X. Chen, B. Hu, M.-X. Huang, G. Shiu, and Y. Wang, "Large primordial trispectra in general single field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 8, article 008, 2009.
- [24] F. Arroja, S. Mizuno, K. Koyama, and T. Tanaka, "Full trispectrum in single field DBI inflation," *Physical Review D*, vol. 80, no. 4, Article ID 043527, 2009.
- [25] S. Mizuno, F. Arroja, K. Koyama, and T. Tanaka, "Lorentz boost and non-Gaussianity in multifield DBI inflation," *Physical Review D*, vol. 80, no. 2, Article ID 023530, 2009.
- [26] K. Koyama, "Non-Gaussianity of quantum fields during inflation," *Class.Quant.Grav.*, vol. 27, Article ID 124001, 2010, <http://arxiv.org/abs/1002.0600>.
- [27] X. Chen, "Primordial non-Gaussianities from inflation models," <http://arxiv.org/abs/1002.1416>.
- [28] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamäki, and A. Väihkönen, "Non-Gaussianity from preheating," *Physical Review Letters*, vol. 94, no. 16, Article ID 161301, 2005.
- [29] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamäki, and A. Väihkönen, "Non-Gaussianity from instant and tachyonic preheating," *Journal of Cosmology and Astroparticle Physics*, vol. 2005, no. 3, article 010, 2005.
- [30] A. Jokinen and A. Mazumdar, "Very large primordial non-Gaussianity from multiple fields: application to massless preheating," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 4, article 003, 2006.
- [31] N. Bartolo, S. Matarrese, and A. Riotto, "Non-Gaussianity in the curvaton scenario," *Physical Review D*, vol. 69, no. 4, Article ID 043503, 2004.
- [32] K. A. Malik and D. H. Lyth, "A numerical study of non-Gaussianity in the curvaton scenario," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 9, article 008, 2006.
- [33] M. Zaldarriaga, "Non-Gaussianities in models with a varying inflaton decay rate," *Physical Review D*, vol. 69, no. 4, Article ID 043508, 2004.
- [34] D. H. Lyth, "Generating the curvature perturbation at the end of inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2005, no. 11, article 006, 2005.
- [35] J. Väliviita and T. Giannantonio, "Constraints on primordial isocurvature perturbations and spatial curvature by Bayesian model selection," *Physical Review D*, vol. 80, no. 12, Article ID 123516, 2009.
- [36] C. Hikage, D. Munshi, A. Heavens, and P. Coles, "Adiabatic versus isocurvature non-Gaussianity," *Monthly Notices of the Royal Astronomical Society*, vol. 404, no. 3, pp. 1505–1511, 2010.



- [37] N. Bartolo, S. Matarrese, and A. Riotto, “Non-Gaussianity and the cosmic microwave background anisotropies,” <http://arxiv.org/abs/1001.3957>.
- [38] C. Pitrou, J.-P. Uzan, and F. Bernardeau, “The cosmic microwave background bispectrum from the non-linear evolution of the cosmological perturbations,” *JCAP*, no. 1007, article 003, 2010, <http://arxiv.org/abs/1003.0481>.
- [39] D. Wands, “Multiple field inflation,” *Lecture Notes in Physics*, vol. 738, pp. 275–304, 2008.
- [40] E. Komatsu, N. Afshordi, N. Bartolo, et al., “Non-Gaussianity as a probe of the physics of the primordial universe and the astrophysics of the low redshift universe,” <http://arxiv.org/abs/0902.4759>.
- [41] D. Babich, P. Creminelli, and M. Zaldarriaga, “The shape of non-Gaussianities,” *Journal of Cosmology and Astroparticle Physics*, vol. 2004, no. 8, article 009, 2004.
- [42] J. R. Fergusson and E. P. S. Shellard, “Shape of primordial non-Gaussianity and the CMB bispectrum,” *Physical Review D*, vol. 80, no. 4, Article ID 043510, 2009.
- [43] L. Alabidi, K. A. Malik, C. T. Byrnes, and K.-Y. Choi, “How the curvaton scenario, modulated reheating and an inhomogeneous end of inflation are related,” <http://arxiv.org/abs/1002.1700>.
