Experimental Test of Special Relativity from a High-γ Electron $g-2$ Measurement

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We report a verification of the theory of special relativity at a value of $\gamma \approx 2.5\times10^4$ based upon a comparison of electron $g-2$ measurements at meV and GeV kinetic energies. Specially we obtain a measure of the equivalence between the quantities $\gamma \equiv (1-\beta^2)^{-1/2}$ and $\tilde{\gamma} \equiv (p/m_0)dp/dE$.

A recent publication[1] has pointed out that an experimental test of special relativity is provided by comparing the values of the electron g-factor anomaly a $[a\equiv\frac{1}{2}(g-2)]$ for electrons with different velocities or γ values. Special relativity predicts that the value of a should be independent of the electron velocity. Newman *et al.*,[1] refer to two measurements of a, one done with electrons of 1 meV kinetic energy ($\gamma-1=10^{-9}$) and the other done with electrons of 100 keV kinetic energy ($\gamma=1.2$). These two measured values agree. We point out here that another measurement of a has been done with electrons of about 12 GeV kinetic energy ($\gamma\sim2.5\times10^4$), which is relevant to this test of special relativity.[2]

The high-γ $g-2$ measurement[3] was obtained as a by-product of the measurement of the polarization of the high-energy longitudinally polarized electron beam at the Stanford Linear Accelerator Center (SLAC). After acceleration to high energy the longitudinally polarized beam was deflected through the beam switchyard by an angle $\theta_c=24.5°$ into the experimental area, with the spin precessing relative to the momentum by an angle

$$\theta_a=\gamma a\theta_c. \qquad (1)$$

The longitudinal component of the beam polarization is then given by

$$P(E)=P_0\cos(\pi E/E_0+\varphi_0), \qquad (2)$$

in which P_0 is the magnitude of the initial vector polarization, $\vec{P}_0$, of the electron beam before the magnetic deflection, φ_0 is projected angle of $\vec{P}_0$ with respect to the electron momentum in the plane of the bent trajectory, E is the electron energy, and E_0 is defined as

$$E_0=\left(\frac{180°}{24.5°}\right)\frac{m_0c^2}{a}\simeq3.2\text{ GeV}, \qquad (3)$$

where m_0 is the electron rest mass.

The longitudinal polarization of the deflected beam was measured by Møller scattering[4] from a Supermendur target foil magnetized to saturation in a 90-G longitudinal magnetic field and inclined at 20° with respect to the beam direction in order to provide a large component of longitudinal polarization. Reversal of the 90-G field reversed the polarization of the target. The Møller-scattered electrons were observed by conventional particle-detection techniques with the SLAC 8-GeV/c spectrometer.[5]

The results of the Møller measurement are shown in Fig. 1 together with the fitted curve $P(E)$ given by Eq. (2) with P_0 and a as free parameters and φ_0 fixed at zero. The data points shown are taken from the earlier publication.[3] From the fit, the value $a=(1.1622\pm0.0200)\times10^{-3}$ is obtained, where the quoted 1.7% uncertainty is the linear contribution of counting statistics (0.7%) and possible systematic effects (1.0%). The systematic contributions are the estimated 0.3% uncertainty in the absolute momentum calibration of the beam switchyard magnet system,[6] and an uncertainty of 82 mrad in the value of φ_0, which

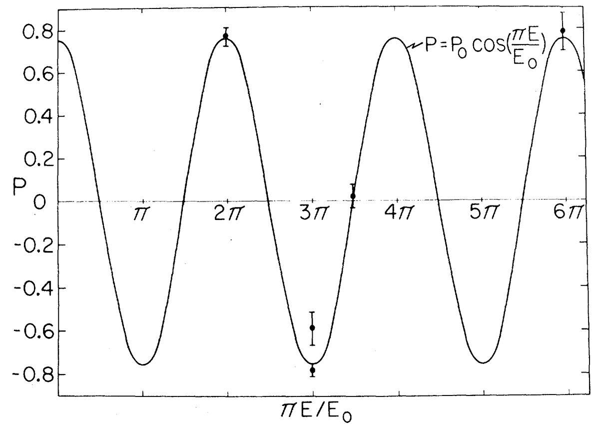

FIG. 1. The longitudinal component P of the electron beam polarization plotted as a function of E/E_0, the angle through which the spin precesses relative to the momentum during the 24.5° magnetic bend into the experimental area. The curve shown is the best fit to the data, with a and P_0 as free parameters.

