

Working Papers

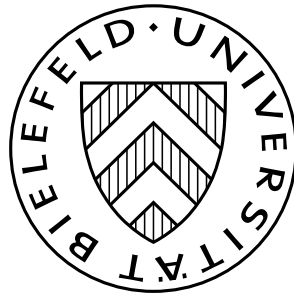
Institute of
Mathematical
Economics

403

November 2008

Culture Formation and Endogenous Cultural Distance

Michael M. Pichler



IMW · Bielefeld University
Postfach 100131
33501 Bielefeld · Germany



email: imw@wiwi.uni-bielefeld.de
<http://www.wiwi.uni-bielefeld.de/~imw/Papers/showpaper.php?403>
ISSN: 0931-6558

Culture Formation and Endogenous Cultural Distance

Michael M. Pichler*

Institut für Mathematische Wirtschaftsforschung (IMW)

Universität Bielefeld

mpichler@wiwi.uni-bielefeld.de

November 2008

Abstract

This paper introduces a new approach to ‘cultural transmission of preferences’ (see Bisin and Verdier, 2000, 2001). It is based on the conceptualization of the culture of a person as a set of cultural values and attitudes, represented as an n -dimensional tuple in Euclidean space. The culture of a person is formed through social learning and imitation from role-models, which correspond to the chosen ‘displayed cultures’ of parents (‘vertical transmission’) and the society at large (‘oblique transmission’). Parents might choose a ‘displayed culture’ that does not coincide with their true culture, since they aim at countervailing negative cultural influences that their children are exposed to in the society at large. Additionally, they can invest into the success that their displayed culture has in the socialization process of their children. We will consider in the present paper an OLG model with two cultural groups, and where in any period, the members of each of the cultural groups have identical culture. We show that if parents have a desire for cultural closeness to their children (e.g. ‘imperfect empathy’), then they will always behave culturally more ‘radical’ relative to the culture of the other cultural group. Furthermore, they will always invest into their socialization success. Nevertheless, these investments are never sufficient to let the distance between the future cultures of the children of both cultural groups be larger or equal than the cultural distance of the parental generation. As a consequence, the cultures of both groups converge to a homogeneous steady state culture, which can be interpreted as a mixture of the two initial cultures. This result corresponds to the ‘melting pot’ theory of integration of cultural groups.

Keywords: Cultural transmission; Socialization

*This research project has been generously supported by the Deutsche Forschungsgesellschaft (DFG). The author thanks Herbert Dawid and Walter Trockel for invaluable advice. Special thanks are due to the members of the International Research Training Group ‘Economic Behavior and Interaction Models’ (EBIM).

1 Introduction

In recent years, the question of successful integration of immigrants with different cultural backgrounds into hosting societies has attained increasing attention, both in media and on the political agenda. In Europe, the focus of the debate has been on an alleged lack of integration among Muslim immigrants, compared to other immigrant groups (compare e.g. Gallis (2005)). This allegation has also been confirmed empirically, see e.g. Bisin et al. (2007) who use a U.K. data-set and find significantly less and slower integration of Muslims compared to non-Muslims. Compared to the European nation states, the debate in the U.S.A. has been less pronounced given its historical pre-disposition as an immigrant country. Still influential is the ‘melting pot’ theory, which assumes that new immigrants will integrate into the society, thereby contributing to the creation of a new cultural identity (Han, 2006, p. 32). While this theory is based on, and successful to describe, the experience of ‘old immigration’ from western and northern European immigrants (which are culturally close to anglo-saxian Americans), it fails to explain the tendency towards cultural pluralism that the U.S. society has experienced thereafter (Gordon, 1964, pp. 115–119, 132–136).

The aim of the present paper is to present a theoretical framework that contributes to the understanding of the process of cultural assimilation (or even dissimilation). Cultural assimilation is defined as a decrease in the distance between the cultures of two persons or groups, underlying the concept of culture as a set of values and attitudes¹. The culture of a person is formed in the youth through the socialization process, shaped by social learning and imitation from role-models (displayed cultures) that the child is confronted with in its social environment. Parents have an incentive to actively engage in the socialization process, given that they want to counteract negative cultural influences that their children are exposed to. The term ‘negative’ applies whenever the displayed culture of a person or group does not coincide with the culture that the child should optimally adopt from the parents’ point of view. What the parents perceive as the ‘optimal culture’ depends on which specific socialization motivation structure is applied. We will consider in this paper the desire of the parents to keep the (future) culture of their offspring close to their own culture.

The present analysis stands in a natural context to the existing literature on cultural transmission of preferences. This literature is based on the work of Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985) in evolutionary anthropology. Bisin and Verdier (2000, 2001) presented a general framework to study the population dynamics of the distribution of preference traits (cultures) under an endogenous intergenerational cultural

¹Cultural assimilation in this context is hence a mutual concept in that it not just presumes the integration of one group into the culture of another; cultural assimilation can rather originate from just one, or both of the cultural groups under scrutiny.

transmission mechanism. In this framework, which is now standard in the literature, the endogeneity stems from the parental choice of socialization intensity, which effectively determines the probability that the child will directly adopt the culture of the parents. If this direct socialization does not succeed, the child will randomly adopt the culture of an adult in the social environment, the probabilities of which are determined by the distribution of the cultural traits in the population. Given that parents perceive a subjective utility loss if their children adopt another culture, the incentives for actively engaging in the socialization process are increasing in the population share of the other cultural group. This means that minorities do socialize more intensely compared to majorities, which under certain conditions prevents their culture from disappearance in the cultural equilibrium. The properties of the model framework have been used in the analysis of several different issues, such as e.g. preferences for social status (Bisin and Verdier, 1998), voting and political ideology (Bisin and Verdier, 2000), corruption (Hauk and Saez-Marti, 2002), development and social capital, hold up problems (Olcina and Penarrubia, 2004).

Nevertheless, the models in this strand of literature are based on intertemporarily fixed cultures, with the cultural distance between the cultural groups left unspecified. As a consequence, the parental assessment of the ‘cultural threat’ that another group exerts on a child in the socialization process has to be introduced as an exogenously fixed degree of ‘cultural intolerance’ (Bisin and Verdier, 2001, p. 307). While as this framework allows the analysis of the population dynamics of the distribution of the fixed preferences (cultures), its construction makes it impossible to study the evolutionary cultural change that results out of the social(ization) interaction of cultural groups. Important issues, such as whether the cultures living together will assimilate or dissimilate over time, must be left unconsidered. Recently, Bisin et al. (2006) introduced parental identity–choice² into the cultural transmission of preferences framework to study the assimilation process of cultural minorities. Parents of the minority group can choose among a continuum of ‘life–styles’, all of which represent different degrees of assimilation to the social norm of the majority. A lack of assimilation causes social costs that positively depend on the population share of the majority group and negatively on the degree of ethnic identity (which is a decision variable for the parents³). Nevertheless, the parental problem of choosing the optimal level of assimilation and ethnic identity is separated from the socialization problem (which is organized along the standard lines of the cultural transmission of preferences literature). Although the assim-

²The concept of identity formation is based on the work of Akerlof and Kranton (2000).

³Bisin et al. (2006) motivate the construction of ethnic identity as a psychological mechanism that is rationally employed to reduce the psychological costs associated with behaving in a non–assimilated way relative to the social norm of the majority cultural group.

ilation choice corresponds to a choice of ‘life–style’, hence a role–model or displayed culture, this has no social–learning impact in the socialization process of the child. Parents still (probabilistically) socialize the children to the fixed cultural specification of the minority group, disregarding of their generically different displayed culture. The dynamic evolution of the degree of assimilation of the cultural minority then derives from the dynamics of the population distribution of cultures and the evolution of the degree of ethnic identity, which determines the parentally perceived utility cost of the anticipated life–style of the child.

Compared to these latter models, the present paper introduces a number of innovations. Most importantly, it drops the assumption of fixed cultures and replaces the ‘cultural transmission (of preferences)’ framework with a new conceptualization of the socialization process that endogenously determines the ‘formation of culture’. This approach is based on the specification of the culture of a person or group as a set of cultural values and attitudes, represented by a tuple in an n –dimensional Euclidean space. The cultural distance between two persons or groups is then defined as the respective Euclidean distance. The culture of a person is formed in its youth period through the socialization process. As Bisin and Verdier (2000, 2001), we impose the ‘tabula rasa’ assumption, i.e. children are born without a pre–defined culture. Rather, culture results out of social learning and imitation of role–models, with the precise specification depending on the relative time–exposures to different ‘displayed cultures’ (viz. ‘role–models’ or ‘life–styles’) in the social environment, and the relative intensities with which the persons incorporating the role–models try to socialize the child. Formally, we also view displayed cultures as tuples in an n –dimensional Euclidean space, and the culture that a child adopts is then an n –tuple in the convex hull of the displayed cultures.

The intuition behind this process is as follows. Assume that a child is exposed to only two different persons in the socialization period. Assume further that one of the persons represents a role–model, or displayed culture, with a strong intensity of a specific cultural value (e.g. family values), and the other person displays a weak intensity of this cultural value. Formally, thus, the entries for this cultural value in the specifications of displayed culture of the two persons take on a high, respectively low, (numerical) value. At this point, we replace the standard assumption in the literature on cultural transmission of preferences, i.e. that the child would be socialized to either of the two cultures that are represented by the role–models in the social environment, with an alternative approach: Children socially learn from both displayed cultures, and all influences experienced in the socialization phase are non–vanishing. Specifically, the ‘success–rates’ of the different displayed cultures in the cultural value–formation process of the child depend on a) how much time both persons have spent demonstrating their role–model to the child, and b) how much effort the persons invested

during that time in convincing the child about the appropriateness of their specific ‘world–view’. In our example, (except for degenerate cases) the child will effectively be socialized to an intermediate stance with respect to the cultural value under scrutiny, i.e. will take on in its own (true) cultural specification a convex combination of the (numerical) values that the two persons socializing it represented. The logic of this form of ‘formation of values’ in the socialization process is easily extended to a framework with multiple socializing persons, and multiple cultural values — the latter case of which we will refer to as the before–mentioned ‘formation of culture’.