- [44] A. Mazumdar and J. Rocher, “Particle physics models of inflation and curvaton scenarios,” <http://arxiv.org/abs/1001.0993>.
- [45] F. Bernardeau, “Gravity and non-gravity mediated couplings in multiple-field inflation,” *Class.Quant.Grav.*, vol. 27, Article ID 124004, 2010, <http://arxiv.org/abs/1003.2869>.
- [46] D. Wands, “Local non-Gaussianity from inflation,” *Class.Quant.Grav.*, vol. 27, Article ID 124002, 2010, <http://arxiv.org/abs/1004.0818>.
- [47] T. Tanaka, T. Suyama, and S. Yokoyama, “Use of delta N formalism—difficulties in generating large local-type non-Gaussianity during inflation,” *Class.Quant.Grav.*, vol. 27, Article ID 124003, 2010, <http://arxiv.org/abs/1003.5057>.
- [48] N. Barnaby and J. M. Cline, “Predictions for non-Gaussianity from non-local inflation,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 6, article 030, 2008.
- [49] D. Langlois and L. Sorbo, “Primordial perturbations and non-Gaussianities from modulated trapping,” *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 8, article 014, 2009.
- [50] J.-L. Lehners and P. J. Steinhardt, “Non-Gaussianity generated by the entropic mechanism in bouncing cosmologies made simple,” *Physical Review D*, vol. 80, no. 10, Article ID 103520, 2009.
- [51] J.-L. Lehners, “Ekpyrotic non-Gaussianity—a review,” *Advances in Astronomy, Special issue “Testing the Gaussianity and Statistical Isotropy of the Universe”*, vol. 2010, Article ID 903907, 19 pages, 2010, <http://arxiv.org/abs/1001.3125>.
- [52] K. Koyama, S. Mizuno, F. Vernizzi, and D. Wands, “Non-Gaussianities from ekpyrotic collapse with multiple fields,” *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 11, article 024, 2007.
- [53] S. Mollerach, “Isocurvature baryon perturbations and inflation,” *Physical Review D*, vol. 42, no. 2, pp. 313–325, 1990.
- [54] A. Linde and V. Mukhanov, “Non-Gaussian isocurvature perturbations from inflation,” *Physical Review D*, vol. 56, no. 2, pp. R535–R539, 1997.
- [55] K. Enqvist and M. S. Sloth, “Adiabatic CMB perturbations in pre-Big-Bang string cosmology,” *Nuclear Physics B*, vol. 626, no. 1-2, pp. 395–409, 2002.
- [56] D. H. Lyth and D. Wands, “Generating the curvature perturbation without an inflaton,” *Physics Letters B*, vol. 524, no. 1-2, pp. 5–14, 2002.
- [57] T. Moroi and T. Takahashi, “Effects of cosmological moduli fields on cosmic microwave background,” *Physics Letters B*, vol. 522, no. 3-4, pp. 215–221, 2001.
- [58] T. Moroi and T. Takahashi, “Erratum: effects of cosmological moduli fields on cosmic microwave background,” *Physics Letters B*, vol. 539, no. 3-4, p. 303, 2002.
- [59] A. Linde and V. Mukhanov, “The curvaton web,” *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 4, article 009, 2006.
- [60] D. H. Lyth, C. Ungarelli, and D. Wands, “Primordial density perturbation in the curvaton scenario,” *Physical Review D*, vol. 67, no. 2, Article ID 023503, 2003.
- [61] M. Sasaki, J. Väliväita, and D. Wands, “Non-Gaussianity of the primordial perturbation in the curvaton model,” *Physical Review D*, vol. 74, no. 10, Article ID 103003, 2006.
- [62] D. Langlois, F. Vernizzi, and D. Wands, “Non-linear isocurvature perturbations and non-Gaussianities,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 12, article 004, 2008.
- [63] K.-Y. Choi and J.-O. Gong, “Multiple scalar particle decays and perturbation generation,” *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 6, article 007, 2007.
- [64] H. Assadullahi, J. Väliväita, and D. Wands, “Primordial non-Gaussianity from two curvaton decays,” *Physical Review D*, vol. 76, no. 10, Article ID 103003, 2007.