results in a 0.7% uncertainty in a. The estimate of the uncertainty in φ_0 is obtained by considering the calculated upper bound to φ_0 for a single electron in the beam[7] as a 3-standard-deviation effect. Had data been taken at more than one zero crossing point, φ_0 and a could have been separately determined. However, with only one zero crossing the two parameters are highly correlated, which requires φ_0 to be estimated independently as we have done above. In this respect the measurement of a could be significantly improved by the addition of data taken at one or more of the five remaining zero-crossing points in the present SLAC energy range. Finally, the fitted value of P_0 is 0.755 ± 0.026, which agrees with both the theoretical expectations and experimental measurements of the polarization of the injected electrons.

We note that our value of a from the high-γ measurement agrees with the more precise values of a determined in the lower-γ measurements. In Table I, we summarize the lepton $g-2$ measurements for electrons and muons. For the high-γ electron $g-2$ measurement reported in this paper, we use the average value of γ over the range covered by the experiment; namely, $\gamma = 1.27\times10^4$ to $\gamma = 3.81\times10^4$. We note that this average value, $\gamma = 2.5\times10^4$, is very close to the only zero-crossing point, $\gamma = 2.22\times10^4$, at which we have obtained data. Since the measured value of $P(E)$ obtained at the crossing point most strongly influences our value of a, the choice of $\gamma = 2.5\times10^4$ to characterize our measurement of a seems well justified.

TABLE I. Lepton $g-2$ measurements.

Lepton	Reference	γ	$a\times10^3$ [a]
e^-	8	$1+10^{-9}$	1.159 652 41(20)
	9	1.2	1.159 657 70(350)
	The present work	2.5×10^4	1.1622(200)
μ^-, μ^+	10	12	1.166 16(31) [b]
	11	29.2	1.165.922(9) [b]

[a] The errors quoted are 1-standard-deviation uncertainties in the last digits.
[b] Average μ^- and μ^+.

Any discussion of the sensitivity of these measurements as a test of special relativity requires, of course, some theoretical model for, or at least a parametrization of, a breakdown of special relativity. As a theoretical problem applied to the $g-2$ measurements, some breakdown of relativistic quantum field theory may be involved. This is a very profound problem which must involve some preferred frame of reference, perhaps determined from cosmological considerations. A systematic phenomenological viewpoint might involve an analysis of the accuracy with which the coefficients of the Lorentz transformation are tested. A recent theoretical model[12,13] for a breakdown of special relativity predicts effects proportional to γ^2.

In their parametrization, Newman *et al.* introduce $\gamma \equiv (P/m_0)dp/dE$ which they allow to be different from $\gamma = (1-\beta^2)^{-1/2}$. Hence the cyclotron, spin, and $g-2$ precession frequencies for motions perpendicular to a magnetic field $\vec{B}$ are given, respectively, by

$$\omega_c = eB/\tilde{\gamma}m_0c, \tag{4}$$

$$\omega_s = geB/2m_0c + (1-\gamma)\omega_c, \tag{5}$$

$$\omega_a = \omega_s - \omega_c = (\tfrac{1}{2}g - \gamma/\tilde{\gamma})eB/m_0c. \tag{6}$$

The term $(1-\gamma)\omega_c$ in Eq. (5) is the Thomas precession frequency and is regarded by Newman *et al.* as of kinematic origin and hence involves the usual γ term, whereas the term $\tilde{\gamma}$ in Eq. (4) is regarded as arising from electron dynamics and hence as possibly different. The $g-2$ experiments determine the quantity $\omega_a(eB/m_0c)^{-1}$ which in the conventional theory equals $\frac{1}{2}(g-2) = a$. Following the parametrization of Newman *et al.*, we

TABLE II. Summary of lepton $g-2$ relativity tests.