Parents have special interest in actively engaging in the socialization process of their children. In the present paper, we will assume that parents experience a level of dis–utility that positively depends on the cultural distance between their (the parents’) own culture and the culture that the children will have adopted after completion of the socialization process. One justification for this assumption centers around the concept of ‘imperfect empathy’, as used by Bisin and Verdier (2000, 2001) and throughout the literature on cultural transmission of preferences. Parents are altruistic and want to maximize the utility that their children generate out of their future social and economic actions. Nevertheless, parents can evaluate this utility only through the filter of their own culture and thus are biased towards favoring their own cultural specification. An alternative justification for endowing parents with a ‘preference for cultural closeness’ to their children is to argue that parents try to avoid frictions in social interactions within the family (under the assumption that those frictions increase in the cultural distance of two persons). Hence, this justification does not necessitate the assumption of altruism.

Given a certain expected distribution of displayed cultures in the population, the parental incentives to engage in the socialization process are increasing in the distance between the displayed culture of any person or group and the parents’ own culture. This stems from the fact that the children’s social interaction with culturally more ‘alienate’ individuals will pull their cultural outcome further away from the parents’ one.

To countervail the negative influences exerted by ‘alienate’ displayed cultures, parents can employ their socialization technique. In this respect, we apply here an adoption of the ‘direct vertical’ and ‘oblique’ transmission framework that has been introduced into the economics literature by Bisin and Verdier (2000, 2001) and is a standard application in the literature on cultural transmission of preferences⁴. In the present paper, parents have

⁴The terminology of ‘direct vertical’ and ‘oblique’ transmission originates from Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985), who also distinguish the latter two cultural transmission channels from ‘horizontal’ transmission, i.e. social learning from other children or peers (which we do not consider in the present paper). As Bisin and Verdier (1998) point out, basing the socialization process on these cultural transmission sources is consistent with results in the sociological literature.

a controlling function over the cultural development of the child. They can demonstrate to their children a role-model (displayed culture) and take additional actions such as to decrease the cultural influence of deviating role-models (that are represented by the general social environment) on the culture of their child. These additional actions concern the time and effort that parents take to convince the child about the superiority of their own world-view compared to the world-views that the child is confronted with in the general society. However, both a deviation from the cultural behavior that would maximize the parents' own utility, as well as the investments into socialization time and effort are costly, which together with the desire for inter-generational cultural closeness constitutes the parental socialization problem.

The central question that this paper addresses concerns the characteristics of the endogenous evolution of the cultural distance between two cultural groups in a dynamic overlapping generations setting. In the present framework, since parents are free to select the optimal displayed culture that they want to transmit to their children, they typically choose to behave more 'radical' relative to the culture of the other group. In other words, the optimal displayed culture features a larger cultural distance to the (displayed and true) culture of the other group, than the true culture of the parents does, and the parents behaviorally dissimilate. This comes in an attempt to countervail the 'cultural pulling' force that the other cultural group exerts on their children. Still, the remaining question then is whether this *behavioral* dissimilation is adjoined by sufficiently intense parental investments into their socialization success such as that the distance between the final cultures of children of both groups will increase compared to that of members of the parental generations (a *cultural* dissimilation).

A central result of this paper is that under the inter-generational cultural closeness socialization motivation, such an outcome can never realize and the cultural distances strictly decline over generations (cultural assimilation). Even, the cultures of both group converge to a homogeneous culture, which is a steady state of the cultural system. This steady state culture can be interpreted as a mixture of the two original cultures, a result of which corresponds to the 'melting pot' theory of integration of cultural groups.

Furthermore, the assimilative process (that is described by cultural convergence) of any cultural group is slower the more intense the desire for inter-generational cultural closeness is. This theoretical result might deliver an explanation for the empirical observation of different successes in the integration process of cultural groups, notably the before mentioned lack of integration of members of the Muslim culture into western societies. Similarly, given the population shares of the cultural groups, cultural groups with a more intense desire for inter-generational cultural closeness are relatively more successful in the socialization process. This also implies that the homogeneous steady state culture will be relatively closer to the initial

culture of that cultural group. .

The remainder of this paper is organized as follows. Section 2 introduces the concept of ‘formation of culture’, which is the adopted version of the cultural transmission of preferences framework, that this paper proposes. Section 3 analyses the properties of the cultural system when culture is reduced to just one endogenous cultural value and when parents have a desire for inter-generational cultural closeness. Finally, section 5 concludes. Note that this working paper does not include the proofs of the Propositions. A separate Appendix with all proofs can be obtained from the author upon request.

2 Culture Formation

2.1 Culture and Cultural Behavior

The central objects of this paper are the concepts of culture and cultural behavior. The culture of a person is defined as the set of values and attitudes that determine the world-view of that person. Cultural values define the valuations that a person (after the completion of the socialization process) assigns to different aspects of life, e.g. the importance of family, friendship, career, or the signaling of status. A cultural attitude defines the world-view on socio-economic or political issues, like how much a person is in favor of equal treatment of women, integration of minority groups or an egalitarian society. The common characteristic of both cultural values and attitudes is that their magnitude can be (subjectively) related to an ordinal scale⁵. Consider a society that is populated by a set of adults, A , consisting of at least two members⁶. Suppose that any adult has n different cultural values and attitudes. Then, we will represent any of the cultural values⁷ of a person $l \in A$ by $\phi_{i,l} \in \Omega_{i,l}$, $i = 1, \dots, n$, where $\Omega_{i,l} := [\underline{\phi}_{i,l}, \bar{\phi}_{i,l}] \subseteq \mathbb{R}$ represents the ordinal scale of the person (which we assume to be closed). The (self-perceived) culture of person l is then defined as the n -dimensional vector $\Phi_l := (\phi_{1,l}, \dots, \phi_{n,l})' \in \Omega_l$, with $\Omega_l := \times_{i=1}^n \Omega_{i,l} \subseteq \mathbb{R}^n$.

Closely related to the ordinal scaling of cultural values is also the assumption that individuals can choose and identify a behavior with respect to different cultural values. The socio-economic actions that a person takes (which also includes the direct communication of cultural values or atti-

⁵‘How important is family to you on a scale from 0 to 10?’, ‘On a scale from 0 to 10, where 0 means ‘not at all’ and 10 means ‘totally in favor’, what is your attitude towards the equal treatment of women in our society?’.

⁶A further specification of the population size will not be needed before the introduction of the model framework in section 3.

⁷Subsequently, we will by default only refer to (cultural) values and will no longer mention (cultural) attitudes for brevity. Nevertheless, all representations and results do equally apply for the concept of attitudes.

tudes) always imply the revelation of a life–style, which in the present context exactly corresponds to the display of a culture. We assume that any individual knows which socio–economic actions it has to take in order to demonstrate a certain life–style or culture (according to its subjective assessment). We will represent the displayed values of a person $l \in A$ as $\phi_{i,l}^d \in \Omega_{i,l}$, $i = 1, \dots, n$ and the corresponding cultural behavior or displayed culture as the n –dimensional vector $\Phi_l^d := (\phi_{1,l}^d, \dots, \phi_{n,l}^d)' \in \Omega_l$. Given that individuals can establish a relationship between socio–economic actions and displayed cultures, this also means that upon observing the socio–economic actions of another person, an individual can relate them to the cultural values that the actions represent and hence to its subjective ordinal scaling (‘This person behaves as if family would be very important to it.’)⁸. But the use of the term ‘subjective’ already points to the fact that in general, the subjective ordinal scalings of two persons will not coincide. To guarantee a meaningful analysis of the dynamical evolution of endogenous cultural distance in later sections, the following assumption of interpersonal comparability will be useful.

Assumption 1 (Interpersonal Comparability). *For any two persons $l, m \in A$, who set own cultural behavior and observe the cultural behavior of the other, we have that $\forall i = 1, \dots, n$ $\phi_{i,l}^d / \phi_{i,hl}^d = \phi_{i,lh}^d / \phi_{i,h}^d$, with $\phi_{i,l}^d, \phi_{i,hl}^d \in \Omega_l$, and $\phi_{i,lh}^d, \phi_{i,h}^d \in \Omega_h$, and where the subscript vw denotes the assessment of the displayed culture of person v by (the ordinal scale of) person w , and where we define $\phi_{i,v}^d := \phi_{i,vv}^d$, $v, w \in \{l, h\}$.*

This assumption means that (subjective) absolute scalings may differ, but the relative scalings are equal across individuals. The next assumption establishes cardinal meaning for (the numerical values assigned to) cultural values. It will be essential for the representation of ‘culture formation’ in the following two sections.