- [65] Q.-G. Huang, “The N-vaton,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 9, article 017, 2008.
- [66] C. Gordon and A. Lewis, “Observational constraints on the curvaton model of inflation,” *Physical Review D*, vol. 67, no. 12, Article ID 123513, 2003.
- [67] M. Beltrán, “Isocurvature, non-Gaussianity, and the curvaton model,” *Physical Review D*, vol. 78, no. 2, Article ID 023530, 2008.
- [68] K. Nakayama and J. Yokoyama, “Gravitational wave background and non-Gaussianity as a probe of the curvaton scenario,” *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 1, article 010, 2010.
- [69] B. A. Bassett, S. Tsujikawa, and D. Wands, “Inflation dynamics and reheating,” *Reviews of Modern Physics*, vol. 78, no. 2, pp. 537–589, 2006.
- [70] A. V. Frolov, “Non-linear dynamics and primordial curvature perturbations from preheating,” *Class.Quant.Grav.*, vol. 27, Article ID 124006, 2010, <http://arxiv.org/abs/1004.3559>.
- [71] G. Dvali, A. Gruzinov, and M. Zaldarriaga, “New mechanism for generating density perturbations from inflation,” *Physical Review D*, vol. 69, no. 2, Article ID 023505, 2004.
- [72] L. Kofman, “Probing string theory with modulated cosmological fluctuations,” <http://arxiv.org/abs/astro-ph/0303614>.
- [73] G. Dvali, A. Gruzinov, and M. Zaldarriaga, “Cosmological perturbations from inhomogeneous reheating, freeze-out, and mass domination,” *Physical Review D*, vol. 69, no. 8, Article ID 083505, 5 pages, 2004.
- [74] C. W. Bauer, M. L. Graesser, and M. P. Salem, “Fluctuating annihilation cross sections and the generation of density perturbations,” *Physical Review D*, vol. 72, no. 2, Article ID 023512, 7 pages, 2005.
- [75] T. Suyama and M. Yamaguchi, “Non-Gaussianity in the modulated reheating scenario,” *Physical Review D*, vol. 77, no. 2, Article ID 023505, 2008.



- [76] K. Ichikawa, T. Suyama, T. Takahashi, and M. Yamaguchi, "Primordial curvature fluctuation and its non-Gaussianity in models with modulated reheating," *Physical Review D*, vol. 78, no. 6, Article ID 063545, 2008.
- [77] M. Bastero-Gil, V. Di Clemente, and S. F. King, "Preheating curvature perturbations with a coupled curvaton," *Physical Review D*, vol. 70, no. 2, Article ID 023501, 2004.
- [78] J. R. Bond, A. V. Frolov, Z. Huang, and L. Kofman, "Non-Gaussian curvature spikes from chaotic billiards in inflation preheating," *Physical Review Letters*, vol. 103, no. 7, Article ID 071301, 2009.
- [79] K. Kohri, D. H. Lyth, and C.A. Valenzuela-Toledo, "On the generation of a non-Gaussian curvature perturbation during preheating," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 2, article 023, 2010.
- [80] G. Felder, L. Kofman, and A. Linde, "Instant preheating," *Physical Review D*, vol. 59, no. 12, Article ID 123523, 7 pages, 1999.
- [81] E. W. Kolb, A. Riotto, and A. Vallinotto, "Curvature perturbations from broken symmetries," *Physical Review D*, vol. 71, no. 4, Article ID 043513, 10 pages, 2005.
- [82] C. T. Byrnes and D. Wands, "Scale-invariant perturbations from chaotic inflation," *Physical Review D*, vol. 73, no. 6, Article ID 063509, 8 pages, 2006.
- [83] T. Matsuda, "Generating the curvature perturbation with instant preheating," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 3, article 003, 2007.
- [84] T. Battefeld, "Modulated perturbations from instant preheating after new ekpyrosis," *Physical Review D*, vol. 77, no. 6, Article ID 063503, 2008.
- [85] C. T. Byrnes, "Constraints on generating the primordial curvature perturbation and non-Gaussianity from instant preheating," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 1, article 011, 2009.