Method	References	$\gamma^{(1)}$	$\gamma^{(2)}$	C_1
μ^-, μ^+ g factor	10 and 11	12	29.2	$(1.4\pm1.8)\times10^{-8}$
e^- g factor	8 and 9	1	1.2	$(-2.6\pm1.8)\times10^{-8}$
	8 and the present work	1	$2.5\times10^{4\text{a}}$	$(-1.0\pm8.0)\times10^{-10}$

[a] The g factor was measured over the γ interval $(1.3-3.8)\times10^4$.

set

$$\omega_a(eB/m_0c)^{-1} = \tfrac{1}{2}g - \gamma/\tilde{\gamma} = a \tag{7}$$

and regard the various $g-2$ measurements done at different electron velocities as determining $\gamma/\tilde{\gamma}$.

In the accompanying Letter by Combley *et al.*,[14] a more general phenomenological viewpoint of a breakdown of special relativity is taken and four distinct γ factors—γ_t, γ_E, γ_M, and γ_T—are introduced for the transformation of time, electromagnetic fields and mass, and for determining the Thomas precession. With certain assumptions about relations among these four γ factors, the parametrization of Combley *et al.* reduces to that of Newman *et al.*

We use the phenomenological model of Newman *et al.*, and, in addition, as do Combley *et al.*, and, in addition, as do Combley *et al.*,[14] assume a power-series expansion for $\gamma/\tilde{\gamma}$ of the form

$$\gamma/\tilde{\gamma} = 1 + C_1(\gamma-1) + \dots, \tag{8}$$

which preserves the nonrelativistic equivalence of γ and $\tilde{\gamma}$ in the limit $\gamma \to 1$. In order to define a figure of merit, we retain only the leading nonconstant term in Eq. (8). Then for each lepton, $g-2$ measurements at two values of γ suffice to determine C_1 according to

$$C_1 = (a^{(2)} - a^{(1)})/(\gamma^{(1)} - \gamma^{(2)}) \tag{9}$$

for measurements $a^{(1)}$ and $a^{(2)}$ at $\gamma^{(1)}$ and $\gamma^{(2)}$, respectively. In Table II, we present the values of C_1 derived from various pairs of lepton $g-2$ measurements given in Table I. Implicit in this parametrization are the assumptions that any violation of special relativity vanishes as one approaches the nonrelativistic limit, and that g is a constant independent of γ. We note that although our measurement of a is relatively imprecise, our value of γ is comparatively very large. Thus our experiment provides a sensitive determination of the coefficients in a power-series expansion such as given by Eq. (8).

As can be seen from Table II, the limit on $|C_1|$ of $<1.7\times10^{-9}$ measurement is the most sensitive upper limit obtained to date. Within the framework of the relativity-breaking model expressed by Eq. (8), we have thus demonstrated the equivalence of γ and $\tilde{\gamma}$. Of course, a linear dependence on $\gamma-1$ is but one possible choice. Indeed, Rédei,[12,13] in a discussion of the validity of special relativity at small distances and the existence of a universal length, suggests that for the lifetime of the muon a modification with a leading term quadratic in γ should be introduced. In the context of higher-order terms we wish to point out that the relative sensitivity of our measurement is enhanced by any higher-order dependence on $\gamma-1$.

In conclusion, we emphasize that we have included in our discussion only those tests of special relativity which are directly comparable to ours. For reference to other tests see Newman *et al.*[1] and Bailey *et al.*[15]

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