Assumption 2 (Cardinality). *If any person $l \in A$ identifies with two different sets of socio–economic actions $x \in X$ and $x' \in X$ the displayed cultures $\Phi_l^d := (\phi_{1,l}^d, \dots, \phi_{n,l}^d)' \in \Omega_l$ and $\Phi_l^{d'} := (\phi_{1,l}^{d'}, \dots, \phi_{n,l}^{d'})' \in \Omega_l$, we have that $\forall i = 1, \dots, n$ the first cultural behavior represents a $\phi_{i,l}^d / \phi_{i,l}^{d'}$ times as high valuation of cultural value i than the second cultural behavior.*

The cardinality assumption states that any individual can compare different sets of (own and observed) socio–economic actions in terms of the

⁸What stands behind the formulation of choice and identification of cultural behavior is the assumption that any set of actions in the socio–economic choice set, $x \in X$, represents the demonstration of a unique culture $\Phi_l \in \Omega_l$ (as perceived by person l). Also, for any culture $\Phi_l \in \Omega_l$, person l knows which unique set of socio–economic choices, $x \in X$, it has to choose to display it. This means that we have a bijective mapping $X \rightarrow \Omega_l$ and we can directly identify any possible set of socio–economic actions as a displayed culture $\Phi_l^d \in \Omega_l$, and vice versa.

relative intensities for all displayed cultural values. Assumption 1 further assures that the relative assessments by all individuals coincide⁹. By assumptions 1 and 2, we can subsequently, without further loss of generality, consider a normalized space of possible (true) and displayed cultures, $\Omega := (\Omega_1, \dots, \Omega_n) \subseteq \mathbb{R}^n$, which is equal for all individuals. We are now also ready to define the cultural distance between the cultures of two persons, as well as the displayed cultural distance between the displayed cultures of the persons as the respective Euclidean distances. Consider two persons $l \in A$ and $h \in A$. Then, their cultural distance and displayed cultural distance are defined as $\Delta_{lh}^\Phi := \|\Phi_l, \Phi_h\|$, respectively $\Delta_{lh}^{\Phi^d} := \|\Phi_l^d, \Phi_h^d\|$, with $\Phi_v, \Phi_v^d \in \Omega$, $v = l, h$.

2.2 Formation of Culture in a Nutshell

This section introduces the basic representation of the psychological processes that lead to the formation of specific cultural values (and with them a specific culture) in the socialization of a child. As in Bisin and Verdier (2000, 2001), we assume that children are born without a predefined culture. Culture is the final outcome of a process of social learning from, and the adoption of, role-models that the children are confronted with in their social environment. Different to Bisin and Verdier (2000, 2001), we do not assume that one, and only one, of the different role-models succeeds in the socialization process and the child exactly adopts the respective culture. This assumption would imply that all socialization influences from other persons or role-models fully vanish after the socialization process has been completed. But this stands in stark contrast to standard results in the research of child psychology and sociology. Although there is yet no uniformly agreed on single theory of the formation of a child's personality (hence the culture of a person in our context), the literature agrees on the fact that the formation of personality is the collective outcome of all socialization influences. We present here a formal approach to the formation of cultural values that accounts for all socialization influences and hence is coherent with the latter result¹⁰.

The basic intuition of this approach shall be explained by means of an example. Suppose that the social environment of a child consists of just two persons. This then means that the child is confronted with two (assumingly) different role-models, or displayed cultures. The two role-models

⁹The cardinality assumption might appear strong. Nevertheless, the use of cardinal values or attitudes is very common in economics, especially in the formulation of utility functions, e.g. the time preference rate, leisure/consumption weights, weights on status through education, consumption or wealth in the status literature, etc..

¹⁰The analysis in this paper is restricted to the socialization influences that the human members of the society exert on a child. However, the logic of the socialization process that we present here could equally be applied to the socialization influence of the society's institutions, like the legal and educational system or the media and marketing sector.

then represent two competing world-views with respect to the underlying displayed cultural values. Suppose that one person displays through its socio-economic actions (and communications) a very high valuation of family, while the contrary holds for the other person. Then the child socially learns both to value family very high, and to value it very low. The central assumption that we take now is that both role-models are mutually mediating as far as the world-view, i.e. culture, that the child will adopt is concerned. This means that the child will be socialized to an intermediate stance with respect to family values. Whether the child's family value will be closer to either the very high valuation of the one person, or to the very low valuation of the other, depends on how much time the persons spent with the child and how much effort (especially in a psychological sense) during that time they put into convincing it that their respective world-view is appropriate.

Let us now consider the formation of culture in the socialization process. Suppose a child is 'exposed' to K (finite) different socialization sources, all of which are characterized by a displayed culture¹¹, $\Phi_k^d := (\phi_{1,k}^d, \dots, \phi_{n,k}^d)' \in \Omega$, $k = 1, \dots, K$. Then the culture (i.e. the set of cultural values) that the child will adopt, $\Phi_c := (\phi_{1,c}, \dots, \phi_{n,c})'$, realizes according to the *time-impact-weighted* displayed culture of the finite set of socialization sources

$$\Phi_c = \frac{\sum_{k=1}^K \sigma_k \Phi_k^d}{\sum_{k=1}^K \sigma_k} \quad (1)$$

where $\sigma_k := \sigma(\tau_k, \chi_k)$ denotes the socialization success of the specific socialization source, which is a function of the time that the child has been 'exposed' to it, $\tau_k \in \mathbb{R}_+$, and the credibility, $\chi_k \in \mathbb{R}_+$, with which the respective world-view has been transmitted to the child. It is immediate to assume that if a child has no contact to a person, the respective displayed culture has no socialization influence, $\sigma(0, \chi_k) = 0$, $\forall \chi_k \in \mathbb{R}_+$, and also that the socialization success can not be negative, $\sigma(\tau_k, \chi_k) \in \mathbb{R}_+$, $\forall (\tau_k, \chi_k) \in \mathbb{R}_+^2$. Furthermore, we assume that if a child merely observes the cultural behavior of a person (and hence this person invests no specific physical and psychical effort to socialize the child), this also yields social learning effects, and that in this case, these effects are linear in the child's pure observation time, $\sigma(\tau_k, 0) = a\tau_k$, $\forall \tau_k \in \mathbb{R}_+$, where $a \in \mathbb{R}_{++}$. Note finally that the representation of the socialization process in equation (1) entails the assumption that the influences of the socialization sources are exerted globally on the culture of the child, i.e. there is no differentiated socialization success for single cultural values.

¹¹One single socialization source could thus concern a single person or a group of persons (or any institution) that feature(s) a homogeneous displayed culture.

2.3 Formation of Culture under Vertical and Oblique Transmission

The last section introduced in a general way the approach to the formation of values and culture that this paper proposes. The next step is to slightly adopt the logic and apply it to an environment where children are socialized through role-models displayed by their parents, and through role-models of unrelated adults that constitute the society at large. These two distinct socialization-channels have been introduced into the economics literature by Bisin and Verdier (2000, 2001) as ‘direct vertical transmission’ and ‘oblique transmission’.

Throughout the whole socialization process, where the child resides with the parents, the parents have a controlling function over the development of the child’s personality (culture). Not only do they directly participate in the social learning process of their child through their displayed culture, but they furthermore can permanently countervail (from their perspective) negative cultural influences exerted by the social environment. This means that during the time that parents spend with the child (which then also corresponds to a distraction of the child from social interaction with other members of the society), they can take effort to convince the child about the superiority of the world-view that they propose via their displayed culture, compared to the world-views that the child is confronted with when not with the parents. We will represent this special role of the parents in the socialization process by an adopted version of the socialization success function of section 2.2.

First, let us normalize the time-span that is constituted by the youth period to one. Further, we assume that only the parents of a child have interest in actively socializing it to a certain culture, while as the unrelated society at large merely displays role-models (hence not adjoined by a specific psychical or physical devotion to the child). According to the basic properties of the socialization success function, it follows that the time-weighted socialization impact of the general social environment is given as $\sigma_s := \sigma(1 - \tau_p, 0)$, where $\tau_p \in [0, 1]$ denotes the time-share of the youth period that the parents spend with the child. Hence, parents can control this time-weighted ‘oblique’ socialization impact by a choice of the ‘physical distraction’ of the child from the general social environment. But then, through their additional choice of the credibility level that they can achieve with respect to the role-model they propose, they can also govern their socialization success rate, i.e. their share in the total time-weighted socialization impact on the child in its youth period

$$\hat{\sigma}(\tau_p, \chi_p) := \frac{\sigma(\tau_p, \chi_p)}{\sigma(\tau_p, \chi_p) + \sigma(1 - \tau_p, 0)}$$

Note that from the basic properties of the socialization success function it

follows that $\hat{\sigma}(\tau_p, \chi_p) \in [0, 1]$, $\forall(\tau_p, \chi_p) \in [0, 1] \times \mathbb{R}_+$.

With a choice of displayed culture and the socialization success rate, parents can choose the location of the child's final culture, subject to the representative displayed culture of the general social environment. Let Φ_p^d denote the displayed culture that the parents of a child choose, and define $\hat{\sigma}_p := \hat{\sigma}(\tau_p, \chi_p)$. Further, let Φ_s^d denote the representative displayed culture of the society at large (see below for its construction). Then the culture of a child, Φ_c , is formed according to

$$\Phi_c = \Phi_p^d + (1 - \hat{\sigma}_p)(\Phi_s^d - \Phi_p^d) \quad (2)$$

Equation (2) corresponds to the view that parents set a 'cultural benchmark' (Φ_p^d), and invest into their socialization success rate ($\hat{\sigma}_p$) to countervail the influence of deviating displayed cultures (Φ_s^d) on the final culture of the children.