- [86] Q.-G. Huang, "A geometric description of the non-Gaussianity generated at the end of multi-field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 6, article 035, 2009.
- [87] F. Bernardeau and J.-P. Uzan, "Non-Gaussianity in multifield inflation," *Physical Review D*, vol. 66, no. 10, Article ID 103506, 2002.
- [88] F. Bernardeau and J.-P. Uzan, "Inflationary models inducing non-Gaussian metric fluctuations," *Physical Review D*, vol. 67, no. 12, Article ID 121301, 2003.
- [89] F. Bernardeau and T. Brunier, "Non-Gaussianities in extended D-term inflation," *Physical Review D*, vol. 76, no. 4, Article ID 043526, 2007.
- [90] L. Alabidi and D. Lyth, "Curvature perturbation from symmetry breaking the end of inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 8, article 006, 2006.
- [91] M. P. Salem, "Generation of density perturbations at the end of inflation," *Physical Review D*, vol. 72, no. 12, Article ID 123516, 7 pages, 2005.
- [92] M. Sasaki, "Multi-brid inflation and non-Gaussianity," *Progress of Theoretical Physics*, vol. 120, no. 1, pp. 159–174, 2008.
- [93] A. Naruko and M. Sasaki, "Large non-Gaussianity from multi-brid inflation," *Progress of Theoretical Physics*, vol. 121, no. 1, pp. 193–210, 2009.
- [94] C. T. Byrnes, K.-Y. Choi, and L. M. H. Hall, "Conditions for large non-Gaussianity in two-field slow-roll inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 10, article 008, 2008.
- [95] C. T. Byrnes, K. Y. Choi, and L. M. H. Hall, "Large non-Gaussianity from two-component hybrid inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 2, article 017, 2009.
- [96] G. I. Rigopoulos and E. P. S. Shellard, "Non-linear inflationary perturbations," *Journal of Cosmology and Astroparticle Physics*, vol. 2005, no. 10, article 006, 2005.
- [97] G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, "Nonlinear perturbations in multiple-field inflation," *Physical Review D*, vol. 73, no. 8, Article ID 083521, 11 pages, 2006.
- [98] L. Alabidi and D. H. Lyth, "Inflation models and observation," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 5, article 016, 2006.
- [99] F. Vernizzi and D. Wands, "Non-Gaussianities in two-field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2006, no. 5, article 019, Article ID 019, 2006.
- [100] G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, "Quantitative bispectra from multifield inflation," *Physical Review D*, vol. 76, no. 8, Article ID 083512, 2007.
- [101] S. Yokoyama, T. Suyama, and T. Tanaka, "Primordial non-Gaussianity in multiscalar inflation," *Physical Review D*, vol. 77, no. 8, Article ID 083511, 2008.
- [102] S. Yokoyama, T. Suyama, and T. Tanaka, "Primordial non-Gaussianity in multiscalar inflation," *Physical Review D*, vol. 77, no. 8, Article ID 083511, 2008.
- [103] H. R. S. Cogollo, Y. Rodríguez, and C. A. Valenzuela-Toledo, "On the issue of the  $\zeta$  series convergence and loop corrections in the generation of observable primordial non-Gaussianity in slow-roll inflation: I. The bispectrum," *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 8, article 029, 2008.
- [104] K.-Y. Choi, L. M. H. Hall, and C. van de Bruck, "Spectral running and non-Gaussianity from slow-roll inflation in generalized two-field models," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 2, article 029, 2007.
- [105] T. Battefeld and R. Easther, "Non-Gaussianities in multi-field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 3, article 020, 2007.
- [106] D. Seery and J. E. Lidsey, "Non-Gaussianity from the inflationary trispectrum," *Journal of Cosmology and Astroparticle Physics*, vol. 2007, no. 1, article 008, 2007.
- [107] J. García-Bellido and D. Wands, "Metric perturbations in two-field inflation," *Physical Review D*, vol. 53, no. 10, pp. 5437–5445, 1996.
- [108] N. Bartolo and A. R. Liddle, "Simplest curvaton model," *Physical Review D*, vol. 65, no. 12, Article ID 121301, 2002.
- [109] D. J. Mulryne, D. Seery, and D. Wesley, "Moment transport equations for non-Gaussianity," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 1, article 024, 2010.
- [110] N. Barnaby and J. M. Cline, "Non-Gaussian and nonscale-invariant perturbations from tachyonic preheating in hybrid inflation," *Physical Review D*, vol. 73, no. 10, Article ID 106012, 2006.
- [111] D. Mulryne, D. Seery, and D. Wesley, "Non-Gaussianity constrains hybrid inflation," <http://arxiv.org/abs/0911.3550>.
- [112] C. T. Byrnes and G. Tasinato, "Non-Gaussianity beyond slow roll in multi-field inflation," *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 8, article 016, 2009.
- [113] D. S. Salopek and J. R. Bond, "Nonlinear evolution of long-wavelength metric fluctuations in inflationary models," *Physical Review D*, vol. 42, no. 12, pp. 3936–3962, 1990.

- [114] W. H. Kinney, “Hamilton-Jacobi approach to non-slow-roll inflation,” *Physical Review D*, vol. 56, no. 4, pp. 2002–2009, 1997.
- [115] D. Battefeld and T. Battefeld, “On non-Gaussianities in multi-field inflation (N fields): bi- and tri-spectra beyond slow-roll,” *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 11, article 010, 2009.
- [116] K.-Y. Choi, J.-O. Gong, and D. Jeong, “Evolution of the curvature perturbation during and after multi-field inflation,” *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 2, article 032, 2009.
- [117] M. Sasaki, “A note on nonlinear curvature perturbations in an exactly soluble model of multi-component slow-roll inflation,” *Classical and Quantum Gravity*, vol. 24, no. 9, pp. 2433–2437, 2007.
- [118] J. P. Conlon and F. Quevedo, “Kähler moduli inflation,” *Journal of High Energy Physics*, vol. 2006, no. 1, article 146, 2006.
- [119] J. R. Bond, L. Kofman, S. Prokushkin, and P. M. Vaudrevange, “Roulette inflation with Kähler moduli and their axions,” *Physical Review D*, vol. 75, no. 12, Article ID 123511, 2007.
- [120] A. Misra and P. Shukla, “Large volume axionic Swiss cheese inflation,” *Nuclear Physics B*, vol. 800, no. 1-2, pp. 384–400, 2008.
- [121] M. Badziak and M. Olechowski, “Volume modulus inflation and a low scale of SUSY breaking,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 7, article 021, 2008.
- [122] E. Palti, G. Tasinato, and J. Ward, “Weakly-coupled IIA flux compactifications,” *Journal of High Energy Physics*, vol. 2008, no. 6, article 084, 2008.
- [123] H.-X. Yang and H.-L. Ma, “Two-field Kähler moduli inflation in large volume moduli stabilization,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 8, article 024, 2008.
- [124] J. J. Blanco-Pillado, D. Buck, E. J. Copeland, M. Gomez-Reino, and N. J. Nunes, “Kähler moduli inflation revisited,” *JHEP*, vol. 2010, no. 1, article 081, 2010, <http://arxiv.org/abs/0906.3711>.
- [125] N. Barnaby, J. R. Bond, Z. Huang, and L. Kofman, “Pre-heating after modular inflation,” *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 12, article 021, 2009.
- [126] E. Komatsu, J. Dunkley, M. R. Nolte et al., “Five-year wilkinson microwave anisotropy probe observations: cosmological interpretation,” *Astrophysical Journal*, vol. 180, no. 2, pp. 330–376, 2009.
- [127] C. T. Byrnes, M. Sasaki, and D. Wands, “Primordial trispectrum from inflation,” *Physical Review D*, vol. 74, no. 12, Article ID 123519, 2006.
- [128] J. Smidt, A. Amblard, A. Cooray, A. Heavens, D. Munshi, and P. Serra, “A measurement of cubic-order primordial non-Gaussianity ( $g_{NL}$  and  $\tau_{NL}$ ) With WMAP 5-Year Data,” <http://arxiv.org/abs/1001.5026>.