Oblique transmission happens through the cultural interaction of the child with the society at large on a *public social space* (e.g. schools and leisure facilities)¹². Assume that any generation consists of a continuum of individuals. Each of these individuals belongs to one of G different cultural groups and the population shares are denoted $q_g \in (0, 1)$, $g = 1, \dots, G$, such that $\sum_{g=1}^G q_g = 1$. The adults (parents) of each cultural group are culturally homogeneous and make identical behavioral choices (see section 3), $\Phi_g^d \in \Omega$. Let us assume that the share (relative density) that the members of each cultural group have on the public social space correspond to the population shares of the cultural groups¹³ — and furthermore that these also coincide with the relative social(ization) interaction times of each child with the members of the cultural groups. Then, we obtain for the time–impact–weighted displayed culture on the public social space (oblique transmission), Φ_s^d , by the linearity of the socialization success function for the case of a child's mere observation of a role–model and equation (1)

$$\Phi_s^d = \sum_{g=1}^G \Phi_g^d q_g$$

Substituting this latter equation into equation (2), we end up at the final representation of the formation of the culture of a child of any group

¹²It would be an interesting extension of the basic model, to endow parents with a controlling function of the social composition of the people that their children interact with. This could be e.g. in terms of neighborhood–, friends–, or school–selection. Thus, parents would create (homogeneous) cultural clubs such as to avoid negative cultural influences that are represented by the general social environment. Nevertheless, we refrain in this paper from including this into the analysis, since it would only increase the dimensionality of the solution problem, without changing qualitatively the results for the cultural dynamics.

¹³In the present model, we do not include a separate choice of distraction from the public social space for the adults, which could alter the presence shares.

$l = 1, \dots, G$ in the socialization process¹⁴

$$\Phi_{c_l} = \Phi_l^d + (1 - \hat{\sigma}_l) \left(\sum_{g=1}^G \Phi_g^d q_g - \Phi_l^d \right) \quad (3)$$

Let us now specify the dynamic change in the cultures of a group from one generation to the next. To make the representation more clear, let us here and subsequently denote next period's culture of a group (which is always identified with the culture of its adults) with a tilde, i.e. $\tilde{\Phi}_l \equiv \Phi_{c_l}$.¹⁵ We then obtain the cultural change over two succeeding generations of any group $l = 1, \dots, G$ as the vector of changes in the cultural values

$$\dot{\Phi}_l := \tilde{\Phi}_l - \Phi_l \quad (4)$$

From equation (3), we can also derive the cultural distance between next period's adult cultures of two cultural groups l and h as

$$\Delta_{lh}^{\tilde{\Phi}} := \left\| (\hat{\sigma}_h - \hat{\sigma}_l) \sum_{g \neq l, h} \Phi_g^d q_g + \Phi_h^d (\hat{\sigma}_h (1 - q_h) + \hat{\sigma}_l q_h) - \Phi_l^d (\hat{\sigma}_l (1 - q_l) + \hat{\sigma}_h q_l) \right\|$$

and the dynamical change in the cultural distances between the two cultural groups is then

$$\dot{\Delta}_{lh}^{\Phi} := \Delta_{lh}^{\tilde{\Phi}} - \Delta_{lh}^{\Phi} \quad (5)$$

Now that we have introduced the psychological processes by which the culture of a child is formed, as well as the representations for the cultural distance between two cultural groups and the dynamical cultural change of a group over generations, we will present in the next section a model framework that endogenously determines the parental socialization decisions (displayed culture and socialization success rate), and hence the formation of culture and the dynamics of cultural distances.

3 Endogenous Cultural Distance

This section introduces the model framework for analyzing the parental decision problem and the dynamics of the cultural distance between two cultural groups, subject to the cultural formation mechanism as introduced in section (2.3) above. Although it would be interesting to provide for an analysis of endogenous cultural distance for a general n -dimensional culture, the scope of the present paper is restricted to the analysis of only one endogenous

¹⁴Note that since all adult members of a cultural group display identical cultures, we can use the group-index to represent the parental cultural behavior.

¹⁵Throughout the rest of the paper, we will distinguish contemporaneous state and choice variables of adults of a cultural group from those of next period's adults with a tilde.

cultural value. It is immediate that the advantage of this restriction is a simplified solution procedure, which nevertheless comes at the cost of generality and of an incompatibility with respect to the representation of culture (a point which will become clear below).

Consider a population consisting of overlapping generations, which live for three life periods, but with the focus of the analysis concentrated on the child/youth period and the adult (parent) period. Each generation consists of a continuum of individuals. Reproduction is asexual and each adult has one offspring, so that the overall population size has constant measure. We define a cultural group as a collection of adults with the same culture and let the culture of a person be represented by the single cultural value (thus, we will subsequently drop the subscript for identifying a specific cultural value). Initially (i.e. not in a homogeneous cultural steady state) the socio-economy is populated by two distinct cultural groups indexed $g \in \{l, h\}$. For this setting, it will also be convenient to define $-g := \{l, h\} \setminus g$. In any given period, any of the adults has either culture $\phi_h \in \Omega := [\underline{\phi}, \bar{\phi}] \subseteq \mathbb{R}$ or $\phi_l \in \Omega$.¹⁶ We will require that in any equilibrium of a period, the adults of a cultural group display identical culture, $\phi_g^d \in \Omega$. In the present one-dimensional setting, the cultural distance and displayed cultural distance between two (members of) distinct cultural groups are denoted $\Delta^\phi := |\phi_l - \phi_h|$, and $\Delta^{\phi^d} := |\phi_l^d - \phi_h^d|$ respectively.

As discussed before, the culture of a person determines its world-view. According to this view, the culture of any person of cultural group g ¹⁷ induces a preference relation \succ_g defined over choices of displayed cultures ('life-styles') ϕ_g^d on Ω . We represent these preferences by a continuous utility function $u^{\phi_g} : \Omega \rightarrow \mathbb{R}$, $u^{\phi_g}(\phi_g^d)$. Besides the utility out of their choices of life-style, parents of any cultural group g do also receive a utility component that results out of the future culture, $\tilde{\phi}_g$, and/or socio-economic actions of their children, $\tilde{\phi}_g^d$. The source of this utility depends on which socialization motivation structure we endow the parents with. This can both be of altruistic or of egoistic nature. We leave the specification of the the socialization motives to section 4 and do at this point represent the inter-generational utility of parents of cultural group g as a continuous function $M^{\hat{\phi}_g}(\Psi_g, \Theta_g)$. The superscript $\hat{\phi}_g$ denotes the target for the culture or displayed culture that the children should adopt subject to the specific socialization motivation, $\Psi_g \subseteq \{\tilde{\phi}_g, \tilde{\phi}_g^d\}$, and $\Theta_g \in \mathbb{R}_+$ denotes the intensity of the socialization motivation. Within the logic of this paper, it is immediate that the latter

¹⁶We suppress here (and subsequently) the time-indices for sake of keeping the notation more readable. Note also that the result that in each period any adult belongs to either of the two cultural groups (outside a homogeneous cultural steady state) is an endogenous model outcome, as long as there are two distinct cultural groups in the starting period.

¹⁷Note that all subsequent results are valid for all cultural groups $g = l, h$ (unless otherwise noted). We will nevertheless abstract from explicitly noting this fact at every instance, to keep the text more readable.

must be interpreted as a cultural value. Thus, we actually extend the set of values that underly the culture of a person¹⁸. However, we restricted the analysis of the present paper to just one endogenous cultural value. With this representation, we then end up with two classes of cultural values: On the one hand, the single endogenous cultural value that effectively induces ‘own’ adult utility out of socio–economic actions; and a fixed cultural value that induces parental utility out of family considerations. Actually, thus, we provide here only for an analysis of the endogenous distance in the ‘own utility’ cultural value, subject to exogenously fixed inter–generational values. Nevertheless, we refrain from changing the terminology used so far, and will continue to speak about the dynamics of ‘cultural distance’ or a ‘homogeneous cultural steady state’ — with the before mentioned qualification kept in the back of our minds.

Abstracting from socialization costs, we obtain for the life–time utility of an adult of cultural group g

$$u^{\phi_g}(\phi_g^d) + M^{\hat{\phi}_g}(\Psi_g, \Theta_g)$$

In order to decide upon the optimal socio–economic choices (the displayed culture) and the investments into the socialization success rate, the parents have to form expectations both about the final culture of their children as well as about the resulting displayed culture that they will set. The anticipation of the future culture of the child is based on the expectations of the socio–economic choices of the members of both cultural groups. Adults also have to form expectations about the cultural behavior of members of the own group since we assume that although all adults know which cultural group they belong to, and who are the other members of it, they are not aware that their own choices will be eventually (i.e. in the equilibrium of any period) identical to that of the other group members. Let us assume for simplicity that all adults expect that all (other) members of a cultural group will display an identical culture, and that the expectations are equal for all adults. Denote these common expectations as $\phi_g^{d,e} \in \Omega$. With this latter assumption, we obtain for the expected future culture of a child of a parent of any group g , $\tilde{\phi}_g^e$

$$\tilde{\phi}_g^e = \phi_g^d + (1 - \hat{\sigma}_g) \left(\phi_g^{d,e} q_g + \phi_{-g}^{d,e} (1 - q_g) - \phi_g^d \right) \quad (6)$$

Since parents can effectively choose the expected culture of their children by a choice of the socialization instruments ϕ_g^d and $\hat{\sigma}_g$ (given their expectations about the displayed cultures in the social environment), we will also refer to the latter equation as the parental socialization technique.

¹⁸The complete representation of culture in the present context would be $\Phi_g := (\phi_g, \Theta_g)'$. Also then, the utility function that is induced by the culture of a person should correctly be denoted as $U^{\Phi_g} := u^{\phi_g}(\phi_g^d) + M^{\hat{\phi}_g}(\Psi_g, \Theta_g)$.

Given the future expected culture of their children, parents form a myopic anticipation of the corresponding optimal future displayed culture. Although the life-time utility of the parents includes an inter-generational component, they do not consider the behavior-changing impact of their children's socialization motivation¹⁹. Thus, parents believe that their children will, unrestricted by inter-generational considerations, maximize as their life-time utility the utility that they obtain through their socio-economic choices. In this respect, it is sensible to make the following assumption.

Assumption 3 (Utility Maximizer). *The utility function u^{ϕ_g} has a unique maximizer*

$$\arg \max_{\phi_g^d \in \Omega} u^{\phi_g}(\phi_g^d) = \phi_g$$

Parents correctly assess the (own utility) preferences over socio-economic behavior that results out of any potential future culture of their children, $\tilde{\phi}_g^e \in \Omega$. By Assumption 3, this means that the parents expect the optimal socio-economic choice of their children to be

$$\tilde{\phi}_g^{d,e} = \tilde{\phi}_g^e = \arg \max_{\tilde{\phi}_g^d \in \Omega} u^{\tilde{\phi}_g^e}(\tilde{\phi}_g^d)$$

Under this form of myopia the expected inter-generational utility of the parents then depends only on the expected future culture of their children, $M^{\hat{\phi}_g}(\tilde{\phi}_g^e, \Theta_g)$. The expected culture of the children is subject to the parental socialization technique (6) via the choice of displayed culture and the socialization success rate. As already discussed above, the latter is a function $\hat{\sigma}_g := \hat{\sigma}_g(\tau_g, \chi_g)$ of the time spent with the child, $\tau_g \in \mathbb{R}_+$, and the credibility of the parental role-model, $\chi_g \in \mathbb{R}_+$. Investment in both components is costly. When parents spend time with the child, they encounter opportunity cost (e.g. in form of foregone labor-income which implies lower consumption utility, or in the form of leisure utility). Investments in the parental credibility imply effort-cost in form of the physical and psychological devotion to the child which are necessary to convince the child of the advantages of the parental way of life. We will assume also that the credibility of the parental displayed culture depends negatively on the distance to the parents' true culture, $|\phi_g - \phi_g^d|$. The motivation behind this assumption is that the parents themselves will be less satisfied of their own life, as well as less

¹⁹A corresponding assumption is also taken by Bisin and Verdier (2000, 2001). It effectively implies that parents do not take into account the utility of the whole dynasty.

Given the assumption of this form of parental myopia, the before discussed effective split of the culture of a person into a single endogenous cultural value and a fixed inter-generational (set) of value(s), could be interpreted in such a way that the parents do only consider the own utility value as the basis for cultural assessments, and hence for their socialization decisions. Nevertheless, this form of parental myopic would still allow to also include the inter-generational values into the endogenous socialization process — hence this source of incompatibility is not resolved with the alternative interpretation.

convincing, when they take on a life–style which they do not consider as optimal. Actually, we then have a credibility–function $\chi_g := \chi(e_g, |\phi_g - \phi_g^d|)$ that depends on physical and psychological effort, $e_g \in \mathbb{R}_+$, and the distance of the displayed and true culture of the parents, $|\phi_g - \phi_g^d| \in [0, |\Omega|]$, where $|\Omega| := \bar{\phi} - \underline{\phi}$. We will at this point refrain from an explicit introduction of the direct cost function of a choice $(\tau_g, e_g) \in [0, 1] \times \mathbb{R}_+$. Instead, we will consider only the indirect cost function of a choice $\hat{\sigma}_g \in [0, 1]$ given $|\phi_g - \phi_g^d| \in [0, |\Omega|]$. These indirect costs will be represented by a continuous function $C : [0, 1] \times [0, |\Omega|] \rightarrow \mathbb{R}_+$, $C(\hat{\sigma}_g, |\phi_g - \phi_g^d|)$.

We are now ready to introduce the optimization problem of any parent of cultural group $g \in \{l, h\}$. Parents maximize their expected life–time utility by choosing the optimal displayed culture and socialization success rate²⁰, subject to the socialization costs and their socialization technique (equation (6))

$$\max_{\phi_g^d \in \Omega, \hat{\sigma}_g \in [0, 1]} u^{\phi_g}(\phi_g^d) + M^{\hat{\phi}_g}(\tilde{\phi}_g^e, \Theta_g) - C(\hat{\sigma}_g, |\phi_g - \phi_g^d|) \quad (7)$$

The following assumption ensures (together with previous assumptions) that the solution to optimization problem (7) is well defined.

Assumption 4 (Compact Culture Set). *The set $\Omega := [\underline{\phi}, \bar{\phi}]$ is bounded (and closed), $-\infty < \underline{\phi} < \bar{\phi} < \infty$.*

Since the choice sets of displayed culture and the socialization success rate are compact and all functions are continuous by assumption, it follows from the Theorem of the Maximum that the set of maximizers of problem (7) is non–empty, convex, compact and upper hemicontinuous. The optimization problems of parents of both cultural groups have identical structure, so that any of the solutions to problems (7) is a tuple

$$\left(\phi_l^d(\phi_l^{d,e}, \phi_l^{d,e}), \phi_h^d(\phi_l^{d,e}, \phi_h^{d,e}), \hat{\sigma}_l(\phi_l^{d,e}, \phi_h^{d,e}), \hat{\sigma}_h(\phi_l^{d,e}, \phi_h^{d,e}) \right) \in \Omega^2 \times [0, 1]^2$$

We will require that in the equilibrium of any period, the beliefs about the displayed cultures are consistent. This means that

$$\left(\phi_l^d(\phi_l^{d,e}, \phi_h^{d,e}), \phi_h^d(\phi_l^{d,e}, \phi_h^{d,e}) \right) = \left(\phi_l^{d,e}, \phi_h^{d,e} \right) \quad (8)$$

in the equilibrium of any period. It then follows that all members of a cultural group display identical culture (given our assumption that any adult expected the displayed culture of all (other) members of a cultural group

²⁰Let us also emphasize at this point that the fact that the parents can also choose very low or even zero socialization success rates could be interpreted as non–according with the special role of the parents in the growing up process of their children. This effectively constitutes a minimal time– and effort–level that the parents invest into the socialization of their children. It would be an interesting extension of the present model to analyze the effects of including such a lower bound for the socialization success rate into the model.

to be identical). The left-hand side of the equation above is an upper hemicontinuous, convex- and compact-valued mapping from a compact and convex set into itself, $\Omega^2 \rightarrow \Omega^2$. By Kakutani's Fixed Point Theorem, a fixed point (8) exists and the consistent believes equilibrium concept that we apply here is well defined. Subsequently, we will denote a consistent believes equilibrium of a period as the tuple $(\phi_l^{d*}, \phi_h^{d*}, \hat{\sigma}_l^*, \hat{\sigma}_h^*) \in \Omega^2 \times [0, 1]^2$. We will refer to the latter also as a 'temporary equilibrium'.

The following is a list of additional assumptions that is needed for the characterization of a temporary equilibrium under consistent believes.

Assumption 5 (Slope).

- (a) The utility function u^{ϕ_g} is \mathcal{C}^1 , quasi-concave and has zero slope only at the point $\phi_g^d = \hat{\phi}_g$.²¹
- (b) The socialization motivation function $M^{\hat{\phi}_g} : \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$ is \mathcal{C}^1 , quasi-concave, and $\frac{\partial M^{\hat{\phi}_g}(\hat{\phi}_g^e, \Theta_g)}{\partial \hat{\phi}_g^e} = 0$ only at the point $\hat{\phi}_g^e = \hat{\phi}_g$ (for $\Theta_g \in \mathbb{R}_{++}$).
- (c) The map C is \mathcal{C}^1 , strictly increasing and quasi-convex in the first argument, and increasing and quasi-convex in the second argument. Moreover, $\frac{\partial C(\hat{\sigma}_g, 0)}{\partial |\phi_g - \hat{\phi}_g^d|} = 0$, $\forall \hat{\sigma}_g \in [0, 1]$, and $\frac{\partial C(0, |\phi_g - \hat{\phi}_g^d|)}{\partial \hat{\sigma}_g} = 0$, $\forall |\phi_g - \hat{\phi}_g^d| \in [0, |\Omega|]$.

Assumption 5 (b) states that parents receive maximal inter-generational utility if they expect their children to adopt exactly the cultural target $\hat{\phi}_g$ (and display this culture by the parental myopia), and that the expected utility is strictly decreasing on both sides of the target.

Proposition 1 (Characterization of a Temporary Equilibrium). *Let Assumption 5 be satisfied. Then, if $\hat{\phi}_h \geq \phi_h > \phi_l \geq \hat{\phi}_l$,²² the following properties are satisfied in a temporary equilibrium under the consistent believes condition²³*

- (a) $\hat{\sigma}_g^* \in (0, 1]$, $\forall g = l, h$
- (b) $\phi_h^{d*} \geq \phi_h > \phi_l \geq \phi_l^{d*}$ (with equalities iff $\phi_h = \bar{\phi}$ or $\phi_l = \underline{\phi}$)

²¹Note also that Assumption 5 (a) implies Assumption 3.

²²We consider here the case where the cultures are non-homogeneous, $\phi_l \neq \phi_h$. Thus, we can without loss of generality assume that $\phi_h > \phi_l$. We will consider the characterization of a temporary equilibrium under homogeneous cultures (which can then also be a steady state) when we specify the socialization motivation in section 4. Also note that the restriction $\hat{\phi}_h \geq \phi_h > \phi_l \geq \hat{\phi}_l$ holds endogenously for the case of non-homogeneous cultures as long as $\phi_{h_0} > \phi_{l_0}$ for the inter-generational cultural closeness socialization motivation.