- [129] V. Desjacques and U. Seljak, “Signature of primordial non-Gaussianity of  $\phi^3$  type in the mass function and bias of dark matter haloes,” *Physical Review D*, vol. 81, no. 2, Article ID 023006, 2010.
- [130] P. Vielva and J. L. Sanz, “Constraints on  $f_{NL}$  and  $g_{NL}$  from the analysis of the N-pdf of the CMB large-scale anisotropies,” *Monthly Notices of the Royal Astronomical Society*, vol. 404, no. 2, pp. 895–907, 2010.
- [131] T. Okamoto and W. Hu, “Angular trispectra of CMB temperature and polarization,” *Physical Review D*, vol. 66, no. 6, Article ID 063008, 2002.
- [132] N. Kogo and E. Komatsu, “Angular trispectrum of CMB temperature anisotropy from primordial non-Gaussianity with the full radiation transfer function,” *Physical Review D*, vol. 73, no. 8, Article ID 083007, 2006.
- [133] A. Cooray, C. Li, and A. Melchiorri, “Trispectrum of 21-cm background anisotropies as a probe of primordial non-Gaussianity,” *Physical Review D*, vol. 77, no. 10, Article ID 103506, 2008.
- [134] D. Jeong and E. Komatsu, “Primordial non-Gaussianity, scale-dependent bias, and the bispectrum of galaxies,” *Astrophysical Journal*, vol. 703, no. 2, pp. 1230–1248, 2009.
- [135] J. Smidt, A. Amblard, C. T. Byrnes, A. Cooray, and D. Munshi, “CMB constraints on primordial non-Gaussianity from the bispectrum and trispectrum and a consistency test of single-field inflation,” *Phys.Rev.*, Article ID 123007, 2010, <http://arxiv.org/abs/1004.1409>.
- [136] P. Creminelli, S. Dubovsky, A. Nicolis, L. Senatore, and M. Zaldarriaga, “The phase transition to eternal inflation,” *Journal of High Energy Physics*, vol. 2008, no. 9, article 036, 2008.
- [137] Y. Rodríguez and C. A. Valenzuela-Toledo, “On the Issue of the  $\zeta$  series convergence and loop corrections in the generation of observable primordial non-Gaussianity in slow-roll inflation. II. the trispectrum,” *Physical Review D*, vol. 81, no. 2, Article ID 023531, 2010.
- [138] K. Enqvist and S. Nurmi, “Non-Gaussianity in curvaton models with nearly quadratic potentials,” *Journal of Cosmology and Astroparticle Physics*, vol. 2005, no. 10, article 013, 2005.
- [139] Q.-G. Huang and Y. Wang, “Curvaton dynamics and the non-linearity parameters in the curvaton model,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 9, article 025, 2008.
- [140] Q.-G. Huang, “A curvaton with a polynomial potential,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 11, article 005, 2008.
- [141] K. Enqvist and T. Takahashi, “Signatures of non-Gaussianity in the curvaton model,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 9, article 012, 2008.
- [142] K. Enqvist, S. Nurmi, O. Taanila, and T. Takahashi, “Non-Gaussian fingerprints of self-interacting curvaton,” *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 4, article 009, 2010.
- [143] C. T. Byrnes, S. Nurmi, G. Tasinato, and D. Wands, “Scale dependence of local  $f_{NL}$ ,” *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 2, article 034, 2010.
- [144] E. Sefusatti, M. Liguori, A. P. S. Yadav, M. G. Jackson, and E. Pajer, “Constraining running non-Gaussianity,” *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 12, article 022, 2009.
- [145] M. LoVerde, A. Miller, S. Shandera, and L. Verde, “Effects of scale-dependent non-Gaussianity on cosmological structures,” *Journal of Cosmology and Astroparticle Physics*, vol. 2008, no. 4, article 014, 2008.
- [146] X. Chen, “Running non-Gaussianities in Dirac-Born-Infeld inflation,” *Physical Review D*, vol. 72, no. 12, Article ID 123518, 7 pages, 2005.
- [147] J. Khoury and F. Piazza, “Rapidly-varying speed of sound, scale invariance and non-Gaussian signatures,” *Journal of Cosmology and Astroparticle Physics*, vol. 2009, no. 7, article 026, 2009.