²³For the results (a) and (b) to hold, we need the additional assumption that $\Theta_g \in \mathbb{R}_{++}$, $\forall g = l, h$. Otherwise, if $\Theta_g = 0$, we would trivially have the result that $\hat{\sigma}_g^* = 0$ and $\phi_g^{d*} = \phi_g$, since the parents have no socialization motivation.

$$(c) \hat{\phi}_h > \tilde{\phi}_h^* > \tilde{\phi}_l^* > \hat{\phi}_l$$

By the results of Proposition 1 it follows that in any temporary equilibrium, parents of both groups will dissimilate behaviorally (except for the boundary case), which implies $\phi_h^{d^*} - \phi_l^{d^*} =: \Delta^{\phi^{d^*}} > \Delta^{\phi}$. This comes in attempt to countervail the negative cultural influence that the other cultural group exerts on the child on the public social space. Of course, such a deviation from the parents' optimal (own utility) socio-economic choice ($\phi_g^d = \phi_g$) must be accompanied with strictly positive investments into the socialization success rate. Since parents do not perceive inter-generational utility loss for marginal deviations from the socialization target cultures, the future cultures of the children of both cultural groups will lie strictly in the interior of the cultural space that is formed by the target cultures. Furthermore, the relative position of the cultures of the two groups (in terms of a lower or higher endogenous cultural value) is preserved over generations. This is a result of the strictly positive socialization success rate that the parents choose to guarantee that the final culture of their children is closer to the target culture than the representative displayed culture of the general social environment is.

Under the following list of assumptions, the behavior of the cultural system can be further specified.

Assumption 6 (Curvature).

- (a) The maps u^{ϕ_g} , $M^{\hat{\phi}_g}$ and $-C$ are \mathcal{C}^2 and concave.
- (b) The optimization problem 7 is strictly concave, i.e. at least one of the maps $M^{\hat{\phi}_g}$ and $-C$ is strictly concave.
- (c) The marginal cost of a deviation of the future expected (displayed) culture of the children from the target culture is increasing in the socialization motivation intensity, $\text{sign}(\hat{\phi}_g - \tilde{\phi}_g^e) \frac{\partial^2 M^{\hat{\phi}_g}(\tilde{\phi}_g^e, \Theta_g)}{\partial \tilde{\phi}_g^e \partial \Theta_g} \geq 0$.
- (d) At any temporary equilibrium the two socialization instruments are complements in the socialization problem of the parents, i.e. for $g = l/h$

$$\frac{\partial^2 M^{\hat{\phi}_g}(\tilde{\phi}_g, \Theta_g)}{\partial \tilde{\phi}_g^2} \frac{\partial \tilde{\phi}_g}{\partial \phi_g^d} \frac{\partial \tilde{\phi}_g}{\partial \tilde{\sigma}_g} + \frac{\partial M^{\hat{\phi}_g}(\tilde{\phi}_g, \Theta_g)}{\partial \tilde{\phi}_g^e} + \text{sign}(\phi_g - \phi_g^d) \frac{\partial^2 C(\tilde{\sigma}_g, |\phi_g - \phi_g^d|)}{\partial |\phi_g - \phi_g^d| \partial \tilde{\sigma}_g} \leq / \geq 0$$

Note that Assumption 6 (d) is sufficient but not necessary for the results of the following Proposition 2 to hold. The point that the results would also hold if the two socialization instruments are not too strong substitutes is developed in the proof of the Proposition.

Proposition 2 (Comparative Statics). *Let Assumptions 5 and 6 be satisfied. Then, in any temporary equilibrium, the following comparative statics results hold (the entries correspond to $g = l/h$)*

$$\begin{aligned} \text{sign} \begin{pmatrix} \frac{\partial \phi_g^{d*}}{\partial \Theta_g} & \frac{\partial \phi_g^{d*}}{\partial q_g} & \frac{\partial \phi_g^{d*}}{\partial \phi_g^{d*}} & \frac{\partial \phi_g^{d*}}{\partial \phi_g} \\ \frac{\partial \hat{\sigma}_g^*}{\partial \Theta_g} & \frac{\partial \hat{\sigma}_g^*}{\partial q_g} & \frac{\partial \hat{\sigma}_g^*}{\partial \hat{\phi}_g} & \frac{\partial \hat{\sigma}_g^*}{\partial \phi_g} \end{pmatrix} \\ = \\ \begin{pmatrix} -/+ & +/- & -/- & +/+ \\ +/+ & -/- & +/- & -/+ \end{pmatrix} \end{aligned}$$

If parents aim at moving the (expected) final culture of their children in a certain direction relative to the displayed culture of the other cultural group, a weak complementarity of the two socialization instruments in the socialization problem (Assumption 6 (d)) is sufficient to ensure that the parents will employ the two instruments in a symmetric way. This means that an increase (decrease) of the parents' displayed culture relative to the displayed culture of the other group will be adjoined by an increase (decrease) of the socialization success rate. If the complementarity assumption (together with the other Assumptions 6) is fulfilled, then, first, if a cultural group has a stronger socialization motivation, its parents will invest more into their socialization instruments to guarantee that the future (expected) displayed culture of their children is closer to the socialization target culture. Second, the investments in the socialization instruments are decreasing in the population share of the own cultural group. This comes from the fact that in any temporary equilibrium, all parents of a cultural group display identical behavior. Hence, the parents can use the more favorable cultural composition of the public social space as a substitute for their own socialization investments. Third, if the target culture for the socialization decision of the parents increases relative to the displayed culture of the other cultural group, the latter constitutes a more negative cultural influence which induces the parents to increase both of their socialization instruments. Finally, for a larger distance between the own culture of the parents and the displayed culture of the members of the other cultural group, parents increase their investments into the socialization instruments to countervail the more negative 'cultural pulling force' on the final culture of their children. Since this holds for both cultural groups, we also have that the displayed cultural distance as well as the socialization success rates are increasing in the (true) cultural distance, $\frac{\partial \Delta \phi^{d*}}{\partial \Delta \phi} > 0$, $\frac{\partial \hat{\sigma}_g^*}{\partial \Delta \phi} > 0$, $g = l, h$.

The following section considers the behavior of the model presented so far under an inter-generational cultural closeness related socialization motivation. The analysis is extended to provide for a characterization of a temporary equilibrium when the system features homogeneous cultures, as well

as it derives properties of the dynamics of the endogenous cultural distance, and a result that shows that minorities invest more into the socialization instruments under certain conditions.

4 Inter-generational Cultural Closeness

It is an undisputed fact that family is one of the most central aspects in human life. This section introduces a class of socialization motives that is based on intra-family considerations of the parents. Specifically, this class represents a desire of the parents for the future culture of their children to be close to their own culture — which hence serves as the target culture in the socialization process. Subsequently, we will shortly introduce three justifications for this assumption.

The first justification is based on a special form of parental altruism called ‘imperfect empathy’. This concept has been introduced into the economics literature by Bisin and Verdier (2000, 2001). Parents are altruistic to their children and want to maximize the utility out of the children’s future socio-economic choices, based on the expectations of their culture. Nevertheless, parents can assess this utility only through the filter of their own preferences (culture), which in the present framework introduces a desire for cultural closeness, since the parents are myopic and expect their children to choose a life-style that accords to their culture (Assumption 3). A second interpretation of family related values is what we call ‘family cohesion’. This concept is based on social interactions within the family, thus on the displayed cultures of the generations. If the displayed cultures of parents (in their third life-period) and children (in their adult-period) are more distant, then this causes more frictions, misunderstandings and (mutual) disappointment. Under the parental myopia, the expected displayed culture of the children coincides with their (true) culture. Furthermore, we assume that also the grand-parental displayed culture will equal their true culture, $\phi_g^d = \phi_g$ (we abstract here from introducing a separate notation for the grand-parental choices), since in the third life-period, they have no more need to engage in socialization, and thus can unrestrictedly maximize their utility out of socio-economic choices. This then justifies the assumption of the parental desire for inter-generational cultural closeness. The third, and purely philosophical, interpretation is based on the assumption that parents simply have an intrinsic desire for their children to develop a personality, hence culture, that is similar to their own one (e.g. if they view the life of their children as a continuation of their own life).

Given these motivations for a desire for inter-generational cultural closeness, we have that $\hat{\phi}_g \equiv \phi_g$ and we will represent the socialization motivation function as $M^{\phi_g}(\tilde{\phi}_g^e, \hat{\beta}_g)$. In the present case, $\hat{\beta}_g$ is a composed value out of the usual time-discount rate (since the relevant socio-economic actions

of the children take place one period after the parents take the socialization actions), and a value that represents the strength of the parental desire for inter-generational cultural closeness²⁴. It follows that under the inter-generational cultural closeness related class of socialization motivation, the parental optimization problem (7) is

$$\max_{\phi_g^d \in \Omega, \hat{\sigma}_g \in [0,1]} u^{\phi_g}(\phi_g^d) + M^{\phi_g}(\tilde{\phi}_g^e, \hat{\beta}_g) - C(\hat{\sigma}_g, |\phi_g - \phi_g^d|) \quad (9)$$

subject to the parental socialization technique (6). Let ϕ_{l_0}, ϕ_{h_0} denote the starting cultures of the two cultural groups. Then, the cultural system satisfies the following Proposition.

Proposition 3 (Characterization of a Temporary Equilibrium). *Let Assumption 5 be satisfied. Then if $\phi_{h_0} > \phi_{l_0}$, the following properties are satisfied in any temporary equilibrium*

1. Case $\phi_h > \phi_l$ (outside homogeneous cultural steady state)

(a) $\hat{\sigma}_g^* \in (0, 1), \forall g = l, h$

(b) $\phi_h^{d*} \geq \phi_h > \tilde{\phi}_h^* > \tilde{\phi}_l^* > \phi_l \geq \phi_l^{d*}$, with equalities iff $\phi_h = \bar{\phi}$ or $\phi_l = \underline{\phi}$

2. Case $\phi_h = \phi_l$ (homogeneous cultural steady state)

(a) $\hat{\sigma}_g^* = 0, \forall g = l, h$

(b) $\phi_h^{d*} = \phi_h = \tilde{\phi}_h^* = \tilde{\phi}_l^* = \phi_l = \phi_l^{d*}$

Compared to the general results of Proposition 1, in case of the inter-generational cultural closeness socialization motivation, where the socialization target coincides with the culture of the parents, they would never choose a combination of socio-economic actions and socialization success rate such that their children adopt exactly the parental culture ($\tilde{\phi}_g = \phi_g$). This comes from the fact that if the future culture of the children only marginally deviates from the parents' culture, this is not perceived as costly. As a result, the final cultures of the children of both cultural groups lie in the interior of

²⁴In the case of imperfect empathy, it is immediate to specify $M^{\phi_g}(\tilde{\phi}_g^e, \hat{\beta}_g) := \hat{\beta}_g u^{\phi_g}(\tilde{\phi}_g^e)$, with $\hat{\beta}_g := \beta_g \alpha_g$ and where $\beta_g \in \mathbb{R}_{++}$ represents the usual time-discount rate and $\alpha_g \in \mathbb{R}_{++}$ denotes the 'degree of parental altruism' (i.e. the relative valuation that parents put on own utility versus the future utility of the children; altruism literature reference here). The formulations in the literature on cultural transmission of preferences, notably Bisin and Verdier (2000, 2001), correspond to the special case $\beta_g \alpha_g = 1$.

Given imperfect empathy, the expected utility loss that parents perceive if their children do not exactly adopt the parental culture is then $V^{\phi_g}(\tilde{\phi}_g^e) := \alpha_g (u^{\phi_g}(\phi_g) - u^{\phi_g}(\tilde{\phi}_g^e))$. Compared to Bisin and Verdier (2000, 2001), this formulation has the advantage, that it does not have to be interpreted as an exogenous degree of cultural intolerance of the parents, but is endogenous within the present model framework (but subject to the still exogenous degree of parental altruism).

the interval that is constituted by the cultures of the parental generation, $\tilde{\phi}_g \in (\phi_l, \phi_h)$. This then implies that over generations, the cultural groups assimilate (but not completely) $0 < \tilde{\phi}_h - \tilde{\phi}_l := \Delta^{\tilde{\phi}} < \Delta^\phi$, i.e. for any two succeeding periods $\Delta^{\tilde{\phi}} < 0$ (equation 5). Asymptotically, thus, the cultural system converges to a homogeneous culture, $\phi_h = \phi_l$, and parents have no more need to engage into active socialization, so that this homogeneous culture is also a rest point. While as the exact location of the homogeneous steady state depends on the initial location of the cultures (and the socialization motivation intensities), the corresponding cultural distance of zero in a homogeneous steady state is globally asymptotically stable.

This result will be stated more formally in a Corollary. First, consider the present overlapping generations model in the continuous time limit²⁵. Let the paths of cultures that evolve out of the parental problems (9) under the condition for a temporary equilibrium be denoted $\phi_h(t, \phi_{h_0})$ and $\phi_l(t, \phi_{l_0})$, where t is the time-index. The corresponding path for the endogenous cultural distance is denoted $\Delta^\phi(t, \Delta_0^\phi) := \phi_h(t, \phi_{h_0}) - \phi_l(t, \phi_{l_0})$.

Corollary 1 (Dynamics of Cultural Distance). *If Assumption 5 is fulfilled, then $\lim_{t \rightarrow \infty} \Delta^\phi(t, \Delta_0^\phi) = 0$, $\forall \phi_{h_0} - \phi_{l_0} =: \Delta_0^\phi \in [0, |\Omega|]$. Moreover, $\forall g \in \{l, h\}$, $\lim_{t \rightarrow \infty} \phi_g(t, \phi_{g_0}) = \phi^* \in (\phi_{l_0}, \phi_{h_0})$.*

Proof. By Proposition 3, 1.(b), the dynamics of the cultural distance is a contraction mapping and the system converges to a homogeneous cultural steady state for any initial cultural distance. By Proposition 3, 2., any homogeneous cultural steady state is a rest point. \square

Corollary 1 states that under the inter-generational cultural closeness related socialization motivation, the cultural system converges to an interior homogeneous steady state. The social(ization) interactions of the two cultural groups induce a dynamic cultural assimilation process, which results in the creation of a new equilibrium culture that can be interpreted as a mixture of the two initial cultures. This theoretical result corresponds to the ‘melting pot’ theory of cultural assimilation (see Han (2006), p. 32).

Assumption 7 (Symmetry). *The maps u^{ϕ_g} and M^{ϕ_g} are symmetric, i.e. $\forall \phi_v^d, \phi_w^d \in \Omega$ s.t. $|\phi_v - \phi_v^d| = |\phi_w - \phi_w^d|$, it holds that $|u^{\phi_v}(\phi_v) - u^{\phi_v}(\phi_v^d)| = |u^{\phi_w}(\phi_w) - u^{\phi_w}(\phi_w^d)|$, and $\forall \hat{\beta}_v = \hat{\beta}_w \in \mathbb{R}_+$, $\forall \tilde{\phi}_v^{d,e}, \tilde{\phi}_w^{d,e'} \in \Omega$ s.t. $|\phi_v - \tilde{\phi}_v^{d,e}| = |\phi_w - \tilde{\phi}_w^{d,e'}|$, it holds that $|M^{\phi_v}(\phi_v, \hat{\beta}_v) - M^{\phi_v}(\tilde{\phi}_v^{d,e}, \hat{\beta}_v)| = |M^{\phi_w}(\phi_w, \hat{\beta}_w) - M^{\phi_w}(\tilde{\phi}_w^{d,e'}, \hat{\beta}_w)|$, $v, w \in \{l, h\}$.*

Assumption 7 states that parents evaluate losses out of own utility and utility losses out of the inter-generational cultural closeness motivation only

²⁵We take the derivation of the continuous time limit from Bisin and Verdier (2001). Consider an OLG economy where the agents live Δ units of time and have children $1 - h$ units of time after birth. We get the continuous time limit by taking the limit for $\Delta, h \rightarrow 0$ with $\frac{h}{\Delta} \rightarrow 0$.

with respect to the absolute distance from the utility optimum, i.e. they are indifferent if the deviation concerns lower or higher than optimal (displayed and future expected) cultural values. Furthermore, for equal deviations from the utility optimum, given equal values for the inter-generational cultural closeness desire, the evaluation of the implied utility loss equals across individuals (and cultures). Under this additional condition, the following result holds.

Proposition 4 (Relative Socialization Investments). *Let Assumptions 6–7 be satisfied. Then $\forall q_g < q_{-g}, \exists \beta(q_g, \hat{\beta}_{-g}, \Delta^\phi) \in (0, \hat{\beta}_{-g})$ such that $\forall \hat{\beta}_g \geq \beta(q_g, \hat{\beta}_{-g}, \Delta^\phi)$ it holds that $|\phi_g - \phi_g^{d*}| > |\phi_{-g} - \phi_{-g}^{d*}|$ and $\hat{\sigma}_g^* > \hat{\sigma}_{-g}^*$.*

Compared to the majority cultural group, the members of the minority face a more unfavorable cultural composition of the public social space. This implies that they have more incentives than the members of the majority to countervail the cultural impact of the public social space on the (expected) culture of the future adult generation, and that they invest more into the socialization instruments. This effect can, in a certain range, even be dominating if the minority cultural group features a lower value for inter-generational cultural closeness (and/or is less patient) than the majority does.

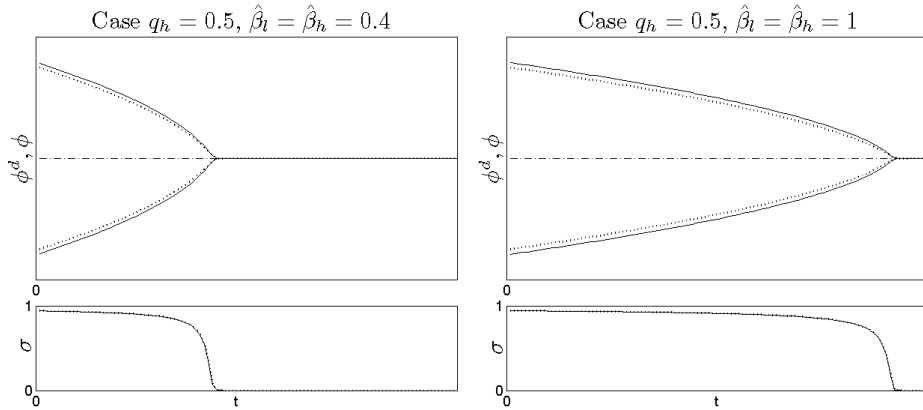


Figure 1: Dynamics of the Cultural System (Symmetric Cases)

To close the analysis of the cultural system under the inter-generational cultural closeness socialization motivation, we illustrate the analytical results by means of a numerical simulation²⁶. Figure 1 shows the dynamics

²⁶For the numerical simulation, we used the following specifications: $u^{\phi_g}(\phi_g^d) = -|\phi_g - \phi_g^d|^2$, $M^{\phi_g}(\tilde{\phi}_g^e, \hat{\beta}_g) = -\hat{\beta}_g|\phi_g - \tilde{\phi}_g^e|^2$, $C(\hat{\sigma}_g, |\phi_g - \phi_g^d|) = \hat{\sigma}_g^2(1 + |\phi_g - \phi_g^d|^2)$. It is immediate that these functions satisfy all relevant Assumptions of this section. Furthermore, we used an initial cultural distance of 30 units, and the total length of the time-axis in all graphs corresponds to 100 periods. Nevertheless, the choice of the initial cultural distance and the resulting length of the convergence path could be arbitrarily changed, so that they are not indicated in the graphs.

of the cultural system for two symmetric cases. In the upper graph of each case, the solid lines represent the displayed culture of the groups and the dotted lines represent the (true) culture of the groups (and the dash-dotted line locates the population-share weighted convex combination of the initial cultures; this would equate to the cultural steady state if parents of both cultural groups would not invest into the socialization instruments). In both pair of graphs of the Figure, one can see that since the optimization problems of parents of both cultural are fully symmetric (and strictly concave), they choose identical investments in the socialization instruments, i.e. both the choices of the distance of displayed and (true) culture as well as of the socialization success rate are equal. Moreover, the investments in both socialization instruments are strictly decreasing along the transitory path (since it features a strictly declining cultural distance). This implies that the cultural system converges to a homogeneous steady state that is located exactly at the population-share weighted convex combination of the two initial cultures, i.e. in the middle of the two initial cultures in the present cases. Compared to the left pair of graphs, the right case features a higher value for the inter-generational cultural closeness for both cultural groups. This higher socialization incentive results in higher investments in the socialization instruments for any given cultural distance, and the speed of convergence is accordingly lower.

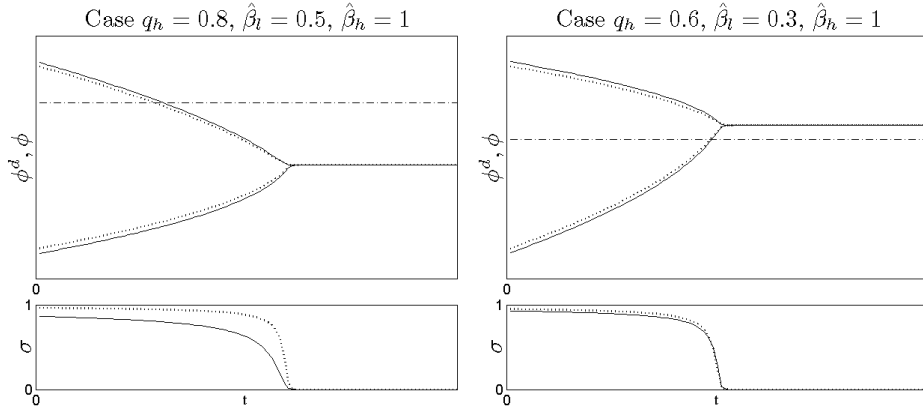


Figure 2: Dynamics of the Cultural System (Asymmetric Cases)

Figure 2 illustrates the dynamics of the cultural system for two asymmetric cases. In both cases, the ‘low value’ cultural group is the minority, whose path of the socialization success rate is represented by the dotted lines in the lower graph of each case. In the left pair of graphs, although the value for inter-generational cultural closeness of the minority cultural group is only half as high as the majority’s, the ‘minority effect’ (i.e. the higher socialization incentives given the more unfavorable cultural composition of the public social space) is strong enough such that its parents invest

more into both socialization instruments throughout the transitory path. Moreover, although the share of the minority group is only one fifth of the total population, the socialization investments of its parents succeed those of the majority’s parents to a high enough extent for the cultural system to converge to a steady state culture that is located in the interior of the population–share weighted convex combination of the two initial cultures and the initial culture of the minority.

These results do not hold for the other asymmetric case, which features a larger population share of the minority, but an even lower relative value for inter–generational cultural closeness. Although the ‘minority effect’ is strong enough for the minority parents’ investments into their socialization success rate to exceed those of the majority’s parents in initial periods (i.e. where the cultural distance is relatively high), this result reverses in later periods. Additionally, this weak domination of the minority cultural group with respect to the socialization success rate is not adjoined by a dominance in terms of a larger distance between the displayed and true culture in any of the periods along the convergence path. As a result, the cultural system converges to a steady state culture that is located in the interior of the population–share weighted convex combination of the two initial cultures and the initial culture of the majority.

5 Conclusions

This paper presented a new approach to cultural transmission of preferences, based on the work of Bisin and Verdier (2000, 2001). The central innovation that we introduce is that the assumption of the existing literature of inter–temporarily fixed cultures of the cultural groups under scrutiny is replaced with an endogeneization of the ‘formation of culture’. The endogeneization is based on modeling the culture of a person as an n –dimensional set of cultural values and attitudes, represented as tuples in Euclidean space. The cultural distance between two persons is then defined as the respective Euclidean distance between the two cultures. The formation of the culture of a person realizes through social learning and imitation of role–models in the youth period. We model role–models as ‘displayed cultures’ of the parents (‘vertical transmission’) and the general social environment (i.e. the other adults of the society; ‘oblique transmission’). If the displayed culture, i.e. the life–style or role–model that an adult takes on, differs from the true culture of a person, then it encounters utility losses, since it pursues a life–style that does not correspond to the subjectively perceived optimal one. Parents are willing to encounter this utility loss, since they have a desire for the (future) culture of their children to be close to the own culture. The justification for this assumption is that parents have either ‘imperfect empathy’ or want to avoid frictions within the family that result out of the differ-

ing world-views of the generations. Parents can support the life-style that they display to their children by investing into their socialization success rate. This involves additional socialization costs in terms of time and effort. The parental optimization problem is then to choose the optimal displayed culture and investments into their socialization success rate given their expectations of the displayed cultures of the general social environment.

The present paper analyzed the dynamics of the cultural distance between two cultural groups with one endogenous cultural value. The central results are as follows. Parents always display a culture that is more distant to the culture of the other cultural group, than their true culture is, complemented with a strictly positive socialization success rate. Nevertheless, the socialization investments of the parents are never strong enough such as that the distance between the future cultures of the children of the two groups is larger or equal than the cultural distance of the parental generation has been. This means that the cultural distance strictly declines over generations and the cultures of both groups converge to a homogeneous steady state culture that is located in the convex hull of the two original cultures. This steady state culture can thus be interpreted as a mixture of the two original cultures, which corresponds to the ‘melting pot’ theory of integration of cultural groups.

We also showed that the assimilative process towards a homogeneous equilibrium culture is slower the stronger the desire for inter-generational cultural closeness is. Furthermore, if the two socialization instruments are complements in the socialization problem of the parents, the socialization investments are decreasing in the population share of the own cultural group, which under certain conditions guarantees that parents of a minority cultural group invest more into the socialization instruments than parents of the majority do.

There are a number of directions, in which the present theoretical analysis could be extended. First, it might be interesting to identify whether there are different classes of socialization motivations under which non-decreasing cultural distances, and a cultural equilibrium with strictly positive cultural distance, are possible. Second, the behavior of the cultural system under different classes of socialization motivations should also be analyzed in an n -dimensional culture framework. Third, the empirical validity of the theoretical results should be investigated.

References

- Akerlof, G. A. and Kranton, R. E. (2000). Economics and identity. *The Quarterly Journal of Economics*, 115(3):715–753.
- Bisin, A., Patacchini, E., Verdier, T., and Zenou, Y. (2006). ‘bend it like

- beckham': Identity, socialization and assimilation. CEPR Discussion Papers 5662, C.E.P.R. Discussion Papers.
- Bisin, A., Patacchini, E., Verdier, T., and Zenou, Y. (2007). Are muslim immigrants different in terms of cultural integration? IZA Discussion Papers 3006, Institute for the Study of Labor (IZA).
- Bisin, A. and Verdier, T. (1998). On the cultural transmission of preferences for social status. *Journal of Public Economics*, 70(1):75–97.
- Bisin, A. and Verdier, T. (2000). A model of cultural transmission, voting and political ideology. *European Journal of Political Economy*, 16(1):5–29.
- Bisin, A. and Verdier, T. (2001). The economics of cultural transmission and the dynamics of preferences. *Journal of Economic Theory*, 97(2):298–319.
- Boyd, R. and Richerson, P. J. (1985). *Culture and the Evolutionary Process*. University of Chicago Press.
- Cavalli-Sforza, L. L. and Feldman, M. W. (1981). *Cultural Transmission and Evolution: A Quantitative Approach*. Princeton University Press, Princeton.
- Gallis, P. E. (2005). Muslims in europe: Integration policies in selected countries. Crs report, University of North Texas.
- Gordon, M. M. (1964). *Assimilation in American Life*. Oxford University Press, New York.
- Han, P. (2006). *Theorien zur internationalen Migration*. Lucius & Lucius.
- Hauk, E. and Saez-Marti, M. (2002). On the cultural transmission of corruption. *Journal of Economic Theory*, 107(2):311–335.
- Olcina, G. and Penarrubia, C. (2004). Hold up and intergenerational transmission of preferences. *Journal of Economic Behavior & Organization*, 54(1):111–132.